

# MAGNETOSPHERE AND IONOSPHERE AS A COUPLED SYSTEM: THEORY AND OBSERVATIONS

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**Abstract:** The coupling of the terrestrial magnetosphere and ionosphere is fundamental to understanding the behaviour of each region. Strong coupling is often associated with the closure of intense electrical currents flowing along magnetic fields lines. The Alfvén wave is a natural agent for carrying these currents, and is known to modify signatures of coupling such as optical auroral emissions when magnetospheric electrons are precipitated in the ionosphere (in so-called 'upward' current regions) as shown in Figure 1. In 'downward' current regions, cold ionospheric electrons are removed and flow into the magnetosphere [Wright *et al.*, 2008]. The redistribution of plasma by currents is rarely taken into account in models of magnetosphere-ionosphere coupling, and traditionally it is more likely that the enhancement of ionospheric conductivity (associated with energetic electron precipitation) is regarded as the dominant effect.

Radar observations [Aikio, 2004] show that ionospheric plasma depletion in downward current regions can be substantial (by up to 90% over 1 minute). This will cause a significant drop in conductivity and have important implications for current flow and hence magnetosphere-ionosphere coupling. This has received little attention in modelling, except by members of the proposed team [Streltsov and Lotko, 2004; Karlsson *et al.*, 2005; Russell, Wright and Hood, 2010]. Recent optical observations of a downward current region (sitting between two auroral arcs) reveal how it widens in an effort to carry the current required by the magnetosphere [Michell *et al.*, 2008]. This behaviour also extends along field lines into the magnetosphere, where the Cluster spacecraft have seen the spatial width of a strong downward current broaden in time [Marklund *et al.*, 2001].

The proposed team will focus on the self-consistent nonlinear coupling of the magnetosphere and ionosphere through observational signatures and theory. We will bring together several observational techniques: The combined use of radar providing coverage of the ionosphere up to 200 km altitude, the FAST satellite providing coverage through the transition region from 400 km up to 4000 km, and the Cluster spacecraft allowing further coverage out to several RE allows the study of magnetosphere and ionosphere coupling in a holistic manner, rather than through the use of assumed inputs into each component part. We will also develop a physical appreciation of the signatures seen in data through the development of theory and modelling. This will take the form of analytical theory and state of the art multi-fluid numerical simulations and kinetic theory.

The team that will carry out this work comprises leading international scientists from around the world who have specialities covering the complementary aspects of our proposal. The success of our research may lead to a significant re-evaluation of the role of the ionosphere in the global picture of magnetosphere-ionosphere coupling at high latitudes and hence add a significant value to the publicity of research programs performed under the auspices of the International Space Science Institute.

**Proposal History:** This proposal is a resubmission from the 2010 and 2011 calls. In light of the feedback from ISSI we have made the following changes:

- Some topics from the previous proposal have been omitted, and the remaining ones reworked into a more focussed research plan that is less ambitious, but realistic.
- The revised **Team Members** section and CV Appendix make it clear (unlike the previous proposal) that the team has extensive expertise with regard to *in situ* particle measurements.
- A detailed account of realistic anticipated outputs (publications, conference sessions, etc.) is provided.
- A new section has been added explaining how the different aspects of the programme will be integrated together.
- ISSI's added value is stated explicitly.

The present proposal has grown out of an existing ISSI team. During the final meeting of that team a new participant attended (Prof. Josh Semeter), who is an expert on ionospheric plasma physics and radar observations. Some of the ideas emerging from the team provided alternative interpretations to the conventional views held by ionospheric scientists and is leading to a very fruitful exchange of ideas that we plan to continue. Indeed, as a result of the previous ISSI team we already have (1) our own observing campaign running on the PFISR radar facility in the US; (2) a Ph.D. student successfully graduated through participating in ionospheric modelling related to the previous ISSI programme; (3) a second Ph.D. successfully graduated through successful theory completed directly as result of the previous programme; (4) applications submitted to several funding agencies to fund Postdoctoral Researchers and Postgraduate students to support the proposed future research programme.

The make up of the new team is different to that of the original ISSI team to reflect the shift of focus and goals of the present proposal.

