

Magnetic drift in molecular clouds and protoplanetary disks

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Physical conditions

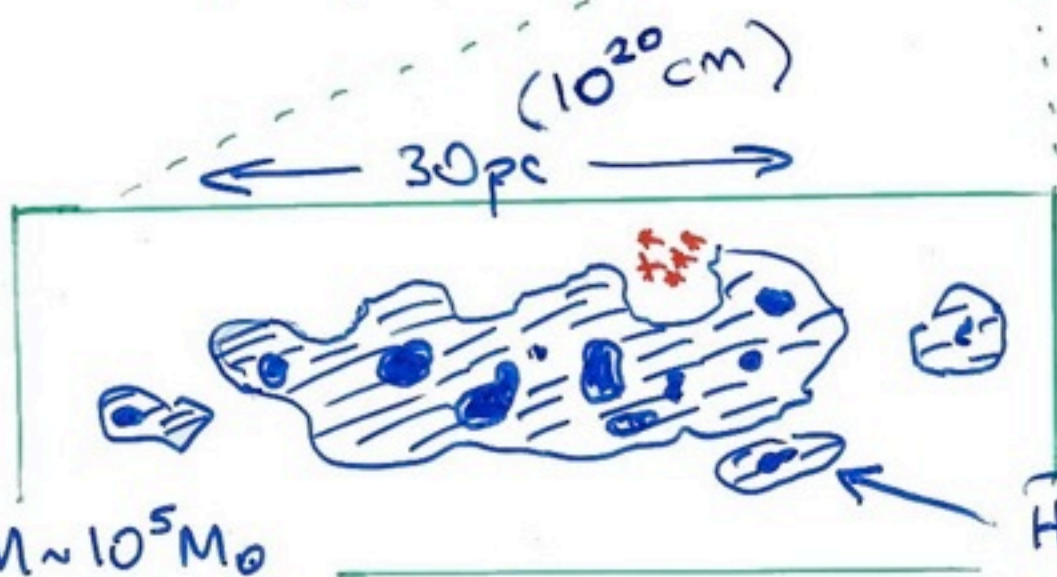
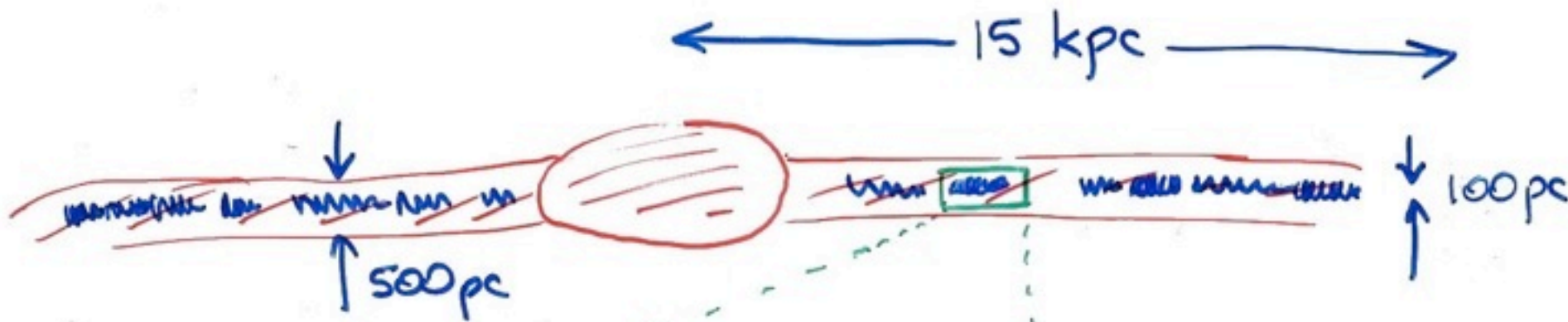
Magnetic diffusion

Shock waves

Gravitational collapse

Magnetorotational instability

Jet launching



$M \sim 10^5 M_{\odot}$
 clumps 1-3000 M_{\odot}
 $T \sim 20 \text{ K}$
 $\Delta v \sim 10 \text{ km s}^{-1}$

gas

$\text{H}_2, \text{He}, \text{CO}, \text{dust}$
 90%, 10%, 0.01%,
 $n_{\text{H}} \sim 10^2 - 10^8 \text{ cm}^{-3}$
 $v_A \sim 2 \text{ km/s}$
 $c_s \sim 0.2 \text{ km/s}$

Star formation disperses cloud after $\approx 30 \text{ Myr}$

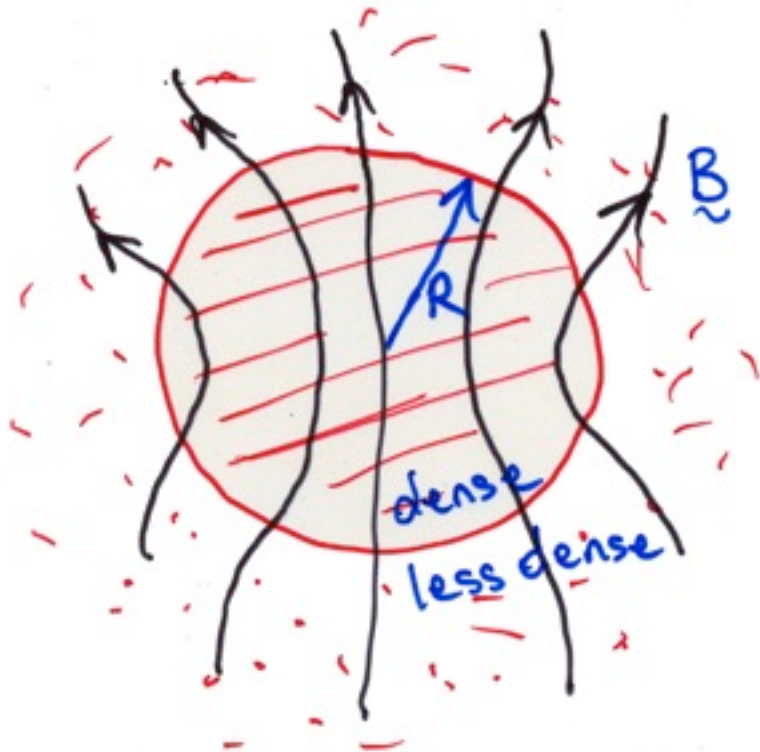
- Magnetic fields play a critical role

- P_{mag} is 30–100 times P_{gas} in molecular clouds

- $v_A \sim 2 \text{ km/s}$, $c_s \sim 0.2 \text{ km/s}$

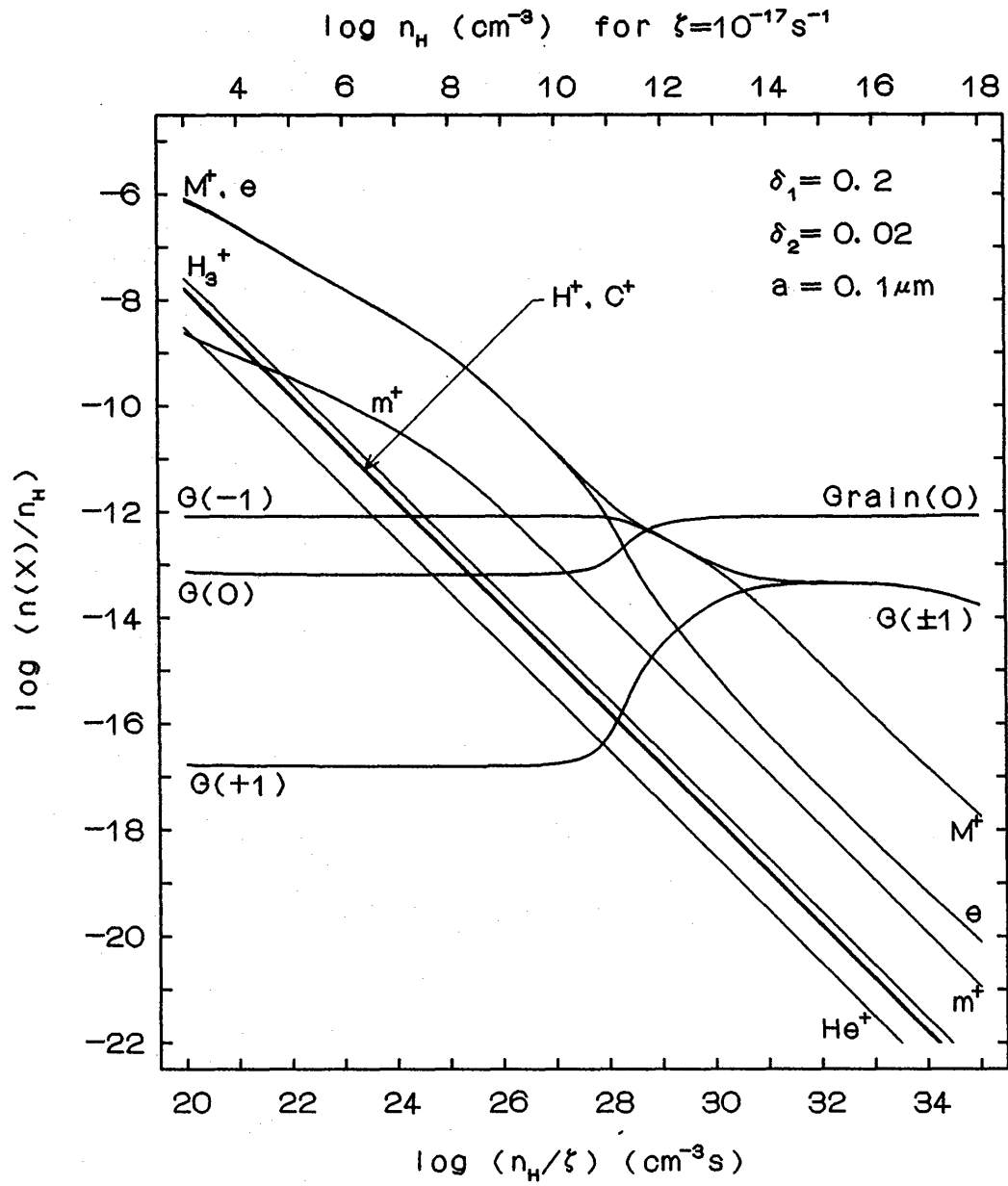
- energy density of magnetic field, fluid motions and self-gravity are similar

- field removes angular momentum from cloud cores and protostellar/ protoplanetary disks



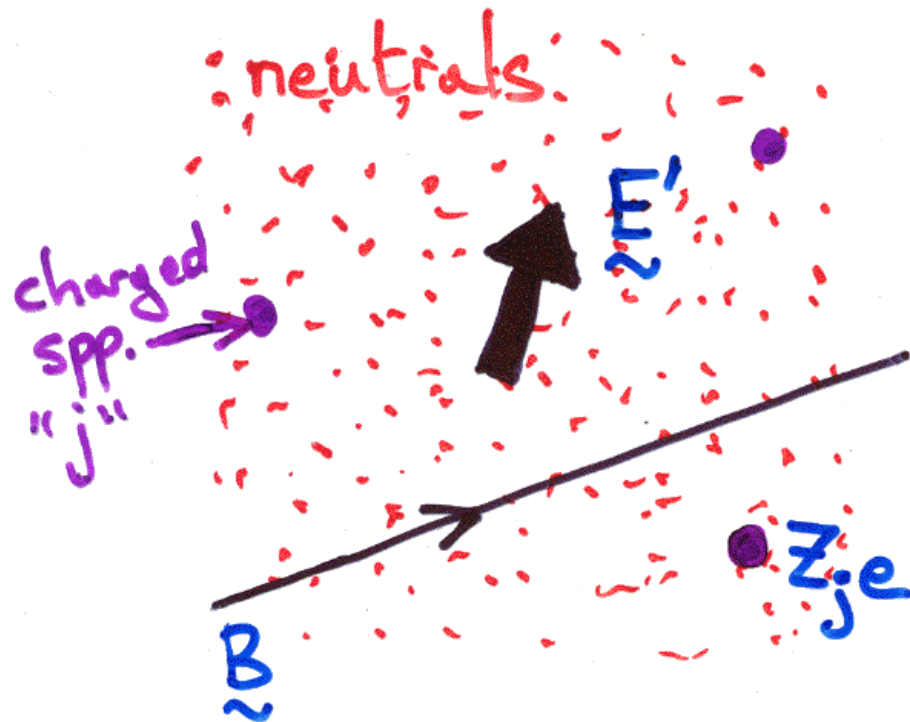
$R: 10^{17} \text{ cm} \rightarrow 10^{11} \text{ cm}$

Low fractional ionisation



0.1 μm grains

Microphysics



$$Z_j e \mathbf{E}' + Z_j e \frac{\mathbf{v}_j}{c} \times \mathbf{B} - \gamma_j m_j \rho \mathbf{v}_j = 0$$

$$\gamma_j = \frac{\langle \sigma v \rangle_j}{m_j + m}$$

$$\beta_j = \frac{Z_j e B}{m_j c} \frac{1}{\gamma_j \rho} = \frac{\text{gyrofrequency}}{\text{collision frequency}}$$

$$|\beta_j| \gg 1 : Z_j e \mathbf{E}' \approx -Z_j e \frac{\mathbf{v}_j}{c} \times \mathbf{B} \quad \text{particles tied to field}$$

$$|\beta_j| \ll 1 : Z_j e \mathbf{E}' \approx \gamma_j m_j \rho \mathbf{v}_j \quad \text{particles tied to neutral fluid}$$

	fully ionized	weakly ionized
Ideal MHD	ions and electrons tied to field	ions, electrons and neutrals tied to field
Ambipolar diffusion	–	neutrals decoupled
Hall drift	ions decoupled $\omega > \frac{eB}{m_i c}$	ions and neutrals decoupled $\omega > \frac{eB}{m_i^* c} \quad m_i^* = m_i \frac{\rho}{\rho_i}$
Ohmic diffusion	ions and electrons decoupled	ions, electrons and neutrals decoupled

Magnetic drift

regime	magnetised component	unmagnetised component	B drift through neutrals
Ideal MHD	neutrals, ions, electrons	—	0
Ambipolar	ions, electrons	neutrals	$\mathbf{v}_i - \mathbf{v}_n = \frac{\mathbf{J} \times \mathbf{B}}{c\gamma\rho_i\rho}$
Hall	electrons	neutrals, ions	$\mathbf{v}_e - \mathbf{v}_i = - \frac{\mathbf{J}}{en_e}$
Ohmic	—	neutrals, ions, electrons	$c \frac{\mathbf{E}' \times \mathbf{B}}{B^2} = \frac{4\pi\eta}{c} \frac{\mathbf{J} \times \mathbf{B}}{B^2}$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \left[\eta \nabla \times \mathbf{B} + \eta_H (\nabla \times \mathbf{B}) \times \hat{\mathbf{B}} + \eta_A (\nabla \times \mathbf{B})_{\perp} \right]$$

- If the only charged species are ions and electrons,

$$n_i Z_i = n_e \quad \eta_H = |\beta_e| \eta$$

$$|\beta_e| \gg \beta_i \quad \eta_A = \beta_i |\beta_e| \eta$$

$$\beta_j = \frac{Z_j e B}{m_j c} \frac{1}{\gamma_j \rho} = \frac{\text{gyrofrequency}}{\text{collision frequency}}$$

