Non-thermal radiative processes in Astrophysics
Leptonic & hadronic elementary processes

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Non-thermal processes in Astrophysics: A great variety of objects (see the different lectures)

1. **Compact sources**: Sources associated with residues of massive stars
   - Stellar size compact sources: black holes and X-ray binaries, pulsars, gamma-ray binaries, gamma-ray bursts.
   - Galactic size compact sources: Central black-hole (Sagitarus A*), Active galactic nuclei.

2. **Diffuse or extended sources**: Sources linked with a compact object but spread over larger scales
   - Stellar size extended objects: Pulsar nebula, supernova remnants, massive star clusters.
   - Interstellar medium.
   - Galactic size extended objects: Galaxy starburst, Clusters of galaxy.

3. **Special effects associated with relativity (special or general)**
   All share a common property: to emit a large fraction of their bolometric luminosity into non-thermal radiation.
Preliminaries-II

Non-thermal processes in Astrophysics: A great variety of processes

1. Processes related to leptons (electrons, electron-positron pairs)
   - Interaction with matter: Coulomb or ionization losses and Bremsstrahlung radiation, pair creation/annihilation.
   - Interaction with magnetic fields: Cyclo-synchrotron radiation, curvature radiation.
   - Interaction with radiation: Inverse Compton, Comptonization.

2. Processes related to hadrons (mostly protons and helium)
   - Interaction with matter: Coulomb or ionization losses and Bremsstrahlung radiation - Pion production - neutrinos.
   - Interaction with radiation: Pion production - neutrinos.
   - Interaction with magnetic field: cyclo-Synchrotron radiation.

3. Impact of losses over particle distribution

4. Radiative transfer and radiation in a plasma
The three different approaches to the problem of radiation.

1. Classical radiation theory (CRT), or classical electrodynamics (CED). Based on a classical treatment of fields/particles. Consider the interaction between particle and fields using interaction kinematics. Derive the radiation intensity solving a radiative transfer equation.

2. Quantum radiation theory (QRT), or quantum electrodynamics (QED)/quantum chromodynamics (QCD) depending we are considering leptons or hadrons. Based on a quantum treatment of fields and particle-field interaction theory. Mandatory to derive the cross sections.

3. Quantum Plasma Dynamics (QPD). A theory that describes plasma kinetics using a semi-classical approach. Largely developed by D.B. Melrose. I will not discuss it (see Melrose D.B. Springer Verlag books).
Bibliography

Books

- Bekefi G., 1966, Radiation processes in plasmas, Wiley. (B66)
- Ginzburg, V.L., 1979, Theoretical Physics and Astrophysics (Oxford :Pergamon)(G64)
- Jackson, J.D., 1962, Classical Electrodynamics, Wiley. (J62)
- Landau, L. & Lifschitz E., 1971, Field theory, Mir. (LL71)
- Schlickeiser R., 2002, Cosmic Ray Astrophysics, Springer. (S02)

Articles

- Blumenthal G.R. & Gould R.J., 1970, Rev. of Mod. Phys., 42, 237 (BG70)
Some recurrent notations

See NRL plasma formulary

- These lectures are given in CGS units.
- For a particle we define $\beta = v/c$ and its Lorentz factor $\gamma = (1 - \beta^2)^{-1/2}$.
- Electromagnetic processes: The classical electron radius $r_e = e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm, Thomson cross section: $\sigma_T = 8\pi/3r_e^2 = 6.65 \times 10^{-25}$ cm$^2$.
- Hadronic processes: The cross section are in units of milli barns=$10^{-27}$ cm$^2$.
- Energy units 1erg =1/1.60 TeV
Outlines

1 Preliminaries
2 Elements of radiative transfer
3 Elements of classical Electrodynamics
   • Definitions
   • The dipolar approximation and the Larmor formula
4 Non-thermal leptonic processes
   • Interaction with matter
   • Interaction with magnetic fields
   • Interaction with radiation
5 Non-thermal hadronic processes
   • Hadron-matter interaction
   • Hadron-radiation interaction
   • Hadron-magnetic field interaction
The radiative transfer theory is the macroscopic theory of propagation of light through a medium. This theory involves the description of systems with size $L \gg \lambda$. It involves the concept of rays.

Rays are described in the eikonal approximation: curves whose tangent at each point lie along the direction of the propagation of the wave. If the wave function $W(\vec{x}, t) = A(\vec{x}, t) \exp(i\phi(\vec{x}, t)) \to A$ is a slowly varying function over a wavelength, $\phi$ varies rapidly over the wavelength.

The radiative transfer and ray approximation are intrinsically limited by the uncertainty principle of quantum mechanics.

Associated to rays different quantities characterize the radiation field.
Specific intensity and its moments

1. Specific intensity or brightness: \( I_\nu(\vec{r}, \Omega, \nu) = \) energy per unit of time frequency and solid angle traversing a surface element [\( \text{erg/s cm}^2 \text{ St Hz} \)], a quantity conserved along one ray (RL79 §1)

\[
I_\nu = \frac{dW}{dtdSd\Omega d\nu}
\]

2. Mean intensity: \( J_\nu = (1/4\pi) \int I_\nu d\Omega \) [\( \text{erg/s cm}^2 \text{Hz} \)].

3. Energy flux (1st moment): \( F_\nu = \int I_\nu \cos(\theta) d\Omega \) [\( \text{erg/s cm}^2 \text{Hz} \)].

4. Radiative pressure (2nd moment, flux of momentum): \( P_\nu = (1/c) \int I_\nu \cos(\theta)^2 d\Omega \) [\( \text{dynes/cm}^2 \text{Hz} \)].

\( \theta = (\vec{n}, \vec{d}) \) \( \vec{n} \) is the normal to \( dS \) and \( \vec{d} \) is the ray direction.
Radiative transfer equation

In vacuo $dI_\nu/ds = 0$, $s$ is the ray path. However in a medium $I_\nu$ varies through the combined effect of:

1. Source term or spontaneous emission coefficient $j_\nu = dW/dVdt d\nu$ [erg/cm$^3$/s St Hz] (synchrotron, bremsstrahlung ...)

2. Absorption coefficient $\alpha_\nu$ [cm$^{-1}$] (synchrotron, bremsstrahlung ...)

The radiative transfer Eq. reads:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

(1)

Also useful is the source function $S_\nu = j_\nu / \alpha_\nu$ = a quantity towards which $I_\nu$ tends to relax.

3. Diffusion coefficient $\sigma_\nu$ [cm$^{-1}$] (Comptonization (lect.2) ...). For isotropic scattering $j_\nu = \sigma_\nu J_\nu$. 

Radiative processes in Astrophysics.
Thermal processes

In case of thermal ($\neq$ Blackbody $I_\nu = B_\nu$) radiation:

$$S_\nu = \frac{B_\nu(T)}{\Delta\Omega} = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{k_B T}\right) - 1\right),$$
$$h\nu_{\text{max}} = 2.82 k_B T. \quad (2)$$

The transfer Eq. can be written as the Kirchoff law for thermal emission

$$j_\nu = \alpha_\nu B_\nu(T). \quad (3)$$

- The brightness temperature is the temperature of a blackbody having the same specific intensity at $\nu$: $I_\nu = B_\nu(T_b)$. In the Rayleigh-Jeans regime $h\nu \ll k_B T$ and $T_b = \frac{c^2}{2h\nu k_B} I_\nu$, $I_\nu = S_\nu/\Delta\Omega$. Important diagnostic in compact sources with resolved solid angle $\Delta\Omega$.

- The blackbody Wien regime ($h\nu \gg k_B T$) leads to $I_\nu = (2h\nu^3/c^2) \exp(-h\nu/k_B T)$.

All these macroscopic quantities can be obtained from a microscopic analysis.
Kirchoff law relies on a detailed balance on a microscopic level (reversibility) between emission and absorption processes. Consider two discrete energy levels \((E, \text{ with a statistical weight } g_1, \text{ population } n_1)\) and \((E + h\nu_0, g_2, n_2)\).

3 coefficients with probabilities/s: spontaneous (incoherent) emission \(A_{2\rightarrow1}\), absorption \(B_{1\rightarrow2}J\), stimulated emission \(B_{2\rightarrow1}J\).

\[ J = \int_{0}^{\infty} J_\nu \phi(\nu) d\nu \ , \phi(\nu) \text{ is the line profile centered on } \nu_0 \]

At thermodynamic equilibrium (but the relations are valid out of thermodynamical eq.) the detailed balance gives:

\[ n_1 B_{12} J = n_2 A_{21} + n_2 B_{21} J \] (4)

Also \(J_\nu = B_\nu\) which provides relations between the coefficients

\[ g_1 B_{12} = g_2 B_{21} \] (5)

\[ A_{21} = \frac{2h\nu^3}{c^2} B_{21} \] (6)
Relation to macroscopic coefficients

- **Spontaneous emission**

  \[ j_\nu = \frac{h\nu}{4\pi} n_2 A_{21} \phi(\nu) \]

- **Absorption+stimulated emission**

  \[ \alpha_\nu = \frac{h\nu}{4\pi} \times (n_1 B_{12} - n_2 B_{21}) \phi(\nu) \]

Hence at the TE \( n_1 / n_2 = g_1 / g_2 \exp(h\nu / kT) \) gives \( S_\nu = B_\nu(T) \), the Kirchoff law.
In the case of a monochromatic wave elliptically polarized. We transform the electric field $\vec{E} \rightarrow \vec{E}'$ where $(x,y)$ is the frame of the observer (instrument) and $(x',y')$ is the frame of the polarization ellipse. We apply a rotation of an angle $\chi$ $E_1, E_2, \phi_1, \phi_2 \rightarrow E_0, \beta, \chi$ (see RL section 2.4). $\beta$ describes $\vec{E}$ in $(x',y')$.

