A mixed SOC-turbulence model for nonlocal transport and space-fractional Fokker-Planck equation

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ISSI International Space Science Institute, Bern, Switzerland

16-09-2013, ore 11:00

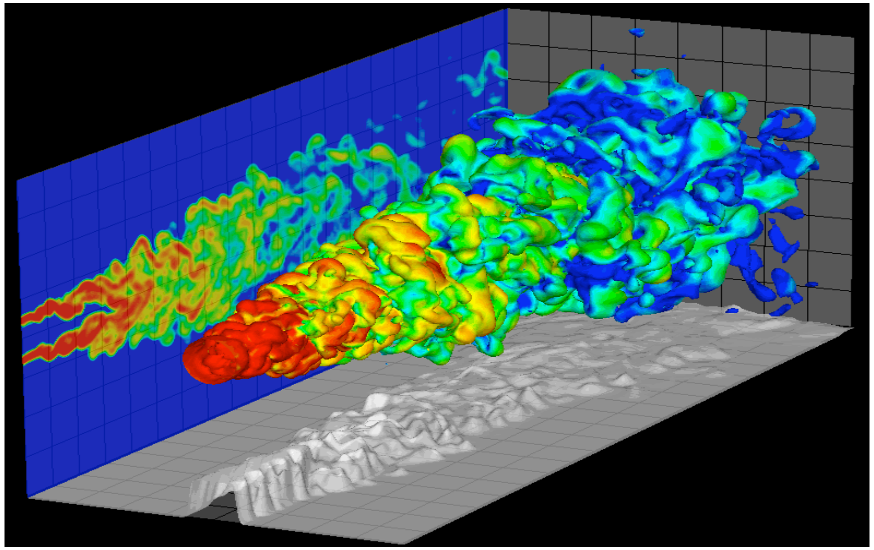
Outline and Challenges

- Turbulence vs SOC paradigm
  - Transport is intermittent
  - Bursts are important at all scales
  - Particle density flux described by PDF
- Fractional diffusion equation (FDE)
  - with fractional time derivative
  - with fractional space derivative

1. SOC (BTW) will produce a time-fractional FDE
2. SOC (BTW, DP) will NOT produce space-fractional FDE
3. DW turbulence (HW) will NOT produce space-fractional FDE

An additional amplification mechanism is needed to produce a FDE with fractional derivative in space!

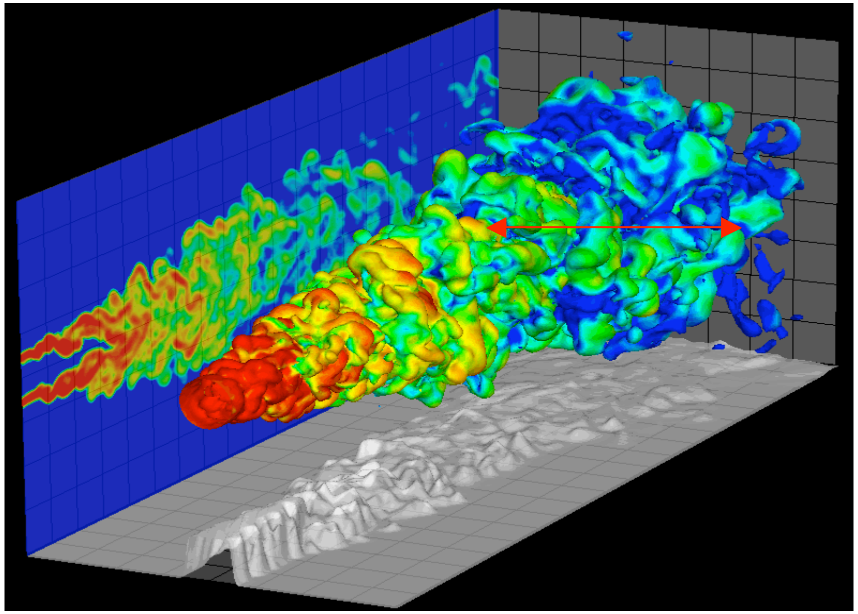
Turbulence or Self-organized criticality?

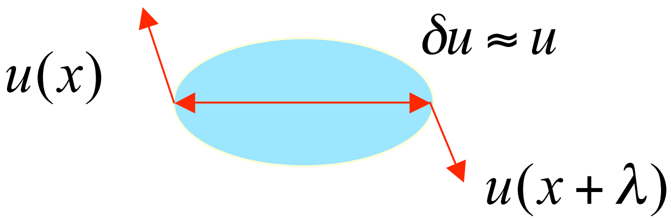


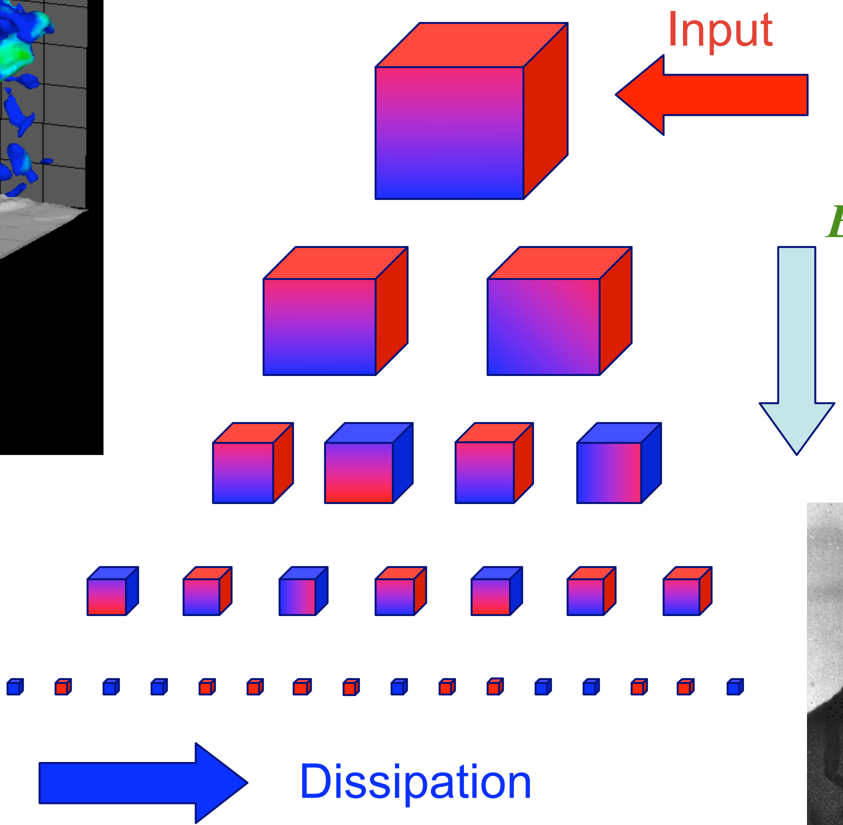
SOC shares with turbulence implications of multi-scale behavior and nonlinear interaction

Both involve many degrees of freedom and scale-free size-distributions of the dynamical entities: vortices in turbulence, avalanches in SOC. No wonder that the notions of turbulence and SOC have been considered difficult to reconcile.

