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ADIABATIC AND NONADIABATIC PROCESSES IN THERMAL MODELS OF SOLAR HARD X-RAY BURSTS

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ABSTRACT

There is growing interest in the idea that solar hard X-ray bursts are due to bremsstrahlung radiation from hot plasma in a segment at the top of a solar coronal loop. We examine the temporal evolution of such a model under the action of the following processes: (i) time variation of the confining toroidal magnetic field $B(t)$; (ii) longitudinal expansion of the source through the motion of a pair of collisionless conduction fronts; (iii) unspecified heating or cooling processes, which supply energy to the plasma at a net rate $\epsilon(t)$ (ergs cm^{-3} s^{-1}). The behavior of the emission measure and temperature of the bremsstrahlung emitting plasma in response to these processes is examined, and, as a result, analytic expressions for the behavior of $B(t)$ and $\epsilon(t)$ necessary to reproduce a given behavior of emission measure and temperature with time are derived. These expressions are in terms of B_0 and n_0, the initial magnetic field strength and density of the heated region, and τ, a characteristic longitudinal expansion time for the source under the action of process (ii). The results are applied to two impulsive solar hard X-ray bursts, reported by Mätzler *et al.* Our analysis indicates that the contribution of process (iii) is negligible after the first few seconds of an event, but that significant nonadiabatic heating (possibly associated with magnetic reconnection inside the source) occurs early in the events.

Subject headings: hydromagnetics — plasmas — Sun: corona — Sun: flares — Sun: X-rays

I. INTRODUCTION

Although it is generally accepted that solar hard X-ray (photon energy $\gtrsim$ 10 keV) bursts are due to collisional bremsstrahlung of energetic electrons, there is currently much controversy as to whether these electrons form part of a thermally relaxed high-temperature plasma ("thermal" model) or instead a superthermal beam ("nonthermal" model) (see, e.g., Emslie and Rust 1980 and references therein). There is at present a growing interest in thermal models of hard X-ray bursts and their associated radiation signatures at other wavelengths (Crannell *et al.* 1978; Mätzler *et al.* 1978; Brown, Melrose, and Spicer 1979; Smith and Lilliequist 1979; Brown, Craig, and Karpen 1980; Emslie and Brown 1980; Emslie and Vlahos 1980; Smith and Auer 1980; Smith and Brown 1980), largely due to the recognition of the importance of collective plasma effects within a distribution of high-energy thermal electrons (e.g., Vlahos and Papadopoulos 1979).

Mätzler *et al.* (1978, hereafter MBCF) examined the intensity-time profiles and spectral evolution of two hard X-ray bursts observed by the *OSO*-5 satellite on 1969 March 1 (2253 UT) and 1970 March 1 (1127 UT), respectively, and concluded that their behavior could be explained by the action of reversible (adiabatic) thermal processes, such as would result from the confinement of a distribution of thermal bremsstrahlung emitting electrons in a magnetic bottle with a time-varying field strength. They based this conclusion on the observed locus of points traced out in emission measure versus temperature (EM, T) space, noting that a three-dimensional adiabatic compression and expansion of a fixed mass of material yields EM $\propto T^{3/2}$ (for a gas with adiabatic index $\gamma = \frac{5}{3}$) and that "the good agreement [of the data] with . . . the predicted correlation due to an adiabatic process shows that the dynamic X-ray spectrum . . . is consistent with an adiabatic compression and subsequent expansion." Further evidence for adiabatic heating has been adduced by Wiehl and Mätzler (1980) from examination of periodicities in microwave burst time profiles. However, in neither of these papers are physical processes which could give rise to such a three-dimensional adiabatic confinement of hot plasma discussed. (Indeed, it is difficult to envisage a situation leading to compression parallel to the containing magnetic field of a toroidal loop, which is generally recognized as the ubiquitous flare magnetic field geometry [e.g., Cheng and Widing 1975].) Further, neither group of authors considers the expansion of the hard X-ray emitting region along these toroidal field lines as a result of thermal conduction, which, for the high temperatures and correspondingly long collisional mean free paths appropriate, is effected by the motion of a pair of collisionless thermal conduction fronts, propagating symmetrically away from the energy release region at approximately the local ion-sound speed $c_s = (kT/m_p)^{1/2}$, where k is Boltzmann's constant, T the electron temperature, and m_p the proton mass (Brown, Melrose, and Spicer 1979; Smith and Lilliequist 1979; Smith and Auer 1980).