The 4 Stokes parameters express this link:

\begin{align*}
I &= E_1^2 + E_2^2 = E_0^2 > 0, \equiv \text{Intensity} \\
Q &= E_1^2 - E_2^2 = E_0^2 \cos(2\beta) \cos(2\chi) \\
U &= 2E_1 E_2 \cos(\phi_1 - \phi_2) = E_0^2 \cos(2\beta) \sin(2\chi) \\
V &= 2E_1 E_2 \sin(\phi_1 - \phi_2) = E_0^2 \sin(2\beta), \equiv \text{degree of circularity}.
\end{align*}
Preliminaries
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Non-thermal hadronic processes

Polarization and Stokes parameters for an ensemble of waves

1. Monochromatic wave case:
   - Pure elliptical polarization $I^2 = Q^2 + U^2 + V^2$.
   - $V = 0$ is the condition for the linear polarization.
   - $Q = U = 0$ is the condition for circular polarization.

2. In fact light $\equiv$ sum of wave packets hence the Stokes parameters are averaged $\langle . \rangle_T$ over the time $T$ of the signal (i.e. $I_{ensemble} = \sum_k \langle I_{mono,k} \rangle_T$).
   - A wave is the sum of an unpolarized $I - \sqrt{Q^2 + U^2 + V^2}$ and a polarized $\sqrt{Q^2 + U^2 + V^2}$ part. The latter can be cast into two oppositely polarized parts $I_\pm = 1/2 (I \pm \sqrt{Q^2 + U^2 + V^2})$.
   - Degree of polarization of an ensemble of waves

   $$\Pi = \frac{I_{pol}}{I} = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} = \frac{(I_+ - I_-)}{I}. \quad (7)$$
Useful spectral quantities: power-law radiation

- Photon differential spectrum (or also per unit of energy keV,...,TeV)
  \[ \frac{dn}{dS dt d\nu} \propto \nu^{-s} \text{ [nb/cm}^2 \text{s Hz]} \].

- Flux or energy differential spectrum
  \[ 4\pi J_\nu = \frac{dW}{dS dt d\nu} = \int I_\nu d\Omega \equiv \frac{\nu dn}{dS dt d\nu} \propto \nu^{1-s} \text{ [erg/cm}^2 \text{s Hz]} \].

- Energy spectrum per unit log of frequency (or energy)
  \[ \nu F_\nu = \frac{dW}{dS dt d\log(\nu)} \propto \nu^{2-s} \text{ [erg/cm}^2 \text{s]} \].

- Differential luminosity
  \[ L_\nu = \int dS \frac{dW}{dS dt d\nu} \text{ [erg/s Hz]} \].

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The luminosity reported at Earth is an observed flux
\[ F_\nu = L_\nu / 4\pi R^2. \]

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1. In radio the Jansky unit = \(10^{-23}\) erg/cm\(^2\)s Hz = \(10^{-26}\) W/m\(^2\)Hz.
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Definition: Power spectrum

The radiation spectrum is given by the time variation of the electric field of the electromagnetic waves. The energy in the spectrum per unit of time and per unit of surface can be obtained from the Poynting vector \( \vec{\Pi} = c \vec{E} \wedge (\vec{B}/4\pi \mu) \).

\[
\frac{dW}{dSdt} = \frac{c}{4\pi} |E(t)|^2.
\]

- The spectrum is expressed using the Fourier transform of the electric field:

\[
\frac{dW}{dSd\omega} = c|E(\omega)|^2. \tag{8}
\]
If $\vec{r} = \vec{r}_0(t)$ is the position of the particle at time $t$, the solution of Maxwell Eq. using the retarded potential method gives the electric field produced by a charge with a velocity $\vec{v} = \vec{\beta}(t_{ret})c$ and with an acceleration $\vec{a} = \vec{v}(t_{ret})$ (see J62 §14, LL71 §8, RL79 §3). The retarded time is $t_{ret} = t - R(t_{ret})/c$. $R(t') = |\vec{r}_0(t) - \vec{r}_0(t')|$, $\vec{n} = \vec{R}/R$. 

![Diagram](image)
Retarded potentials: electric field solution

- The electric field has two components: generalization of a Coulomb field and radiation field.

\[
\vec{E}(\vec{r}, t) = q \left( \frac{\vec{n} - \vec{\beta}}{\gamma^2 d^3 R^2} + \frac{q}{c} \frac{\vec{n}}{d^3 R} \right) \wedge (\vec{n} - \vec{\beta}) \wedge \vec{\beta} |_{\text{ret}},
\]

\[
= \vec{E}_{\text{Coul},v} + \vec{E}_{\text{rad}}, \quad \vec{B}(\vec{r}, t) = \vec{n} \wedge \vec{E} |_{\text{ret}}
\]

It differs to the static case by two effects (i) retarded times (ii) beaming effects: \(d = 1 - \vec{n}.\vec{\beta}\) accounts for angular effects. If the particle is non-relativistic \(t_{\text{ret}} = t(O(\beta))\) and \(d \sim 1\). If the particle is relativistic one can have \(d \ll 1\).
Spectrum radiated by one particle

Using Eq. 8 one gets the energy radiated per unit of solid angle and frequency

$$\frac{dW}{d\omega d\Omega} = \frac{q^2}{4\pi^2c} \times \left| \int \text{Re}(E(t)) \exp(i\omega t) dt \right|^2 . \quad (10)$$

• Including the radiation part (second one) of Eq. 9 (still evaluated at retarded times $t' = t - R(t')/c$) gives:

$$\frac{dW}{d\omega d\Omega} = \frac{q^2}{4\pi^2c} \times \left| \int \frac{\vec{n}}{d^2} \wedge \left( (\vec{n} - \vec{\beta}) \wedge \vec{\beta} \right) \exp(i\omega(t' - \vec{n}.\vec{r}_0(t')/c)) dt' \right|^2 . \quad (11)$$
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The electric dipolar approximation

We consider the sources of the electromagnetic field as an ensemble of charges displayed over a region of size $a$. We consider the radiation produced at an observer place by this ensemble of charges at large distances $r \gg a$.

In this configuration we only retain a part of the solution of the electromagnetic field produced by one charge, we discard the Coulomb contribution which scales as $1/r^2$ and has a Poynting flux vanishing at infinity (see Jackson, Rybicki & Lightman).

The electric dipole approximation consists in writing $|\vec{r} - \vec{r}'| \sim |\vec{r}|$. It can be translated into a condition over the wavelength of the radiation

$$a \ll \lambda.$$  

(12)
Total power radiated by an ensemble of particle: The Larmor formula

The total power emitted by an ensemble non-relativistic particles is obtained by integrating the Poynting flux over a sphere of radius $r$ using solutions 9 for one particle and summing over the ensemble contribution. One gets the Larmor formula (with $\vec{D} = \sum_i q_i \vec{r}_i$)

$$P_{em} = \frac{2}{3} \frac{\dddot{D}^2}{c^3}. \quad (13)$$

For one charge $e$ with an acceleration $\vec{A} = \frac{\dddot{D}}{e}$ one gets:

$$P_{em} = \frac{2}{3} \frac{e^2 A^2}{c^3}. \quad (14)$$

- The power radiated per unit of solid angle $P(\theta) = 3/8\pi P_{em} \sin^2(\theta)$ is not a relativistic invariant ($\theta = (\vec{A}, \vec{n})$).
Total power radiated by relativistic particles: The relativistic Larmor formula

We apply the Larmor formula in the instantaneous rest frame of the radiating particle and hence proceed with a Lorentz transformation to get the emitted power in the observer frame. In fact the total power emitted $P_{em} = \Delta E/\Delta t$ is a scalar and a relativistic invariant. We can express the total power emitted as $P_{em} \propto A.A$; in terms of a scalar product of the quadri-acceleration $\vec{A}$. In the observer frame:

$$P_{em} = \frac{2}{3} \frac{-e^2 A.A}{4\pi\epsilon_0 c^3} = \frac{2}{3} \frac{e^2\gamma^4 c^3}{c^3} \times \left( A^2_{\parallel} + \gamma^2 A^2_{\perp} \right).$$  \hspace{1cm} (15)

With $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$. $A_{\parallel, \perp}$ are the components of the acceleration vector parallel and perpendicular to the boost $\vec{v}$ in the observer frame.
Angular power

We use the Lorentz transformation of the energy-momentum 4 vector from the particle rest frame (R') to the observer frame (R). The energy emitted dW in a direction dΩ = d cos(θ)dφ is

$$\frac{dW}{dΩ} = (\gamma(1 - \beta \cos \theta))^{-3} \times \frac{dW'}{dΩ'}$$  \hspace{1cm} (16)

We distinguish among the emitted power in frame R :

$$P_e = \frac{dW}{dt'dΩ} = \gamma^{-4}(1 - \beta \cos \theta)^{-3} \times \frac{dW'}{dt'dΩ'}$$

and the received power by a fix observer in frame R :

$$P_r = (\gamma(1 - \beta \cos \theta))^{-4} \times \frac{dW'}{dt'dΩ'}$$

The latter includes a retardation effect due to the motion of the source. The term \(dW'/dt'dΩ'\) is deduced from Eq. 14.