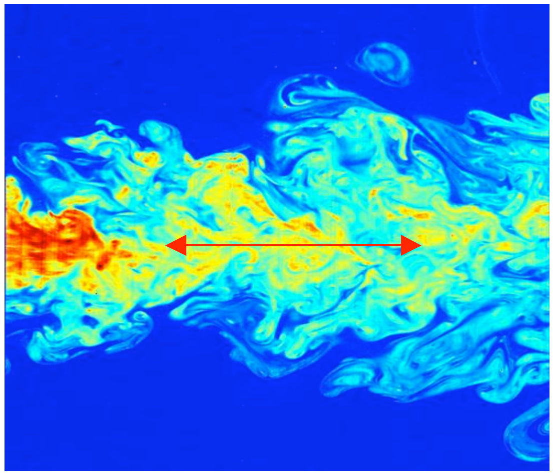
Turbulence: K41








A.N.Kolmogorov



Spectral energy density:

$$\left\langle \left[u(x+\lambda)-u(x)\right]^2 \right\rangle = 2\int_{k_{\min}}^{k_{\max}} E(k')\left[1-\frac{\sin k'\lambda}{k'\lambda}\right]dk'$$

K41: Inertial range

Kinetic energy density of the turbulent flow:

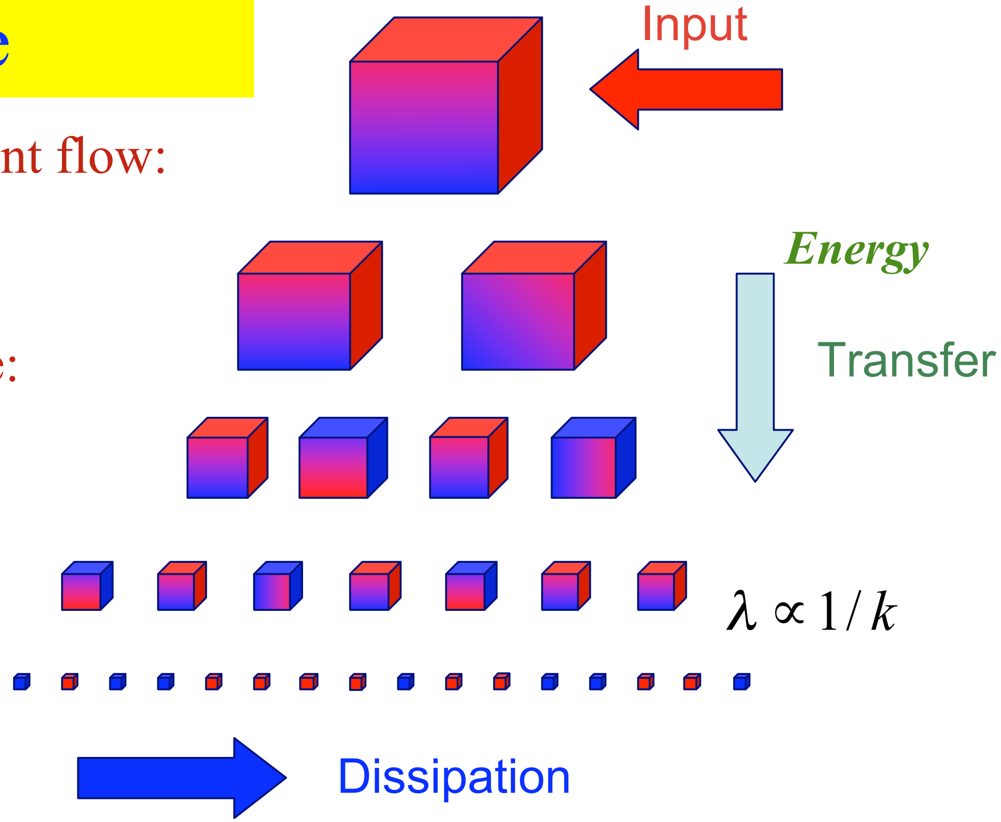
$$\varepsilon \propto \tfrac{1}{2}\rho(\delta u)^2$$

Nonlinear (eddy turnover) time scale:

$$\tau_{NL} \propto \lambda / \delta u$$

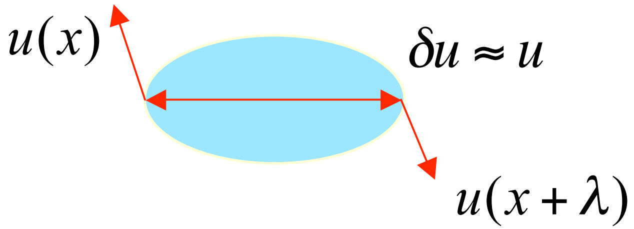
The energy cascade rate:

$$\dot{\varepsilon} \propto \varepsilon / \tau_{NL} \propto \tfrac{1}{2}\rho(\delta u)^3 / \lambda$$



Assumptions:

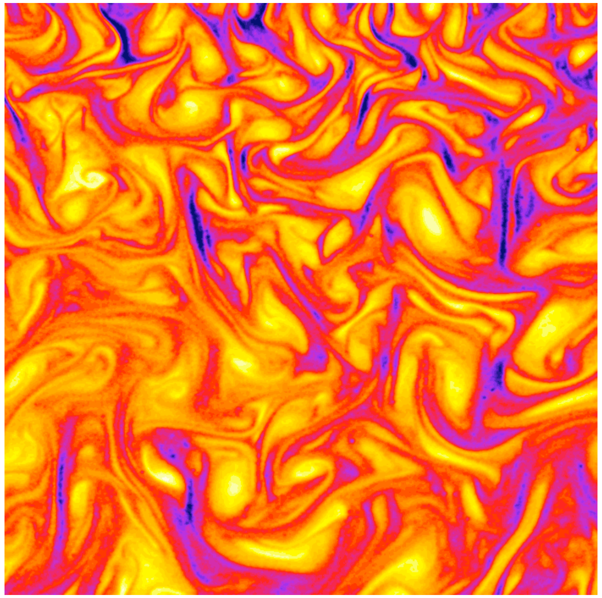
1. **space filling**, homogeneous, isotropic turbulence
2. **k-independent** energy cascade rate

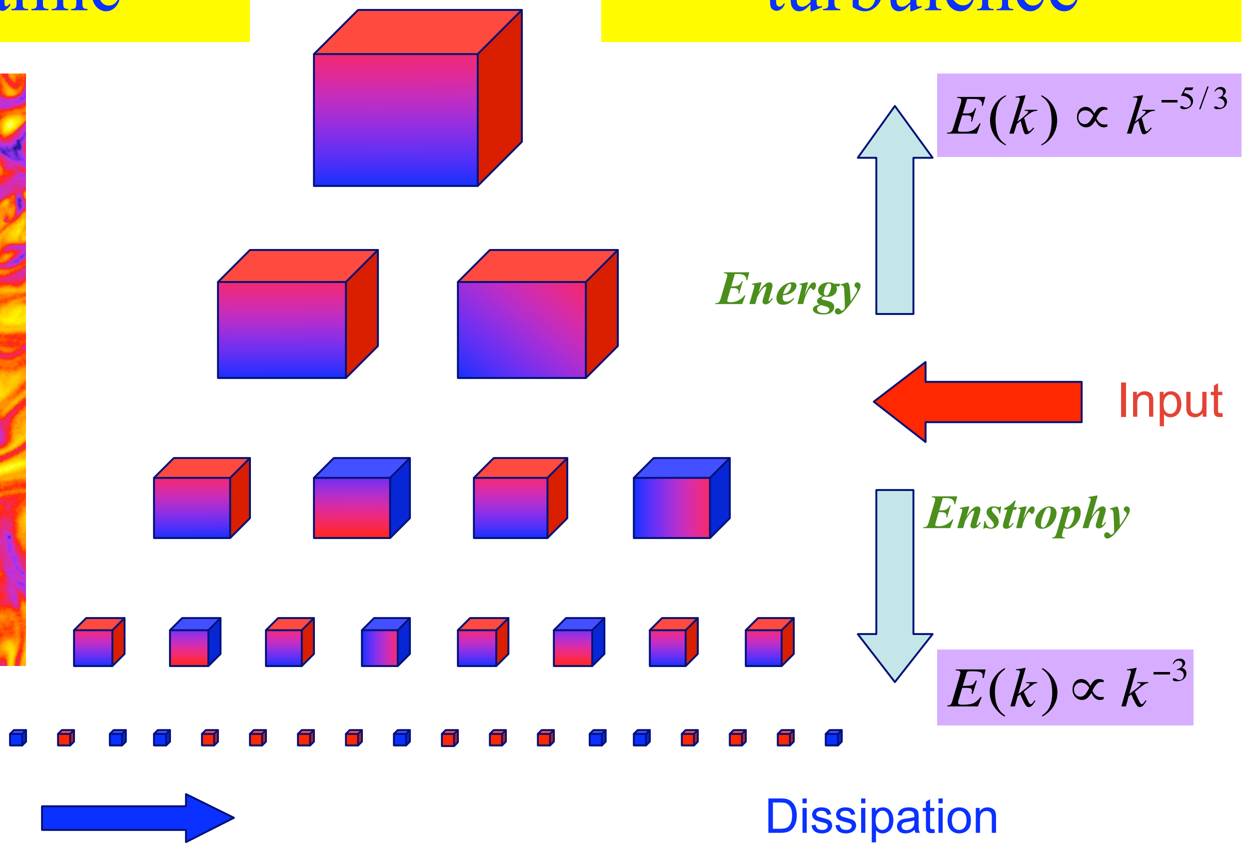


$$\left\langle \left[u(x+\lambda) - u(x)\right]^2 \right\rangle = 2\int_{k_{\min}}^{k_{\max}} E(k')\left[1 - \frac{\sin k'\lambda}{k'\lambda}\right]dk'$$

$$(2\dot{\varepsilon}/\rho)^{2/3} \times k^{-2/3} \quad \Rightarrow \quad E(k) \propto k^{-5/3}$$

2D Hydrodynamic turbulence





Robert H. Kraichnan

- 2 inviscid constants of motion
- 2 formal inertial ranges
  - kinetic energy (-5/3, backward)
  - mean-square vorticity (-3, direct)
- The phenomenon of vortex stretching is forbidden

Why -3 spectrum?