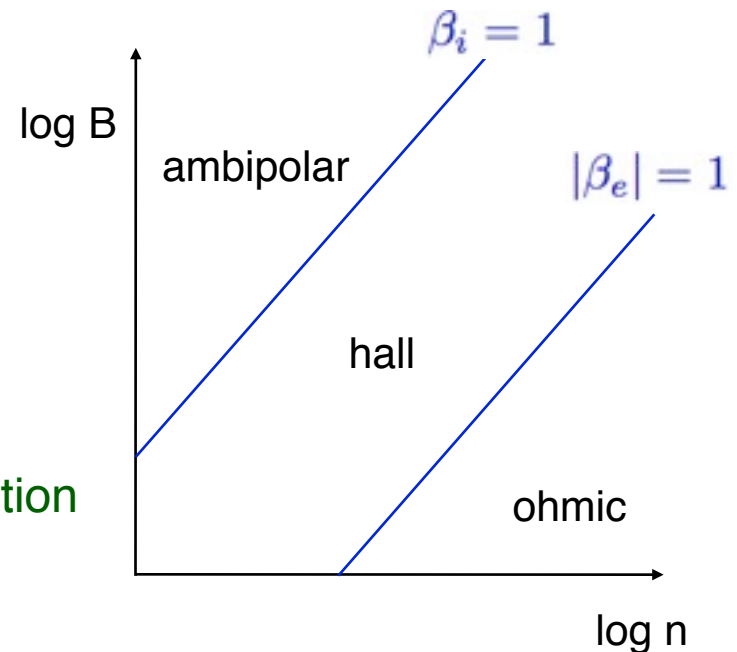
- Three distinct diffusion regimes:

$$\beta_i \ll |\beta_e| \ll 1 \quad - \text{Ohmic (resistive)}$$

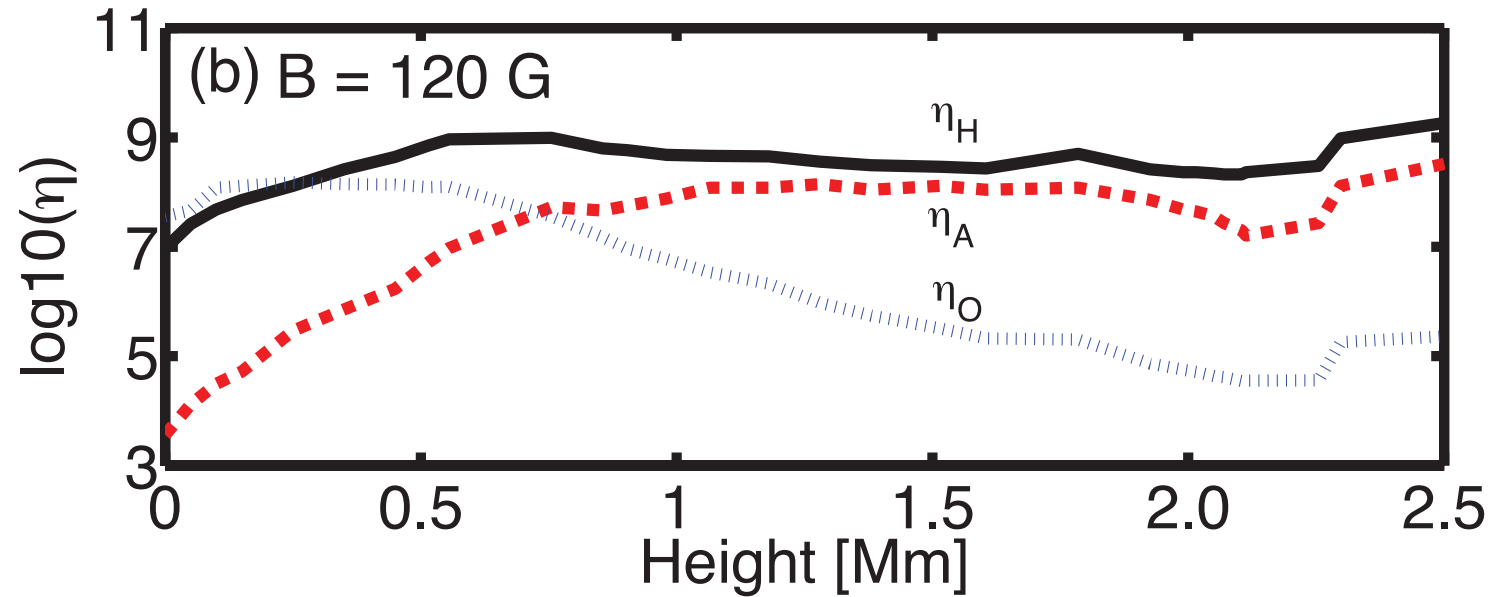
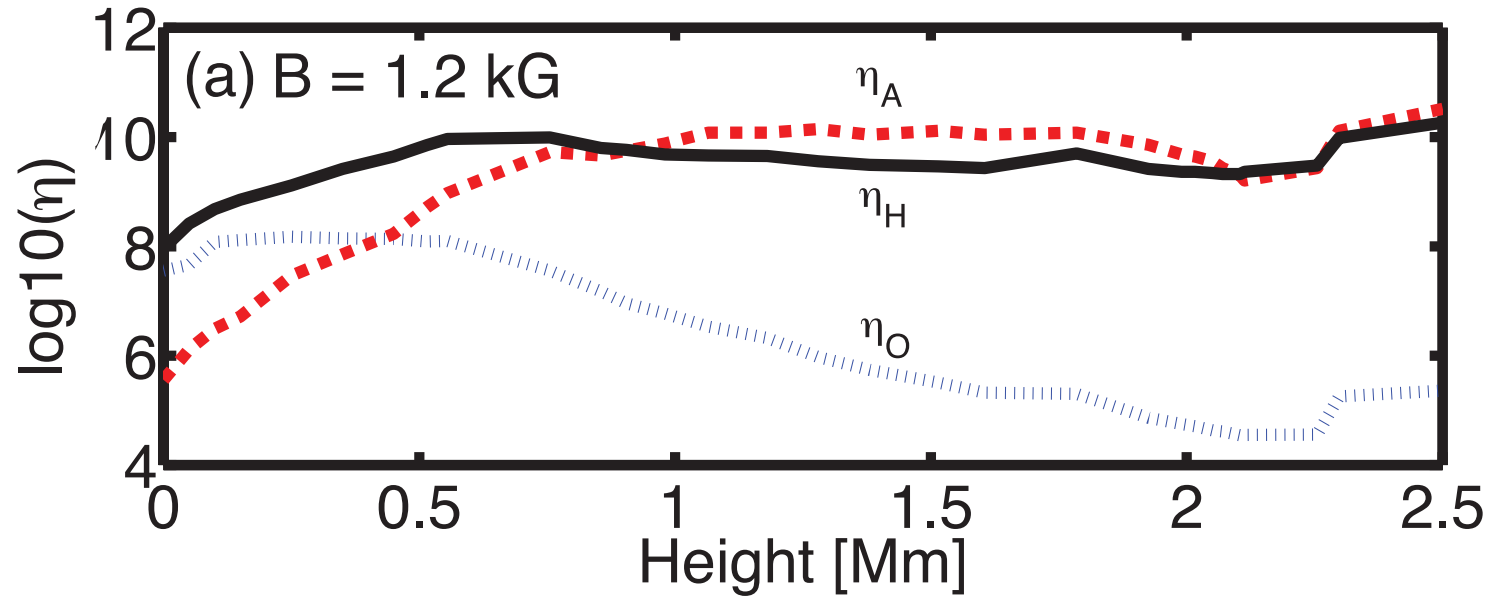
$$\beta_i \ll 1 \ll |\beta_e| \quad - \text{Hall}$$

$$1 \ll \beta_i \ll |\beta_e| \quad - \text{Ambipolar}$$

– see Pandey & Wardle (2008) for generalisation to all levels of ionisation



The solar chromosphere



$$\beta_j = \frac{Z_j e B}{m_j c} \frac{1}{\gamma_j \rho} = \frac{\text{gyrofrequency}}{\text{collision frequency}}$$

- For ions,

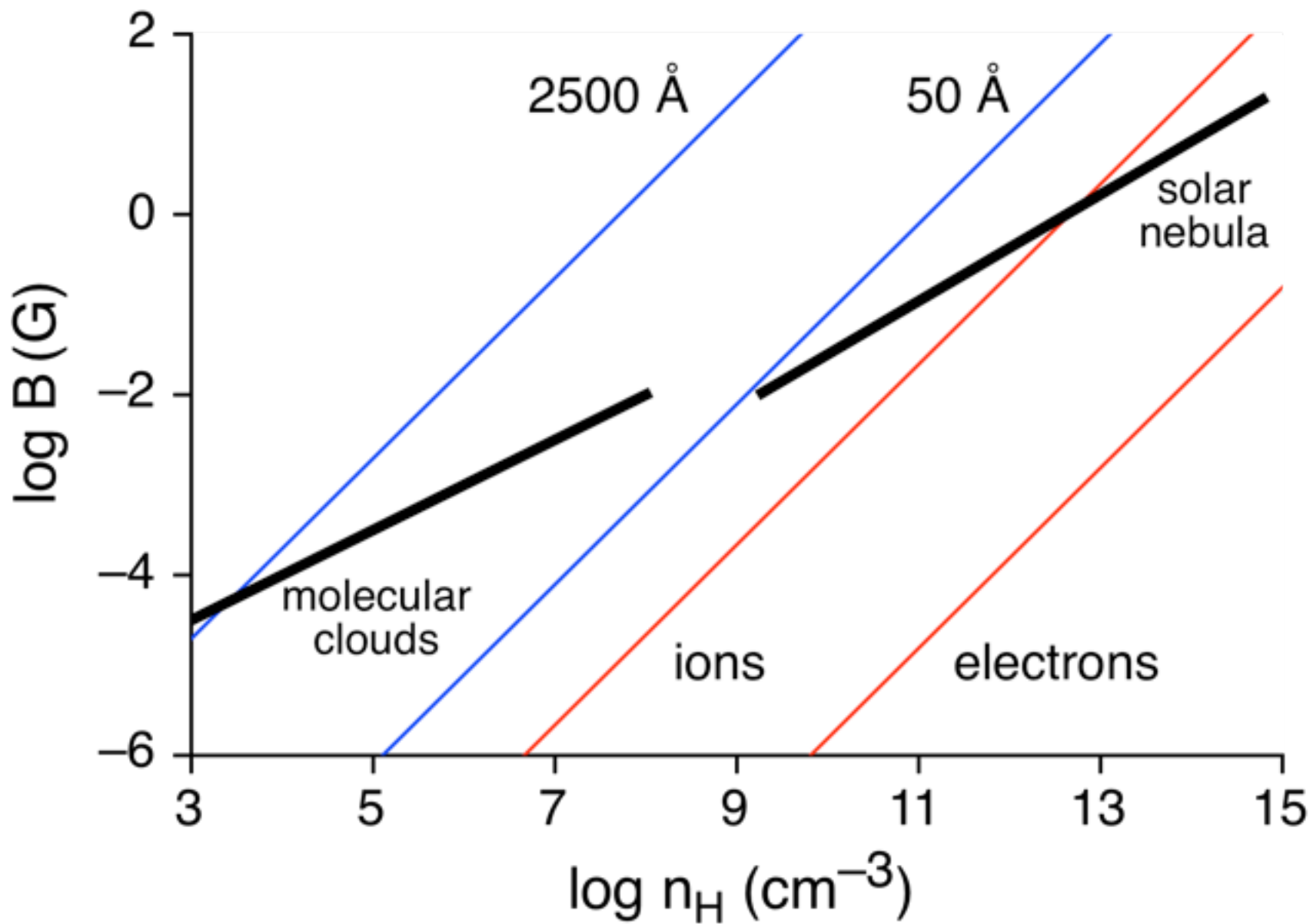
$$m_i \approx 30 m_H \quad \langle \sigma v \rangle_i \approx 1.6 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$$

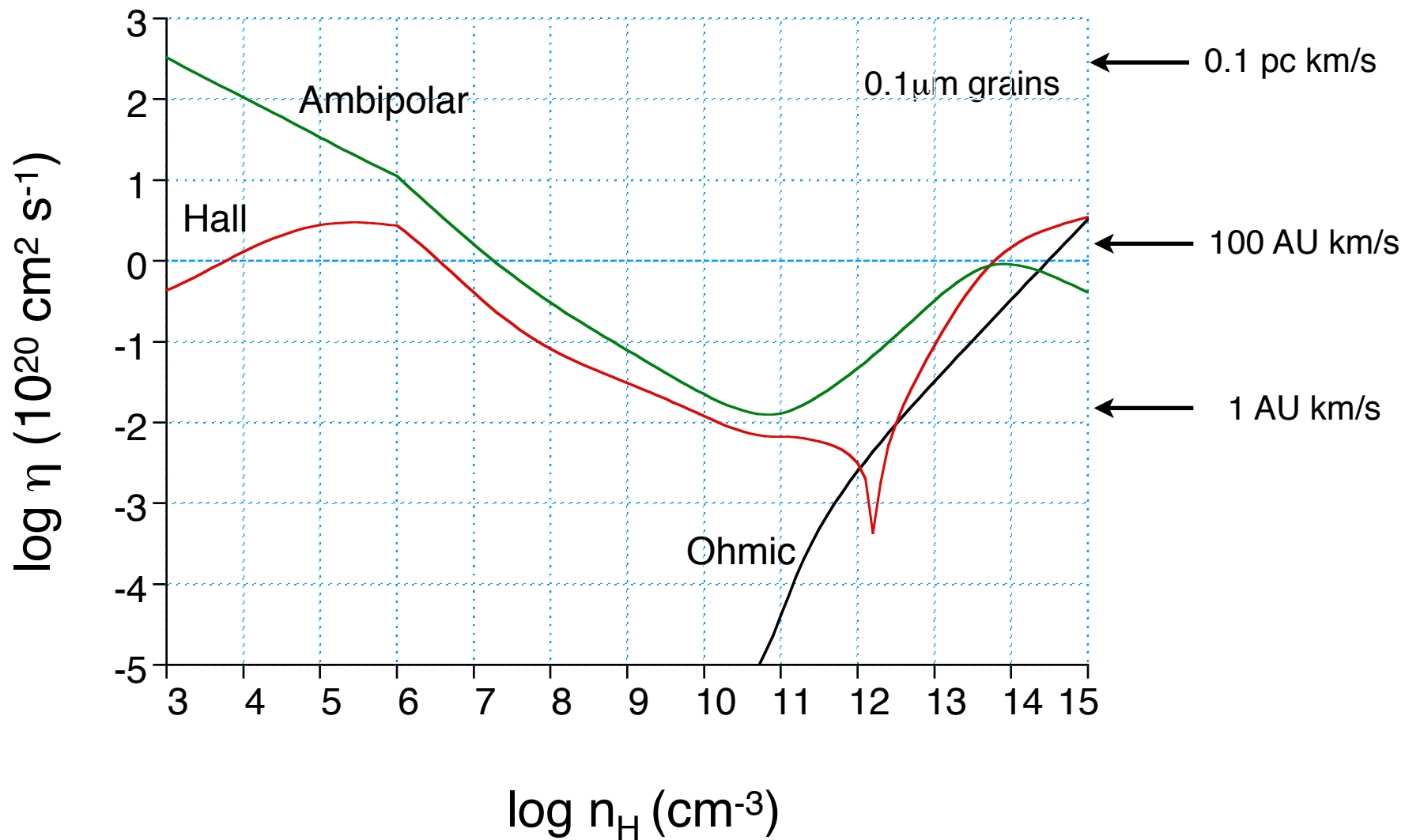
$$\beta_i \approx 4.6 \times 10^{-3} \left(\frac{B}{1 \text{ G}} \right) \left(\frac{n_H}{10^{15} \text{ cm}^{-3}} \right)^{-1}$$

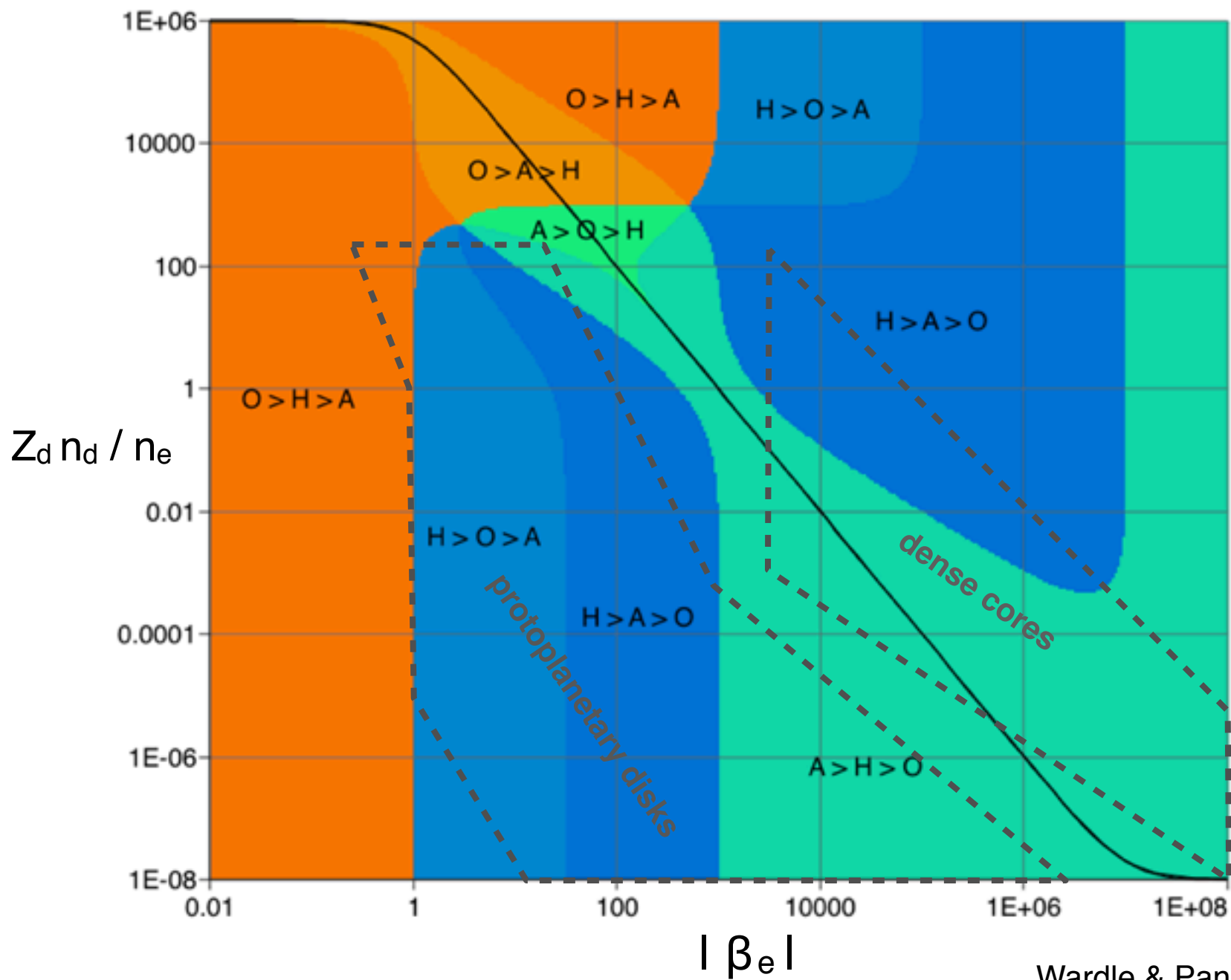
- For electrons,

$$\langle \sigma v \rangle_e \approx 1 \times 10^{-15} \text{ cm}^2 \left(\frac{128 k T}{9 \pi m_e} \right)^{1/2}$$

$$\beta_e \approx -3.5 \left(\frac{B}{1 \text{ G}} \right) \left(\frac{n_H}{10^{15} \text{ cm}^{-3}} \right)^{-1} \left(\frac{T}{100 \text{ K}} \right)^{-1/2}$$







