A Simplified Treatment of Ambipolar Diffusion?

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- Ambipolar diffusion (AD) is the process of ions and neutrals drifting apart in a partially ionized plasma (PIP),
  - Allowing the magnetic flux to diffuse out of compressed, low-ionization regions.

- AD is generally thought to be important for the collapse of magnetically supported molecular cloud cores.

- Its numerical calculation often involves very short timesteps.
  - Making numerical simulations expensive.

  - Often solved by advancing the induction equation alone on its own timestep (e.g., Mac Low+95)

- Is there a way to circumvent this technical difficulty?

AD basics:

- The relevant equations:
  - Momentum equation:

$$\frac{\partial \mathbf{u}_n}{\partial t} + \mathbf{u}_n \cdot \nabla \mathbf{u}_n = -\nabla P_n + \underbrace{(\nabla \times \mathbf{B}) \times \mathbf{B}}_{\text{Lorentz force}}$$

  - Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u}_n \times \mathbf{B})}_{\text{Flux freezing}} + \underbrace{\nabla \times \left\{ \frac{\mathbf{B}}{4\pi\gamma\rho_n\rho_i} \times [\mathbf{B} \times (\nabla \times \mathbf{B})] \right\}}_{\text{AD}}$$

with ρ_n = mass density of neutrals; ρ_i = mass density of ions

u_n = neutrals bulk velocity.

- AD has two simple limiting behaviors:
  - Flux-freezing (ideal MHD) when AD is negligible.

$$\frac{\partial \mathbf{u}_n}{\partial t} + \mathbf{u}_n \cdot \nabla \mathbf{u}_n = -\nabla P_n + \left(\nabla \times \mathbf{B}\right) \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times (\mathbf{u}_n \times \mathbf{B})$$

  - Decoupled HD + uniform B when AD is strong.

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times \left\{ \frac{\mathbf{B}}{4\pi\gamma\rho_n\rho_i} \times \left[\mathbf{B} \times \left(\nabla \times \mathbf{B}\right)\right] \right\} \Rightarrow \mathbf{B} \to \mathrm{cst.}$$

$$\frac{\partial \mathbf{u}_n}{\partial t} + \mathbf{u}_n \cdot \nabla \mathbf{u}_n = -\nabla P_n + \cancelto{0}{\left(\nabla \times \mathbf{B}\right) \times \mathbf{B}}$$

- *In the latter case, one is integrating frantically* (timescale for AD is << than flux-freezing term) *just to decouple the field!*

- Is it possible to take advantage of these limiting behaviors to avoid direct integration with small timesteps?

- A case where it works: cooling (VS et al. 2007, ApJ, 657, 870)

- The internal energy equation per unit mass including cooling ($n\Lambda$) and heating (Γ) specific rates is:

$$\frac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e = -(\gamma - 1) e \nabla \cdot \mathbf{u} + \Gamma - n\Lambda$$

- When heating and cooling dominate, this becomes

$$\frac{de}{dt} \approx \Gamma - n\Lambda \quad \Rightarrow \quad n\Lambda \rightarrow \Gamma$$

(approach to thermal equilibrium)

- The thermal equilibrium condition

$$n\Lambda(n,e)=\Gamma(n,e)$$

together with the equation of state

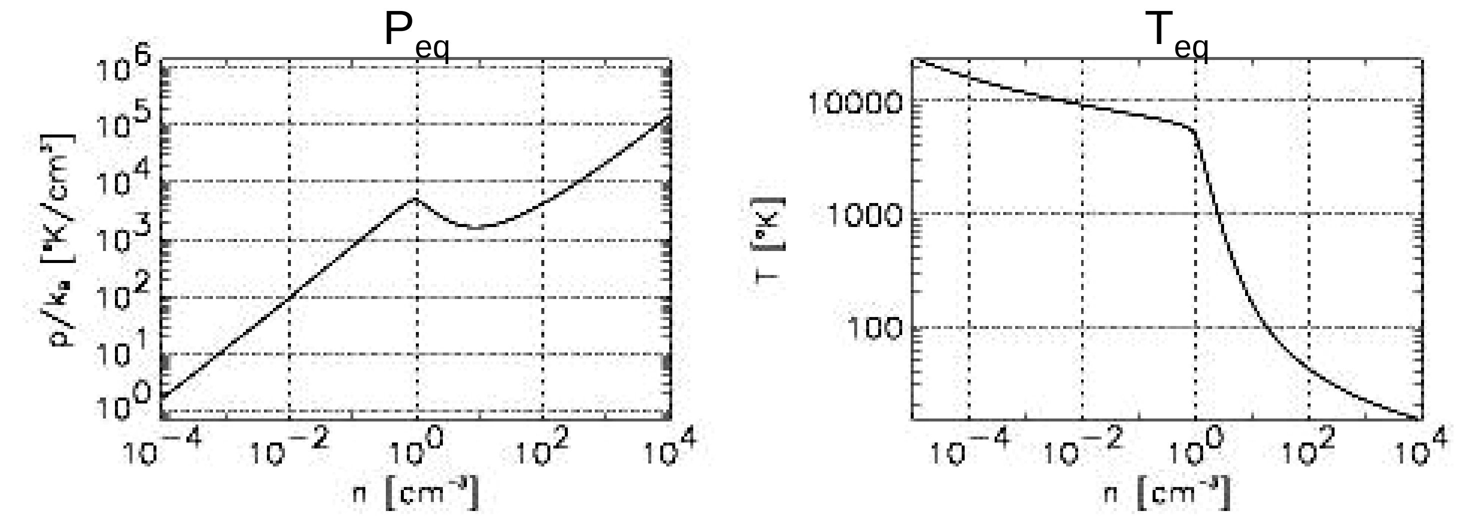
$$P=(\gamma-1)\rho e,$$

allow one to eliminate P to get

$$e_{\mathrm{eq}}=e_{\mathrm{eq}}(\rho).$$

$$\boxed{\text{Thermal equilibrium; } (e\propto T_{\mathrm{eq}}).}$$

  - Can use a (perhaps piecewise) polytropic approximation:



- The timescale to reach equilibrium is

$$\tau = \frac{e - e_{\mathrm{eq}}}{n\Lambda - \Gamma}$$

Cooling time

  - Can be *much* shorter than dynamical timescale, especially at high *n*.
  - But only effect is to approach thermal equilibrium.
    - A polytropic approximation would suffice.

- Then can “pre-integrate” over HD step Δt:

$$\left(e - e_{\mathrm{eq}}\right)^{-1} d(e - e_{\mathrm{eq}}) = -\frac{n\Lambda - \Gamma}{e - e_{\mathrm{eq}}} dt = -\frac{dt}{\tau}$$

$$\Rightarrow$$

$$e(t + \Delta t) = e_{\mathrm{eq}} + \left[e(t) - e_{\mathrm{eq}}\right]\exp\left(-\frac{\Delta t}{\tau}\right).$$

- This has the correct limiting behavior:

$$\text{If } \Delta t >> \tau, \quad e(t+\Delta t) \to e_{\text{eq}};$$

$$\text{if } \Delta t << \tau, \quad e(t+\Delta t) \to e(t) - \Delta t(n\Lambda - \Gamma)$$

- Is a similar procedure possible for the AD?
  - How to interpolate between ideal MHD and HD + uniform B?
  - Can introduce weight factors into the Lorentz force and AD terms?

Thank you!