Enstrophy density of 2D turbulent flow:

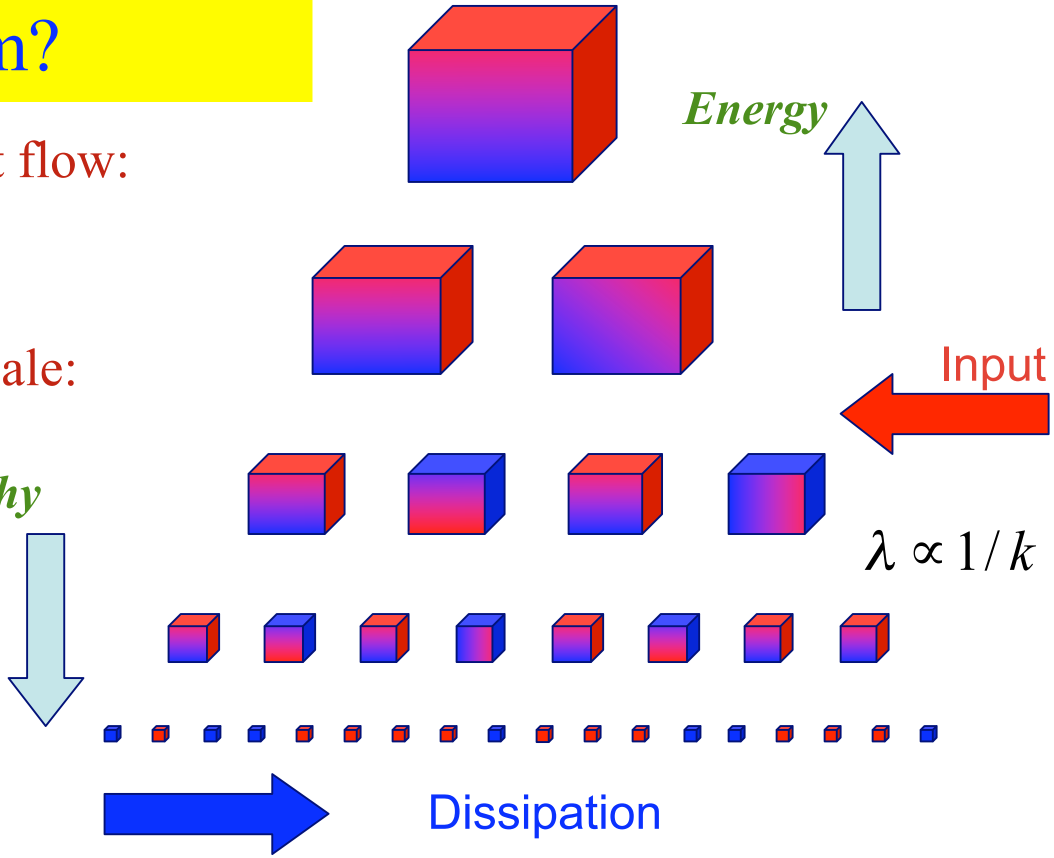
$$\eta \propto (\mathrm{rot}\ \delta u)^2 \propto (\delta u / \lambda)^2$$

Nonlinear (eddy turnover) time scale:

$$\tau_{NL} \propto \lambda / \delta u$$

The enstrophy cascade rate:

$$\dot{\eta} \propto \eta / \tau_{NL} \propto (\delta u / \lambda)^3$$



Assumptions:

1. **space filling**, homogeneous, isotropic turbulence
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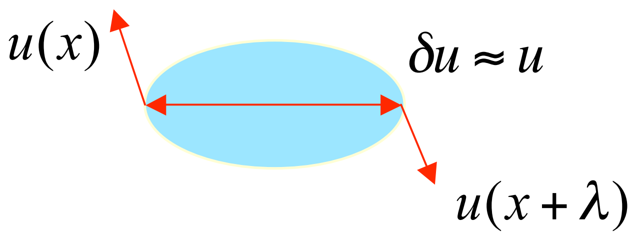


$$\left\langle \left[u(x+\lambda) - u(x) \right]^2 \right\rangle = 2 \int_{k_{\min}}^{k_{\max}} E(k') \left[1 - \frac{\sin k'\lambda}{k'\lambda} \right] dk'$$

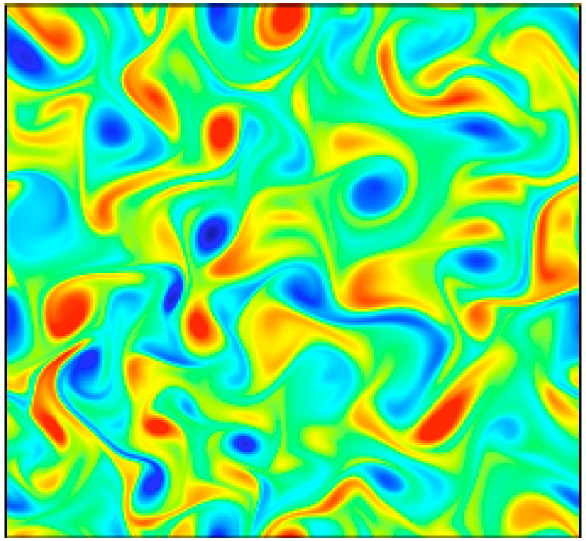
$\propto \mathrm{const}\, k^{-2}$ $\Rightarrow$ $E(k) \propto k^{-3}$

2D drift-wave turbulence: Hasegawa-Wakatani Eqs

$$\frac{\partial n}{\partial t} + \{\phi, n\} + \frac{\partial \phi}{\partial y} = \frac{1}{\delta}(\phi - n) + \mu_n \nabla^2 n$$

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} = \frac{1}{\delta}(\phi - n) + \mu_\omega \nabla^2 \omega$$

$$\omega = \nabla^2 \phi$$



The Poisson bracket is used to denote the nonlinear terms originating from advection with the ExB drift

$$\{\phi, f\} = (\mathbf{z} \times \nabla \phi) \cdot \nabla f$$

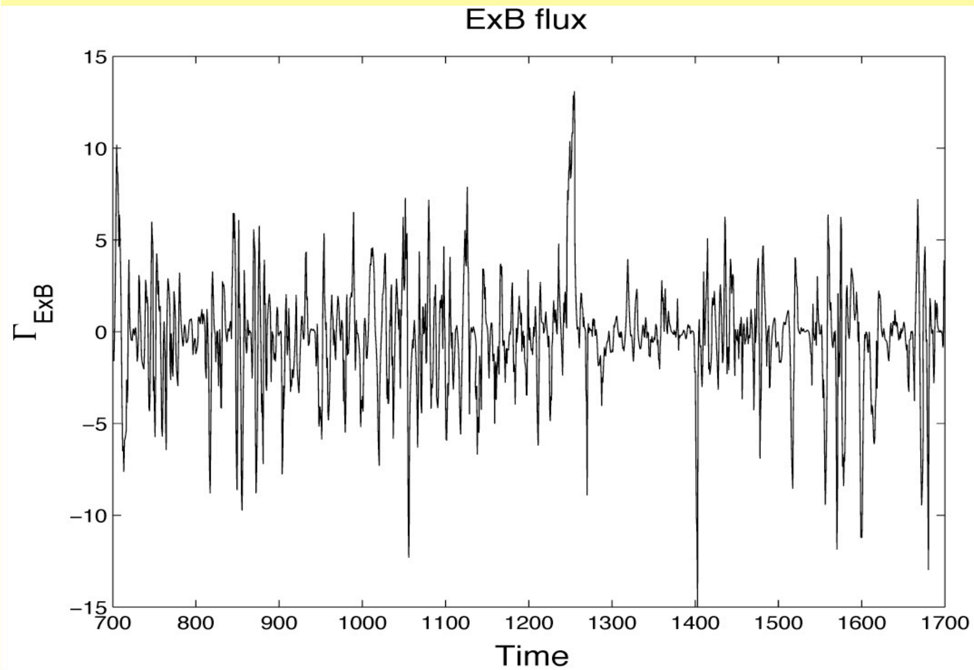
$$\delta = 1 / k_\parallel L_\parallel < 1$$

$$L_\parallel = \sqrt{L_n T_e / m_e c_s \nu_{ei}}$$

$$L_n^{-1} = \nabla n / n_0$$

The nonadiabaticity parameter δ characterizes deviation between the potential and the density fluctuations and absorbs the parameters of the parallel dynamics. It constitutes an internal drive for the turbulence, offering a max growth rate δ/8