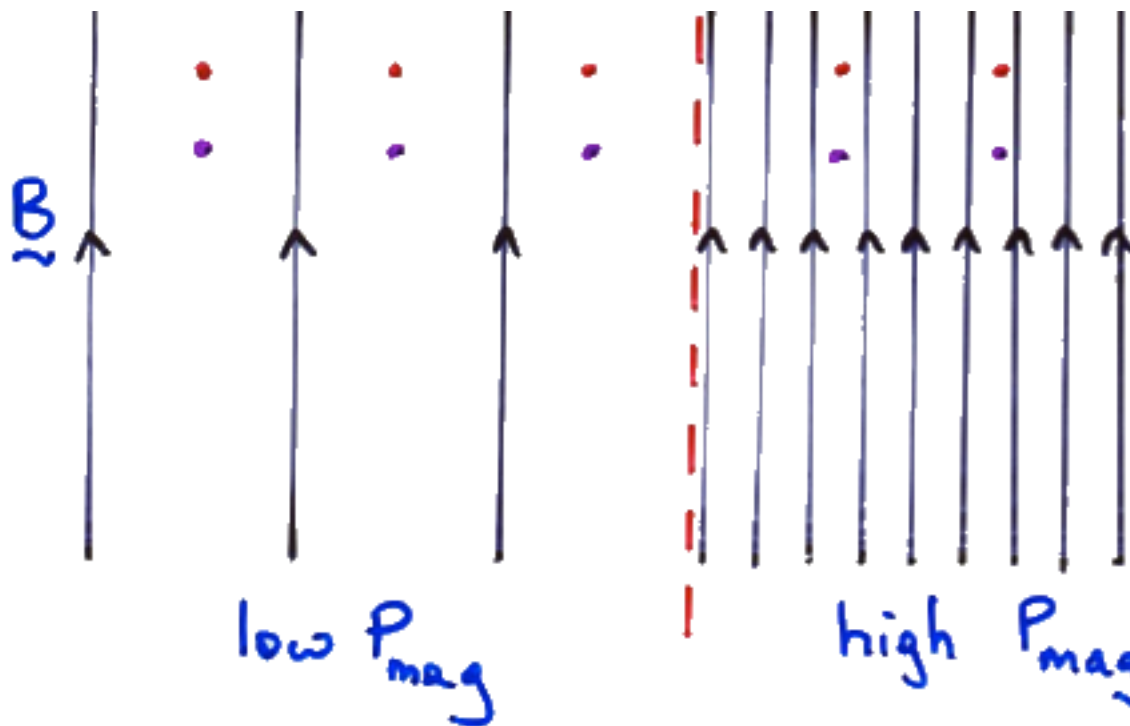
$$\eta_A = \frac{1 + \beta_g^2 + (1 + \beta_i \beta_g)P}{(1 + P/P_0)^2 + (\beta_g + \beta_i P)^2} \frac{B^2}{4\pi\gamma\rho_i\rho}$$

$$\eta_H = \frac{1 + \beta_g^2 - \beta_i^2 P}{(1 + P/P_0)^2 + (\beta_g + \beta_i P)^2} \frac{cB}{4\pi en_e}$$

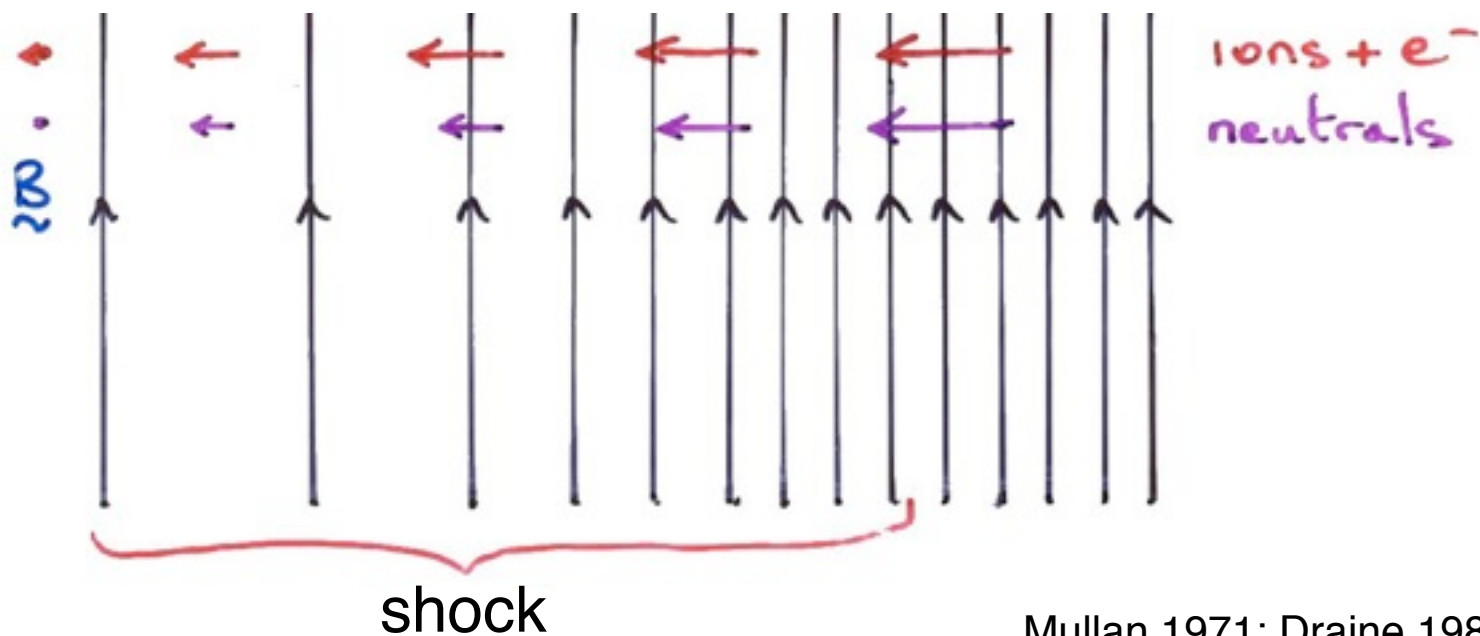
$$\eta = \frac{1}{1 + P/P_0} \frac{c^2}{4\pi\sigma_e}$$

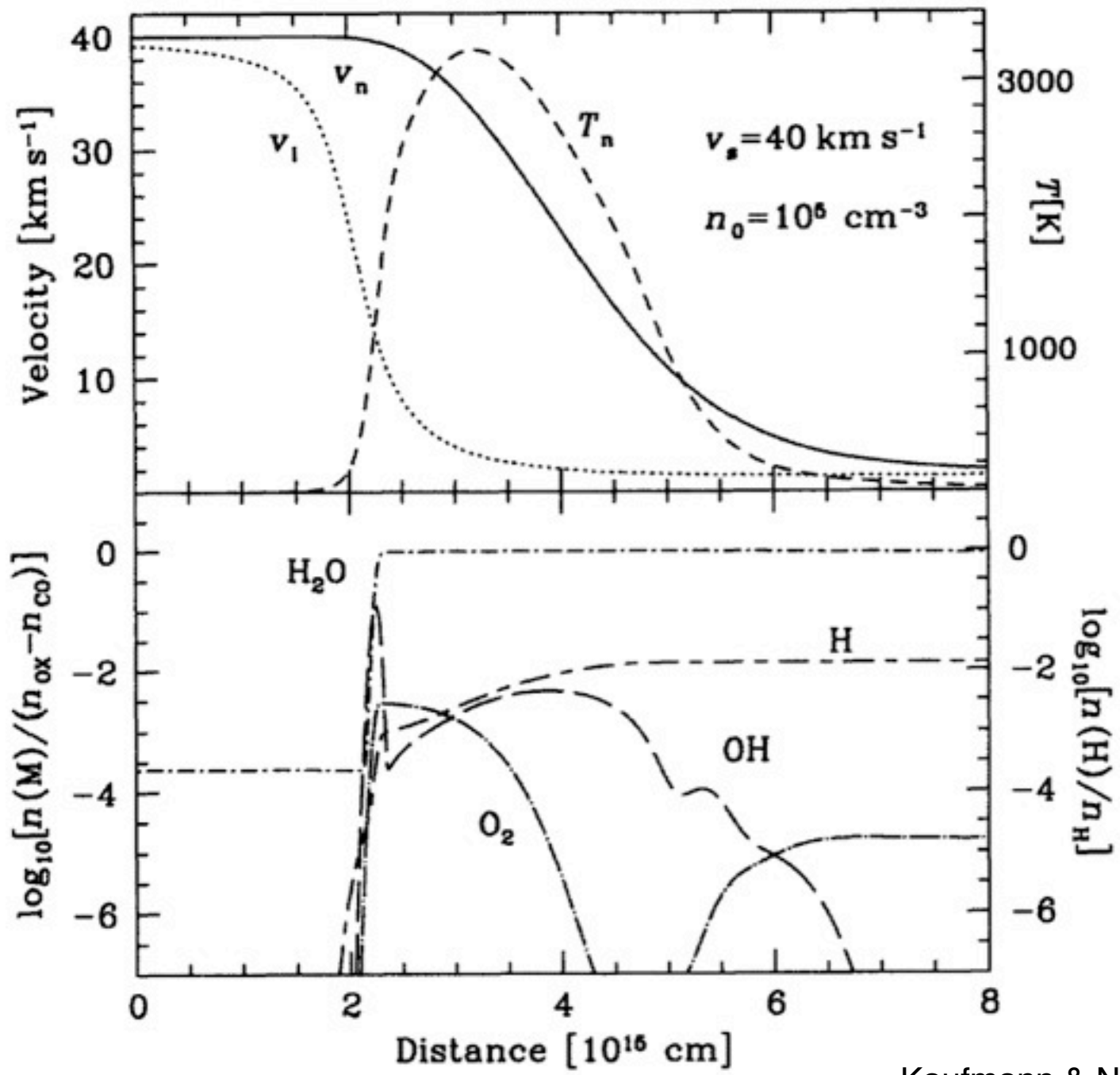
C-type shock

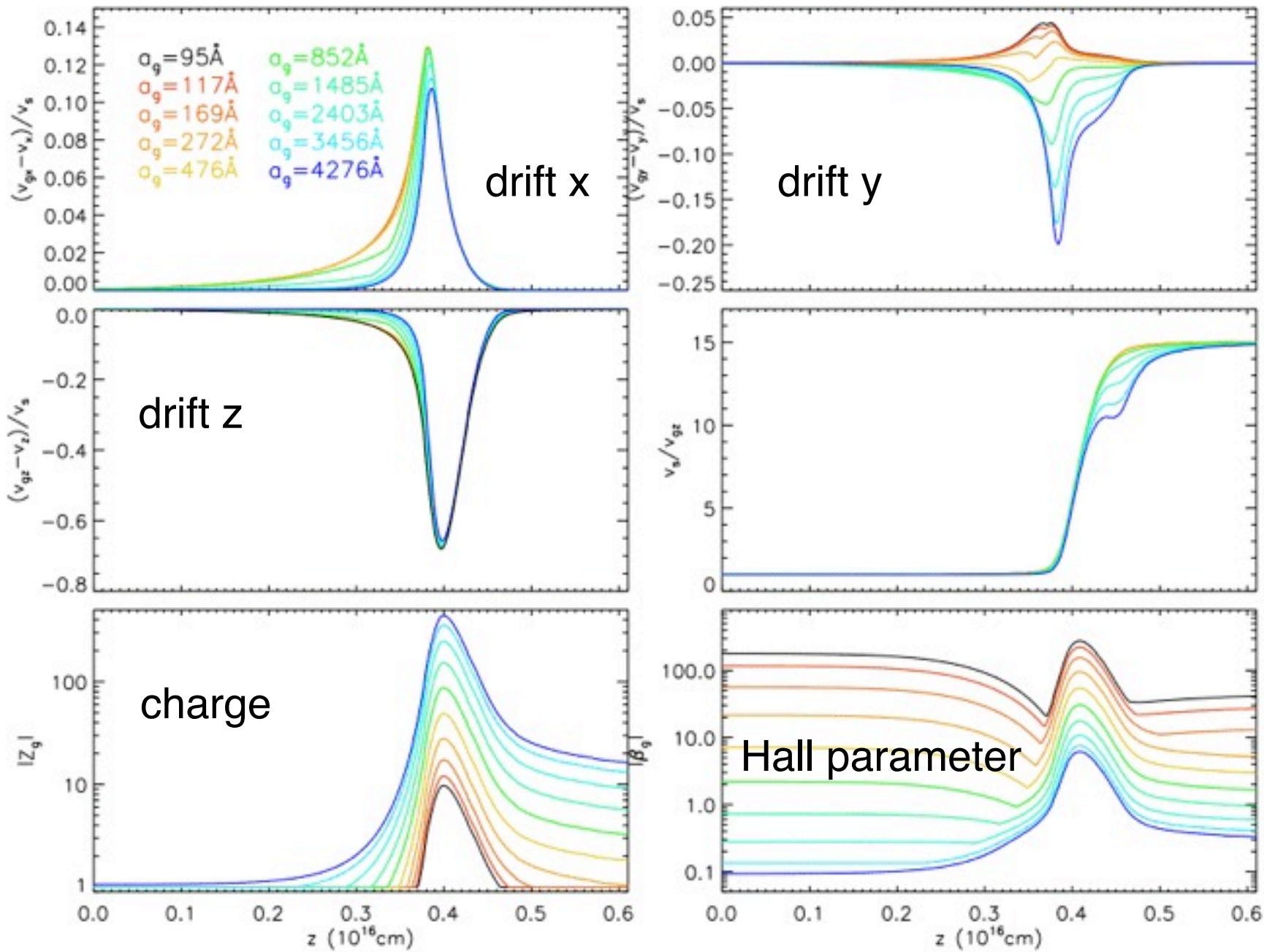
$t = 0$



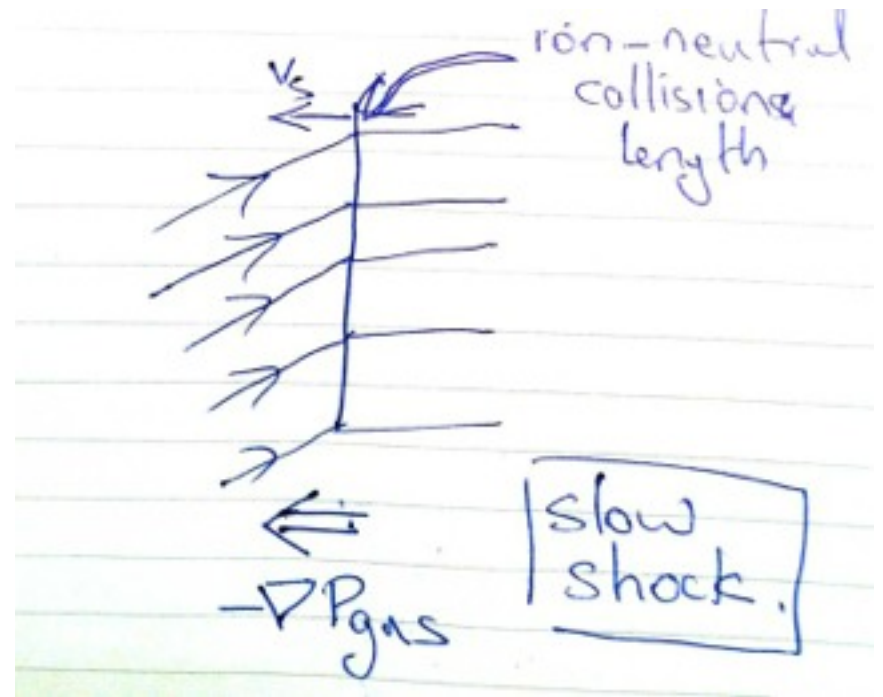
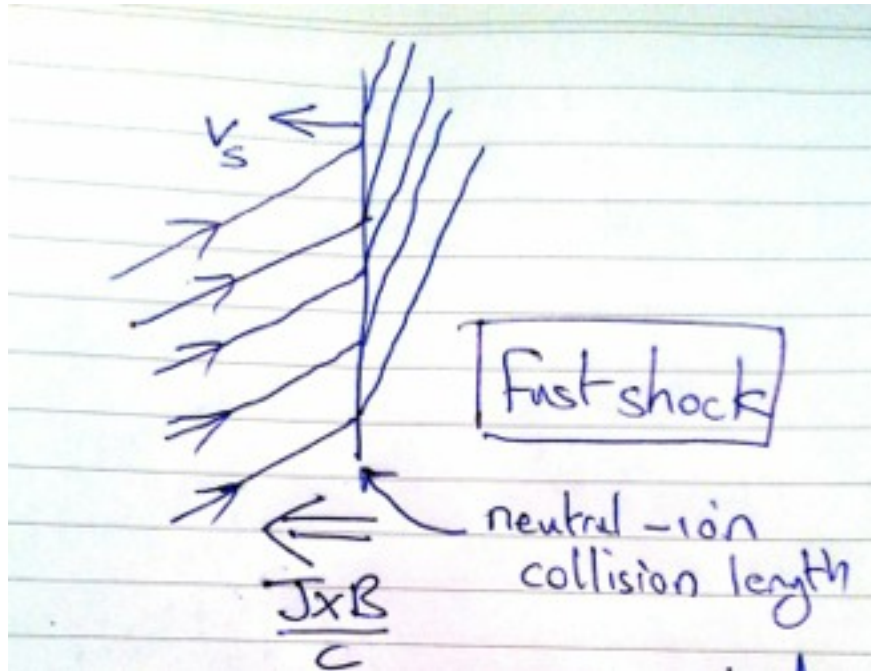
$t > 0$