In this paper we therefore examine the evolution of thermal bremsstrahlung emitting material confined in a toroidal loop. We consider the effects of (i) a time-varying toroidal confining magnetic field $B(t)$ (i.e., betatron acceleration; see Brown and Hoyng 1975 for an application of this process to *nonthermal* X-ray burst modeling); (ii) expansion of the source along the field lines as a result of the motion of the above mentioned collisionless conduction fronts; and (iii) heating or cooling of the plasma by unspecified processes (e.g., heating by magnetic reconnection and losses due to radiation and the escape of high energy electrons; see Brown, Melrose, and Spicer 1979; Smith and Brown 1980) resulting in a net heating rate $\epsilon(t)$ (ergs cm^{-3} s^{-1}). Modeling involving process (i) has been carried out by MBCF, and a model involving process (ii) has been investigated by Brown, Craig, and Karpen (1980), but neither of these papers considers both processes simultaneously, and both neglect the effect of process (iii) except as a means of heating the plasma to hard X-ray temperatures at the very beginning of the burst. In § II we explain the model in greater detail and derive the expected behavior of emission measure and temperature as a function of time (in terms of the contributions from the above three processes), and we also show how the expressions for this behavior may be analytically inverted to yield $B(t)$ and $\epsilon(t)$ for a given observed EM(t), $T(t)$ behavior. In § III we apply these results to the two events studied by MBCF, in order to test whether a close fit to an adiabatic (EM, T) behavior necessarily implies that the mechanism responsible for modulating the observed hard X-ray bursts is strongly adiabatic. In § IV we briefly state our conclusions.

II. SOURCE MODEL AND THE BEHAVIOR OF EMISSION MEASURE AND TEMPERATURE WITH TIME

By neglecting the effects of magnetic field fluctuations on the plasma density, Brown, Craig, and Karpen (1980) found it necessary to introduce a large number of small heated regions, each with a very short lifetime, in order to explain the EM(t), $T(t)$ behavior found by MBCF. In our more general model this "multiple kernel" approach is unnecessary to explain the observational behavior of EM and T; this is because the observed decrease in emission measure as the plasma cools (MBCF)—the fundamental difficulty with models involving a single heated region confined by a static magnetic field (see Brown, Craig, and Karpen 1980)—can be accomplished by a reduction in the density of the source as a result of a reduction in the strength of the confining magnetic field (process [i]). In order to make the calculation as simple as possible, therefore, we shall here address only the behavior of a *single* heated region.

The geometry of the X-ray source model is shown in Figure 1. The X-ray emitting material is confined transversely by a strong (in general time-dependent) magnetic field $B(t)$ (gauss) and longitudinally by the ion-acoustic turbulence associated with the collisionless conduction fronts (Brown, Melrose, and Spicer 1979; Smith and Brown 1980), which move along the toroidal axis of the loop at the local ion-sound speed $(kT[t]/m_p)^{1/2}$. The density of the gas is $n(t)$ (cm^{-3}) and the length and area of the bremsstrahlung emitting column of material are $L(t)$ (cm) and $A(t)$ (cm^2), respectively.