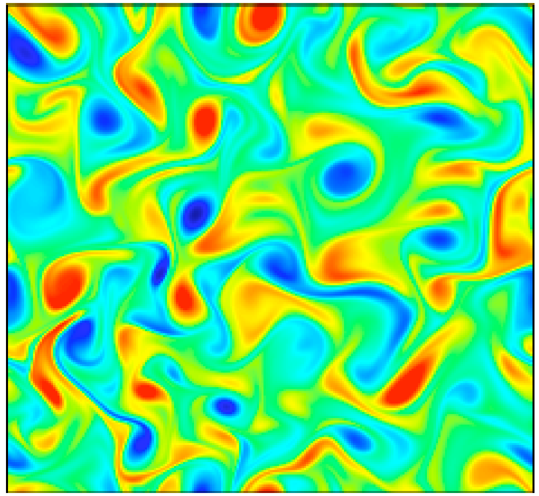
2D drift-wave turbulence



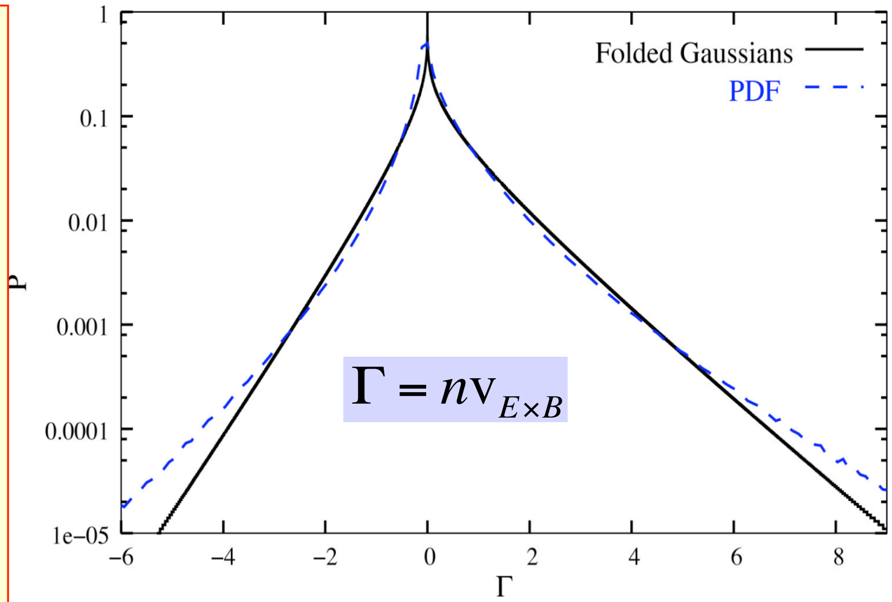
Properties:

- **Plasma turbulence** and transport strongly intermittent
- **No** easy parametrization
- **Not** simple diffusion in Fickian sense
- **PDF's** of radial displacements of tracer particles are non-Gaussian with exponential tails and the moments indicating **superdiffusive transport**

Bursting flux!



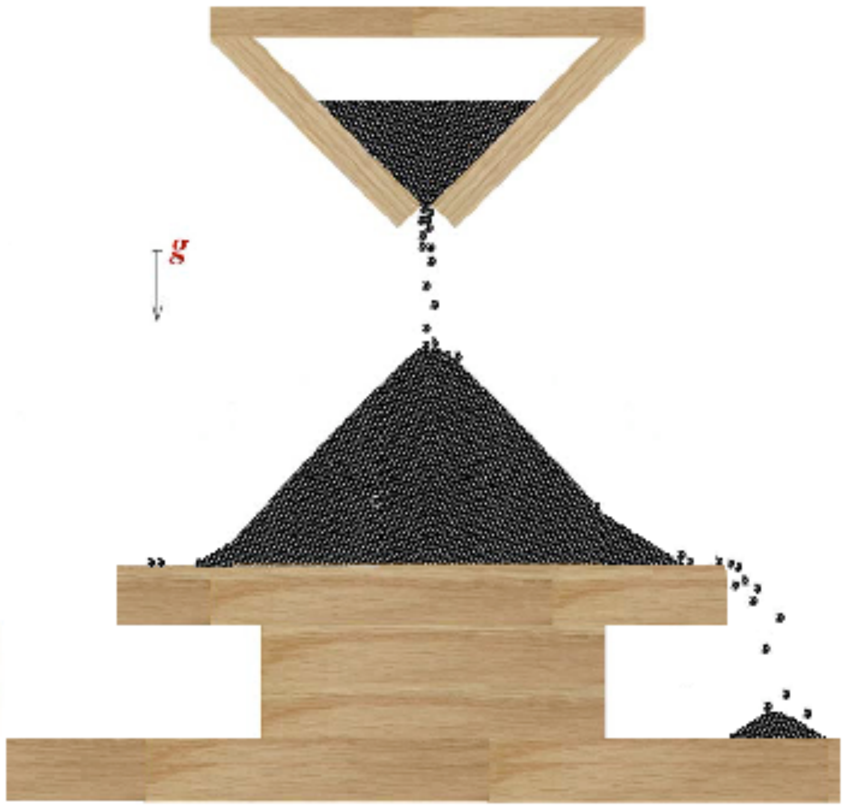
Flux PDF



V. Naulin et al., Phys. Lett. A 321, 355 (2004)

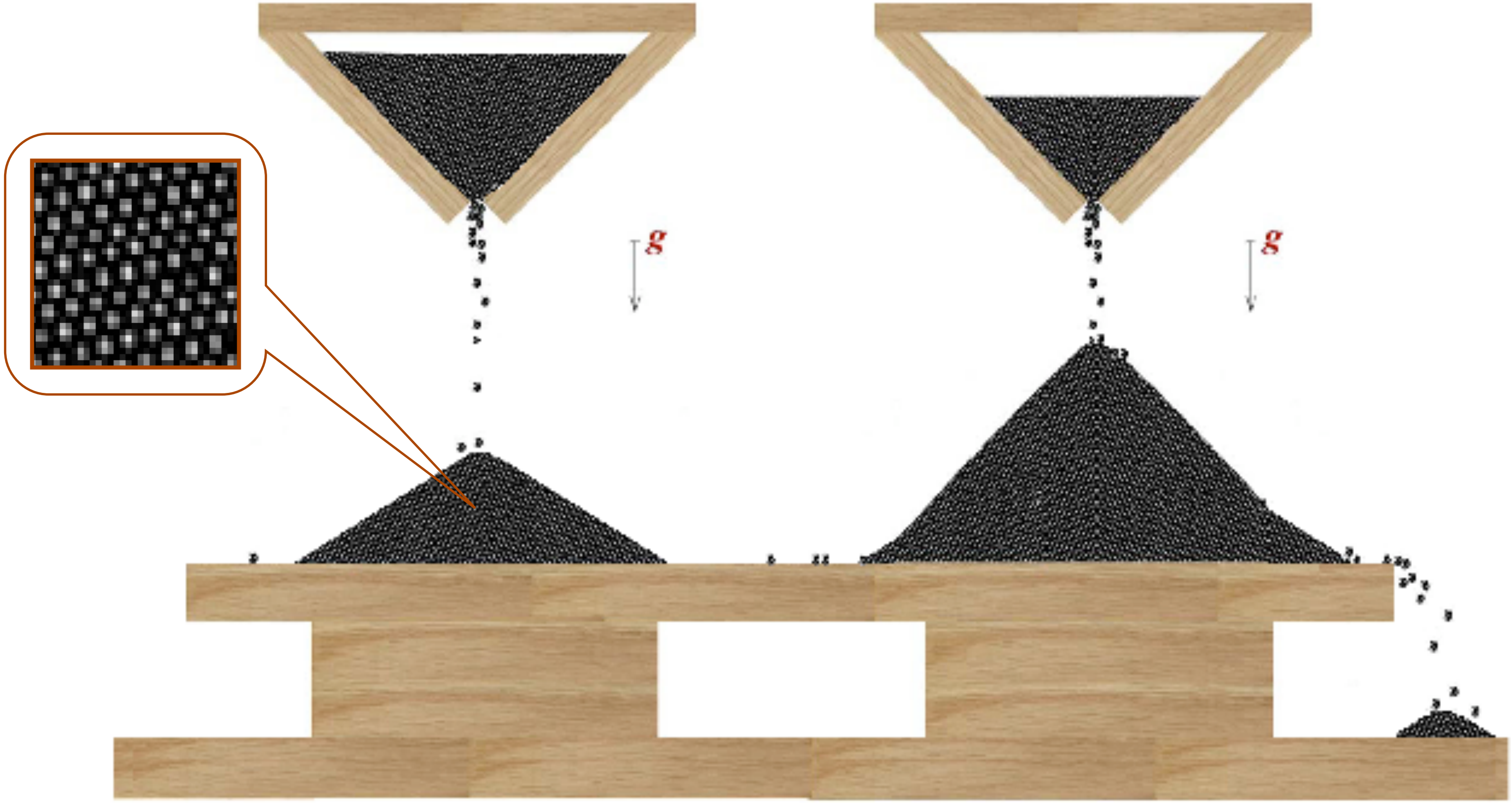
Self-organized criticality (SOC) -- NESS…