- The Doppler factor \(D = (\gamma(1 - \beta \cos \theta))^{-1}\) is \(\gg 1\) if \(\gamma \gg 1\) and \(\cos \theta \sim \beta\). The power emitted is highly amplified in the particle direction of motion in a cone of size \(\sim 1/\gamma\).
Reaction force

- As the particle is radiating a part of its own energy hence there must be a force acting on the particle due to its radiation (J62, LL71, RL74).
  - Braking force due to radiation (Abraham-Lorentz force)

\[
\vec{F} = \frac{2e^2}{3c^3} \ddot{u} + (\vec{F}_{\text{ext}}).
\]  

(17)

\( \vec{F}_{\text{ext}} \) is an external force; e.g. Lorentz force by an EM field. The reaction force is a perturbation if the condition \( \lambda \gg r_e \) is fulfilled, \( \lambda \) is the wavelength of the radiation in the particle rest-frame (see LL71).

- In the case of relativistic particles a 4-vector formulation is used:

\[
g^\mu = \frac{2e^2}{3c} \frac{d^2 u^\mu}{ds^2} - \frac{P_{\text{rad}}}{c^2} u^\mu.
\]  

(18)

And \( P_{\text{rad}} \) is given by Eq. 15 and \( \vec{U} = (\gamma, \gamma \vec{v}/c) \). In fact as \( \gamma \gg 1 \) the second term is dominant (LL71) and the force can be seen as a friction.
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Lepton-matter interaction : Generalities

The relevant processes here are of two kinds depending if the lepton do emit or not a photon during the interaction.

- **Process with no photon**: Coulomb interaction in a fully ionized plasma - Ionization losses in a partially ionized plasma.

- **Process with photon production**: Bremsstrahlung (active in fully/partially ionized plasma).
  - Bremsstrahlung process of a non-relativistic electron in the Coulomb field of a charged ion using a classical approach- Thermal Bremsstrahlung.
  - Bremsstrahlung process of a relativistic electron in interaction with atoms. This involves at least some connection with QED as the energy of the emitted photon may be $\sim m_e c^2$. NB : Thermal Bremsstrahlung can be treated using the methods developed in QED.
Coulomb and ionization losses

Loss rates (see G64, L81, SM98):

- **Coulomb losses in a cold ionized plasma** \(^2\) \((n_e\) is the background electron density\)

\[
- \frac{dE}{dt} = 2\pi cr_e^2 m_e c^2 \frac{1}{\beta} n_e \ln \left( \frac{\pi E m_e c^2}{r_e h^2 c^2 n_e} - \frac{3}{4} \right) \text{ [erg/s]} . \tag{19}
\]

- **Ionization in a neutral plasma** \((I_H = 13.6 eV, I_{He} = 24.6 eV\) are the ionization potentials\)

\[
- \frac{dE}{dt} = 2\pi cr_e^2 m_e c^2 \frac{1}{\beta} \times \sum_{s=H,He} Z_s n_s \times \ln \left( \frac{(\gamma - 1) \beta^2 E^2}{2 I_s^2} + \frac{1}{8} \right) \text{ [erg/s]} . \tag{20}
\]

Both processes dominate at energies < GeV in the interstellar medium (S02).

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2. The case of hot plasma \(k_B T_e \sim m_e c^2\) is treated in Moskalenko I. & Jourdain E., 1997, A&A, 325, 401 and the relativistic case in L81.
At higher energies (but still non-relativistic for the moment) the interaction with the Coulomb field of a charge induces an acceleration of the electron and the emission of a photon. The acceleration is perpendicular to the velocity.

The power radiated by the particle is given by Eq. 8 expressed in terms of the Fourier transform of the electric field with the FT of $\ddot{D}$:

$$\frac{dW}{dSd\omega} = \frac{8\pi \omega^4}{3c^3} D(\omega)^2$$  \hspace{1cm} (21)
Bremsstrahlung radiation by one non-relativistic particle

The timescale of the interaction is $\tau = b/v$.

Interaction of an electron of charge $e$ with an ion of charge $Ze$ at an impact parameter $b$

The dipole moment $\vec{D} = -e\vec{v}$ leads to: $|D(\omega)| = e\Delta v(b)/2\pi \omega^2$ for $\omega\tau \ll 1$.

The velocity variation along the trajectory is due to the centripetal acceleration produced by the normal electric Coulomb field of the ion (see RL79 Eq.4.70b (CGS), L81 Eq.3.35 (SI)). The flux of incident electron and the density of targets leads to the total power radiated per unit time and volume:

$$\frac{dW}{d\omega dt dV} = (n_e v) \times n_i \times \int_{b_{\min}}^{b_{\max}} 2\pi b \frac{dW(b)}{db d\omega} db = \frac{16\pi e^6}{3m_e^2 c^4 \beta} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right) \quad (22)$$
The Gaunt factor : $b_{\text{max}}$ and $b_{\text{min}}$

We have followed a classical treatment but include quantum corrections for $b_{\text{min}}$.

- $b_{\text{max}}$ is obtained for $\omega \tau = \omega b / v = 1$ thus $b_{\text{max}} = v / \omega$.
- $b_{\text{min}}$ is obtained either in the limit $\Delta v = v$ or in the quantum limit where the classical trajectories cease to be valid.
  - Condition 1 gives : $b_{\text{min}} = 4Z^2e^2 / \pi m_e v^2$. It dominates in the low energy limit $E_k = mv^2 / 2 \ll Z^2 \text{Ry}$, $\text{Ry} = 4\pi^2 m_e e^4 / 2\hbar^2$.
  - Condition 2 gives : applying the uncertainty principle : $b_{\text{min}} = h / mv$. It dominates in the high energy limit $T = mv^2 / 2 \gg Z^2 \text{Ry}$.

We can define the Gaunt factor

$$g_{ff}(\nu, \omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{\text{max}}}{b_{\text{min}}}\right)$$  (23)
Bremsstrahlung spectrum by a population of thermal particles

We consider the case of an isotropic non-relativistic Maxwellian distribution of electrons with a temperature $T$

$$F(\nu)4\pi v^2 dv = 4\pi v^2 \times \exp(-m_e v^2 / 2k_B T) dv$$

We get ($K_{Th} = 32\pi e^6 / 3(m_e^2 c^4) = 32\pi / 3 \times r_e^3 m_e c^2$):

$$\frac{dW}{dt d\nu dV} = K_{Th} \times n_e \times n_i \times Z^2 \left( \frac{2\pi m_e c^2}{3k_B T} \right)^{1/2} \exp(-h\nu / k_B T) \bar{g}_{ff}(T, \nu) \text{[erg/s Hz cm}^3] \text{].}$$ (24)

The power radiated is almost independent ($\bar{g}_{ff}$ is not strongly dependent on $\nu$) of the frequency except if $h\nu \gg k_B T$ where it is cut off exponentially. The factor $\bar{g}_{ff}$ is in the range (1-5). It can be found in RL79 in figure 5.2 and in references therein.
Bremsstrahlung self-absorption

An electron can either absorb a photon in the field of an ion with an absorption coefficient $\alpha_\nu$ (Kirchoff’s law)

$$
\alpha_\nu = \frac{dW}{dtd\nu dV} \times (4\pi B_\nu(T))^{-1} \left[ cm^{-1} \right], \quad B_\nu(T) = \frac{2\nu^3/c^2}{\exp(h\nu/k_B T) - 1}.
$$

The effect of absorption is strong at $\nu \ll k_B T$ if the opacity $\alpha_\nu R \gg 1$ (R is the size of the emitting region).

Typical thermal free-free spectrum as produced in HII regions (in radio wavebands). The flux is $\propto \nu^2$ at low frequencies (Rayleigh-Jeans spectrum) and exponentially cut (Wien spectrum) off at high frequencies.
Bremsstrahlung in the relativistic regime

In that regime $h\nu$ may be not negligible wrt $m_ec^2$. Three approaches are possible:

- Derive an equivalent expression as Eq.22 in the relativistic case from the radiated spectrum by an accelerated particle (L81).
- Uses the method of virtual quanta (Weiszäcker & Williams method, see BG70 section 3.2).
- Use QED calculations to derive the differential Bremsstrahlung cross section (BG70 section 3.3, 3.5). This calculation give more accurate results.