## Scientific Rationale:

### Overview

Figure 1 shows the current circuit involved in Magnetosphere-Ionosphere coupling. The *upward* current leg is carried by accelerating magnetospheric electrons into the ionosphere and causes the visible aurora. The return (*downward*) current region is fed by accelerating ionospheric electrons out into the magnetosphere. These 'field-aligned' currents are naturally carried by Alfvén waves, which are the primary agent in coupling the magnetosphere and ionosphere. Indeed, information from the distant magnetosphere can be communicated to the ionosphere by the propagation of an Alfvén wave. There will then be a reflected Alfvén wave, which will communicate the response of the ionosphere back to the magnetosphere.

The self-consistent interaction of the magnetosphere and ionosphere as a single coupled system has been studied by several of our team member to date [Russell *et al.*, 2010; Russell and Wright, 2011; Streltsov and Lotko, 2004], and shows the process to be rich in complex physics: The flow chart in Figure 2 depicts the processes, which is continually driven by an incident Alfvén wave from the magnetosphere carrying a field-aligned (parallel) current. The value of this current at the base of the magnetosphere is central to understanding the behaviour and, depending upon its sign, will either precipitate electrons in the ionosphere (upward current) or remove electrons from the ionosphere (downward current). Whatever the case, the current will change the ionospheric density and hence electrical conductivity. The ionosphere will launch a reflected Alfvén wave into the ionosphere whose properties depend upon this conductivity. Subsequently the waves at the base of the magnetosphere are a superposition of incident and reflected Alfvén waves which *together* determine the local parallel current, and hence density and conductivity evolution. This process feeds back on itself in a self-consistent fashion that is nonlinear, and can give rise to the formation of small scales that introduce new physics such as electron inertial effects.

Figure 3 illustrates (in the right panels) the drop in ionospheric electron number density as a downward current flux tube moves overhead and evacuates electrons. The left panel is a snapshot from simulations showing the depressed conductivity in the downward current region, and the small scale electron inertial Alfvén waves that fill the magnetosphere above it.

The formation of low density ionospheric cavities, and the associated signatures in the magnetosphere are the central theme of the proposal. There are a number of ways to study the interaction illustrated in Figures 2 and 3, and we propose to bring together a number of complementary approaches:

- **Radar Observations** of the ionospheric density cavity and *conjugate* low altitude **satellite data**.
- **Satellite Observations** of the reflected Alfvén wave which may exhibit electron inertial features and strong field aligned currents.
- **Modelling and Theory** to assist with understanding and interpreting the above data.

### Radar-Satellite Conjunctions

#### Radar Data:

Understanding the propagation and reflection of Alfvénic power within the magnetosphere-ionosphere system, and its consequences for ionospheric structuring and outflow, requires consideration of three-dimensional, time-dependent physics. Such models are inherently difficult to evaluate, but recent advances in ionospheric remote sensing, exploited in our proposed research, can contribute significantly to this debate.

The ionosphere has long been interpreted as a projection of magnetospheric dynamics. The optical aurora provides a direct measure of particle kinetic energy flux, which can be used to evaluate physical mechanisms for auroral acceleration, including wave-particle coupling in small-scale dispersive Alfvén waves [Semeter, *et al.*, 2008]. Incoherent Scatter Radars (ISRs) measure variability in the ionospheric state parameters ( $Ne$ ,  $Te$ ,  $Ti$ ,  $Vi$ ) as a function of space and time. When these measurements are coupled, a powerful diagnostic emerges for understanding magnetospheric forcing and the ionospheric response. This is exemplified in the figure 4, which illustrates the dramatic changes in ionospheric state at the boundary of an Alfvénic auroral arc at the poleward boundary of the auroral oval [Semeter *et al.*, 2005].

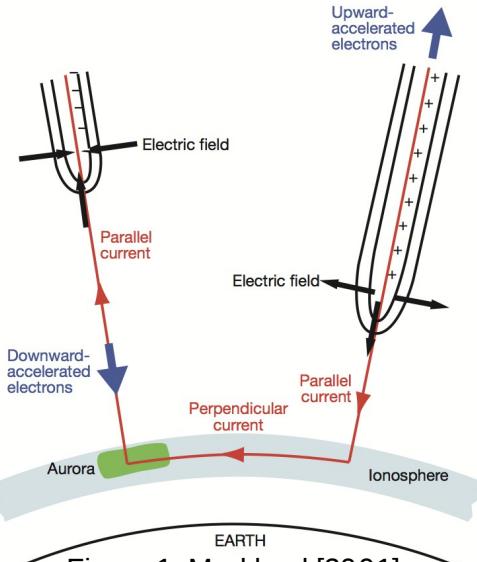


Figure 1: Marklund [2001], auroral current circuit.