- A paradigm for complex dissipative systems that relax through bursts
- Self-organized state ➨ critical state (at the border of chaos) reached without fine tuning of any external or control parameters
- Critical state ➨ attractor, robust with respect to variations of parameters and with respect to randomness



SANDPILE: PROTOTYPE MODEL OF SOC (BTW 1987)

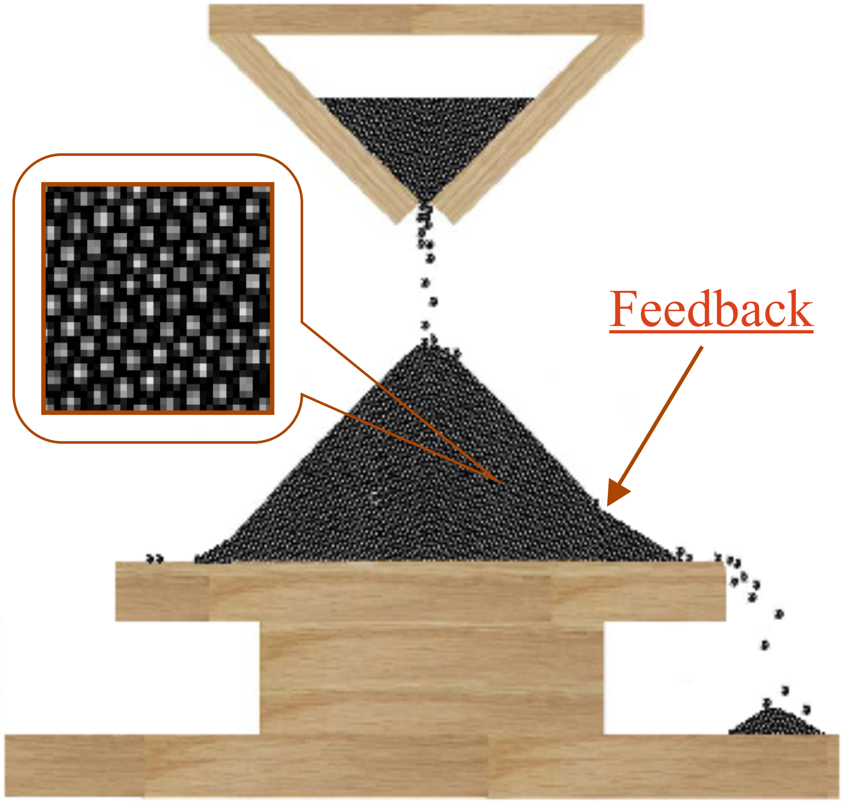
The fundamental physics of the SOC state



Near equilibrium and far from the critical state, the system produces no avalanches

Near criticality and far from equilibrium, series of relaxations of widely varying size are generated

SOC: Cellular-automation (CA) models



If:

the local slope or pressure $z_{i,j}$ exceeds the critical value z_c

Then:

at the next time step (in two dimensions)

$$z_{i,j} \to z_{i,j} - 4, \qquad z_{i,j\pm1} \to z_{i,j\pm1} + 1,$$

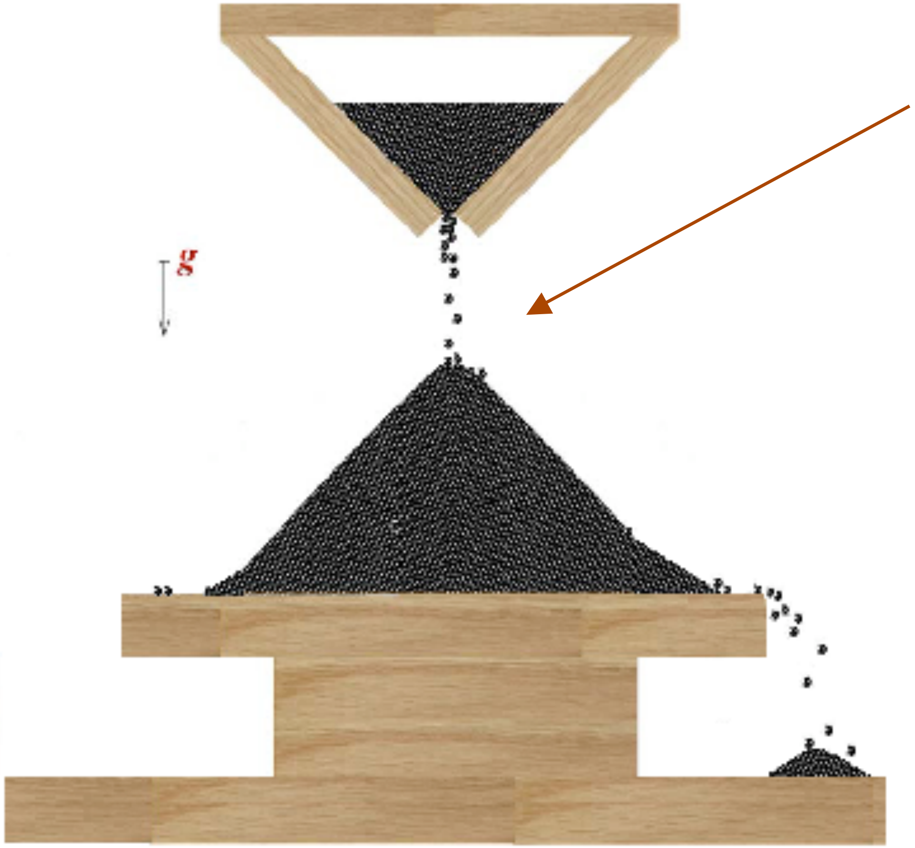
$$z_{i\pm1,j} \to z_{i\pm1,j} + 1$$

In model 2 the slope is increased by repeatedly letting

$$z_{i,j} \to z_{i,j} + 1$$

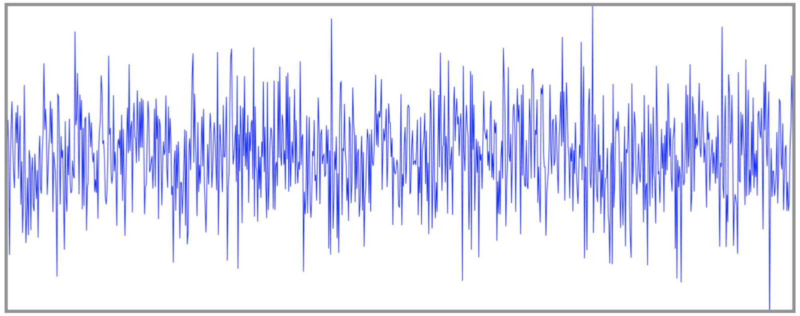
at random sites (i, j) and allowing the system to relax following the dynamical rule above. The boundary condition is $z = 0$.