We have the following chain to get the loss timescale

$$\frac{dN}{dtd\nu} = c\Sigma_i n_i \frac{d\sigma}{d\nu} \rightarrow \frac{-dE}{dt} = \int d\nu h\nu \frac{dN}{dtd\nu} \rightarrow t_{loss} = \frac{E}{|dE/dt|}.$$  

Hereafter we will note $\bar{E} = E/m_ec^2$ and the Bremsstrahlung photon energy $\epsilon = h\nu/m_ec^2$. 

$36/97$
Bremsstrahlung cross section

- Naked ion of charge $Ze$ (see BG70, eq.3.1 and references therein):
  \[ d\sigma = 4Z^2 \alpha r_e^2 \frac{d\epsilon}{\epsilon E^2} \left( \frac{E^2 + (E - \epsilon)^2}{\epsilon} - \frac{2}{3} E(E - \epsilon) \right) \left( \ln \left( \frac{2E(E - \epsilon)}{E - \epsilon} \right) - \frac{1}{2} \right) \]

- An atom:
  \[ d\sigma = \alpha r_e^2 \frac{d\epsilon}{\epsilon E^2} \left( \frac{E^2 + (E - \epsilon)^2}{\epsilon} \phi_1 - \frac{2}{3} E(E - \epsilon) \phi_2 \right) \]

$\phi_{1/2,s}$ are the screening functions given in BL70 (Fig9, table 2). At low energies $\phi_{1/2,s} \rightarrow 4(Z^2 + Z)(\ln(2E(E - \epsilon)/\epsilon) - 1/2)$, (weak shielding case $\equiv$ ion + $Z$ electrons).
Low energy means $\Delta = 1/(2\alpha)\epsilon/(E(E - \epsilon)) > 1$.  

(25)

(26)
Bremsstrahlung cross section

\[ \frac{d\sigma}{d\epsilon} \] on atomic hydrogen for different electron energies \(^3\).

Bremsstrahlung loss timescale

1. Weak shielding case (ionized medium or an atom with low energy electrons) : (i=ions of charge $Z+$, $Z$ electrons)

$$-\frac{d\bar{E}}{dt} = 4\alpha r_e^2 c \sum_i n_i Z(Z + 1) \ln(2\bar{E} - 1/3)\bar{E} \text{ [s}^{-1}! \text{]} \tag{27}$$

2. Strong shielding case : (s=species : bound electrons, ions and high-energy electrons)

$$-\frac{d\bar{E}}{dt} = \alpha r_e^2 c \sum_s n_s \left( \frac{4}{3} \phi_{1,s} - \frac{1}{3} \phi_{2,s} \right). \tag{28}$$

The typical loss timescale is (ionized case) in a plasma composed of hydrogen + 10% of helium.

$$t_{loss} \simeq 1.45 \times 10^8 \times n_{H,cm}^{-1} [\ln(2\bar{E} - 1/3)]^{-1} \text{ [years]} \tag{29}$$
Bremsstrahlung spectrum by a population of non-thermal relativistic particles

We consider the case of an isotropic relativistic power-law distribution of electrons with total density $n_e$.

$$N(E)dE = K_e \bar{E}^{-s}dE \quad , \quad E_{\text{min}} < E < E_{\text{max}} \quad , \quad n_e = \int_{E_{\text{min}}}^{E_{\text{max}}} N(E)dE$$

The differential flux is (weak shielding) : $(\int d\sigma/d\epsilon(E)N(E)dE)$

$$\frac{dW}{dt d\epsilon dV} = 4\alpha r_e^2 c K_e m_ec^2 \Sigma n_i(Z^2 + Z) \times I(\epsilon, \bar{E}_L, s) \quad , \quad \bar{E}_L = \text{max}(\bar{E}_{\text{min}}, \epsilon) \quad (30)$$

$$I(\epsilon, \bar{E}_L, s) = 4 \times \frac{3}{3} \times \left( \frac{\bar{E}_L^{-(s-1)}}{(s-1)} - \frac{\epsilon \bar{E}_L^{-s}}{s} + \frac{3 \epsilon^2 \bar{E}_L^{-s-1}}{4 (s+1)} \right) \times \phi \propto \epsilon^{-s+1} \quad (\bar{E}_L = \epsilon)$$

$$\phi_{\text{LE}} = \ln(2\bar{E}_{\text{min}}(\bar{E}_{\text{min}} - \epsilon)/\epsilon - 1/2) \quad , \quad \epsilon < \bar{E}_{\text{min}}$$

$$\phi_{\text{HE}} = \ln(4\epsilon - 1/2) \quad , \quad \epsilon > \bar{E}_{\text{min}}$$
Summary

1. Thermal Bremsstrahlung. Self-absorption produces $F \propto \nu^2$ at high opacity and low frequencies and $F \propto \exp(-h\nu/k_B T)$ at high optically thin frequencies.

2. Loss timescale is large in the interstellar medium, but Bremsstrahlung usually dominates loss processes at energies $\sim GeV$ (see lesson II). Important loss in denser media (HII regions, supernova remnant in interaction with molecular clouds ...)

3. Non-thermal Bremsstrahlung. A distribution $E^{-s}$ do produce an energy (photon) spectrum in $\epsilon^{-s+1} \ (\epsilon^{-s})$.
Electron-positron pair production

Reaction: $\gamma + \gamma \rightarrow e^- + e^+$

- Pair creation threshold: $\epsilon_1 \epsilon_2 (1 - \cos \chi) \geq 2$, $\chi = (\vec{k}_1, \vec{k}_2)$: angle of interaction.

<table>
<thead>
<tr>
<th>$\epsilon_1$</th>
<th>$\epsilon_2$</th>
<th>keV</th>
<th>MeV</th>
<th>GeV</th>
<th>TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GeV</td>
<td>MeV</td>
<td>UV</td>
<td>IR</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Electron-positron pair production cross section

\[
\sigma_{\gamma\gamma}^{CM} = \frac{3\sigma_T}{16} (1 - \beta_{CM}^2) \left[ 3 - \beta_{CM}^4 \right] \ln\left( \frac{1 + \beta_{CM}}{1 - \beta_{CM}} \right) - 2 \beta_{CM} (2 - \beta_{CM}^2) \nonumber
\]

\[
\beta_{CM} = \sqrt{1 - \frac{2}{\epsilon_1\epsilon_2 (1 - \cos \chi)}}. 
\] (31)

See 4. One uses it to derive the photon-photon pair production opacity.

\[
\tau_{\gamma\gamma}(\epsilon_2, \Omega_2) = \int dZ \int d\epsilon_1 \frac{d \cos \chi_1 d\phi_1}{4\pi} n_{ph,1}(\epsilon_1, \Omega_1) \sigma_{\gamma\gamma}^{CM}(\epsilon_1, \epsilon_2, \chi)(1 - \cos \chi(\chi_{1/2})) . 
\] (32)

Integral over Z the length along the line of sight, \(\epsilon_1, \Omega_1\) the energy and solid angle of the low energy photons ... 5 integrals needed to get \(\tau_{\gamma\gamma}(\epsilon_2)\)!

In-flight reaction: $e^- + e^+ \rightarrow \gamma + \gamma$: This produces an annihilation line that peaks at 511keV if the particles annihilate at rest.

Other processes: positronium formation. 2 ground states.

1. Para-positronium p-Ps (fraction 1/4): Anti-parallel spins produce a singlet state $^1S_0$. Lifetime = 125ps in vacuum. Decay into 2 gamma-rays at 511keV.

2. Ortho-positronium o-Ps (fraction 3/4): Parallel spins produce a multiplet state $^3S_1$. Lifetime = 142.05 ± 0.02ns in vacuum. Decay into 3 gamma-rays through a continuum spectrum.
Contribution to the galactic annihilation line

Fit of the diffuse annihilation as observed by SPI instrument on board INTEGRAL satellite. See Prantzos, N. et al (AM) 2011, Rev.Mod.Phys. 83, 1001 for a review about diffuse galactic annihilation line.

(integrated) Annihilation cross section

\[ \sigma_{an}^{CM} = \frac{3\sigma_T}{32\beta_{CM}^2\gamma_{CM}^2} \left[ (2 + \frac{2}{\gamma_{CM}^2} - \frac{1}{\gamma_{CM}^4}) \ln\left( \frac{1 + \beta_{CM}}{1 - \beta_{CM}} \right) - 2\beta_{CM}(1 + \frac{1}{\gamma_{CM}^2}) \right] \]

\[ \gamma_{CM} = \sqrt{\frac{1 + \gamma_+\gamma_- (1 - \beta_+\beta_-\mu)}{2}} = (1 - \beta_{CM}^2)^{-1/2}. \quad (33) \]

With \( \mu = (\vec{\beta}_+, \vec{\beta}_-) \). The electron-positron production rate can be deduced \(^6\).

\[ R_\pm = 2 \int d\vec{\beta}_+ d\vec{\beta}_- f(\vec{\beta}_+)f(\vec{\beta}_-)\beta_{CM} c\sigma_{an}^{CM} \times \left( \frac{\gamma_{CM}^2}{\gamma_+\gamma_-} \right) \left[ \text{cm}^{-3}\text{s}^{-1} \right]. \quad (34) \]

\[ f(\vec{\beta}_\pm) : e^\pm \text{ distribution normalized wrt to the densities } n_\pm = \int f(\vec{\beta}_\pm) d\vec{\beta}_\pm. \]


\(^7\) To derive a spectrum one must turn back to the differential cross section...
Outlines

1 Preliminaries
2 Elements of radiative transfer
3 Elements of classical Electrodynamics
   - Definitions
   - The dipolar approximation and the Larmor formula
4 Non-thermal leptonic processes
   - Interaction with matter
   - Interaction with magnetic fields
   - Interaction with radiation
5 Non-thermal hadronic processes
   - Hadron-matter interaction
   - Hadron-radiation interaction
   - Hadron-magnetic field interaction
Lepton-magnetic field interaction : Generalities

The relevant process name here depends on the energy of the radiating particle. We speak about cyclotron radiation for a non-relativistic particle ($v < 0.5c$). We speak about gyromagnetic radiation at mildly relativistic speeds. The synchrotron radiation is produced by relativistic particles.