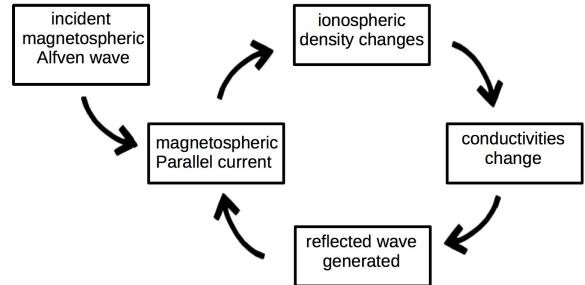


Figure 2: Self-consistent interaction of the driven magnetosphere-ionosphere system.

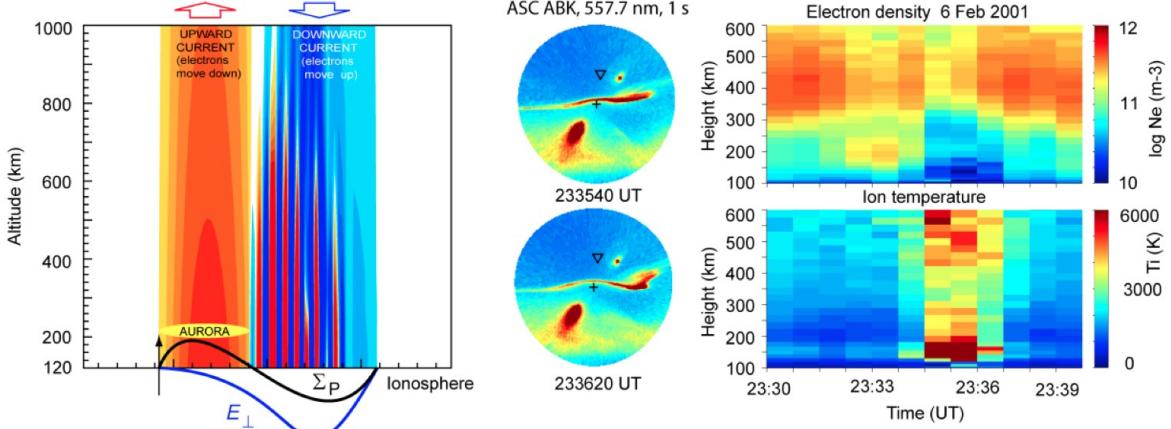


Figure 3: Modelling and observations of magnetosphere-ionosphere coupling [Streltsov and Lotko, 2004; Aikio *et al.*, 2004].

The measurements in this figure were acquired by mechanically steering a 30-ton antenna across an auroral boundary. A major step forward in this capability came about in 2007, with the commissioning of the first electronically steerable ISR. The so-called Poker Flat ISR (PFISR) can be re-pointed over a dense grid of beam positions on a pulse-by-pulse basis, allowing the construction of three-dimensional images of the ionosphere at rapid cadence [Semeter *et al.*, 2009]. The implications of this new diagnostic capability for studies of magnetosphere-ionosphere coupling are still being explored. The proposed working group will serve as a valuable focal point for these efforts.

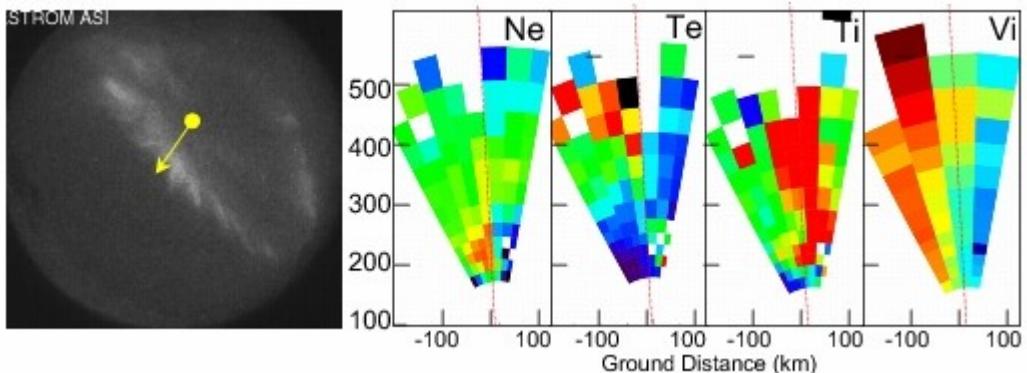


Figure 4: All-Sky auroral image and 2D profiles of ionospheric parameters [Semeter *et al.*, 2004].