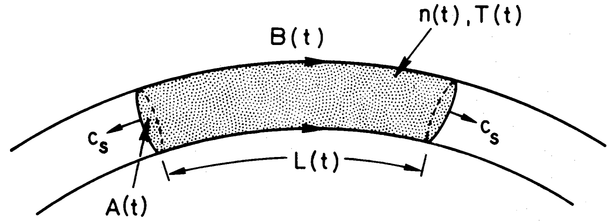

FIG. 1.—The hard X-ray source model considered, at time t in its evolution. Hot ($\gtrsim 10^8$ K) plasma at electron temperature $T(t)$ is confined in a toroidal arch by a strong magnetic field $B(t)$ and by two ion-acoustic turbulent fronts, which form as a result of the instability of the reverse current associated with the free streaming of the hot electrons (see Brown, Melrose, and Spicer 1979) and are separated by a distance $L(t)$. These fronts move down the arch at the ion-acoustic speed c_s (see eq. [1]). The cross-sectional area of, and electron density within, the arch are $A(t)$ and $n(t)$, respectively. The plasma is being internally heated and cooled at a rate $\epsilon(t)$ (ergs cm^{-3} s^{-1}).

The confining magnetic field $B(t)$ is assumed to be purely toroidal and to vary smoothly with time along the entire length of the source; any poloidal (i.e., non-potential) component of the field is assumed to be contained within this smooth outer envelope of toroidal field. Thus, although we shall subsequently invoke annihilation of this poloidal component to provide heating of the flare plasma (process [iii]), the toroidal field $B(t)$ discussed below is considered to change by coherent radial oscillations, driven by (for example) external motions. Any heating or cooling caused by time variation of the strength of this toroidal magnetic field envelope will hereafter be referred to as "adiabatic" heating, and heating or cooling due to the expansion of the source (process [ii]) and to processes occurring within this envelope (process [iii]) will be referred to as "non-adiabatic" heating.

The motion of the conduction fronts causes the length of the X-ray emitting column to increase at a rate

$$\frac{dL(t)}{dt} = 2\left(\frac{k}{m_p}\right)^{1/2}(T[t])^{1/2} = \alpha T^{1/2} , \qquad (1)$$

where the factor 2 appears because of symmetry (note that Brown, Craig, and Karpen 1980 do not allow for this factor). The behavior of $T(t)$ is more complicated; it depends on the amount of compression effected by changes in $B(t)$, on the cooling by motion of the conduction fronts into the cool ambient part of the loop (see Brown, Craig, and Karpen 1980), and by the action of any other heating or cooling agents that may be present (see § I). Denoting this net heating rate by $\epsilon(t)$ (ergs cm^{-3} s^{-1}) and writing

$$b(t) = \frac{B(t)}{B_0} = \frac{A_0}{A(t)} = \frac{n(t)}{n_0} \qquad (2)$$

(where the subscript zero denotes conditions at [arbitrary] $t = 0$ and we have used the conservation of magnetic flux and mass respectively), we have that

$$\frac{d \ln T(t)}{dt} = \frac{2}{3}\frac{d \ln b(t)}{dt} - \frac{d \ln L(t)}{dt} + \frac{\epsilon(t)}{n(t)kT(t)} . \qquad (3)$$

In writing equation (3) we have assumed that the time scale for change of T due to betatron action is less than the time scale for longitudinal expansion of the source (i.e., the source "lifetime"). This is justified by noting that the transverse dimension of the source is smaller than its longitudinal extent, except possibly very early in the event (cf. the observed appearances of flare loops—Cheng and Widing 1975) and that the Alfvén velocity v_A is greater than c_s (this is equivalent to the condition that the plasma β be less than unity, as we have tacitly assumed in speaking of a confined plasma). The factor $\frac{2}{3}$ on the first term on the right hand side appears because betatron compression increases only $T_\perp$, the temperature of electrons moving perpendicular to the toroidal magnetic field lines, and leaves $T_\parallel$ unaltered. Thus, to identify a unique $T(t)$ with the plasma (see MBCF), we must either assume rapid temperature isotropization, or alternatively interpret the temperature derived from the bremsstrahlung spectra obtained by MBCF as $T = (2T_\perp + T_\parallel)/3$. In fact, even for *collisional* relaxation (which probably takes longer than relaxation by wave-particle interactions at the ion-acoustic fronts [see Brown, Melrose, and Spicer 1979]), temperature isotropization occurs on a time scale $\tau_i \sim 5 \times 10^{-2} T^{3/2} n^{-1} \sim (3 \times 10^{11}/n)$ seconds (Spitzer 1962), so that for the X-ray kernel densities $n \sim 10^{11}$ cm^{-3} frequently adopted in thermal hard X-ray burst modeling (Smith and Lilliequist 1979; Emslie and Vlahos 1980), τ_i will be of the order of a few seconds. This is comparable to the time resolution of the MBCF data, making the assumption of temperature isotropization a reasonable one. Finally we note that we have neglected any hydrodynamic effects of $\epsilon(t)$. The characteristic velocities associated with hydrodynamic and conductive motions are in the ratio $(T_i/T_e)^{1/2}$, where T_i and T_e are the ion and electron temperatures respectively; since the essence of the ion-acoustic front dissipative model invoked in the discussion above assumes that $T_i \ll T_e$ (see, e.g., Brown, Melrose, and Spicer 1979), we consider the static plasma approximation to be quite a good one. For further discussion of hydrodynamic effects in thermal hard X-ray source models, see Smith and Lilliequist (1979) and Smith and Auer (1980).