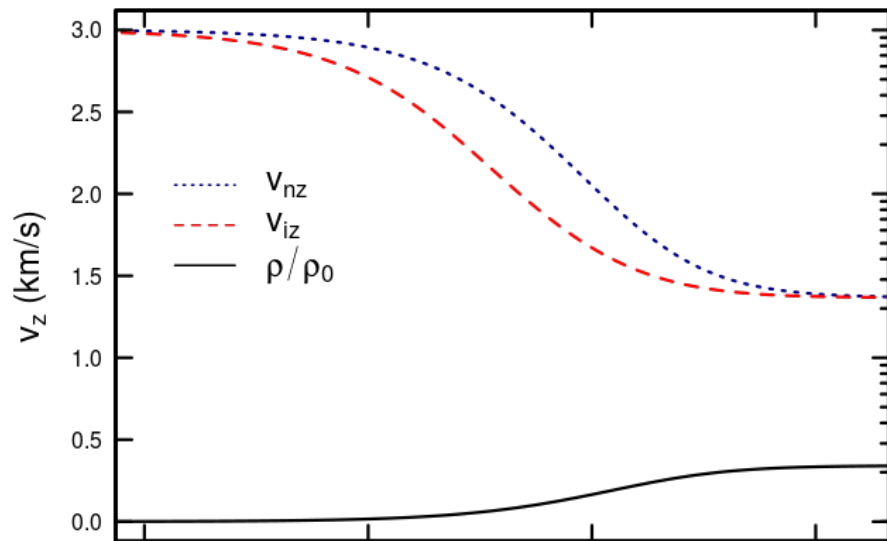




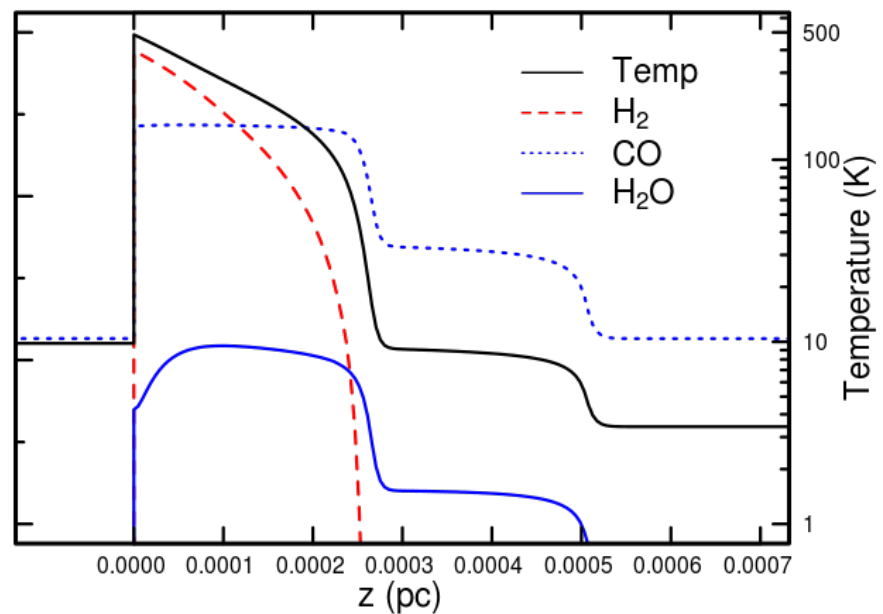
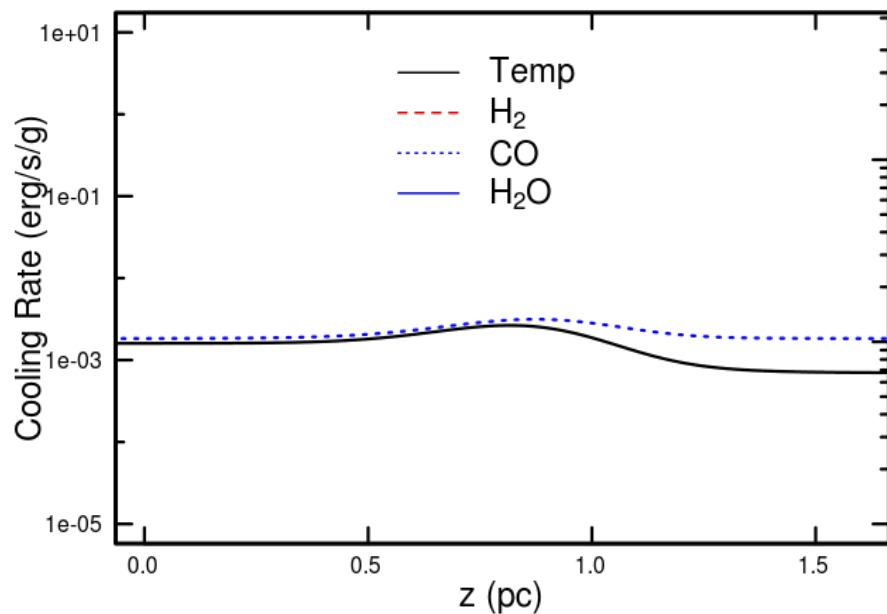
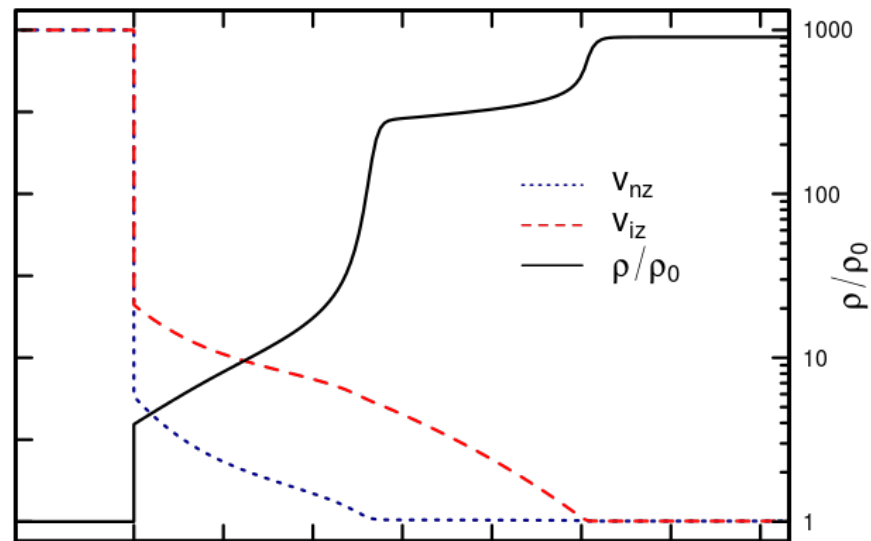
Fast vs slow MHD shocks



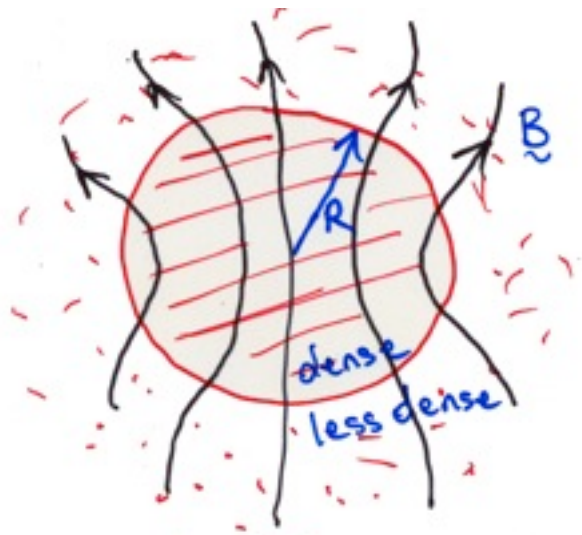
Fast



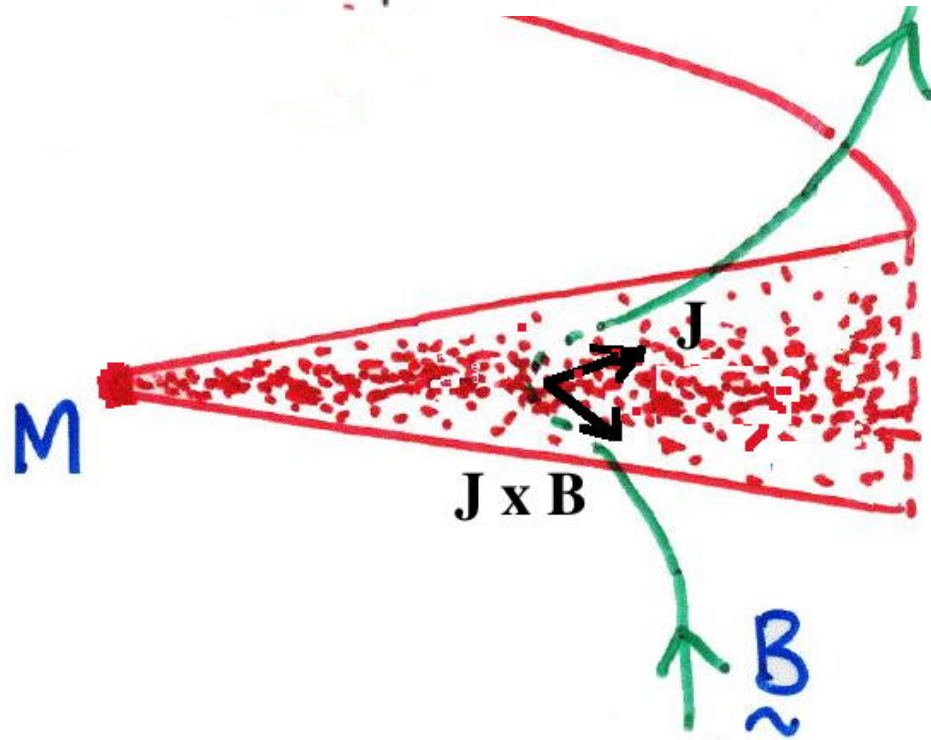
Slow



Field line drift: collapsing cores / protoplanetary disks



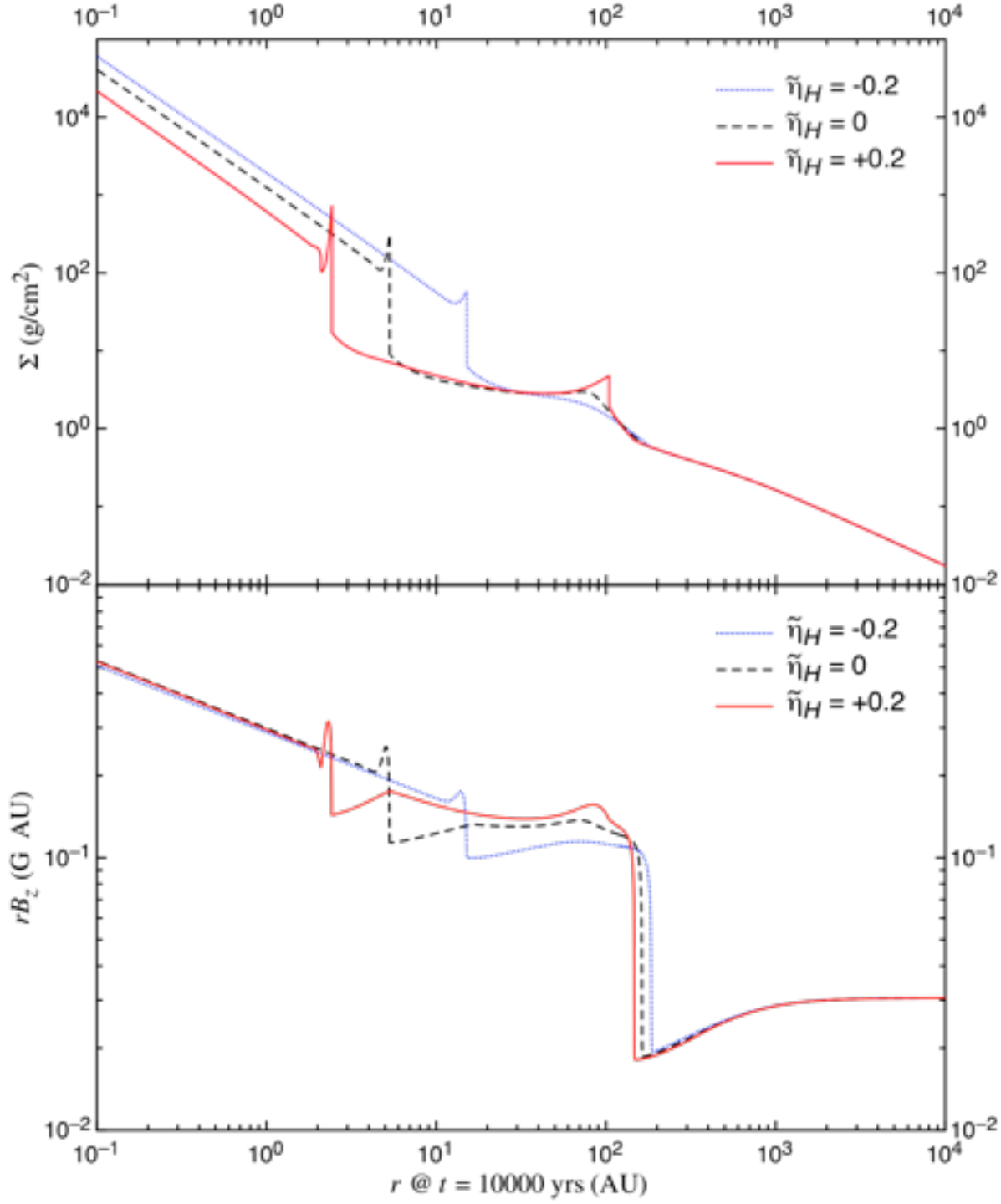
- Hall drift is “sideways”
tends to induce or reduce twisting in B
- Sense depends on sign of B

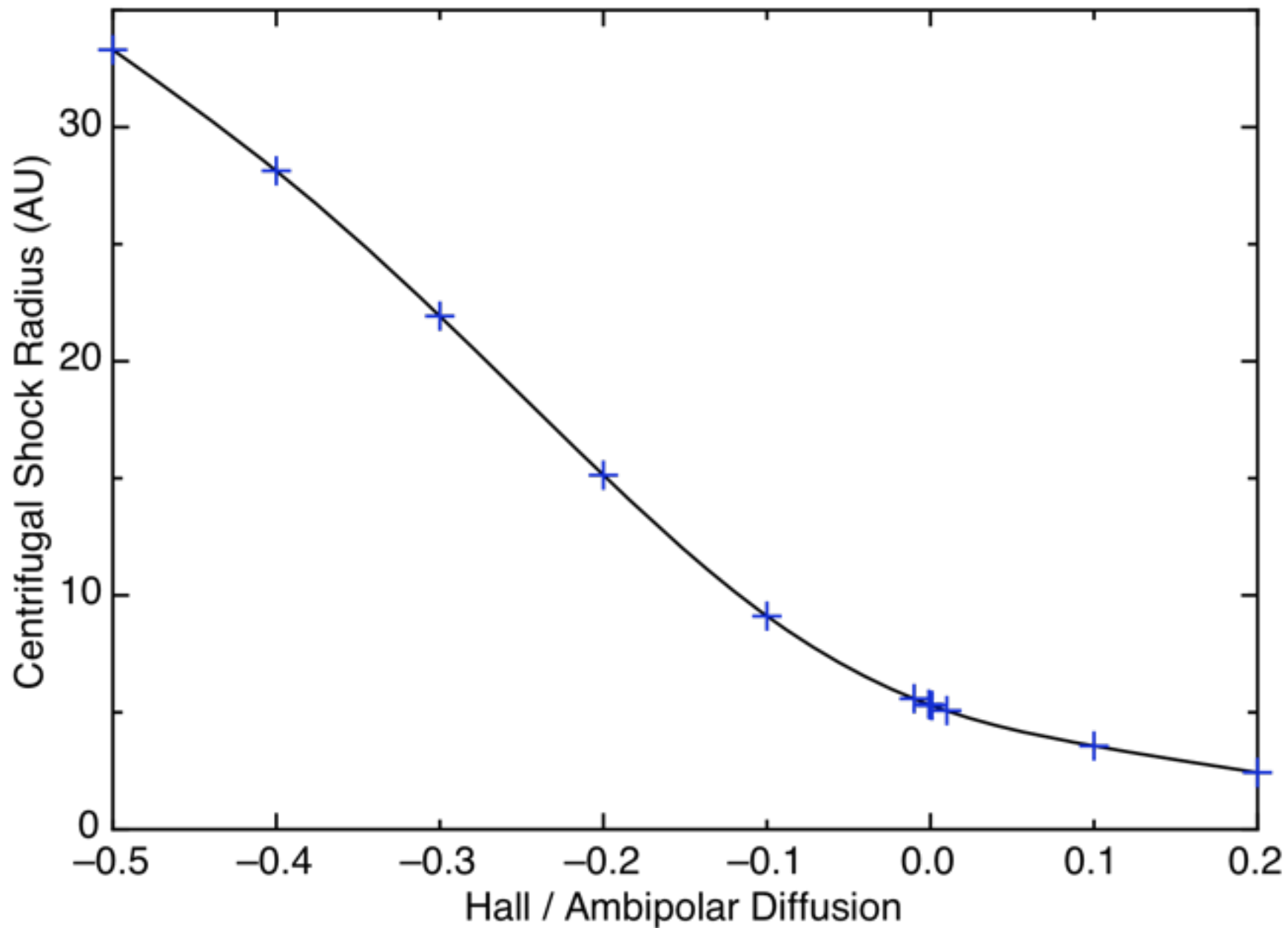


Ambipolar:
$$\frac{\mathbf{J} \times \mathbf{B}}{c\gamma\rho_i\rho}$$

Ohmic:
$$\frac{4\pi\eta}{c} \frac{\mathbf{J} \times \mathbf{B}}{B^2}$$

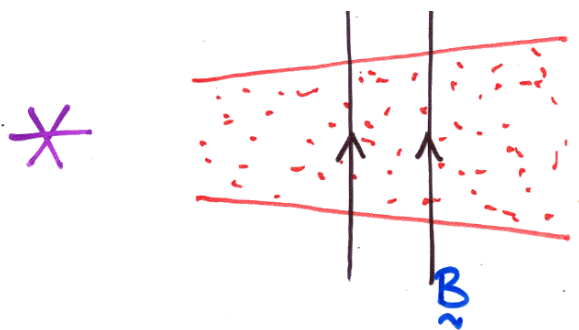
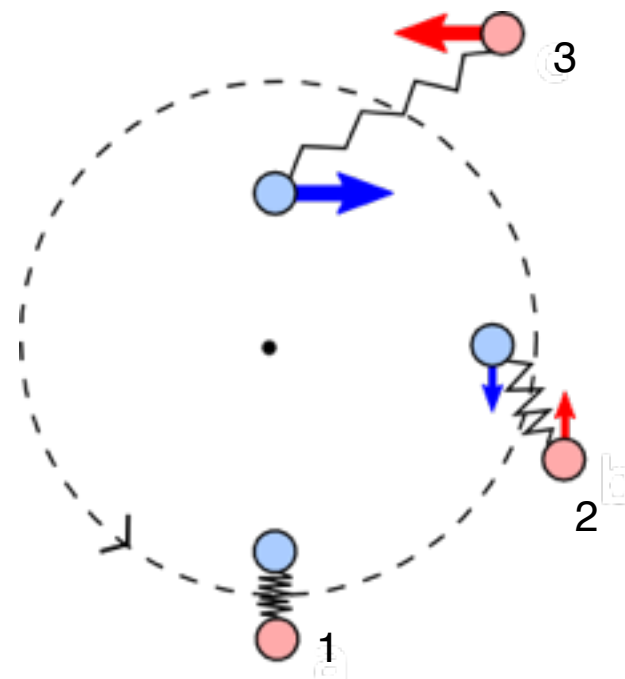
Hall:
$$-\frac{\mathbf{J}}{en_e}$$