Hadrons do also radiate in a magnetic field but at lower rate and at different frequencies. The calculations will be here displayed only for leptons. Protons are rapidly discussed at the end of lecture I.

Here we consider free electrons. When the plasma is tenuous enough then collective charge effects associated with a plasma can be discarded (see lecture II for the case of plasma effects). Hence the results in these sections are those obtained in vacuo.
We consider an *uniform* magnetic field (MF) of strength $B_0$. The electron motion is helical with a pulsation $\Omega_0 = qB_0/\gamma m_e c$ ($\Omega_b = \gamma \Omega_0$) and with a pitch-angle $\alpha = (\vec{B}_0, \vec{v})$ and $\omega_b = qB_0/m_e c$. The Larmor radius is defined as: $r_L = p \sin(\alpha)/qB_0$. We have noted $\theta = (\vec{R}, \vec{B}_0)$: the direction between the observer and the magnetic field.
Equation of motion

The equation of motion (relativistic case) :

\[
\frac{d(\gamma m_e \vec{v})}{dt} = \frac{e}{c} \vec{v} \wedge \vec{B}_0
\]

\[
\frac{d(\gamma m_e c^2)}{dt} = \frac{e}{c} \vec{v}.\vec{E} = 0
\]  (35)

The second Eq. gives \( \gamma = \text{cat} \) and hence \( |\vec{v}| = \text{cst.} \). This leads to :

\[
\gamma m_e \frac{d\vec{v}_\perp}{dt} = \frac{e}{c} \vec{v}_\perp \wedge \vec{B}_0
\]

\[
\frac{d\vec{v}_\parallel}{dt} = 0 ,
\]  (36)

thus \( |\vec{v}_\parallel| = \text{cst} \) hence an uniform motion along the MF and also \( |\vec{v}_\perp| = \text{cst} \) (no work from the MF) hence the helical motion. The particle is subject to an acceleration perpendicular to \( \vec{v} \) of constant intensity. In this circular motion the pulsations at \( m \times \Omega_0 \) are contributing to the radiation.
Cyclotron emissivity: The Schott formula

The procedure is as follows (see B66 chapter 6, ! in MKSA units)

- Use $W(\Omega, \omega)$ the energy emitted per unit solid angle and frequency interval (Eq.11)
- Develop the Fourier exponential term in terms of Bessel function: this produces the development over harmonics.
- The emitted power is obtained by dividing $W(\Omega, \omega)$ by

$$\int_{-\infty}^{+\infty} \exp(-iyt) = 2\pi \delta(y)$$

the time of the radiation has been produced.

Where $y = m\Omega_0 - \omega(1 - \beta_\| \cos(\theta))$ implies that radiation is produced at harmonics $m\Omega_0$ modified by the Doppler term of the motion the electron along the MF.

The cyclotron emissivity is (in [erg/s Hz st]):

$$x = \omega/\Omega_0\beta_\perp \sin(\theta)$$

$$j(\omega, \beta, \theta) = \frac{q^2\omega^2}{2\pi c} \times \sum_{m=1}^{\infty} \left[ \left( \frac{\cos(\theta) - \beta_\|}{\sin(\theta)} \right)^2 J_m^2(x) + \beta_\perp^2 J_m'(x)^2 \right] \delta(y),$$
Emission cyclotron lines

Cyclotron line emissivity as function of $\beta = \nu/c$ for the first 10 harmonics. Dotted lines: synchrotron approximation (Marcowith A. & Malzac J., 2003, A&A., 409, 9. ! use $\nu$ instead of $\omega$ and $\nu_b = \Omega_b/2\pi$).

- The ratio of the total emissivity of two successive harmonics is $\propto \beta^{-2}$ and the interval between two harmonics is $\Omega_0$. 
Radiation by a relativistic particle: The synchrotron limit

Once $v \rightarrow c$ the emission is beamed along $\vec{v}_\perp$ and focused within a cone of angle $\sim 1/\gamma$. In the relativistic (non-relativistic) limit the electric field $E(t)$ is a pulse over a time $T < 2\pi/\Omega_0$ (periodical $2\pi/\Omega_0$) hence $|E(\omega)|^2$ is spread over $\Delta\omega = \gamma^3 \sin \alpha \Omega_0 > \Omega_0$ ($\Delta\omega < \Omega_0$).
Differential synchrotron emissivity

Detailed calculations are provided in RL79 section 6.4 and can be derived from the Schott formula in the limit $\beta \to 1$ as well (see B66).

$$\frac{dW}{d\omega dt} = \frac{\sqrt{3}q^3B\sin(\alpha)}{4\pi m_c^2} \times F(x) ,$$

with

$$F(x) = x \int_x^{\infty} K_{5/3}(u) du , x = \frac{\omega}{\omega_c} , \omega_c = \frac{3qB\sin(\alpha)\gamma^2}{2m_c} .$$

Function $F(x)$ wrt to $x$. It peaks at $0.29x$ and can be approximated by $x^{0.3} \exp(-x)$.
The synchrotron radiation is polarized

In the relativistic limit, two terms remain in the Schott formula corresponding to two electric polarizations 1/ one parallel to $\vec{B}_0$ 2/ one perpendicular to $\vec{B}_0$. The total emissivity can be decomposed into two components either. The power emitted Eq.38 can be re-written as (see B66 section 6.3 or RL section 6.4) :

$$\frac{dW}{dt d\omega} = \frac{dW}{dt d\omega}^{\parallel} + \frac{dW}{dt d\omega}^{\perp},$$

$$P_{\parallel/\perp} = P_{\min/\max} = \frac{dW_{\parallel/\perp}}{dt d\omega} = \frac{\sqrt{3}q^3B \sin(\alpha)}{4\pi m_e c^2} \times (F(x) \mp G(x)),$$

with $G(x) = xK_{2/3}(x)$.

The degree of polarization is (see Eq. 7) :

$$\Pi(\omega) = (P_{\max} - P_{\min})/P_{\text{tot}} = G(x)/F(x).$$
Synchrotron loss timescale

- From the total power radiated by a relativistic particle Eq.15 with $A_\parallel = 0$ for an isotropically distributed particle population so averaged over the pitch-angle $\langle \sin^2(\alpha) \rangle_\Omega = 2/3$ we get:

$$t_{\text{loss}} = \left[ \frac{1}{E} \left| \frac{dE}{dt} \right| \right]^{-1} = \frac{3(m_e c^2)^2}{4\sigma_T c E U_B} \text{[s]} .$$ (40)

The magnetic energy density is $U_B = B^2/8\pi$.

- Interstellar medium estimates:

$$t_{\text{loss}} = 5 \times 10^8 E_{\text{GeV}}^{-1} B_{5\mu G}^{-2} \text{[years]} .$$ (41)
Spectrum produced by a population of particles

The spectrum emitted by a population of particles with a density \( n = \int N(p, \alpha) dp \) is given:

\[
\frac{dW}{d\nu dV dt} = \int \frac{dW}{2\pi d\omega dt}(p, \alpha)N(p, \alpha)dpd\Omega \text{ [erg/Hz cm}^3\text{s]} . \tag{42}
\]

We consider an isotropic power-law distribution of electrons:

\[
N(p, \alpha) = K_e p^{-s} \text{ with } m_e c \ll p_{\text{min}} < p < p_{\text{max}}. \text{ The spectrum hence scales as } \nu^{-(s-1)/2} \text{ in the limit } \nu_c(p_{\text{min}}) \ll \nu \ll \nu_c(p_{\text{max}}).
\]

\[
4\pi j_\nu = \frac{dW}{d\nu dV dt} = (4\pi K_e r_e q B) \times \left(\frac{3\nu_b}{2\nu}\right)^{(s-1)/2} \times E(s) . \tag{43}
\]

The function \( E(s) \) is given by Eq.4.60 in BG70.