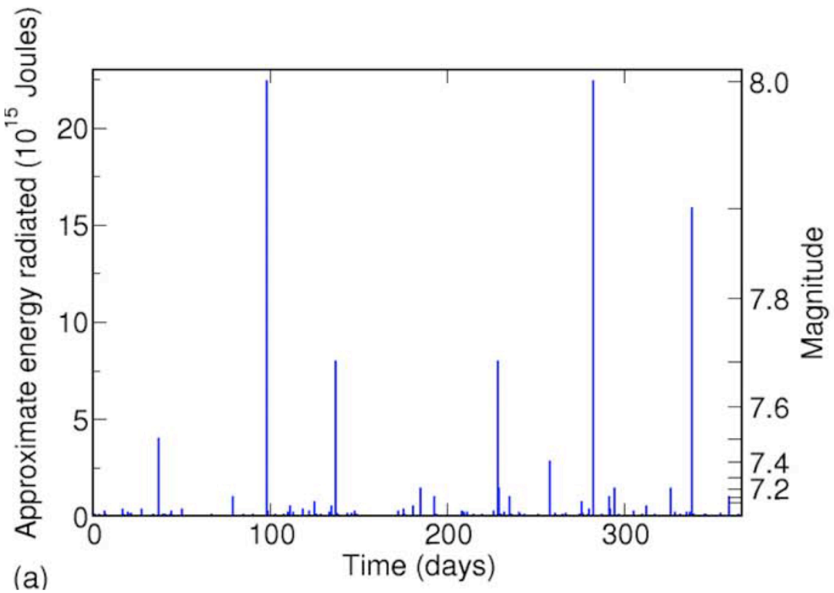
The fundamental physics of the SOC state



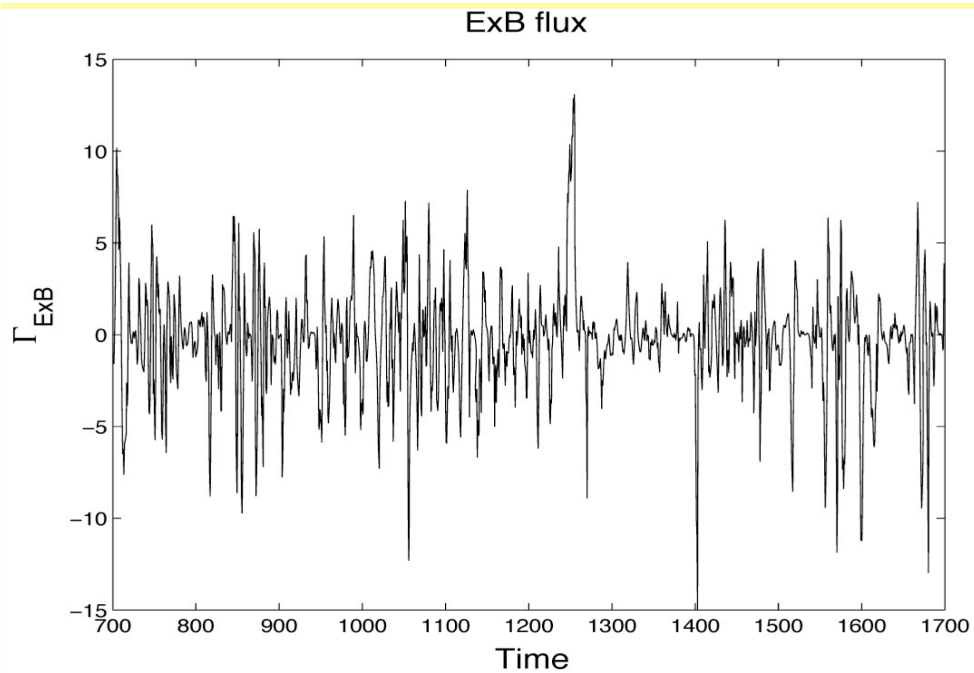
Crackling noise (1/f type noise)

White noise

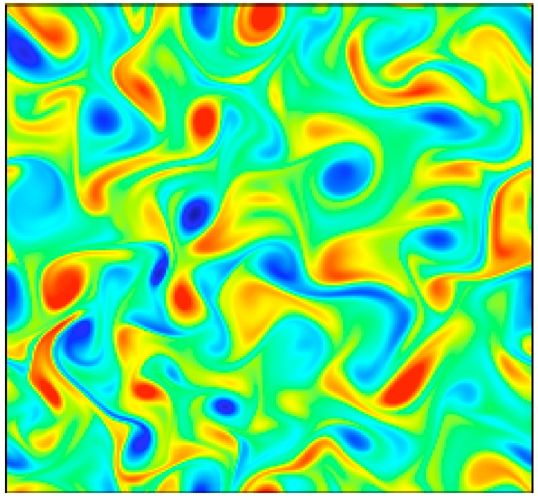




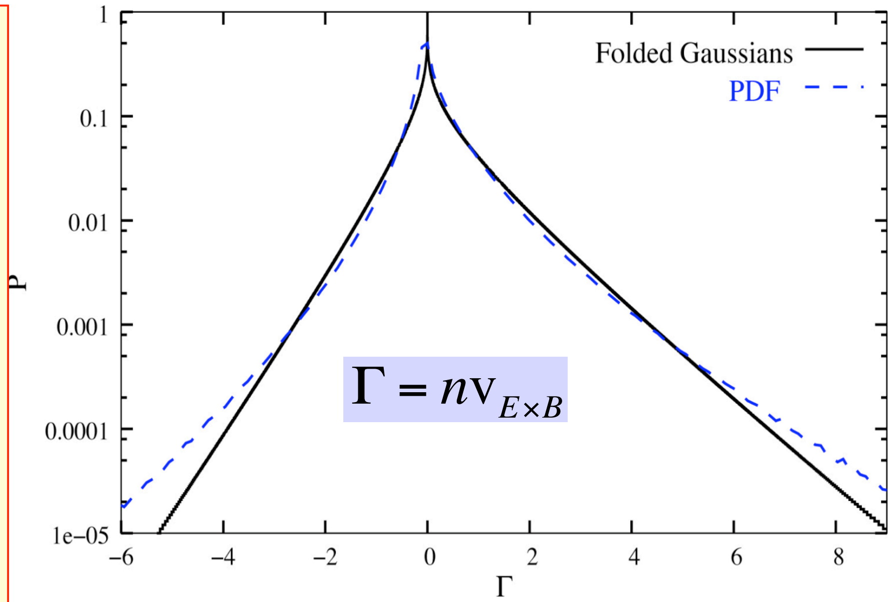
2D drift-wave turbulence



Bursting flux!