#### *Satellite Particle Data:*

A key region for understanding the complex coupling between the ionosphere and the magnetosphere is the transition region between the cold dense plasmas of the ionosphere and the hot tenuous plasmas of the magnetosphere. Since launch in August 1996, the FAST [Carlson *et al.*, 1998] satellite has gathered a wealth of observations of the processes occurring through this region. It is here that the effects of the drag and feedback of the collisional and conducting ionosphere on magnetospheric processes become easily observable. While there has been a considerable body of work performed to advance understanding of the acceleration processes which drive aurora from a number of satellite missions to this region, there has been little done to identify the action of ionospheric effects and how these are manifested in the magnetosphere.

We plan to use incoherent scatter radar measurements (EISCAT and PFISR) and magnetically conjugate measurements from FAST to advance understanding of the coupled system. Since the launch of FAST there have been numerous such conjunctions which, to date, have not been extensively used. Obvious correlations include the measurement of ion outflows, heating, and density depletions and enhancements in regions of downward and upward current [Streltsov and Lotko, 2004].

#### Satellite Observations of Small Scale Alfvén Waves

An important mediator of energy exchange between the magnetosphere and the ionosphere is the Alfvén wave. Many studies have been devoted to the Alfvén wave interaction with a passively responding ionosphere, producing global field line resonances with periods greater than  $\sim 150$  s [e.g. Southwood, 1974; Chen and Hasegawa, 1974], or resonances within the ionospheric Alfvén resonator (0.1 – 10 s) [e.g. Trakhtengertz and Feldstein, 1987; Lysak, 1988]. More rarely has the modification of the ionosphere due to the currents, carried by Alfvén waves in these scenarios, been taken into account, and then mostly modifications due to precipitation in the upward current region [Polyakov and Rapoport, 1981; Sato, 1978]. These models predict the creation of small transverse-scale Alfvén waves within the frequency ranges given above by the respective resonant

system.

Recently small-scale Alfvén waves with periods *in between* those predicted by the above theories have been observed by the Cluster satellites in the lower magnetosphere [Karlsson *et al.*, 2004], and is reproduced in Figure 5. Cluster is a fleet of four identical spacecraft. Electric and magnetic field components are plotted in the figure for each spacecraft. A careful analysis of the data is able to determine the spatial scale of fluctuations and their temporal dependence. Our modelling shows that one candidate for the formation of waves with these properties is the interaction of a large-scale field-aligned current system with the nightside, low-conductivity ionosphere [e.g. Streltsov and Karlsson, 2008; Russell *et al.*, 2010; Russell and Wright, 2011], in particular the evacuation of ionospheric plasma in the downward current region. One prediction of this type of magnetosphere-ionosphere interaction is that the waves will be generated preferentially at the interface between the upward and downward current regions, where in the latter the ionospheric conductivity is strongly modified by the plasma evacuation.

### Modelling and Theory

The above sections discuss observations that reveal how the magnetosphere and ionosphere are coupled together self-consistently. We shall perform mathematical modelling and numerical simulations to deepen our understanding of these processes. To date, we have demonstrated that the region between upward and downward current systems is a natural location for small scales to develop in the conductivity [Streltsov and Lotko, 2004; Russell *et al.*, 2010; Russell and Wright, 2011], and this allows new physics to become important. In the new programme we will study this nonlinear coupling in more detail by introducing electron inertial effects in the magnetosphere to understand, in a fundamental fashion, what governs the period of the waves produced at the upward/downward current interface. The development of this theory will be completed with a view to studying the spatial and temporal scales that Cluster can identify in the investigation described in the section "Satellite Observations of Small Scale Alfvén Waves" above.

