Hilbert–Huang Transform analysis of solar quasi-periodicities

Dmitrii Kolotkov

Centre for Fusion, Space and Astrophysics,
University of Warwick, UK

e-mail: D.Kolotkov@warwick.ac.uk
Hilbert–Huang Transform (HHT)

Empirical Mode Decomposition (EMD)+Hilbert transform

Skeleton of the method:

- Decomposition of the initial data set into a number of intrinsic modes by EMD
- Subsequent Hilbert transformation of the identified modes

Empirical Mode Decomposition (EMD)

1. Identifying of all local extrema of the original data $X(t)$

2. Define upper/lower envelopes connecting all maxima/minima and their mean $m_1$. Then

$$h_1(t) = X(t) - m_1(t).$$
Empirical Mode Decomposition (EMD)

Sufficient number of repetitions allows the residue to become the first intrinsic mode function (IMF):

- Original data
- The first IMF
- Data with the first IMF subtracted
Example of the EMD expansion

Solar flare on 14 May 2013, Kolotkov et al. (2015)

Correlation curve (red) and R+L flux (blue) at 17 GHz detected in solar flare 14 May 2013, with NoRH and NoRP, respectively

Radio map at 17 GHz, 14 May 2013, NoRH

Loop length ∼ 40 Mm
Example of the EMD expansion
Set of intrinsic empirical modes in the correlation curve

- Mode 3: Period ~ 45 s
- Mode 4: Period ~ 15 s
- Mode 5: Period ~ 100 s
- Mode 6: Period ~ 45 s
Hilbert spectrum

\[ Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} \]

\[ a(t) = \left[ X^2(t) + Y^2(t) \right]^{1/2}, \]

\[ Y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{X(t')}{t-t'} \, dt' \]

\[ \omega = \frac{d\theta(t)}{dt}. \]

Hilbert transformation is used to obtain the frequency-power-time distribution for each IMF, designated as the **Hilbert spectrum**
Origination of mode 5

- Period ~ 100 s
- Loop length ~ 40 Mm
- Phase speed ~ 800 km/s

Kink modes: typical speed ~ 1000 km/s, periods ~ a few minutes

Pascoe et al. (2016)
Origination of mode 3

Period ~ 15 s, loop length ~ 40 Mm, phase speed ~ 5.3 Mm/s

Sausage mode:

Period ~ 16 s, Melnikov et al. (2005)
A few more examples on the detection of underlying trends with EMD...

Solar flare on 04 Feb 2014, R+L flux at 17 GHz, NoRP

![Graphs showing signal and derivative of signal over time, with periods ranging from 2 to 10 s.](image)
A few more examples on the detection of underlying trends with EMD...

Solar flare on 25 Feb 2014, R-L flux at 17 GHz, NoRP
Significance of EMD modes in comparison with coloured noises of $S \propto f^{-\alpha}$ (Kolotkov et al. 2016)

- Check the noise colour, i.e. the power law index $\alpha$, in the original data set:

$$E_m P_m^{1-\alpha} = \text{const},$$

where $E_m = \sum X_i^2$ is the total energy of the $m$th mode, and $P_m$ is its main period.

- For the chosen sort of noise, calculation of the confidence intervals using the chi-squared distribution:

$$f(E_m) = \frac{k_m}{\bar{E}_m} \chi^2 \left( \frac{k_m E_m}{\bar{E}_m}, k_m \right),$$

where $\bar{E}_m(\alpha)$ is the average total energy of the $m$th mode, and $k_m(\alpha)$ is the number of degrees of freedom.
EMD analysis of synthetic coloured noises
(Kolotkov et al. 2016)

- white
- pink (flicker)
- red (random walk)

Wu & Huang, (2004)
8 Dec 2010 (NOAA 11131)

ROI 1 – quiet sun region

ROI 2 – sunspot umbra
Quiet sun region

171 Å, $\alpha=1.32\pm0.04$

171 Å, $\alpha=1.30\pm0.02$

304 Å, $\alpha=1.29\pm0.02$

304 Å, $\alpha=1.30\pm0.01$

1600 Å, $\alpha=0.86\pm0.03$

1600 Å, $\alpha=0.85\pm0.03$
Sunspot umbra
Quasi-periodic oscillations of a small-scale magnetic structure (*Kolotkov et al. 2017*)

SDO/HMI line-of-sight magnetogram at 23:00:45 UT on 6 July 2013

Average field (blue) in the structure, and its total area (black) in 100 G contour

In the context of Sun-like stars: e.g. *Karoff et al. (2013).*
EMD modes of the average field signal

$\alpha=0$ (blue lines)  
$\alpha=1$ (purple lines)

Mode 8

Period $\sim$ 80-230 min
Scaling of the instant period of Mode 8

Such a behaviour could be explained e.g. by the dynamical interaction with the boundaries of supergranula cells or in terms of the vortex shedding appearing during the magnetic flux emergence. However, a specific mechanism is still to be revealed.
Conclusions

• HHT (EMD) is a good tool for studying non-stationary and non-regular time-series

• It operates self-adaptively, and hence is readily able to determine natural intrinsic modes, including underlying trends

• It works well on different time scales: from a few seconds to tens of years (e.g. 11 yr solar cycle), and is worth applying for stellar quasi-periodicities

• This “hammer” should be used correctly, e.g. accounting for the appearance of frequency-dependent components.