It is a straightforward matter to use equations (1) through (3) to derive expressions for the evolution of EM and T in terms of $b(t)$, $\epsilon(t)$ and the characteristic source expansion time

$$\tau = \frac{L_0}{\alpha T_0^{1/2}} . \qquad (4)$$

However, since we are primarily interested in the reverse problem, i.e., the determination of $b(t)$ and $\epsilon(t)$ for an observed EM(t), $T(t)$ behavior, we now concentrate on this direction in performing the analysis. Defining

$$x(t) = \frac{T(t)}{T_0} \quad \text{and} \quad y(t) = \frac{\text{EM}(t)}{\text{EM}_0} , \qquad (5)$$

we have that

$$y(t) = \left[\frac{n(t)}{n_0}\right]^2 \frac{A(t)}{A_0}\frac{L(t)}{L_0} = b(t)\frac{L(t)}{L_0} , \qquad (6)$$

where we have used equation (2). Equation (1) directly integrates to give

$$L(t) = L_0\left\{1 + \frac{1}{\tau}\int_0^t [x(t')]^{1/2} dt'\right\} , \qquad (7)$$

so that, by equations (6) and (7),

$$b(t) = y(t)\left\{1 + \frac{1}{\tau}\int_0^t [x(t')]^{1/2} dt'\right\}^{-1} . \qquad (8)$$

Using equations (2) and (7) in equation (3) we further derive

$$\frac{\epsilon(t)}{n_0 k T_0} = b\frac{dx}{dt} - \frac{2}{3}x\frac{db}{dt} + \frac{x^{3/2} b}{\tau\left\{1 + \frac{1}{\tau}\int_0^t [x(t')]^{1/2} dt'\right\}} , \qquad (9)$$

which, using equation (8), gives

$$\frac{\epsilon(t)}{n_0 k T_0} = \left[y^{5/3}\frac{d}{dt}\left(\frac{x[t]}{(y[t])^{2/3}}\right)\left\{1 + \frac{1}{\tau}\int_0^t [x(t')]^{1/2} dt'\right\} + \frac{5[x(t)]^{3/2} y(t)}{3\tau}\right] \Big/ \left\{1 + \frac{1}{\tau}\int_0^t [x(t')]^{1/2} dt'\right\}^2 . \qquad (10)$$

Equations (8) and (10) give $B(t)/B_0$ and $\epsilon(t)/n_0 k T_0$ as functions of τ and the observed X-ray spectral parameters $x(t)$, $y(t)$ (i.e., $T[t]$, EM$[t]$). As $\tau \to \infty$ (corresponding to a source whose relative expansion rate is so slow that it may be considered confined longitudinally in addition to transversely), it is readily verified that $\epsilon(t) = 0$ (i.e., the source is adiabatic) if and only if $y(t) \propto (x[t])^{3/2}$—see MBCF. Similarly, for a static field ($b \equiv 1$) but finite τ, the relation $\epsilon(t) = 0$ yields the behavior