The depletion of ionospheric plasma and resulting loss of conductivity impedes current closure in the ionosphere. To date we have modelled this by representing the ionosphere as a conducting sheet. This has been very successful, but will be of limited use for the more detailed observing campaign the team is running on the PFISR radar. We plan to extend modelling to provide a numerical scheme that resolves the E and F regions and can be coupled to a responsive magnetosphere.

### Integrated Nature of Our Multi-Disciplinary Approach

The central theme of this proposal is the self-consistent interaction of the magnetosphere and ionosphere and the associated production of density cavities. We aim to pursue this by addressing situations where there is strong coupling through intense auroral field-aligned currents. Of particular importance in Figures 3 and 4 is the drop in plasma density at 200 km altitude. This has been traditionally associated with evacuation through ion heating. However, we call into question the reliability the ion temperature supplied by automated routines, and are in the process of selecting and analysing similar events manually. Our modelling work to date indicates that field-aligned currents can play a major role in depleting ionospheric electron density (thus causing cavities) and so provides an alternative to the traditional mechanisms based on ion heating. Moreover, the PFISR experiments will provide a 3D time-dependent view of this process to clarify the dominant physics.

We shall complement radar observations with conjugate satellite data to facilitate the unambiguous identification of the physical processes operating in cavity formation. In particular the ion outflow signature at FAST will allow us to assess the reliability of the ion heating inferred from radar data. Measurement of these processes in the ionosphere using radar and at ~4000 km by FAST will reveal the extent of these phenomena along the field-line and allow an assessment of their importance through feedback into the magnetosphere.

Our modelling has indicated that the formation of highly evacuated cavities are also associated with the production of electron inertial Alfvén waves propagating away from Earth at the edge of the downward current and adjacent to the upward current channel. Indeed, we have also analysed a case study of Cluster data showing such waves (Figure 5). The analysis of more such events will give a clearer idea of the frequencies of these waves which can be used with modelling to infer the properties of the ionosphere that launched the waves. This 'seismological' use of satellite wave data to infer ionospheric parameters is a particularly novel aspect of our modelling work.

The combination of ionospheric radar observations and satellite observations, together with their integration through theoretical modelling will provide this ISSI team a unique opportunity to develop a coherent picture of the coupled magnetosphere-ionosphere system by determining (1) the dominant processes that cause plasma depletion in the ionosphere; (2) how global currents close in the ionosphere; (3) what the effect of these depletions are on the magnetosphere and *in situ* satellite measurements.

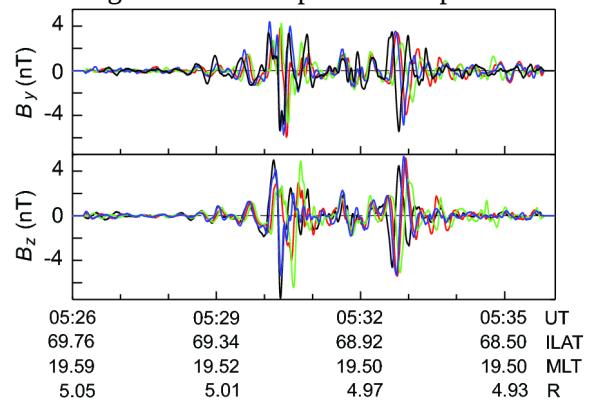


Figure 5: **E** and **B** fields from the four Cluster spacecraft [Karlsson *et al.*, 2004].

**Team Members (comprising 5 nationalities):**

Dr. Christopher Chaston ([ccc@berkeley.edu](mailto:ccc@berkeley.edu)) from the University of California at Berkeley, USA, who is an expert in data analysis of electromagnetic fields and particles measured by Polar, FAST, and THEMIS satellites on auroral magnetic field lines.

Dr. Tomas Karlsson ([tomas.karlsson@ee.kth.se](mailto:tomas.karlsson@ee.kth.se)) from the University of Stockholm, Sweden, who is an expert in data analysis of electromagnetic fields and particles measured by the Cluster satellites in the magnetosphere.

Dr. Hans Nilsson ([hans.nilsson@irf.se](mailto:hans.nilsson@irf.se)), from the Swedish Institute of Space Physics, Kiruna, is an expert in EISCAT radar data analysis, optical auroral and Cluster observations of particles.

Prof. Josh Semeter ([jls@bu.edu](mailto:jls@bu.edu)) from Boston University, USA, who is an expert on radar observations of the ionosphere and modelling of auroral features.