Flux PDF



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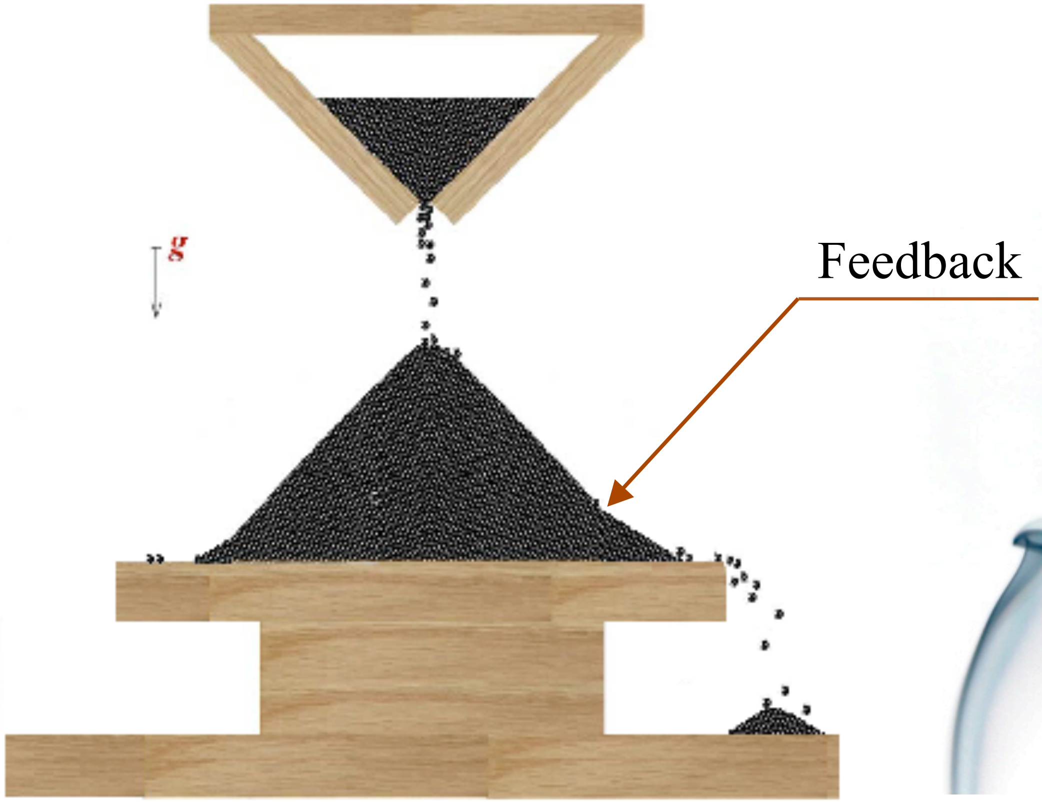
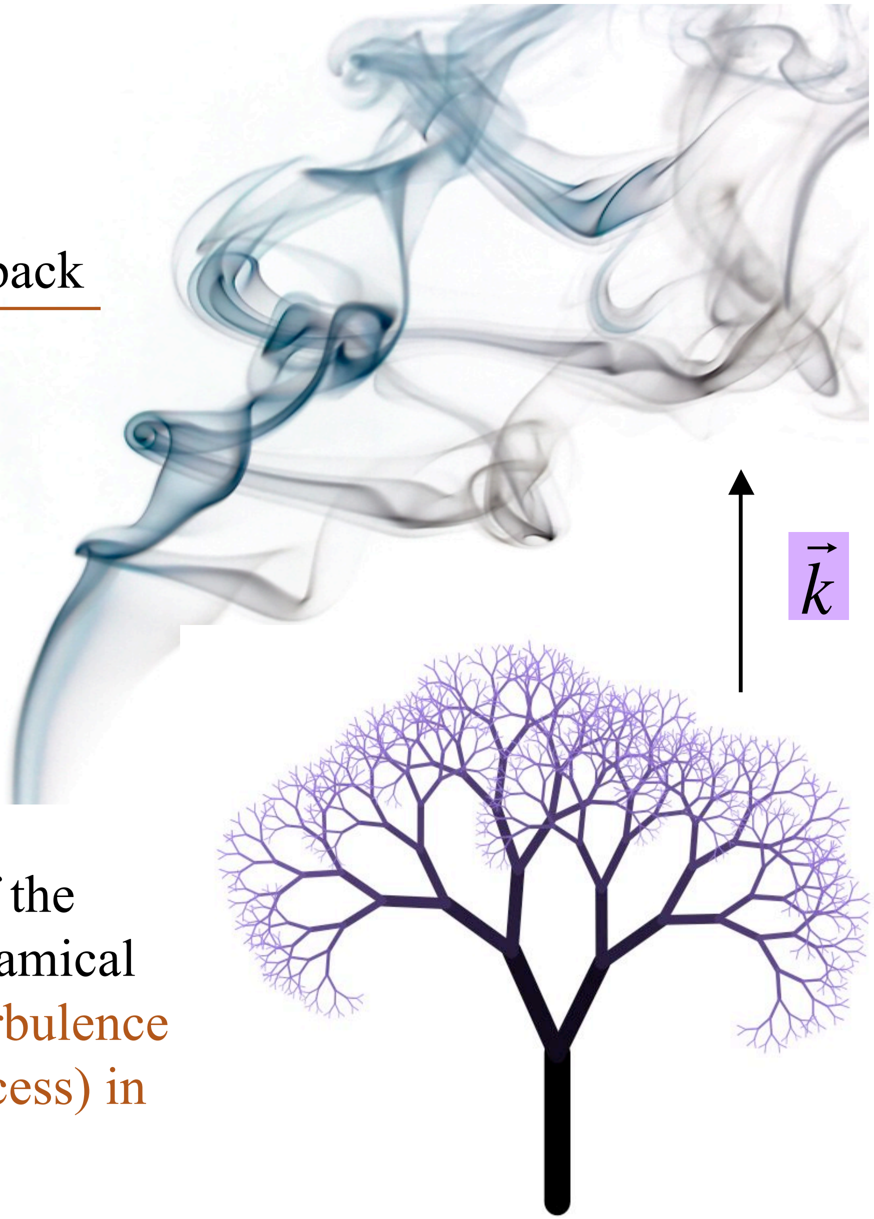
Turbulence vs SOC: Boundaries & Dynamical feedback

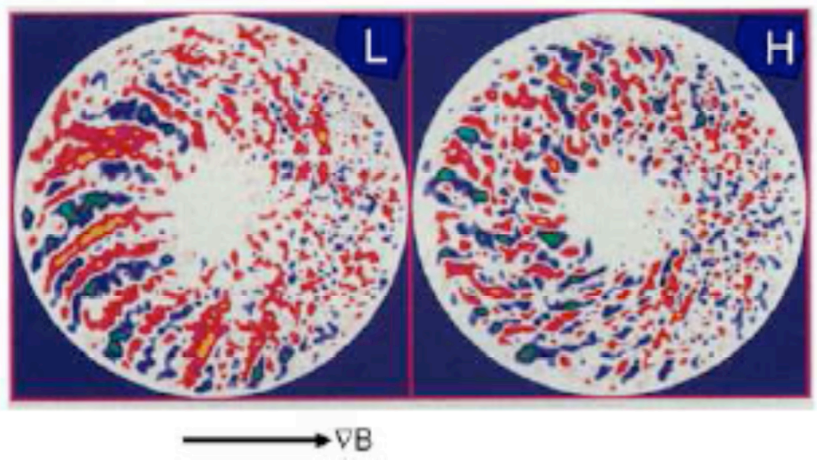




The SOC phenomena involve a feedback of the relaxation (particle loss) process on the dynamical state of the system at criticality, whereas turbulence pursues as a cascade (simple branching process) in reciprocal space.

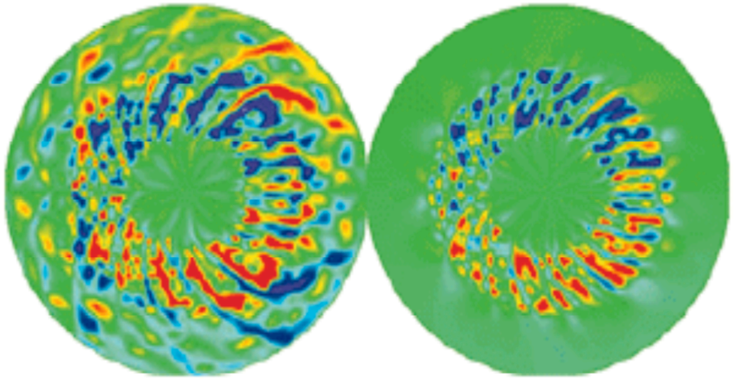
SOC hypothesis for edge turbulence in tokamaks

Toroidal ITG gyro-kinetic simulations



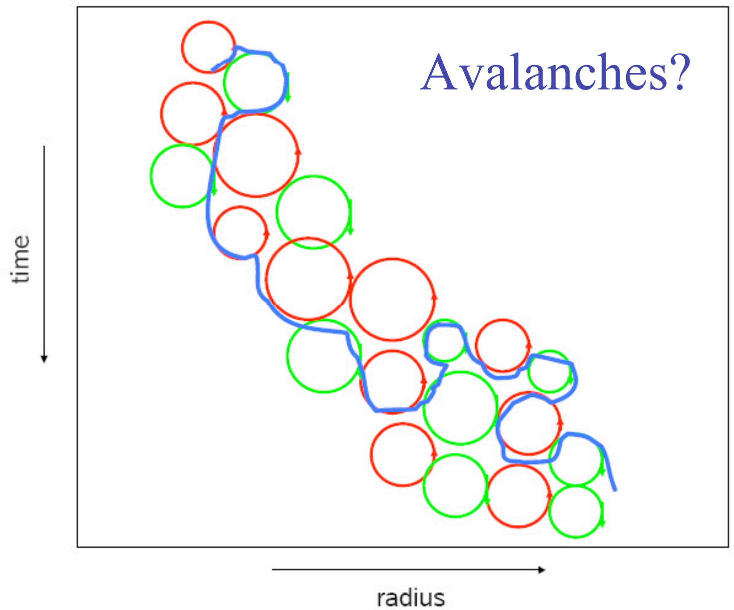
[See: Z. Lin et al, *Science* **281** (1998) 1835]