$$x(t) = \left(1 + \frac{3t}{2\tau}\right)^{-2/3} , \qquad (11)$$

$$y(t) = \left(1 + \frac{3t}{2\tau}\right)^{2/3} , \qquad (12)$$

in agreement with equation (7) of Brown, Craig, and Karpen (1980), who model a longitudinal expansion of the source but in a static field situation. In the following section we shall apply the general formulae (8) and (10) to derive the behavior of $B(t)$ and $\epsilon(t)$ throughout the two events studied by MBCF and so test the validity of the adiabatic approximation inferred by these authors.

III. APPLICATION TO OBSERVATIONS

In Figure 2 we show the results of applying equations (8) and (10) to the observational parameters derived by MBCF for the impulsive events of 1969 March 1 and 1970 March 1, respectively, with time $t = 0$ set to the first X-ray data point plotted by MBCF for each event. (Note that since our analysis deals only with a single heated region, it is more applicable to single-spike impulsive bursts such

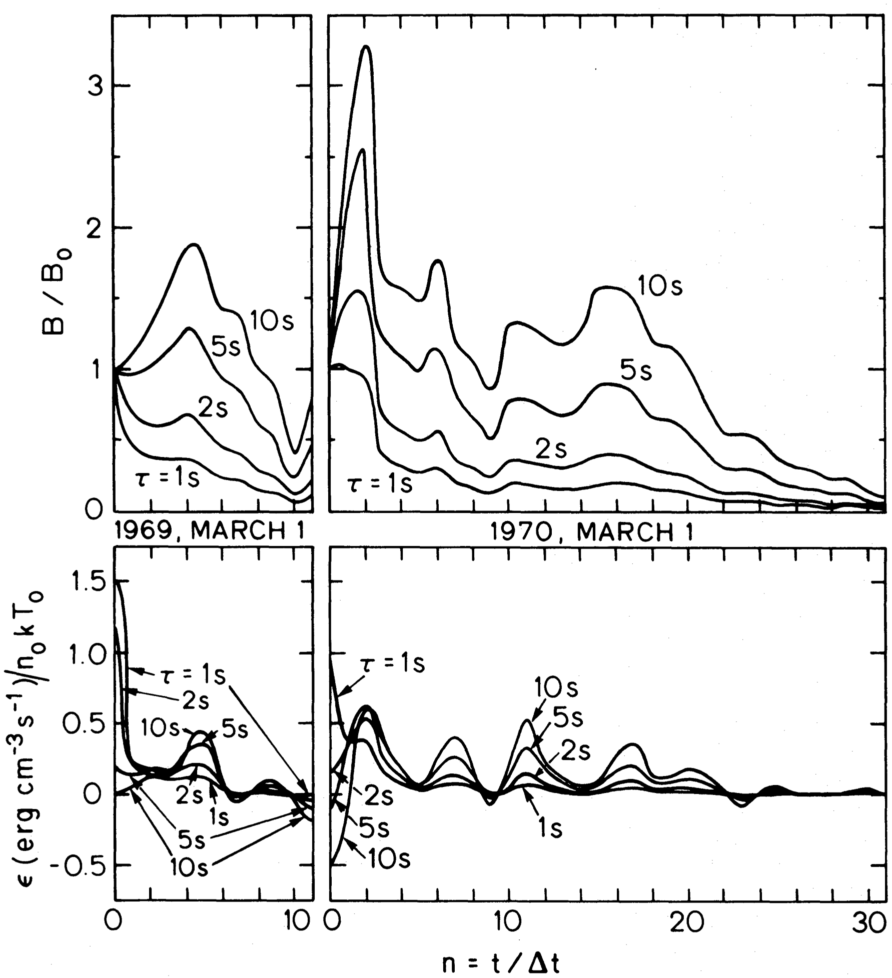

FIG. 2.—Inferred variation of $B(t)$ and $\epsilon(t)$ for the events of 1969 March 1 and 1970 March 1, respectively, based on equations (8) and (10) and the observational (EM, T) parameters quoted by Mätzler *et al.* (1978). The time ordinate is in units of $\Delta t = 1.9$ seconds (data time resolution) past the initial data point ($t = 0$). The curves are labeled with the value of τ (eq. [4]) appropriate.