Prof. Anatoly Streltsov ([streltsa@erau.edu](mailto:streltsa@erau.edu)) from Embry-Riddle Aeronautical University, Florida, USA, who is an expert in numerical simulations of magnetosphere-ionosphere interactions in the auroral zone.

Dr Andrew Wright ([andy@mcs.st-and.ac.uk](mailto:andy@mcs.st-and.ac.uk)) from the University of St Andrews, UK, who is an expert MHD and kinetic theory, numerical simulations, and mathematical modelling in Solar-Terrestrial Physics. Dr Wright will serve as the Team Leader for this project.

**Expected Outputs:** We anticipate that results from this project will lead to one substantial research paper co-authored by the whole team that provides an overview of the progress we have made and summarises the competing ideas that are thought to govern local magnetosphere-ionosphere coupling. In addition we anticipate 3 more specialist papers describing (1) the new model we will develop, and its behaviour; (2) Radar observations of the ionosphere correlated with conjunctions of the FAST satellite to unambiguously identify if radar observations of ion heating and outflow are reliable measurements; (3) Cluster observations of Alfvén waves in the downward current region to identify the spatial and temporal scales of these waves and assess the importance of electron inertial effects. These papers will be suitable for publication in leading peer-review journals such as *Journal of Geophysical Research* or *Annales Geophysicae*.

The previous ISSI team was acknowledged in two Ph.D. theses, and we plan to continue this valuable training aspect for new Ph.D. students. It also convened a session at the AGU Fall meeting in 2010. We plan to convene a similar session after conclusion of the proposed programme, perhaps at EGS this time.

All publications and conference presentations of our research will acknowledge the valuable role played by ISSI.

**Schedule of the Project:** The project will be accomplished within 18 months during which time the group will have 3 one-week meetings at ISSI. The first meeting will take place in August, 2012. The second meeting will take place 9 months later in May, 2013. During these two meetings we will discuss our results and fine tune our plans. During the last meeting (in February of 2014) we will summarize our results and agree the schedule for submitting research papers and proposing conference sessions.

**Facilities Required:** Each team member will conduct his research at his home institutions. All that is required from ISSI is to provide basic computer equipment during team meetings for presentations and connection to the Internet.

**Financial Support Requested of ISSI:** The standard financial support from ISSI as it is described in Section 6 of the Call for Proposals 2012 will be adequate for our team.

**Added Value of ISSI:** The proposed research fits well within the areas supported by ISSI. The results are expected to catalyse a re-evaluation of time-dependent magnetosphere-ionosphere coupling which is broadly relevant to our understanding of stellar-planetary interactions in the universe. To complete the programme it is essential for the team members to meet for extended discussions (as stated in the programme schedule), which will only happen if ISSI support our programme for the following reasons: (1) An ISSI programme provides a focussed programme that all team members commit to for 18 months. (2) ISSI provides an ideal location for an international gathering and a very conducive environment to the style of meeting we aim to hold. (3) Without the financial support of ISSI we will not be able to meet and complete our programme - indeed, we have failed to do so over the last 2 years since our previous Team finished. Hence the focus provided by a sponsored ISSI team is essential.

## Appendix I

References cited in the proposal:

