Magnetometer data analysis and Ionospheric electrodynamics (with SECS)

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MIRACLE network

- Magnetometers
- All-sky cameras
- (STARE radars)
- Also several other instruments in the area
## Selected analysis methods

<table>
<thead>
<tr>
<th>Input</th>
<th>Assumptions</th>
<th>Output</th>
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<td>Field continuation and separation</td>
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Adapted from Amm et al (2003)
Ionospheric electrodynamics

• Primary variables: $E$, $J$, $\Sigma_P$, and $\Sigma_H$

• 6 degrees of freedom:
  
  $$E = -\nabla \Phi_E - \hat{e}_\parallel \times \nabla \Psi_E$$
  
  $$J = -\nabla \Phi_J - \hat{e}_\parallel \times \nabla \Psi_J$$
  
  + 2 conductances

• 2 equations: $\nabla \cdot$ and $\nabla \times$ of

  $$J = \Sigma_P E + \Sigma_H \hat{e}_\parallel \times E$$

• Usually may assume $\nabla \times E = 0 \iff \Psi_E = 0$
  
  (sometimes not: Vanhamäki et al., 2007)

• Need to know 3 input variables
2D vector fields

• Ionosphere is approximately 2D
  (some 3D effects may be important, Amm et al., 2008)

• Vector field $\mathbf{V}$ with 2 scalars
  • potentials: $\mathbf{V} = -\nabla \Phi - \mathbf{e}_\parallel \times \nabla \Psi$
  • divs and curls: $f = \nabla \cdot \mathbf{V}$ and $g = (\nabla \times \mathbf{V})_\parallel$

• $\Phi$ and $\Psi$: Spherical (cap) harmonics, Fourier series ...

• $f$ and $g$: SECS
Spherical Elementary Current Systems

• Green's functions of $\nabla \cdot$ and $\nabla \times$ operators
• Complete set of basis functions

\[ \vec{J}_{el,cf} = \frac{I_{el,cf}}{4\pi R_I} \delta (R_I - r) \cot (\theta'/2) \hat{e}_\theta, \]

\[ \vec{J}_{el,df} = \frac{I_{el,df}}{4\pi R_I} \delta (R_I - r) \cot (\theta'/2) \hat{e}_\phi, \]
Ionospheric equivalent current

• Impossible to determine ionospheric $J$ from ground $B$

• **Definition:** $J_{eq}$ is div-free spherical sheet current that gives correct $B$ below the ionosphere

• $J_{eq}$ always exist and is unique (potential theory)

• If $\chi \geq 70^\circ$ then $J_{eq} \approx$ div-free part of $J$
  
  $\rightarrow$ Get $(\nabla \times J)_{\parallel}$ from ground $B$
\( \vec{J}_{el, cf} = \frac{I_{el, cf}}{\xi \pi R_I} \delta (R_I - r) \cot \left( \frac{\theta'}{2} \right) \hat{e}_\theta' \)

\( \vec{B}_{el, cf} = \frac{\mu_0 I_{el, cf}}{4 \pi r} \left[ \left( \frac{R_I \cos \left( \frac{\theta'}{2} \right) - r}{\sin \left( \frac{\theta'}{2} \right) \sqrt{r^2 - 2r R_I \cos \left( \frac{\theta'}{2} \right) + R_I^2}} - \cot \left( \frac{\theta'}{2} \right) \right) \hat{e}_\theta' + \left( \frac{1}{\sqrt{r^2 - 2r R_I \cos \left( \frac{\theta'}{2} \right) + R_I^2}} - 1 \right) \hat{e}_\theta' \right] \)

**\( \vec{J}_{eq} \) with SECS**

- Amm and Viljanen (1999)
- Need only div-free SECS
- Analytical \( \vec{B} \) for individual SECS
\[(\text{Measured } B_H) = (\text{Transfer matrix}) \cdot (\text{DF SECSs})\]

- Ground induction
  \[\rightarrow\text{ Only horizontal } B\]

\[+ = \text{SECS pole}\]
\[\bullet = \text{magnetometer}\]
Examples

Pulkkinen et al. (2003a)

Huttunen et al. (2002)

THEMIS, Amm 2008
Further developments

- $B_G$ separation into internal and external parts, Pulkkinen et al. (2003b)
  (often not feasible in practise)

- 1-dimensional version for satellite analysis, Juusola et al. (2006)
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<td>AMIE</td>
<td>Optimization method, also with sparse data</td>
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<td>Elementary current method</td>
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<td>Method of characteristics ($J_{eq}$-based)</td>
<td>$\alpha$ assessable from ASC or $B_G$ data</td>
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KRM method

- Now we have \((\nabla \times \mathbf{J})_\parallel = (\nabla \times \mathbf{J}_{eq})_\parallel\) from \(B_G\)

- If we know \(\Sigma_P\) and \(\Sigma_H\)
  \(\rightarrow\) Can solve \(\mathbf{E}\) and \(\mathbf{J}\)

- KRM method by Kamide et al. (1981)
  - Works only globally

- Local KRM with SECS by Vanhamäki and Amm (2007)
Comparison

• Classical KRM
  • Find such $\mathbf{E} = -\nabla \Phi_E$ that $(\nabla \times \mathbf{J})_\parallel = (\nabla \times \mathbf{J}_{eq})_\parallel$
  • Boundary conditions for $\Phi_E$? (Murison et al, 1985)

• SECS-based KRM
  • Find such $j_\parallel = \nabla \cdot \mathbf{J}$ that $\nabla \times \mathbf{E} = 0$
  • Represent $\mathbf{J}$ and $\mathbf{E}$ with SECS
  • Implicit boundary conditions (sources vanish outside)
Example

- $\Omega$-band event
- $J_{eq}$ from MIRACLE
- $\Sigma_P$ and $\Sigma_H$ from UVI
- $E$ and $J$ from KRM
Summary

• 5 variables, 2 equations → Need 3 as input

• Ground $\mathbf{B}$ gives equivalent current, or $(\nabla \times \mathbf{J})_\parallel$

• Spherical Elementary Current Systems (SECS)
  • div-free and curl-free basis functions

• Local KRM with SECS, boundary conditions
References