as those studied by Crannell *et al.* [1978] and MBCF. More complex events should be analyzed in terms of their individual X-ray "spikes," each representing an "elementary flare burst"—see de Jager and de Jonge 1978.) Results are shown for four values of the parameter τ (equation [4]), corresponding to initial heated kernel lengths L_0 of $\sim 2 \times 10^8$, 5×10^8, 10^9, and 2×10^9 cm, respectively (for the source temperatures quoted by MBCF); we feel that this last value of L_0 is a suitable upper limit for the length of the initial heated kernel in a compact flare event (see MBCF, Fig. 2). We do not attempt to quantitatively assess the uncertainties in the values of $B(t)$ and $\epsilon(t)$; these are clearly dependent on the magnitude of the uncertainties in MBCF's estimates of EM and T. We note that there is some debate as to the accuracy and validity of these latter quantities (see Brown 1978); for the present, however, it is clear that since we are analyzing single-spike impulsive bursts, the inferred oscillations in $B(t)$ and $\epsilon(t)$ (Fig. 2) cannot be real and therefore give us some idea of the uncertainties involved (viz., $\sim 50\%$ in B/B_0 and $\sim 0.5 n_0 kT_0$ in ϵ)

Qualitatively, the behavior of $B(t)$ and $\epsilon(t)$ may be ascertained from the (EM, T) loci of MBCF from the following considerations. For static fields, equations (11) and (12) show that the (EM, T) locus of the event should be a straight line with gradient -1 (on a log-log plot), and should be traced in the direction of decreasing T and increasing EM. For infinite τ yet varying B field, the locus (again on a log-log plot) is a straight line of gradient $\frac{3}{2}$, traced "upward" or "downward" according as $B(t)$ is increasing or decreasing. Thus, the path traced out in the (log [T], log [EM]) plane with respect to these two reference lines gives us an indication of the heating and cooling processes at work. For example, for positive ϵ and increasing B, the path traced will be upward and to the right and *below* the gradient $\frac{3}{2}$ reference line (i.e., toward higher T), while for negative ϵ and B still increasing, the path will lie above this reference line. Similarly, for $\epsilon \sim 0$

and slowly varying B, the path will lie approximately on the gradient -1 reference line, slightly above or below it according as B is increasing or decreasing; the directions traced tend to the gradient $\frac{3}{2}$ line in the limit of rapidly varying B.

The dependence of $B(t)$ and $\epsilon(t)$ on τ may also be explained as follows. For large τ, the source expands slowly relative to its original size and so the observed increase in EM must arise from the betatron action of an increasing confining field $B(t)$. On the other hand, for low values of τ the source is expanding so rapidly that $B(t)$ has to *decrease* in order to keep EM small enough to be compatible with observations. In the latter case, since a rapid expansion of the confining field also cools the plasma, $\epsilon(t)$ has to be positive (and larger for smaller τ) to keep the temperature of the plasma acceptable. These trends are clearly evident in the figure.