The degree of polarization is:

\[
\Pi(s) = \frac{(s + 1)}{(s + 7/3)} .
\]
Cyclotron-synchrotron self-absorption

It is possible to define an absorption cross section of a photon of frequency $\nu$ by a particle of momentum $p$ from an analysis based on Einstein coefficients $^8$: ($j(\nu, p)$ is the particle emissivity given by Eq.38.)

$$\sigma(\nu, p) = \frac{1}{2m_e\nu^2} \times \frac{1}{\gamma p} \partial_\gamma (\gamma p j(\nu, p)) \ . \quad (44)$$

In the non-relativistic regime integrating $j(\nu, p)$ around $\nu_b$ gives

$$\sigma(\nu = \nu_b) \sim (\sigma_T/\alpha)B_{cr}/B \text{ as } \gamma \rightarrow 1 \ (B_{cr} = 4 \times 10^{13} \text{ Gauss}).$$

Angle averaged cyclo-synchrotron absorption cross section wrt to $\nu$ for different particle momenta (see $^8$ for details)

---

Case of power-law distribution of relativistic particles

The absorption coefficient is:

$$\alpha_{\nu} = \int dp N(p) \sigma(\nu, p) \text{ [cm}^{-1}\text{]} \quad (45)$$

One gets for $N(p) = Ke p^{-s}$ and $m_c \ll p_{\text{min}} < p < p_{\text{max}}$.

$$\alpha_{\nu} = \sqrt{3} \times \frac{r_{e} c K_{e}}{\nu B} \times \left(\frac{2\nu}{3\nu B}\right)^{-(s+4)/2} \times A(s), \quad (46)$$

$$A(s) = 2\pi \times 2^{(s-2)/2} \Gamma\left(\frac{3s + 2}{12}\right) \Gamma\left(\frac{3s + 22}{12}\right) \times \int_{0}^{\pi} \sin(\alpha)^{(s+4)/2} d\alpha.$$  

- The self-absorbed spectrum is using Eq.43: $S_{\nu} = j_{\nu} / \alpha_{\nu} \propto \nu^{5/2}$. 

The curvature radiation process

The curvature radiation process is similar to the synchrotron radiation process except $r_L \rightarrow r_c$ : the magnetic field curvature radius.

- Synchrotron emission comes from a cone offset by the particle pitch-angle (down) / curvature radiation the radiation is emitted along the magnetic field lines (up).

Similar spectrum emitted per particle $^a$

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{e^2}{r_c} \gamma \times F(x) ; \omega_c = \frac{3}{2} \frac{c}{r_c} \gamma^3 ; x = \frac{\omega}{\omega_c}.$$  

The power radiated by one particle is (see Eq 40):

$$P = \frac{2}{3} \frac{q^2 c}{r_c^2} \times \beta^3 \gamma^4.$$  

---

$^a$ Ochelkov Y.P., Usov V.V., 1980, Ap&ss 69, 4390
Outlines

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3. Elements of classical Electrodynamics
   - Definitions
   - The dipolar approximation and the Larmor formula
4. Non-thermal leptonic processes
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   - Interaction with radiation
5. Non-thermal hadronic processes
   - Hadron-matter interaction
   - Hadron-radiation interaction
   - Hadron-magnetic field interaction
Lepton-radiation interaction : Generalities

The relevant process here is the Compton process: the diffusion of a photon by an electron (or a positron). We speak about **Inverse Compton** radiation when the electron is relativistic. We speak about **Comptonization** when Inverse Compton radiation proceeds in a hot plasma (like the coronal gas (see lesson II) found in stellar corona or the gas in an accretion disk).
The Thomson scattering

If the scattering electron is at "rest", $v \ll c$ and if $\hbar \nu \ll m_e c^2$, the photon energy is conserved.

The power emitted per unit solid angle has been obtained in slide 24. This is the radiation produced by an electron accelerated by the electric field of the incoming wave ($\vec{d} = e\vec{r}$). The flux of the wave being $F = cE^2/8\pi$ the differential cross section is

$$\frac{d\sigma_T}{d\Omega} = \frac{dP}{d\Omega} \times \frac{1}{F} = \frac{r_e^2}{2} \left(1 + \sin(\Theta)^2\right),$$

(48)

with $\Theta = (\vec{d}, \vec{k}_1)$ and $\vec{k}_1$ is propagation direction of the out-coming photon.

Integrated over $\Theta$ this gives $\sigma_T$. 

The Compton scattering

In case photons have a momenta \( h\nu \sim m_e c^2 \). The electron has a recoil and the process involves QED. Using conservation of momentum and energy the incoming \( \epsilon = h\nu / m_e c^2 \) and out-coming \( \epsilon_1 = h\nu_1 / m_e c^2 \) photon energies are linked by:

\[
\frac{\epsilon_1}{\epsilon} = \frac{1}{1 + \epsilon(1 - \cos(\theta))}
\]

Or alternatively: \( \lambda_1 - \lambda = \lambda_c \times (1 - \cos\theta) \), \( \lambda_c = \frac{h}{m_e c} \). Hence if \( \lambda \gg \lambda_c \) or \( h\nu \ll m_e c^2 \) the diffusion is closely elastic \( \lambda_1 \sim \lambda \).
The Compton process cross section

The cross section is given by the Klein-Nishina formula (see RL79 chapter 7 and references therein).

\[
\frac{d\sigma}{d\Omega} = \frac{r_e^2 \epsilon_1^2}{2 \epsilon^2} \times \left( \frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2(\theta) \right)
\] (50)

- Non-relativistic regime (Thomson limit) : \( \epsilon \ll 1, \epsilon_1 = \epsilon \) and \( \sigma = \sigma_T \).
- Ultra-relativistic regime : \( \epsilon \gg 1 \) and the angle-integrated cross section tends to

\[
\sigma = \frac{3}{8} \sigma_T \times \frac{1}{\epsilon} \left( \ln(2\epsilon) + \frac{1}{2} \right) \ll \sigma_T .
\] (51)
The Inverse Compton process

- If now the electron is relativistic $v \sim c$ hence even a small recoil can provide the diffused photon with a lot of energy in contrast to Eq.49: *Inverse Compton process*.

- Formula 49 is now applied in the electron rest-frame (R’) moving at a speed $v$ wrt to the observer frame (R). Applying a double Lorentz transformation one gets in the *Thomson regime* ($\epsilon' < 1$) the maximum energy of the diffused photon (See RL section 7.1):

$$
\epsilon_1 \approx \gamma^2 \epsilon \times (1 - \beta \cos(\theta'_1))(1 - \beta \cos(\theta))
$$

$$
\epsilon_{1,\text{Max}} = 4\gamma^2 \epsilon,
$$

(52)

$\theta$ ($\theta'_1$) is the angle between the incident (diffused) photon and the electron in the R (R’) frame.
Results in the Klein-Nishina regime

- The efficiency of the scattering process depends on the energy of the incoming photon in the electron rest-frame, i.e. if $\epsilon' \sim 1$ hence Klein-Nishina effects become important and the process efficiency is reduced: Scattering process (Thomson regime: $\epsilon' \ll 1$) $\rightarrow$ Catastrophic loss process (Klein-Nishina regime: $\epsilon' \geq 1$).

- In the Klein-Nishina regime ($\epsilon'(1 - \cos(\theta')) > 1$), $\epsilon'_1$ becomes independent of $\epsilon$:

  \[ \epsilon'_1 \simeq \gamma(1 + \cos(\theta'))(1 - \beta \cos(\theta'_1)) \]  

  (53)
Spectrum produced by one relativistic electron: case study in the Thomson regime

The calculation is done in BG70 section 2.6, RL79 section 7.2 is a bit (too) lengthy.

1. The calculation is based on the invariance of the differential photon density $dn/\epsilon$ where $dn(\epsilon, x = \cos(\theta)) = n(\epsilon, x)d\epsilon dx$

2. From this the calculation is performed in the R’ frame we have:

$$d\sigma/d\Omega_1 d\epsilon'_1 = \left(\frac{r_e^2}{2}\right)(1 + \cos^2(\theta'_1))\delta(\epsilon'_1 - \epsilon')$$

From 1/ one can deduce the total emitted power in R’

$$\frac{d\epsilon'_1}{dt'} = c\sigma_T \int \epsilon'_1 dn'(\epsilon') = \text{INV} = -\frac{1}{m_e c^2} \frac{dE_e}{dt}.$$ 

From 1/ and 2/ one can deduce the photon spectrum produced by one relativistic electron.

$$\frac{dN}{dt' d\epsilon' d\Omega'_1 d\epsilon'_1} = dn' c \frac{d\sigma}{d\Omega_1' d\epsilon'_1} [\text{Nb/s st } \epsilon' \epsilon'_1].$$
Inverse Compton loss timescale

We find:

- Loss rate (or radiated power) in the case of an isotropic incident photon distribution in $R$:

\[
- \frac{dE}{dt} = \sigma_T c \left( \gamma^2 \left( 1 + \frac{1}{3} \beta^2 \right) - 1 \right) U_{ph},
\]

\[
U_{ph} = \int \epsilon d\epsilon(n(\epsilon) \quad [\text{erg/cm}^3] \quad (54)
\]

- Loss timescale ($\beta = 1$)

\[
t_{loss} = \frac{E}{|\dot{E}|} = \frac{3(m_e c^2)^2}{4\sigma_T c E U_{ph}} \quad [s],
\]

\[
t_{loss} \simeq [3 \times 10^8] E_{GeV}^{-1} U_{ph, eV/cm^3}^{-1} \quad [\text{years}].
\]
Inverse Compton photon spectrum produced by one particle