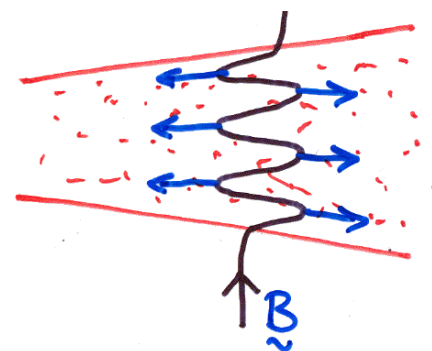
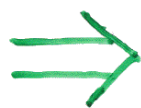


Magnetorotational instability (MRI)

- Satellites joined by a spring
 - angular momentum transferred from inner \rightarrow outer
 - spiral inwards and outwards, respectively
 - stretch spring, increases torque
 - runaway process
- Buckled magnetic field
 - couples fluid elements at different radii
 - tension plays role of spring
 - buckling increases
 - generates MHD turbulence



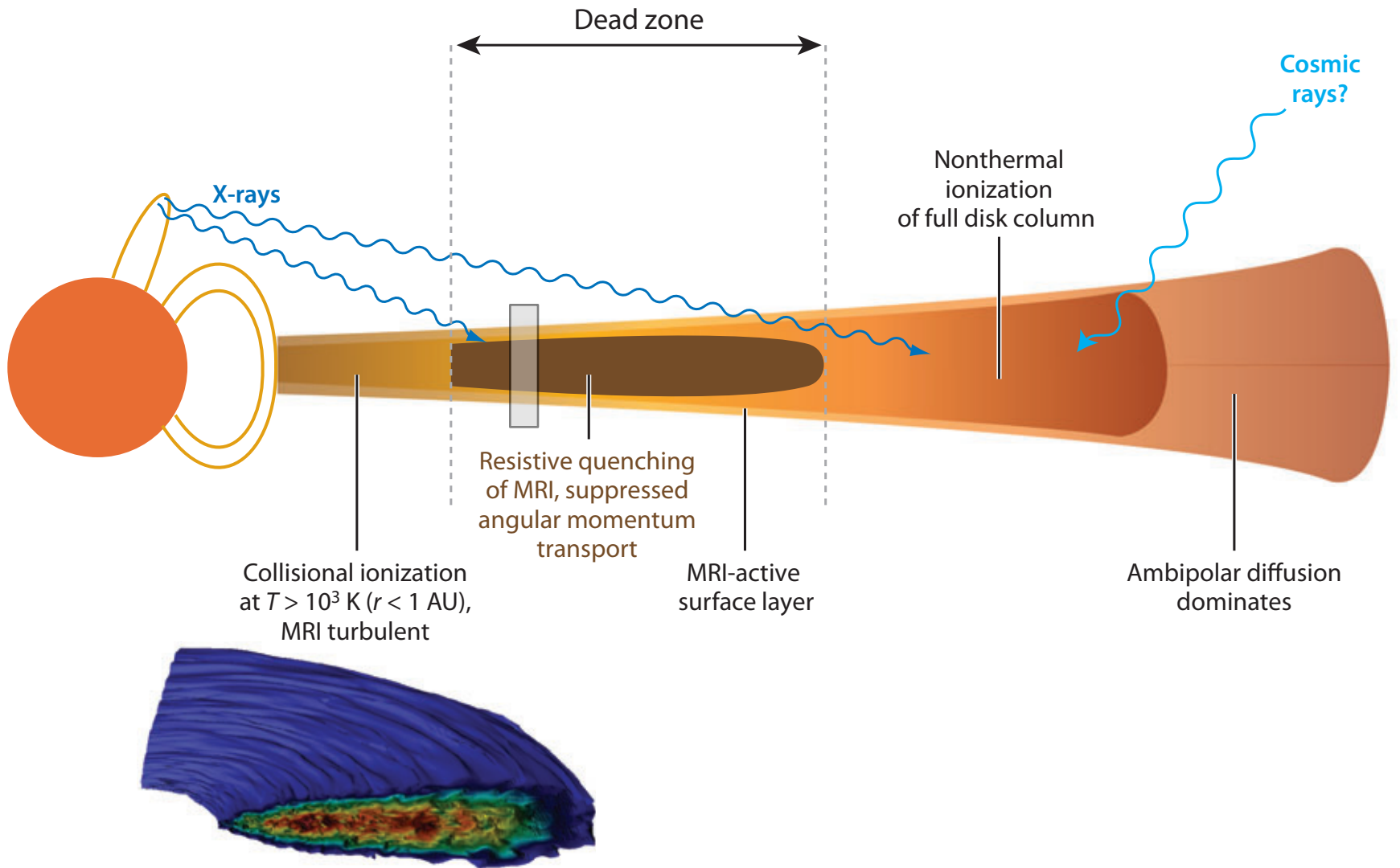
Keplerian disk + weak B_z



timescale $\sim T$

$$\lambda \sim v_A T$$

Dead Zones



MRI – with dead zone

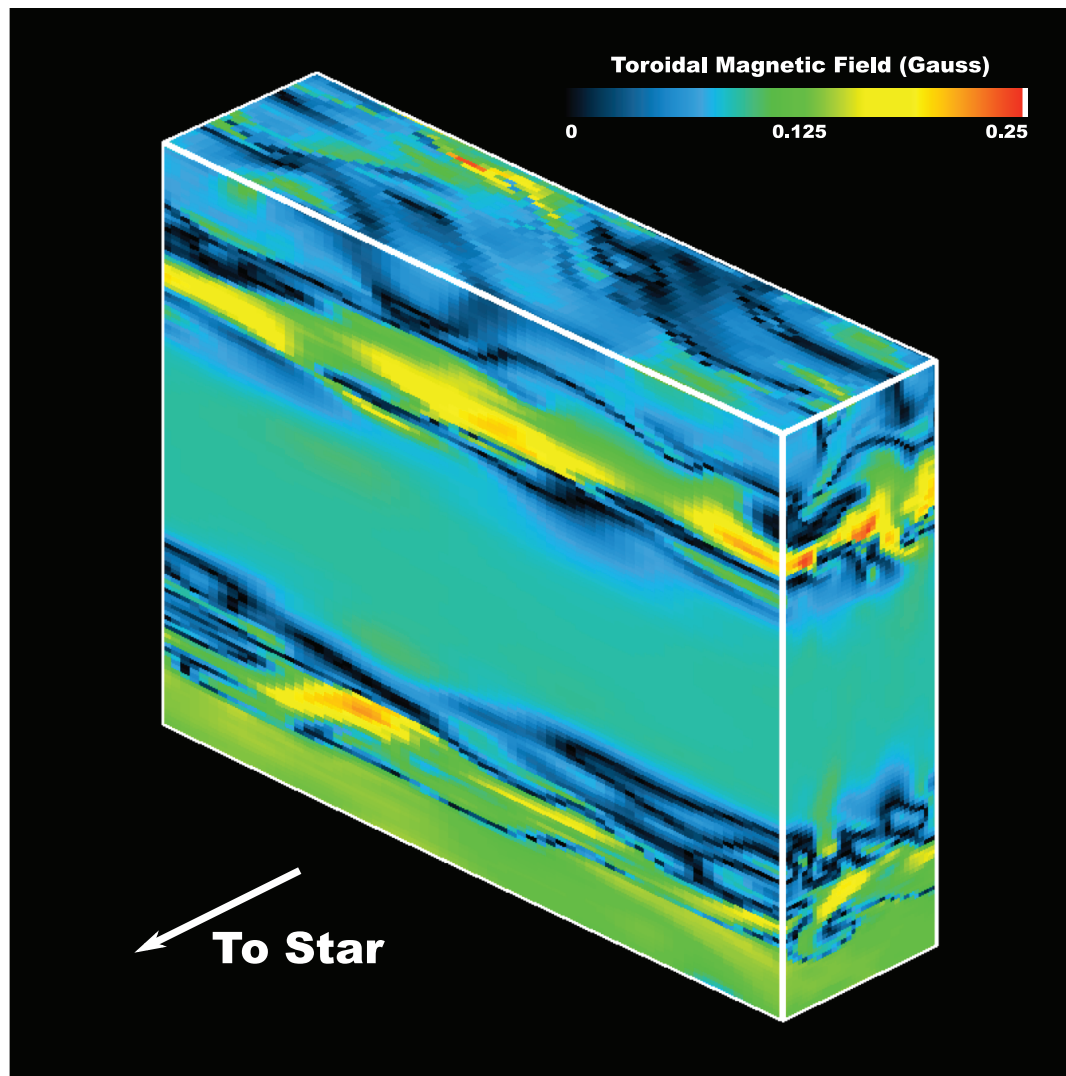
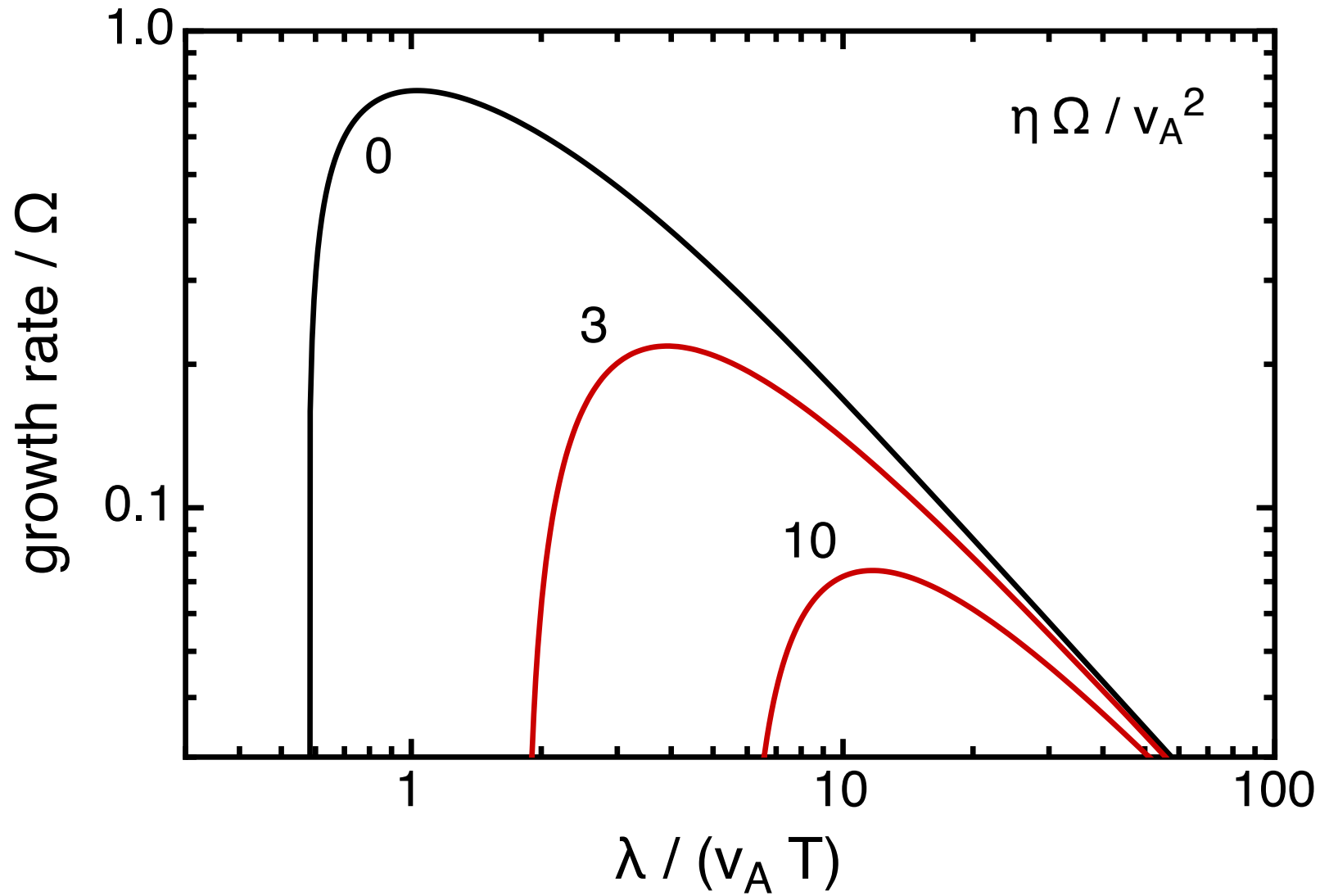


FIG. 2.—Snapshot of the toroidal magnetic field strength at 55 orbits in a resistive MHD calculation of a patch of the protosolar disk at 5 AU including well mixed $1 \mu\text{m}$ grains. The undead zone at center is filled with a uniform, 0.1 G shear-generated toroidal magnetic field while patchy fields are found in the turbulent layers above and below. The star lies off-page to the lower left and the disk midplane is horizontal through the image center.