Concentrating now on the numerical results illustrated in Figure 2, we see that for low values of τ, $\epsilon(t)$ can be quite large (e.g., for $\tau = 1$ s in the 1969 March 1 event, the specific thermal energy nkT of the plasma more than doubles in the first second or so of observing time), although these large values of ϵ are limited to the initial phase of the event (after the first few seconds, yet still on the rise phase of the X-ray burst profile [MBCF], $\epsilon \lesssim 0.3 n_0 k T_0$ in all cases). This shows that there is strong nonadiabatic heating during the first few seconds of observing time; this heating is not associated with magnetic field *compression*, but instead, presumably, with magnetic field *annihilation* inside the source. We also note that $B(t)/B_0$ becomes very small toward the end of the event, decreasing to as little as ~ 0.01 for $\tau = 1$ s in the 1970 March 1 event. (Note the upturn in $B[t]$ and the corresponding negative $\epsilon[t]$ for the last data point in the 1969 March 1 event [also the negative ϵ at large τ for the first data point in the 1970 March 1 event]; this negative ϵ behavior corresponds to the sudden large increase in emission measure and slight cooling reported for these times by MBCF, and we suggest that these [low flux] data points are in fact subject to large uncertainties due to poor X-ray count statistics.) We interpret the small $B(t)/B_0$ toward the end of the events as implying that B_0 is already substantially increased over its "relaxed" preflare value; indeed, the magnetic field required to maintain plasma with a temperature $\gtrsim 2 \times 10^8 K$ (see MBCF) and a density of order 10^{11} cm^{-3} (see Smith and Lilliequist 1979; Smith and Auer 1980) is ~ 300 gauss.

Another characteristic feature of the results of Figure 2 is the large variation in $B(t)$ required to explain the observed hard X-ray characteristics. These large values contrast with the relatively small ($\sim 20\%$) variations in $B(t)$ derived by Brown and Hoyng (1975) in their application of betatron acceleration in a *nonthermal* trap model to the large hard X-ray event of 1972 August 4. This large difference is due to two main reasons. First, the dynamic range of hard X-ray flux in the single-spike bursts analyzed by MBCF is much greater than in the prolonged August 4 event. Second, in a thermal model, the hard X-ray flux is (crudely) proportional to EM(t), in turn roughly proportional to $b(t)$ (equation [6]), while in a *nonthermal* model the hard X-ray flux is proportional to $n(t)$, again proportional to $b(t)$, but also to the nonthermal electron population with energies above the X-ray photon energy under consideration. As pointed out by Brown and Hoyng (1975), this latter quantity can dramatically change for only modest changes in $B(t)$ (and so the nonthermal electron energy), due to the very steep (power-law) form frequently adopted for the nonthermal electron distribution. We note finally, however, that the magnetic field compression ratios required in the present model are significantly less than those required in a model without longitudinal expansion of the source (MBCF); this follows directly from equation (6).

IV. SUMMARY

We have derived a method of inverting the observed (EM$[t]$, $T[t]$) behavior of a single-spike hard X-ray burst to determine the temporal behavior of the physical processes operating in the source (assumed to be emitting by thermal bremsstrahlung). The results (Fig. 2) of applying this procedure to two such events observed by Mätzler *et al.* (1978) show that after the first few seconds of the bursts the source is simply relaxing both by a reduction in the strength of the confining toroidal magnetic field and by dissipating its energy into the cool ambient plasma through the motion of the ion-acoustic turbulent fronts along the flaring loop. In the early stages of the events, however, there is significant nonadiabatic heating $[\epsilon(t)/n_0 k T_0 \sim 1]$, which may be taken as evidence for magnetic field reconnection in islands within the source. There is also an indication of adiabatic heating $[dB(t)/dt > 0]$, if τ (and so the initial source size L_0) is large enough (see eq. [14]). Significant magnetic field changes are necessary to obtain the observed EM(t), $T(t)$ behavior: static field models predict a *negative* correlation of EM with T, generally the opposite to the observed correlation in the bursts studied by Mätzler *et al.* (1978). (As mentioned in § II, Brown, Craig, and Karpen 1980 have attempted to resolve this discrepancy by invoking a superposition of short lifetime kernels, each with a highly asymmetric time profile and a constant thermal energy content [i.e., $\epsilon \equiv 0$]. However, this analysis fails to produce sufficiently hard X-ray spectra [i.e., with sufficiently large color temperatures] without invoking mechanisms of conducting heat from the source which have not yet been supported on a firm theoretical basis.)

As a concluding remark, it is hoped that spatial hard X-ray images from the NASA *Solar Maximum Mission* satellite (which can resolve distances $\gtrsim 5 \times 10^8$ cm) may provide constraints on the parameter L_0 and hence τ, thus removing this degree of freedom from the results.

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