- Aikio, A., T. K. Mursula, S. Buchert, F. Forme, O. Amm, G. Marklund, M. Dunlop, D. Fontaine, A. Vaivads, and A. Fazakerley (2004), Temporal evolution of two auroral arcs as measured by the Cluster satellite and coordinated ground-based instruments, *Ann. Geophys.*, 22, 4089.
- Carlson, C. W., R. F. Pfaff, and J. G. Watzin (1998), The Fast Auroral Snapshot (FAST) mission, *Geophys. Res. Lett.*, 25, 2017.
- Chen, L., and A. Hasegawa (1974), A theory of long-period magnetic pulsations, 1, Steady state excitation of field line resonance, *J. Geophys. Res.*, 79, 1024.
- Karlsson, T., G.T. Marklund, S. Figueiredo, T. Johansson, and S.Buchert (2004), Separating spatial and temporal variations in auroral electric and magnetic fields by Cluster multipoint measurements, *Ann. Geophys.*, 22, 2463.
- Karlsson, T. , G. Marklund , N. Brenning, and I. Axnas, On Enhanced Aurora and Low-Altitude Parallel Electric Fields, *Physica Scripta*. Vol. 72, 419–422, 2005.
- Keiling, A., et al. (2005), Some properties of Alfvén waves: Observations in the tail lobes and plasma sheet boundary layer, *J. Geophys. Res.*, 110, A10S11, doi:10.1029/2004JA010907.
- Lysak, R. L. (1988), Theory of auroral zone PiB pulsation spectra, *J. Geophys. Res.*, 93, 5942.
- Marklund, G. T., et al. (2001), Temporal evolution of the electric field accelerating electrons away from the auroral ionosphere, *Nature*, 414, 724–727.
- Mende, S. B., S. E. Harris, H. U. Frey, V. Angelopoulos, C. T. Russell, E. Donovan, B. Jackel, M. Greffen, and L. M. Petigolas (2008), The THEMIS array of ground based observatories for the study of auroral substorms, *Space Sci. Rev.*, 141, 357- 387, doi:10.1007/s11214-008- 9380-x.
- Michell, R. G., K. A. Lynch, and H. C. Stenbaek-Nielsen (2008), Ground-based observational signature of a downward current channel in an active auroral arc, *Geophys. Res. Lett.*, 35, L08101, doi 10.1029/2008GL033596.
- Polyakov, S.V. and V.O. Rapoport (1981), The ionospheric Alfvén resonator, *Geomagn. Aeron.*, 21, 816.
- Robinson, T. R., et al. (2000), FAST observations of ULF waves injected into the magnetosphere by of modulated RF heating of the auroral electrojet, *Geophys. Res. Lett.*, 27, 3165.
- Russell, A. J. B., A. N. Wright, and A. W. Hood (2010), Self-consistent ionospheric plasma density modifications by field aligned currents: steady state solutions, *J. Geophys. Res.*, 115, A04216, doi:10.1029/2009JA014836.
- Russell, A. J. B., and A. N. Wright (2012), Ionosphere-Magnetosphere waves, *J. Geophys. Res.*, 117, A01202, doi:10.1029/2011JA016950.
- Sato, T (1978), A theory of quiet auroral arcs, *J. Geophys. Res.*, 83, 1042.
- Semeter, J., T. Butler, C. Heinselman, M. Nicolls, J. Kelly, and D. Hampton (2009), Volumetric imaging of the auroral ionosphere: Initial results from PFISR, *J. Atmos. Sol. Terr. Phys.*, 71, 738–743. doi:10.1016/j.jastp.2008.08.014.
- Semeter, J., M. Zettergren, M. Diaz, and S. Mende (2008), Wave dispersion and the discrete aurora: New constraints derived from high-speed imagery, *J. Geophys. Res.*, 113, A12208, doi:10.1029/2008JA013122.
- Semeter, J., C.J. Heinselman, G. Sivjee, H.U. Frey, and J.W. Bonnell (2005), The ionospheric response to wave-accelerated electrons at the polar cap boundary, *J. Geophys. Res.*, 110, A11310, doi:10.1029/2005JA011226.
- Southwood, D. J. (1974), Some features of field line resonances in the magnetosphere, *Planet. Space Sci.*, 22, 483.
- Streltsov, A.V., and W. Lotko, Multiscale electrodynamics of the ionosphere-magnetosphere system, *J. Geophys. Res.*, 109, doi:10.1029/2004JA010457, 2004.
- Streltsov A., T., and T. Karlsson (2008), Small-scale, localized electromagnetic waves observed by Cluster: Result of magnetosphere-ionosphere interactions, *Geophys. Res. Lett.*, 35, 22, L22107, doi: 10.1029/2008GL035956.
- Streltsov A., T. T.R. Pedersen, E.V. Mishin, and A.L. Snyder (2010), Ionospheric feedback instability and substorm development, *J. Geophys. Res.*, in press.
- Trakhtengertz, V. Y., and A. Y. Feldstein (1987), Turbulent regime of magnetospheric convection, *Geomagn. Aeron.*, 27, 221.
- Wright, A. N., C. J. Owen, C. C. Chaston, and M. W. Dunlop, Downward current electron beam observed by Cluster and FAST, *J. Geophys. Res.*, 113, A06202, doi:10.1029/2007JA012643, 2008.

## **Appendix II**

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