MRI growth rate - ohmic and/or ambipolar diffusion

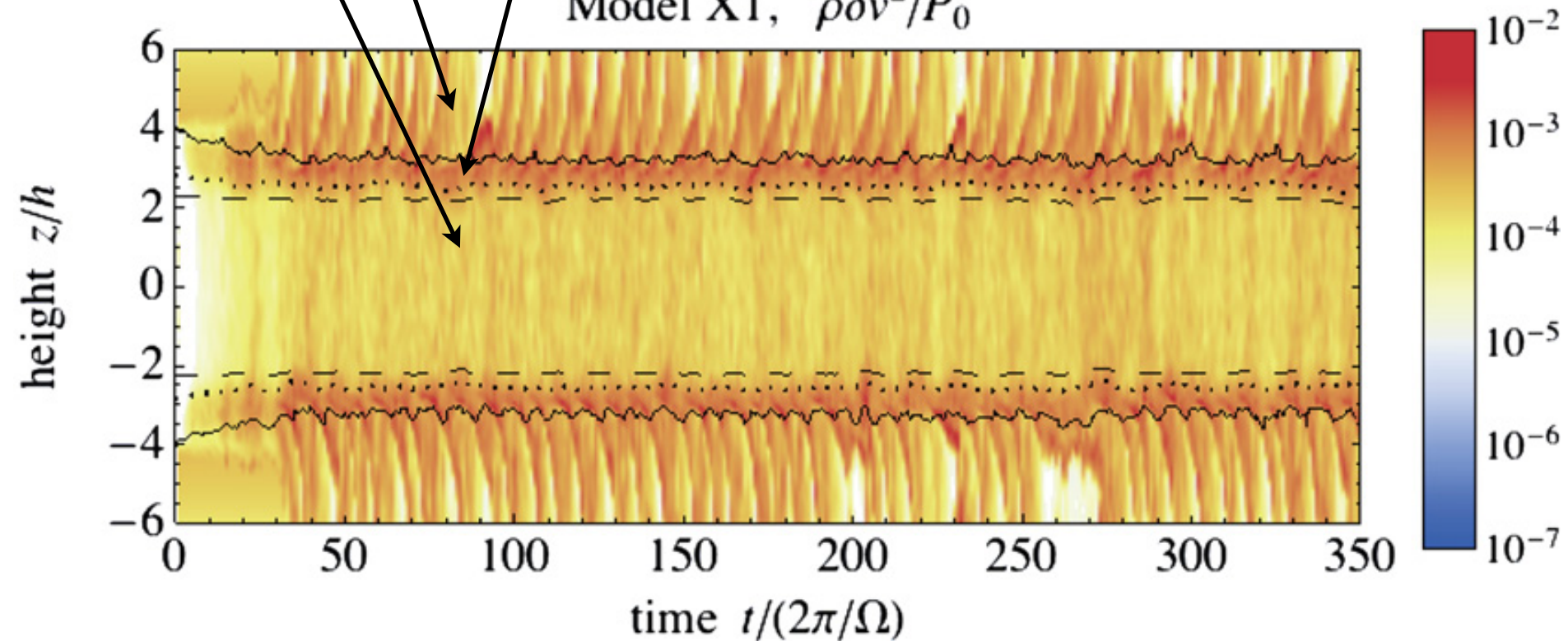


low density:
 v_A too large
 $\lambda \sim v_A T \gg h$

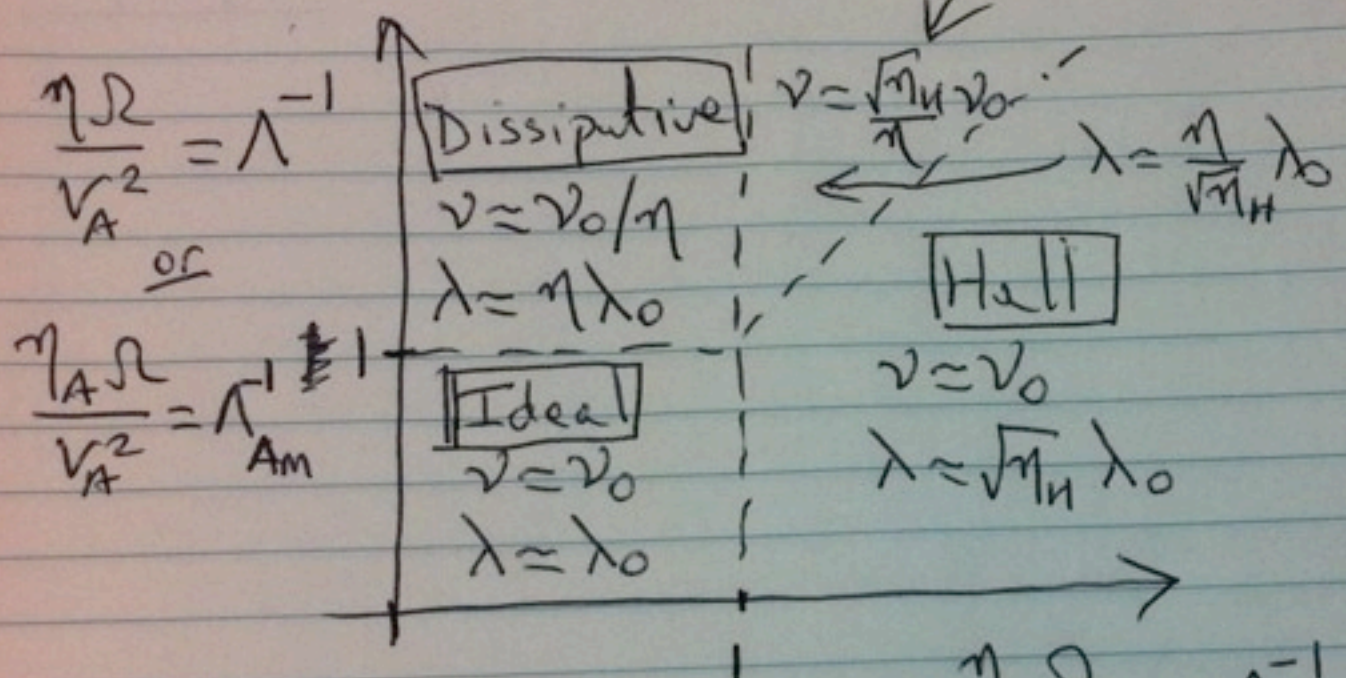
low ionisation:
 η too large
 $\lambda \sim \eta v_A T \gg h$

MRI-active layer:
 $\lambda \sim v_A T < h$

Model X1, $\overline{\rho \delta v^2} / P_0$



Hall-modified

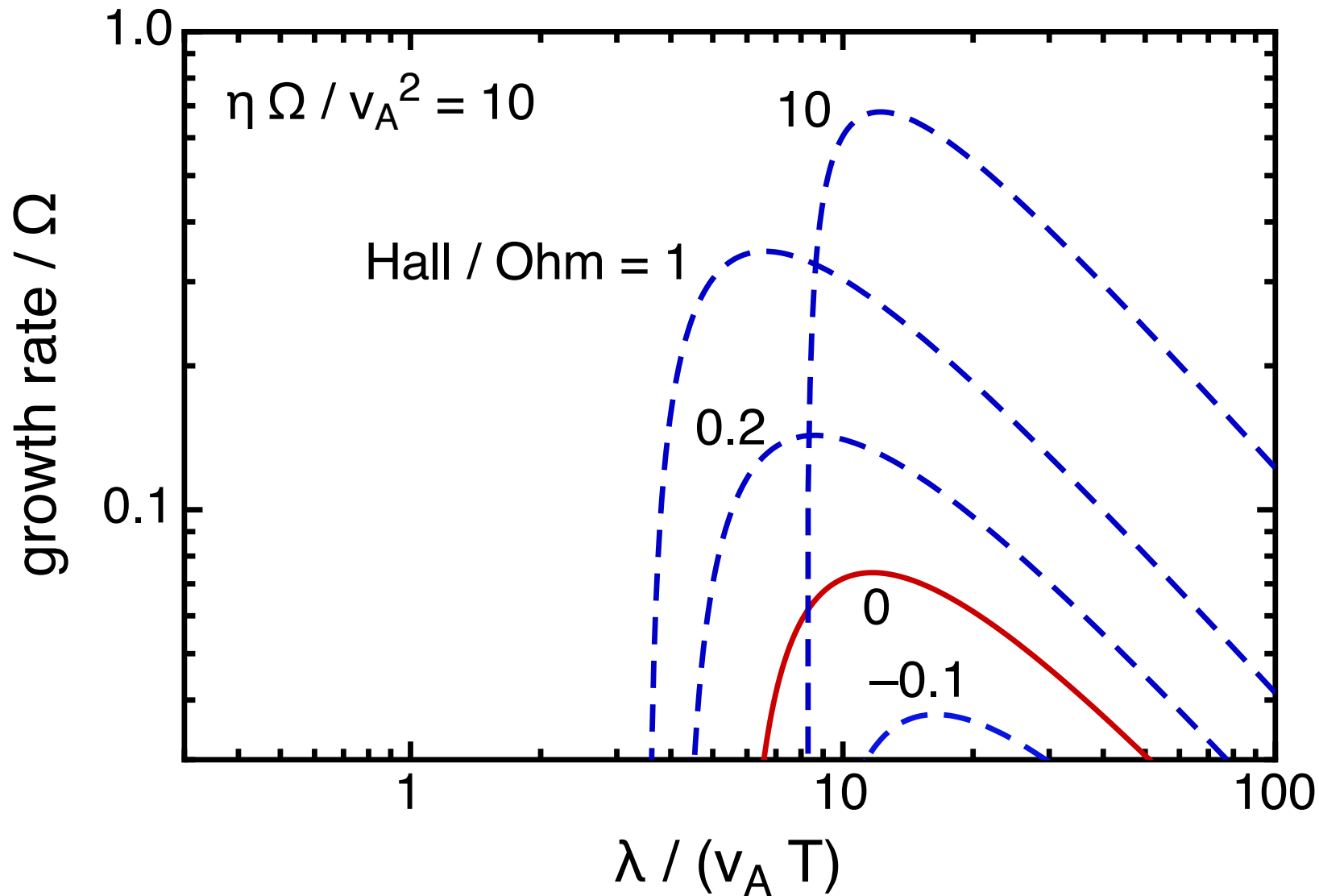


$$v_0 = \frac{3}{4} \Omega$$

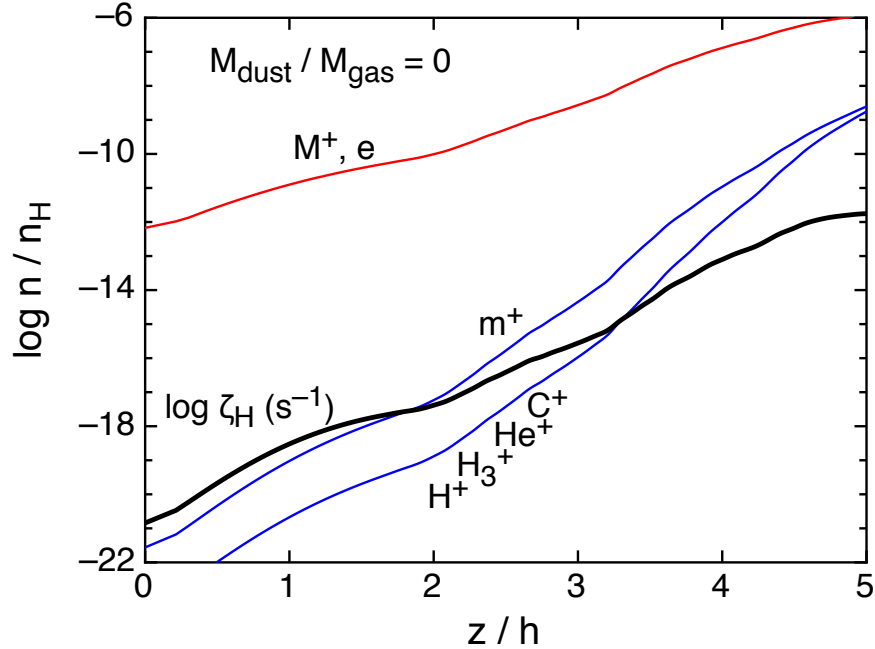
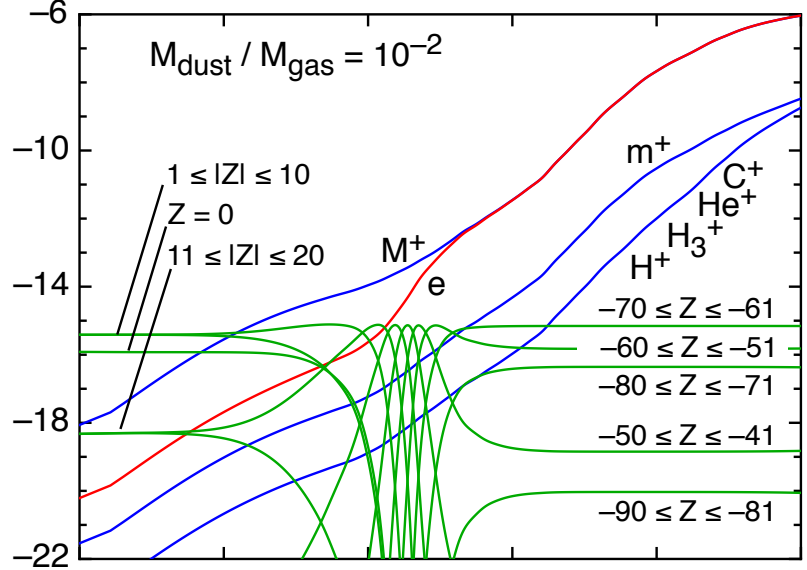
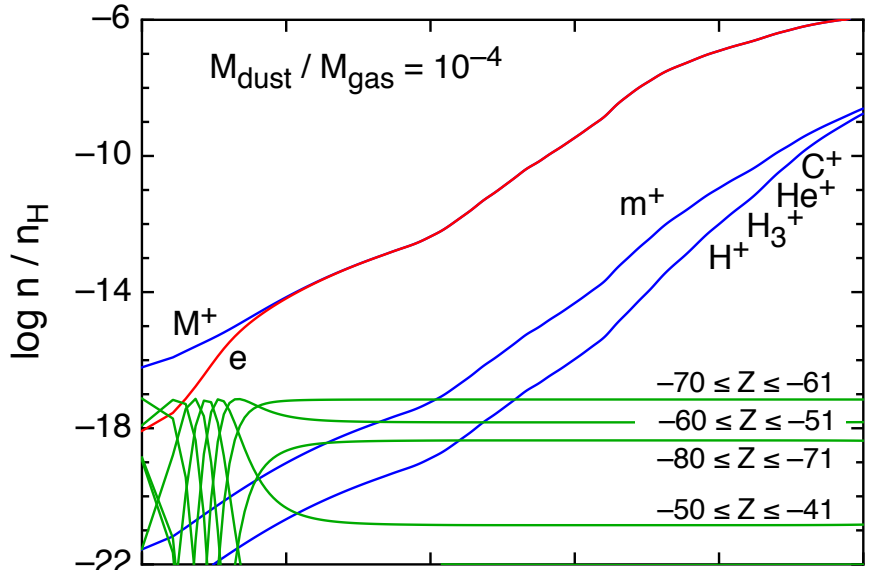
$$\lambda_0 = V_A T$$

$$\frac{\eta_H \Omega}{V_A^2} = \Lambda_H^{-1}$$

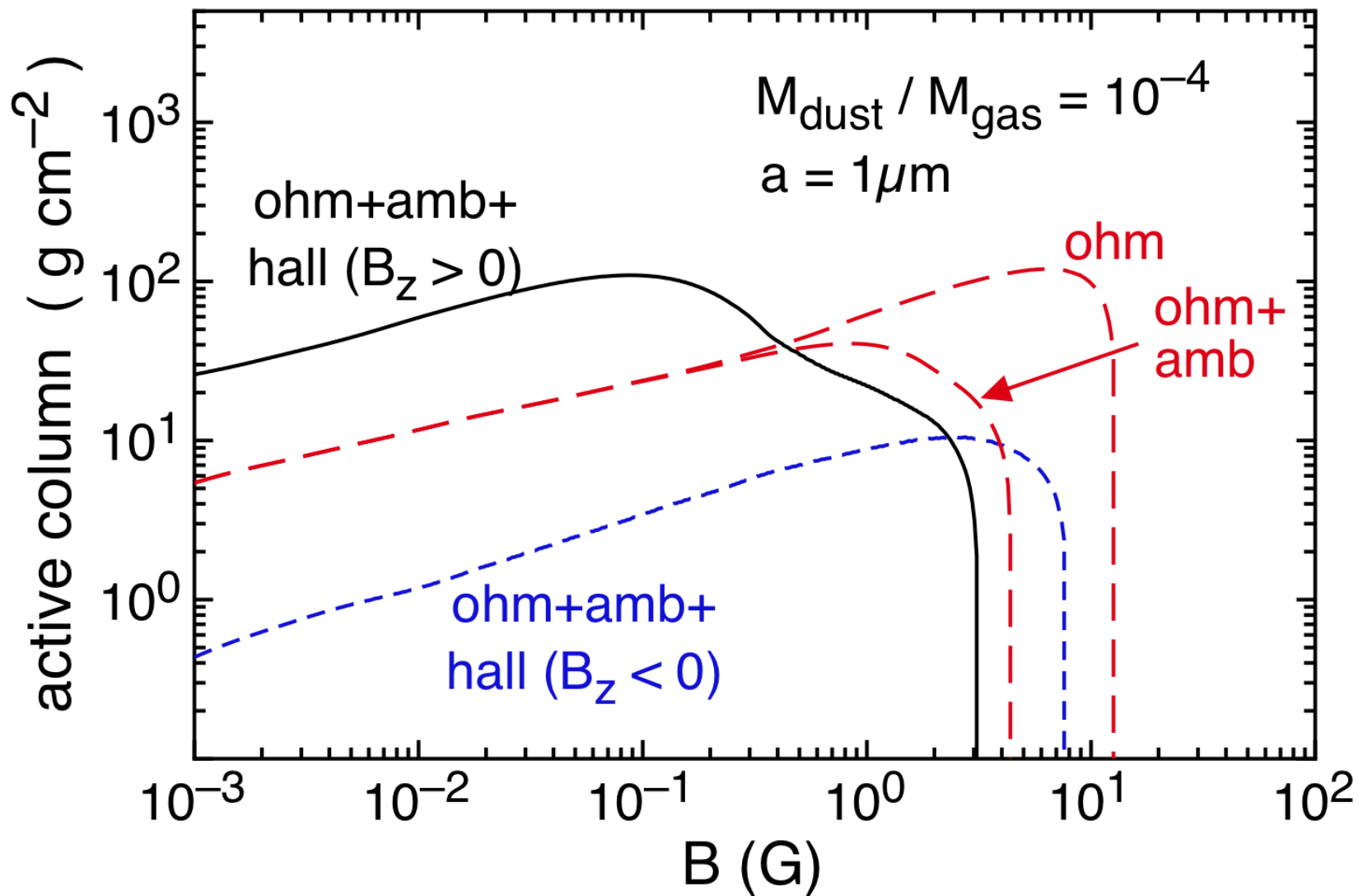
MRI growth rate - ohmic (and/or AD) vs Hall