Here I use the general Klein-Nishina cross section in step 2 of slide 68. The diffused photon spectrum is (BG70 Eq.2.48):

with $\Gamma_e = 4\epsilon\gamma$, $q = E_1/\Gamma_e(1 - E_1)$, $E_1 = \epsilon_1/\gamma$.

$$\frac{dN}{dtd\epsilon_1} = \frac{2\pi r_e^2 c}{\gamma^2} \frac{n(\epsilon)d\epsilon}{\epsilon} \times F(\Gamma_e, q)$$

(56)

$$F(\Gamma_e, q) = 2q \ln(q) + (1 + 2q)(1 - q) + \frac{1}{2} (1 - q) \frac{(\Gamma_eq)^2}{(1 + \Gamma_eq)}.$$

- $F(\bar{E}_1, \Gamma_e)$ as function of $x = \bar{E}_1 = E_1/(\Gamma_e(\Gamma_e + 1)^{-1})$ for different values of $\Gamma_e = 0$ (dotted), 1, 10, 50, 100 (long dashed).
- $\Gamma_e \ll 1 (\Gamma_e \gg 1)$ corresponds to the Thomson regime (the Klein-Nishina regime).
Spectrum produced by a population of particles

We consider the case of an isotropic power-law distribution of electrons:

\[ N_e(\gamma) = K_e \gamma^{-s}, \gamma_{\text{min}} < \gamma < \gamma_{\text{max}} \]  (with \( \int N_e(\gamma)d\gamma = n_e \)). Integrating Eq. 56 over \( N_e(\gamma) \) yields to a complicated expression (BL 70 Eq. 2.75).

- **Thomson limit:**

\[
\frac{1}{m_e c^2} \frac{dW}{dtd\epsilon_1dV} = \frac{\epsilon_1 dN}{dtd\epsilon_1dV} = \pi r_e^2 c K_e(s, n_e) \times \epsilon_1^{-(s-1)/2} \times F(s) \tag{57}
\]

\[
F(s) = \frac{2^{s+3}(s^2 + 4s + 11)}{(s + 3)^2(s + 1)(s + 5)} \int \epsilon^{(s-1)/2} n(\epsilon)d\epsilon.
\]

- **Klein-Nishina limit** (\( C(s) \) is given by Fig6. BL70):

\[
\frac{\epsilon_1 dN}{dtd\epsilon_1dV} = \pi r_e^2 c K_e(s, n_e) \times \epsilon_1^{-(s-1)} \times F(s, \epsilon_1) \tag{58}
\]

\[
F(s, \epsilon_1) = \int \frac{n(\epsilon)}{\epsilon} (ln(\epsilon \epsilon_1) + C(s))d\epsilon.
\]

The Klein-Nishina spectrum is steeper than the Thomson spectrum.
Some spectral examples

Inverse Compton spectrum produced by a power-law electron distribution over a black-body photon source in the Thomson regime (blue) and Klein-Nishina regime (orange)
Synchrotron self-Compton (SSC) process

The soft photons are synchrotron radiation produced by the same (not always the case) electron population. In the Thomson regime: if the synchrotron spectrum extend over $[\epsilon_s, \text{min}, \epsilon_s, \text{max}]$ hence the SSC spectrum extends over $\gamma^2 \times [\epsilon_s, \text{min}, \epsilon_s, \text{max}]$ for each particle, hence SSC is more extended than IC emission produced from a black-body.

- In the Thomson regime the ratio of the power radiated by IC to synchrotron is (see Eqs. 38, 54):
  \[
  \frac{P_{IC}}{P_S} = \frac{U_{ph}}{U_B}.
  \]  
  (59)

- In the case of SSC $U_{ph}$ is produced by the synchrotron radiation and hence if we have several IC generations:
  \[
  \frac{P^2_{IC}}{P^1_{IC}} = \frac{P^1_{IC}}{P_S} = \frac{U_{ph}}{U_B}
  \]  
  (60)
Spectral energy distribution (radio loud quasar): Synchrotron emission (Sy) in red, synchrotron self-compton (SSC) emission in blue, External Comptonization of Direct disk radiation (ECD) in green, and External Comptonization of radiation from the clouds (ECC) in yellow.
The synchro-Compton loss catastrophe

- We note $\eta$ the ratios in Eq. 60. In the case of homogeneous synchrotron self-absorbed source it is possible to calculate this ratio in terms of a the brightness temperature $T_b$ (see slide 11)\(^9\).

  - The brightness temperature such that $\eta = 1$:

    $$ T_{b,\eta=1} = 10^{12} K \times \left( \frac{\nu}{1 \text{Ghz}} \right)^{-1/5}. $$

    (61)

    Hence at 1 Ghz the brightness temperature should not exceed $10^{12}$ K otherwise it induces a loss catastrophe producing huge amount of X and gamma-rays as the power in secondary generations exceeds the power in primary produced photons.

  - The point is that several radio sources do have $T_b > T_{b,\eta=1}$.

---

Induced Compton scattering

- In this process photons scatter off free electrons at rate enhanced with respect to Compton scattering by the photon intensity (occupation number).

- The condition for Induced Compton scattering (ICS) to be important is: \( f \left( \frac{k_B T_B}{m_e c^2} \right) \tau_T \geq 1 \) and \( \tau_T = n_e \sigma_T R f \sim \Omega_0 / 4\pi \) accounts for the anisotropy of the incident photon beam \(^{10}\).

- The transfer Eq. becomes non-linear. In the case of isotropic photon distribution and uniform particle distribution confined in the same region, noting \( y(\nu) = \nu k_B T_B / m_e c^2 \) we have (with \( \bar{t} = n_e \sigma_T c \bar{t} \)):\[\frac{\partial y}{\partial \bar{t}} = 2y \frac{\partial y}{\nu}, \quad (62)\]

- The effect of ICS is to pump photons from high to low frequencies as \( y \) moves along the characteristic \((\nu, \bar{t})\) with a speed \(-2y\). Hence the brightness temperature at low energies increases.

Isotropic homogeneous Induced Compton scattering

Time evolution of an isotropic photon field under the effect of ICS by an homogeneous electron distribution. $y(\nu, 0) \propto \nu^{1.5}(1 - \exp(-\nu^3))$. A series of shocks developed that in nature is controlled by the electron thermal velocity, the shock thickness is $V_e/c = \delta \nu/\nu$ (see Coppi et al (1993)).
Polarization

The degree of polarization depends on the incoming photon polarization. In case of an isotropic electron distribution with $N_e(E) \propto E^{-s}$ one may consider two cases: (i) Incoming photons are not strongly polarized (i.e. (most of) accretion disk, background radiation) (ii) Incoming photons are polarized (i.e. Synchrotron radiation).

The results are\(^\text{11}\): In the case of an incoming beam (in direction and frequency) of photon in the Thomson regime

$$
\Pi_{IC} \simeq \frac{3 + 4s + s^2}{11 + 4s + s^2},
$$

In the case of synchrotron photon spectrum $N_\omega \propto \omega^{-\alpha}$ scattered by the same electrons:

$$
\Pi_{SSC} = \eta(\alpha, p) \times \Pi_{\text{sync}} \times \Pi_{IC}, \; \Pi_{\text{sync}} = \frac{(3s + 3)}{(3s + 7)}.
$$

• (upper panel) IC polarization degree $\Pi$ (s=1.01,1.5,2,3,4 black to magenta) in case of a power-law electron and a beam of photons at an angle of 85° with l.o.s. Klein-Nishina case : $\Pi_{KN} \sim 0.5/(1 + \epsilon')$.

• (lower panel) SSC polarization degree in case of a photon distribution $N_{\omega} \propto \omega^{-(\alpha+1)}$ ($\alpha = 1, p = 3$ red and $\alpha = 0.5, p = 2$ black) with an angle between MF and l.o.s. of 85°. In both figures the proper energy/frequency limits have to be considered.
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Coulomb losses (S02, SM98 and ref. therein): Case of an ion of charge \( Z \) and velocity \( \beta = v/c \) and energy \( E \). The losses are dominated by scattering off thermal electrons:

\[
\frac{dE}{dt} = -4\pi cr_e^2 n_e m_e c^2 Z^2 \ln \Lambda \times \frac{x_m^2 + \beta^2}{\beta^2},
\]

(65)

\( x_m = 0.0286 (T_e/2 \times 10^6 K)^{1/2} \) and \( \ln \Lambda = [30 - 40] \) in the ISM conditions \(^{12}\).

Ionization losses:

\[
\frac{dE}{dt} = -2\pi cr_e^2 m_e c^2 n_e Z^2 \times \frac{1}{\beta} \times \sum_{s=H,He} n_s \times K_s(I_s, \beta, E).
\]

The complicated function \( K_s \) is described in S02 and SM98 and references therein.