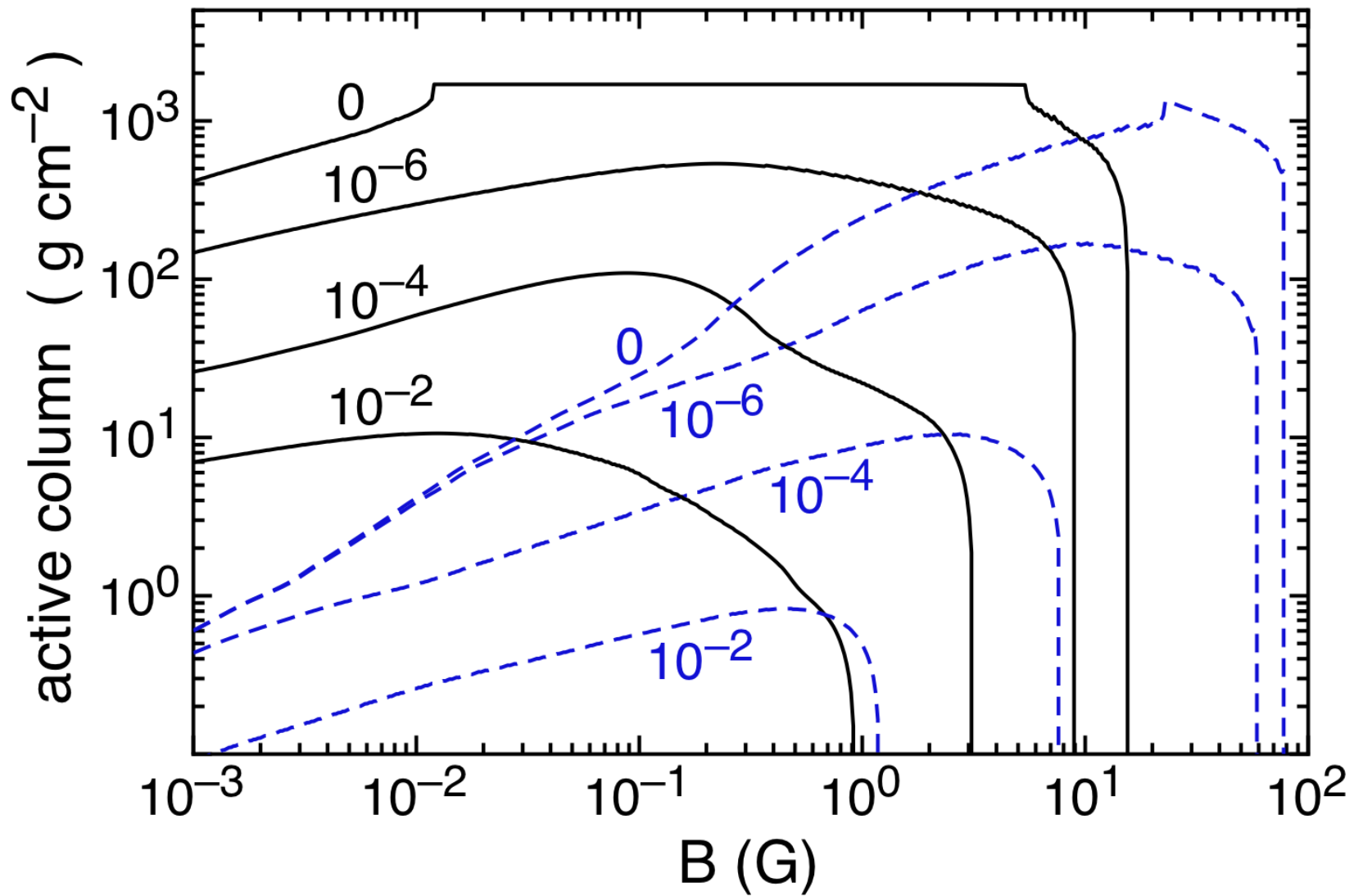
Charged particle abundances

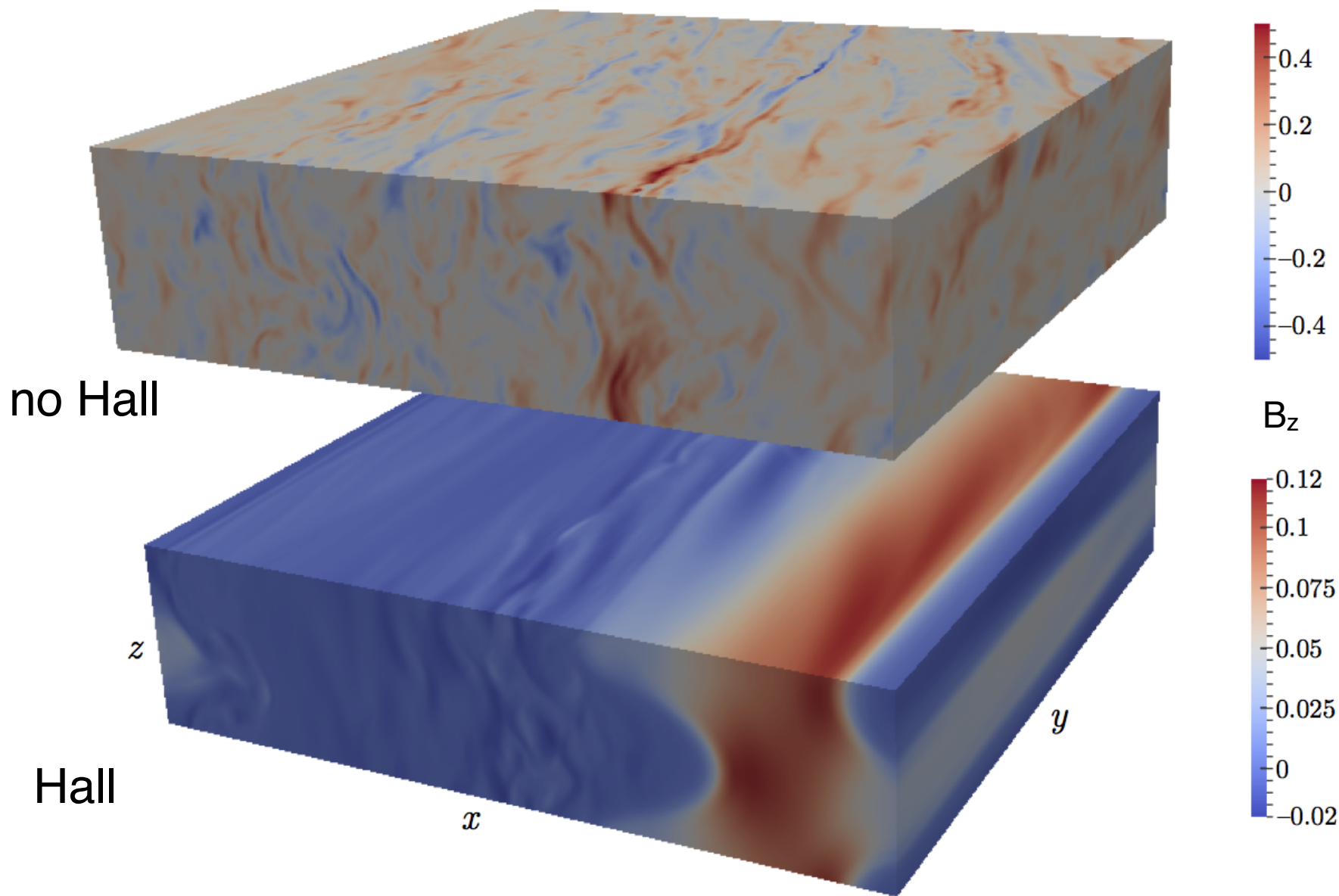


Column density of active layer

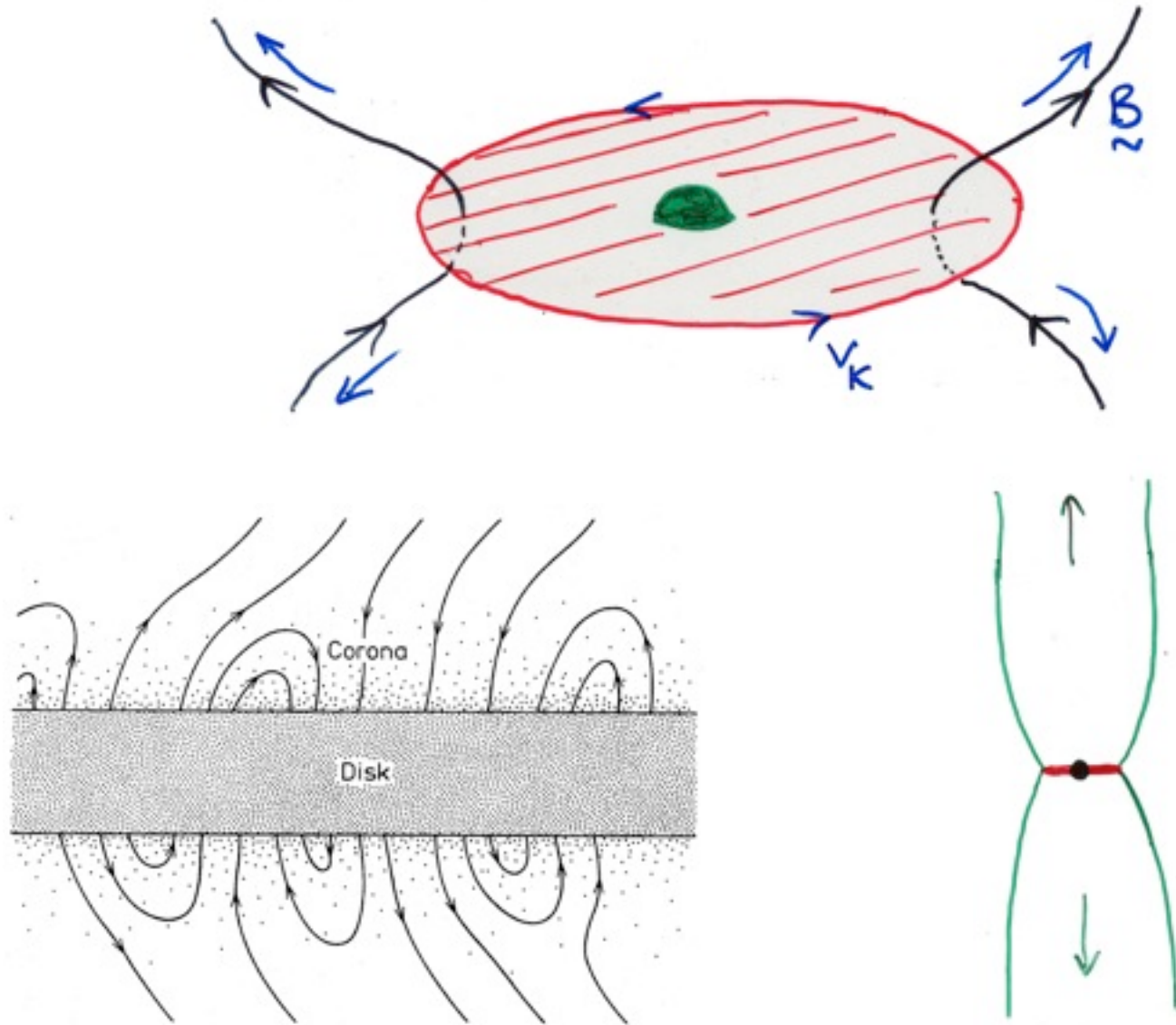


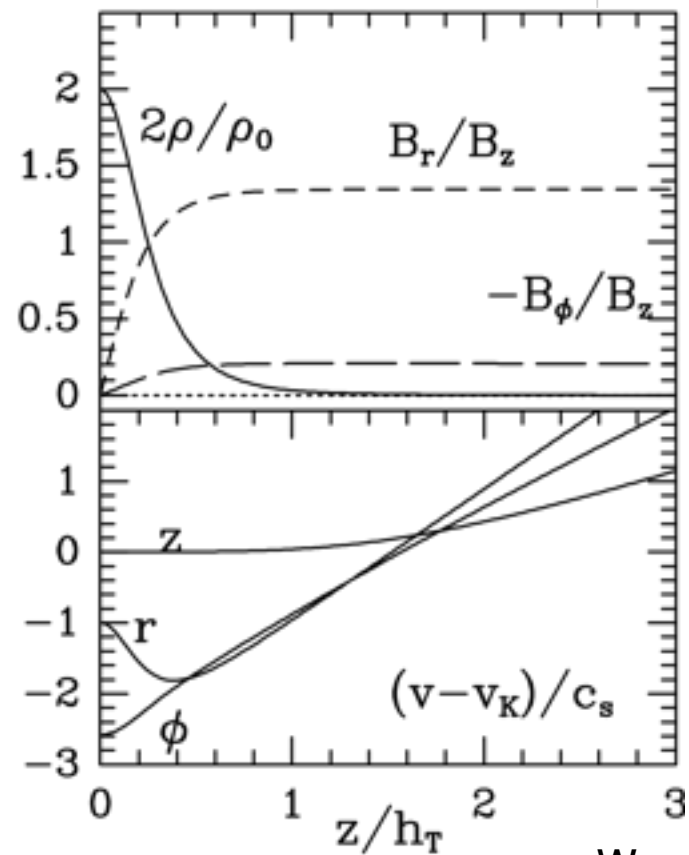
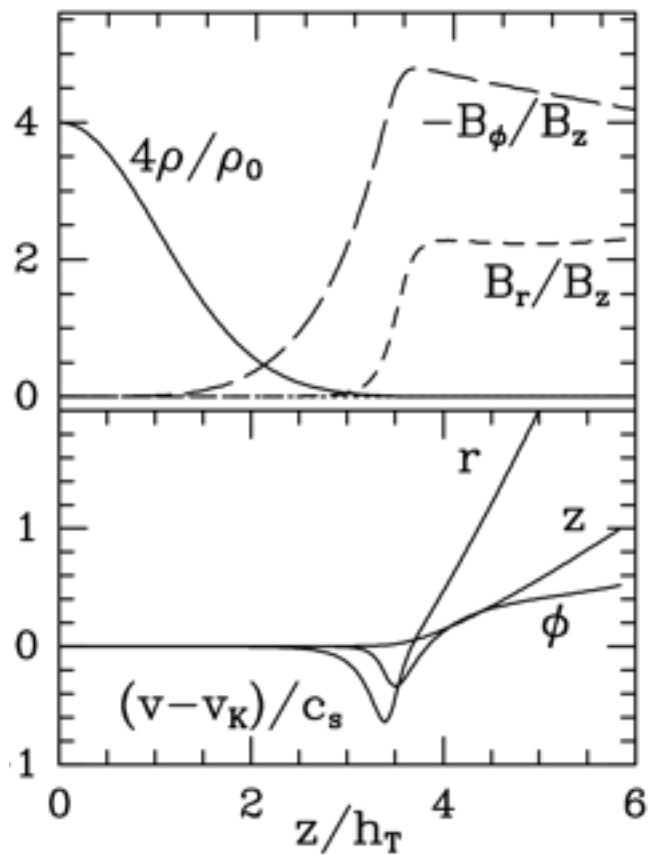
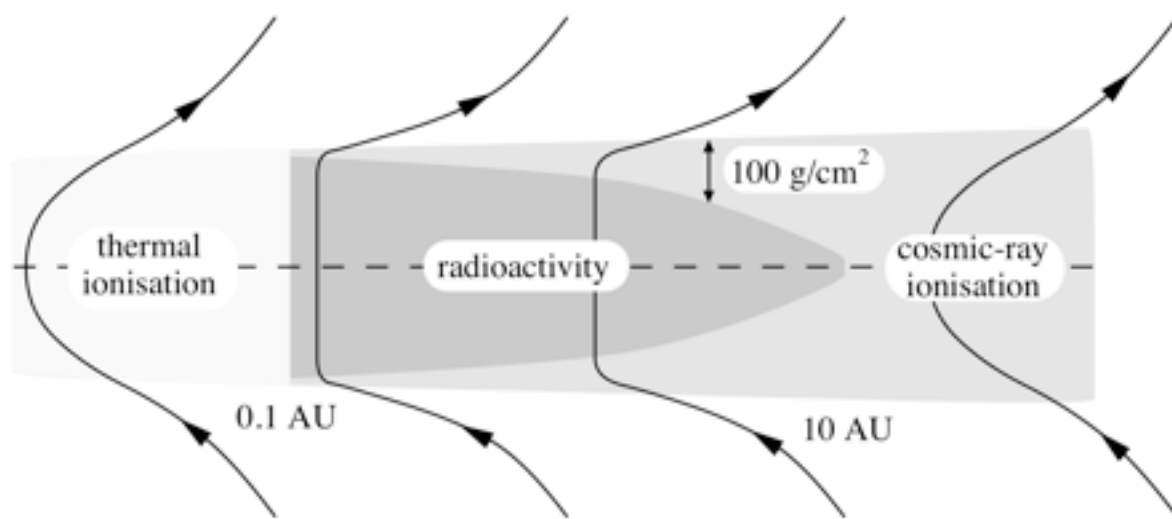
Column density of active layer