---

Inverse Bremsstrahlung

Bremsstrahlung produced by relativistic hadrons over a charged plasma. It has been invoked in the contexts of cosmic X-ray background, diffuse galactic X-ray emission and at shocks\(^ {13} \).

\[
\frac{F_{IB}}{F_B} \propto \left( \frac{m_e T_p}{m_p T_e} \right)^{(s-1)/2}, \quad s : \text{non-thermal particle distribution index.}
\]

see S02

- Proton pion production ($n_1, n_2$=multiplicities).

\[ p + p \rightarrow p + p + n_1 (\pi^+ + \pi^-) + n_2 \pi^0, \]
\[ \pi^\pm \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu, \]
\[ \pi^0 \rightarrow 2\gamma, \]
\[ \mu^\pm \rightarrow e^\pm + \nu_e/\bar{\nu}_e + \bar{\nu}_\mu/\nu_\mu. \] (67)

- Other channels for protons: $n, 2n, \bar{p}, D, K^0, K^\pm, \Sigma^0, \Sigma^\pm, \Lambda^0, \Lambda^\pm...$

- Other hadrons: essentially in the interstellar medium $\alpha$ particles but in particular sources one may be concerned with heavier particles.

All these processes involve inelastic interactions: $a_1 + a_2 \rightarrow \Sigma_i a_i$ with a total mass $M = \Sigma_i m_i$. The threshold condition for the interaction (prime is expressed in the center of mass frame):

\[ E'_{th} = \left( m_1^2 c^4 + m_2^2 c^4 + 2E_1 E_2 - 2p_1 p_2 c^2 \cos(\theta_{1,2}) \right)^{1/2} \geq M c^2. \] (68)
Neutron production

- Neutrons have a short lifetime ($\gamma \times 10^3$ s) but have no charge so are insensitive to electromagnetic fields and can transport a substantial fraction of the energy from the sources\(^\text{14}\).

- Fraction of protons converted into neutrons: 1/4 in pp collisions and 1/2 in p\(\gamma\) collisions

---

Here is shown a parametrization of the inelastic cross section\textsuperscript{15}

\[ L = \ln\left(\frac{E_P}{1\text{TeV}}\right) \]  
and from Eq. 68 the threshold energy is:

\[ E_{th} = 2m_\pi + m_p + \frac{m_\pi^2}{2m_p} = 1.23 \text{ GeV} \]

\( m_\pi \simeq 135 \text{ MeV}, m_p \simeq 0.938 \text{ GeV} \).


Gamma-ray emissivity by one particle

Each pion produces a differential number of gamma-ray photons:

\[
\frac{dN_\gamma}{dE_\gamma}(E_\gamma) = 2 \times \int_{E_\gamma + 4m_\pi c^2 / E_\gamma}^{\infty} \frac{dN_\pi}{dE_\pi}(E_\pi) \frac{dE_\pi}{\sqrt{E_\pi^2 - m_\pi^2 c^4}}.
\]  

(69)

Now each proton produces pions at a differential rate \(^{16}\):

\[
R(E_\pi) \simeq c n_H \sigma_{pp}(E_p(E_\pi)) \times \delta(E_\pi - \langle E_\pi \rangle) H(E_p - E_{th}),
\]  

(70)

- Typical pion energy: \(\langle E_\pi \rangle \sim 1/6 T_p\) (\(E_p \leq 10\) TeV, \(T_p\) is the kinetic energy) and \(E_p(E_\pi) = m_p c^2 + 6E_\pi\), \(K_\pi = 1/6\) is the inelasticity factor

Proton loss timescale

\[ t_{\text{loss}} = 3 \times E_p \times \left[ \int R(E_{\pi})dE_{\pi} \right]^{-1} \sim 5 \times 10^7 n_{H,cm}^{-1} \text{ years} . \quad (71) \]

I have used \( n_H \) as the total hydrogen target density \( n_H = n_{HI} + n_{HII} + 2n_{H2} \).
pp interaction process is anisotropic

- Kinematical effect: the neutral pion and gamma-rays are mostly produced in the direction of the incident proton especially at high energies\textsuperscript{17}. This calculation requires to retain the angles of secondary particles in the cross section.

\[ N_p \propto E_p^{-2} \]

\textsuperscript{17} Karlsson N. & Kamae T., 2008, ApJ, 674, 278

![Gamma-ray spectra at different angles wrt the beam for $N_p \propto E_p^{-2}$.](image)
Gamma-ray emissivity by a population of particles

We define the pion rate integrated over the proton distribution

\[ q_\pi(E_\pi) = \int R(E_\pi)n_p(E_p)dE_\pi = \frac{n_Hc}{K_\pi}\frac{\sigma_{pp}(E_p(E_\pi))}{K_\pi}n_p(E_p(E_\pi)) \text{[nb/cm}^3\text{s E}_\pi] \text{.} \]

(72)

again, \( E_p(E_\pi) = m_p c^2 + E_\pi/K_\pi \) and then use Eq.69. In case of a power law gamma-ray and proton indices are identical.

EF(E) (TeV/cm\(^{-3}\) s) vs E(TeV) gamma-ray spectrum from pion decay for different power-law proton distribution \( n(E_p) \propto E^{-p} : p = 2 \) (3 dotted-dashed), \( p = 2.5 \) dotted-dashed), other lines correspond to fits of the diffuse galactic gamma-ray emissivity (see Aharonian & Atoyan 2000).
Electron-positron and neutrino production

Here are displayed the results from Kelner et al 2006 using parametrization of the gamma-ray, secondary lepton and neutrino production rate.

\[ EF(E)(\text{TeV/cm}^{-3}\text{s}) \text{ vs } E(\text{TeV}) \text{ gamma-ray, lepton(electron and positron), muonic neutrinos emissivities for different distribution of protons } \]
\[ n_p \propto E_p^{-\alpha} \times \exp\left(-\frac{E_p}{E_0}\right)^\beta. \]

Left panel \( \alpha = 2, \beta = 1, E_0 = 1 \text{ PeV.} \) Left panel \( \alpha = 1.5, \beta = 1, E_0 = 1 \text{PeV.} \) In dashed line: delta approximation Eq.72.

- One must account for the radiation (synchrotron, Bremsstrahlung, Inverse Compton) radiation from secondary leptons.
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Hadron-radiation interaction processes

- Photo-pair production
  \[ p + \gamma \rightarrow p + e^- + e^- . \]
  Photon threshold \( E'_\gamma = \Delta mc^2 (1 + \Delta m/m_p) = 1 \text{ MeV} \) (proton restframe)

- Photo-pion production
  \[ p + \gamma \rightarrow p + \pi^0 , \]
  \[ p + \gamma \rightarrow p + \pi^+ + \pi^- . \]
  Photon threshold \( E'_\gamma = \Delta mc^2 (1 + \Delta m/m_p) = 145 \text{ MeV} \) (proton restframe)

- Photo-disintegration
  \[ A + \gamma \rightarrow (A - 1) + n/p . \]

Important process in the context of ultra high-energy cosmic ray propagation in the intergalactic medium\(^\text{18}\)

18. see e.g. Khan E. et al 2005, Aph, 23 191 for further details
Photo-pion cross section as function of the incoming photon energy in the proton restframe. Peak 
\( \sigma \sim 5 \times 10^{-28} \text{cm}^2 \) at \( \epsilon' \sim 2\epsilon'_{th} \).

See \(^{19}\). From this we can deduce the collision rate in the observer frame

\[
R = \frac{c}{\gamma_p} \int n(\epsilon', \Omega') \sigma_\pi(\epsilon') d\epsilon' d\Omega' \ [\text{nb/s}],
\]

(73)

with \( \int n(\epsilon', \Omega') d\epsilon' d\Omega' = n_{ph} \).

---

Loss timescales

The loss timescale is derived using Eq. 73 but accounting for the inelasticity parameter $K(\epsilon') \to 0.5$ (at high energy, $\epsilon' \gg 1$)²⁰

$$t_{loss} = \left[ \frac{c}{\gamma_p} \int n(\epsilon', \Omega') K(\epsilon') \sigma_\pi(\epsilon') d\epsilon' d\Omega' \right]^{-1}.$$  (74)

In case of a power-law energy density of soft photons $U_\nu \propto \nu^{-s}$ one gets :

$$t_{loss} = \left[ U_{rad, \text{erg/cm}^3} \times E_p^{s, \text{GeV}} \times P(s) \right]^{-1},$$  (75)

where $P(s)$ are tabulated in²⁰ for both photo-pion and photo-pair processes also in case of black body radiation. Typically for $s = 1$, $t_{loss} \sim 10^8 U_{rad, \text{erg/cm}^3}^{-1} \times E_p^{-1} \text{ years}$.

Loss timescales over the cosmic microwave background

We have $U_{CMB} \simeq 4 \times 10^{-13}$ erg/cm$^3$.

Inverse of loss timescale with respect to the cosmic ray energy.

Typically for $\sim 10^{20}$ eV protons $t_{loss} \sim 10^8$ years$^{21}$.

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Synchrotron radiation

- Loss timescale

\[ t_{\text{syn},p} = \left[ \frac{4}{3} \left( \frac{m_e}{m_p} \right)^3 \times \frac{c \sigma T U_B}{m_e c^2 E_{p, GeV}} \right]^{-1} = 1836^3 \times t_{\text{syn},e} . \]  (76)

- Peak of particle radiation

\[ \nu_{c,p} = \frac{m_e}{m_p} \times \nu_{c,e} = \frac{1}{1836} \nu_{c,e} . \]  (77)