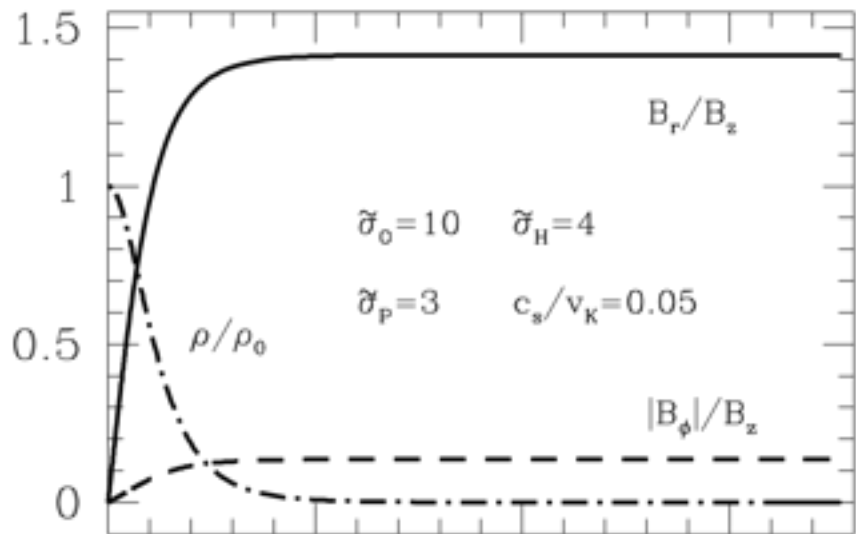


Magnetically-driven jets

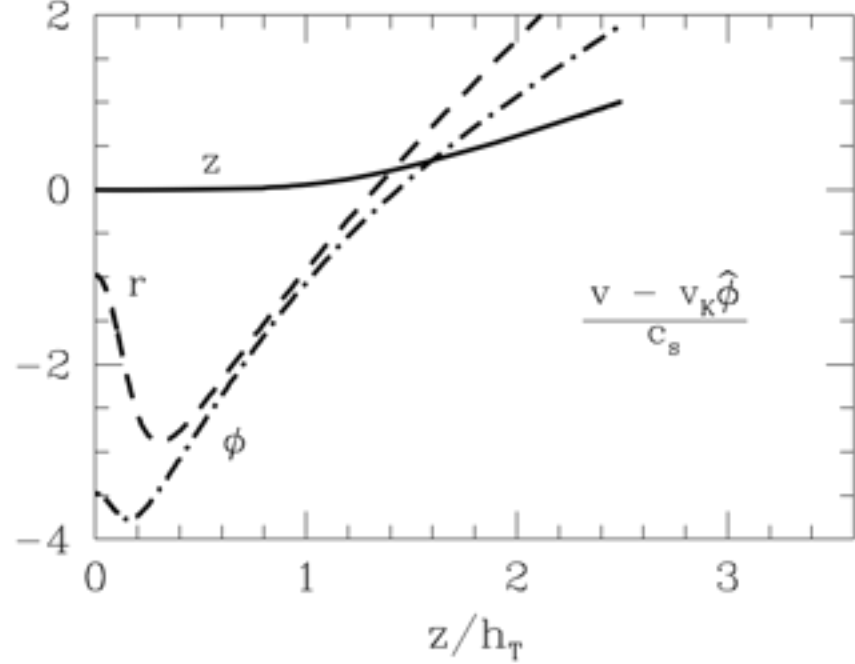
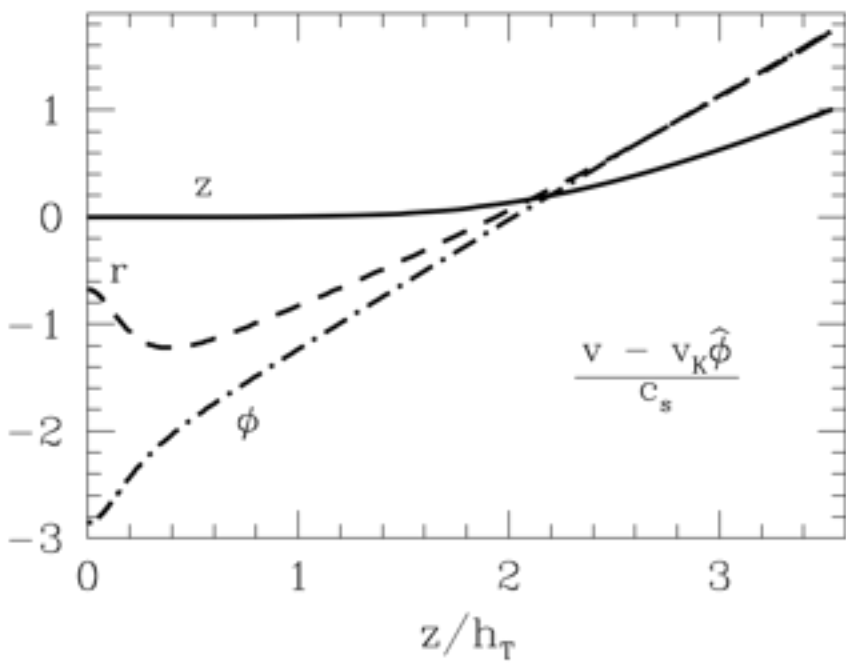
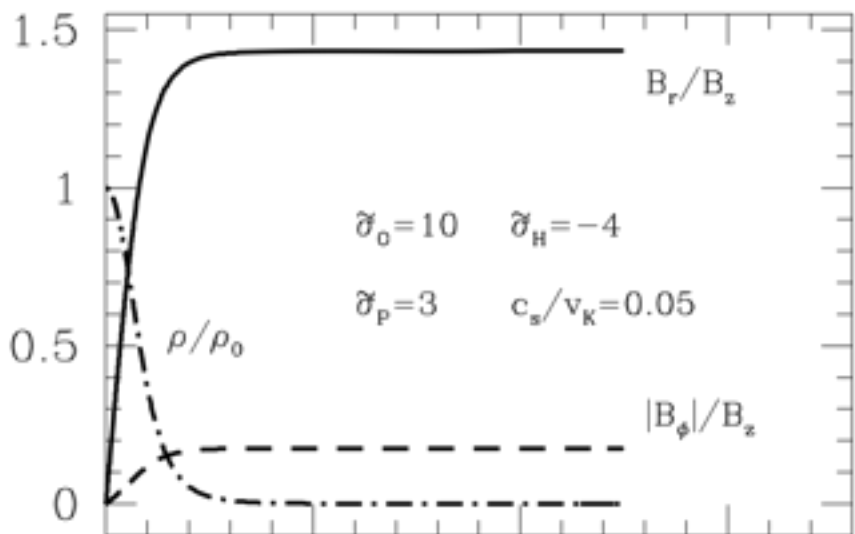


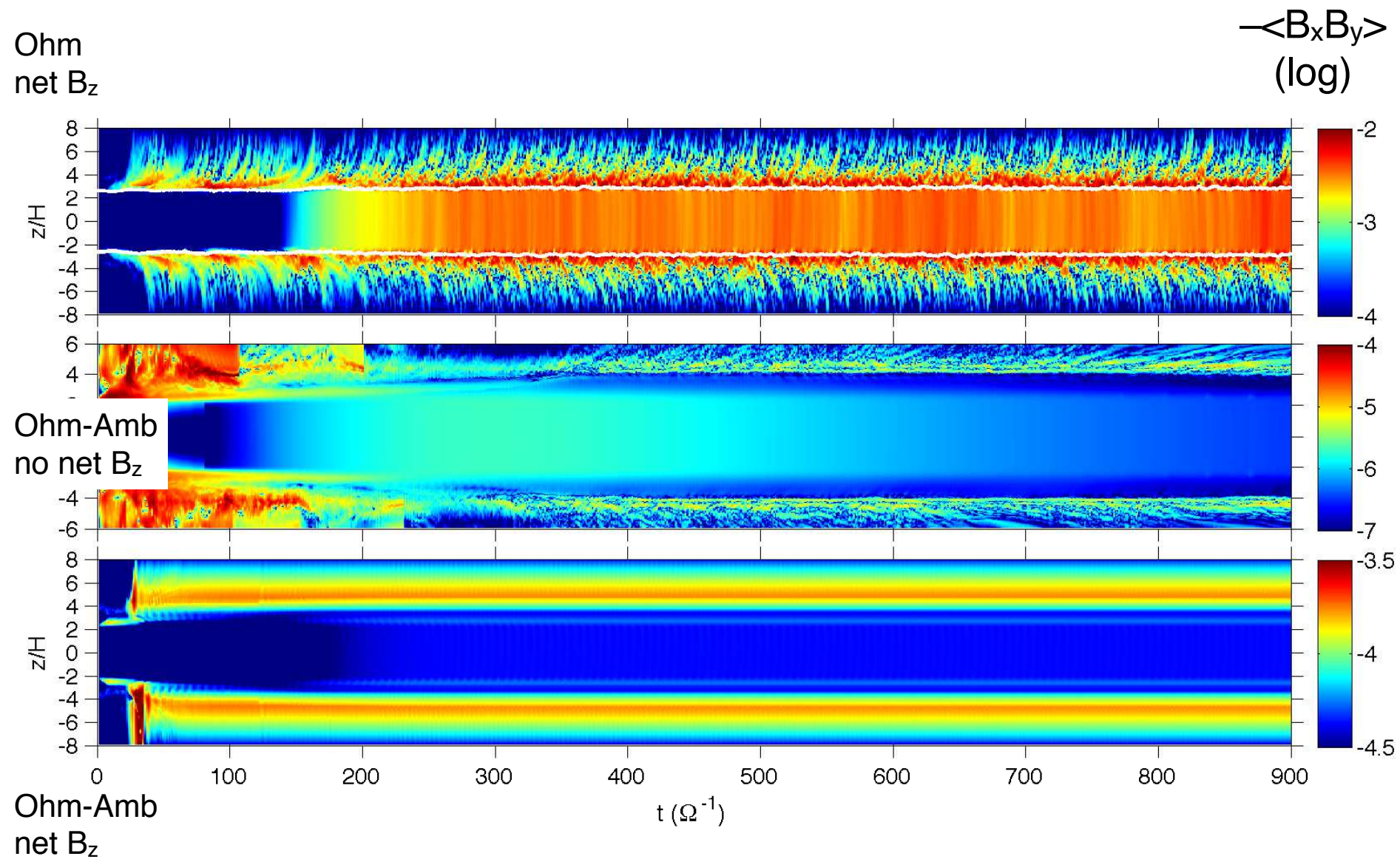


$a_0=0.97$ $\Lambda_0=4.7$ $\epsilon=0.68$ $\epsilon_B=0$

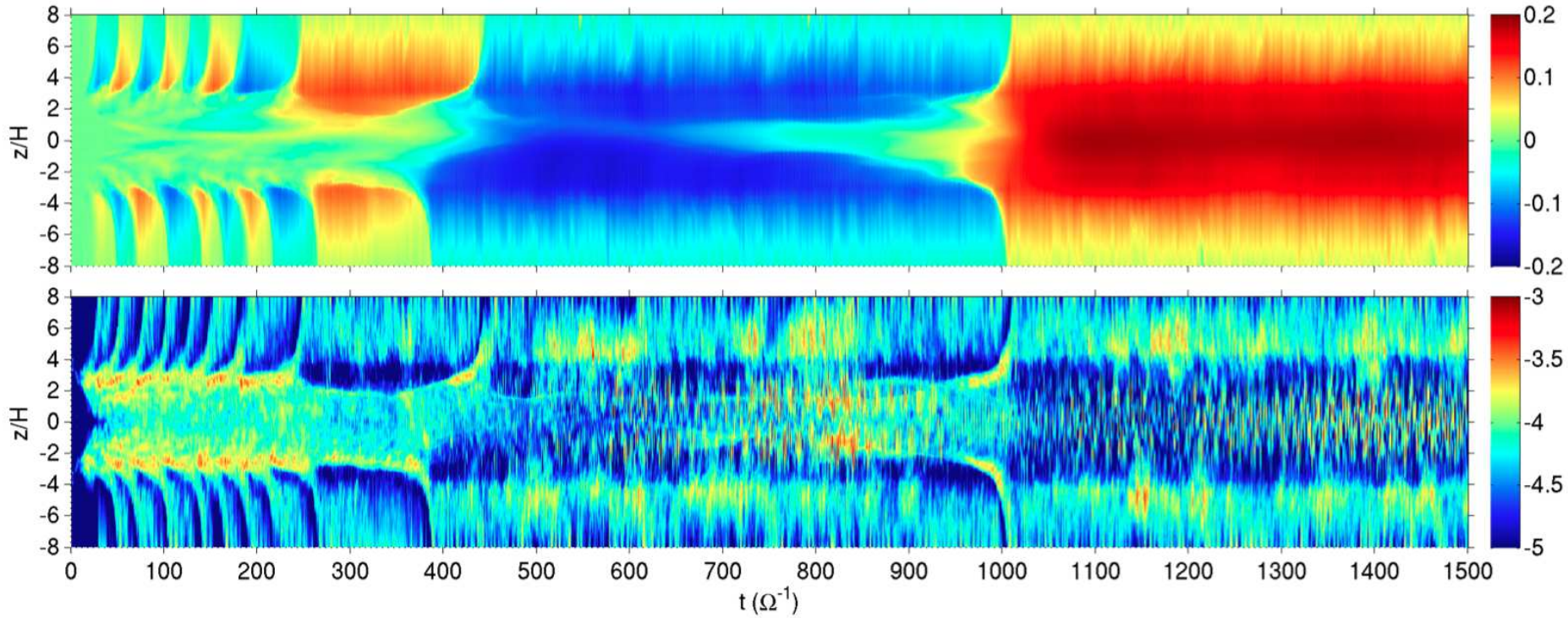


$a_0=0.97$ $\Lambda_0=4.7$ $\epsilon=0.98$ $\epsilon_B=0$

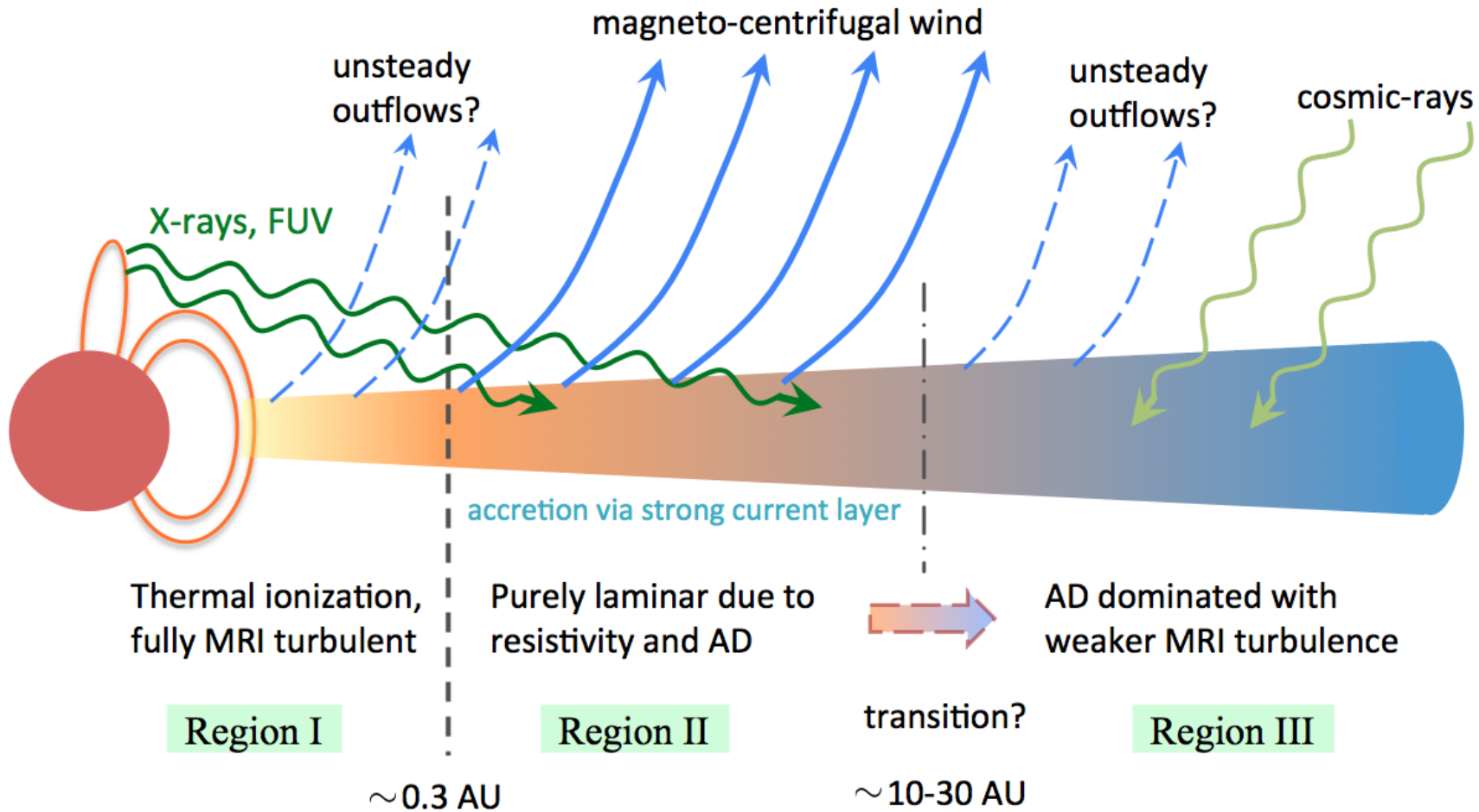




$\langle B_y \rangle$
(log)



$\langle \rho v_z^2 / 2 \rangle$
(log)



Summary

- Ideal MHD breaks down on scale of molecular cloud cores and protoplanetary disks (not just in localised current sheets!)
 - dissipation of fluid motions, structure of shock waves
 - core collapse: angular momentum, magnetic flux
 - protoplanetary disks: distribution and nature of MHD turbulence
 - disk-driven jets: launching, coupling between jet and disk
- Determined by well-defined microscopic processes
 - “low” densities (clouds, cores): ambipolar diffusion (Mestel & Spitzer 1956)
 - high densities (protoplanetary disks): ohmic resistivity (e.g. Hayashi 1981)
 - Hall effect overlooked (shocks, collapsing cores and disks) (Wardle 1998, 1999, Wardle & Ng 1999)
 - uncertainties: grain population, ionizing sources, “turbulent” transport
 - Amb: $v_B \sim J \times B \sim B^2$
 - Hall: $v_B \sim \pm J \sim B$; depends on sign of B ; no dissipation
 - Ohm: $v_B \sim J \times B / B^2 \sim B^0$; important only for high density, weak fields
- cf solar chromosphere
 - Hall \sim Ambipolar \gg Ohm