Fluid resistive interchange turbulence (cylinder)

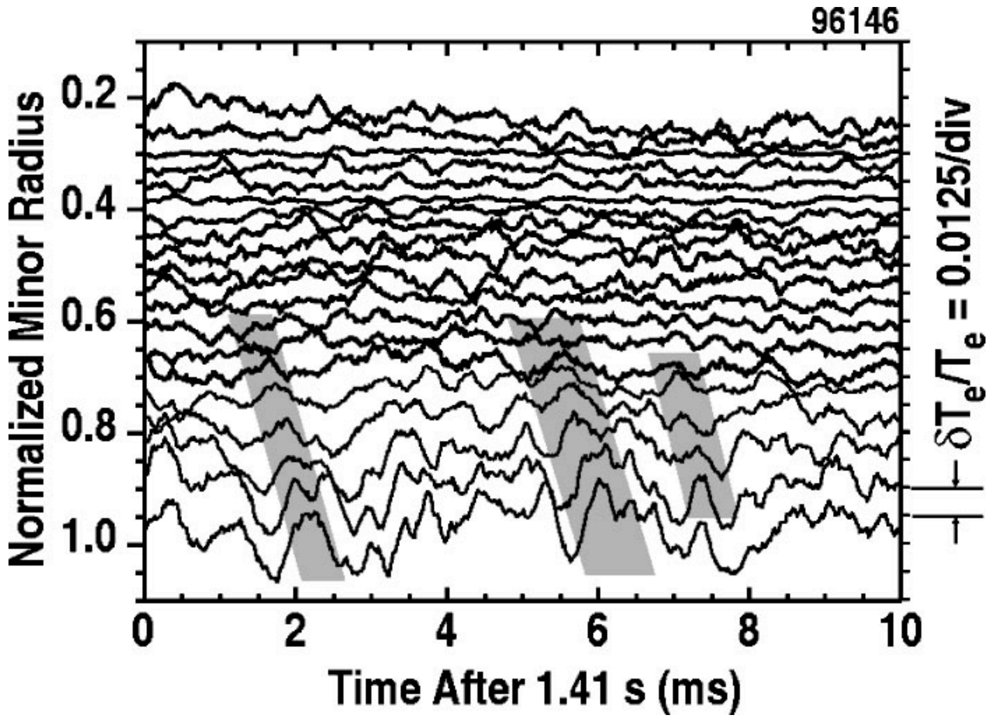


[See: B.A. Carreras et al, Phys. Fluids B 5 (1993) 1491]

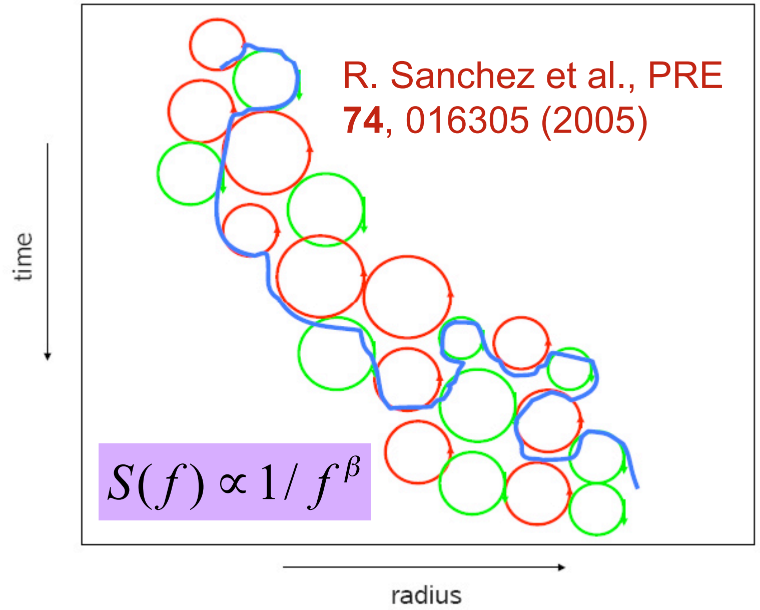
Classic measurements of avalanche-type transport include the analysis of edge plasma turbulence, yielding non-Gaussian probability distribution functions that are long-tailed. One unresolved problem here is the complex nature of coexistence of collective effects due to turbulence, turbulent transport, and SOC.



L-mode tokamak plasmas



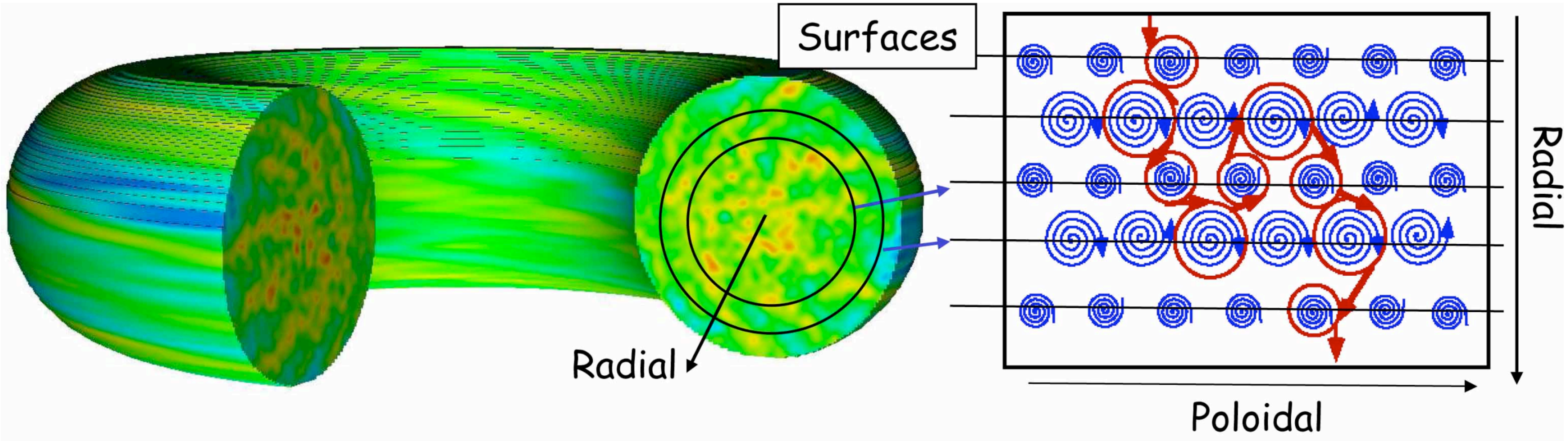
P. Politzer et al., Phys. Plasmas **9**, 1962 (2002)



SOC hypothesis
suggested as an explanation?

The normalized electron temperature fluctuations ($\delta T/T$) for L-mode plasma discharges vs time in the DIII-D tokamak. ***The highlighted bands*** indicate examples of ***avalanche-like events,*** outwardly propagating disturbances, moving at 300 m/s.

Fractional diffusion models for radial transport in tokamaks



1. D. del-Castillo-Negrete, Phys. Plasmas **13**, 082308 (2006).
2. D. del-Castillo-Negrete, B. A. Carreras, and V. E. Lynch, Phys. Plasmas **11**, 3854 (2004).
3. R. Sanchez, B. A. Carreras, D. E. Newman, V. E. Lynch, and B. Ph. van Milligen, Phys. Rev. E **74**, 016305 (2006).

Bi-fractional diffusion equation

$$\frac{\partial^\beta}{\partial t^\beta} n(x,t) = \frac{\partial^q}{\partial |x|^q}\left[D_{q,\beta}\, n(x,t)\right]$$

q-moment $\psi(\Delta x)$

$$D_{q,\beta} = \boxed{a^q} \big/ \boxed{\tau^\beta}$$

β-moment $\phi(\Delta t)$

$$\frac{\partial^\beta}{\partial t^\beta} n(x,t) = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial t} \int_0^t dt' \frac{n(x,t')}{(t-t')^\beta}$$

$$0 < \beta \le 1$$

G.F. Riemann
(1826-1866)

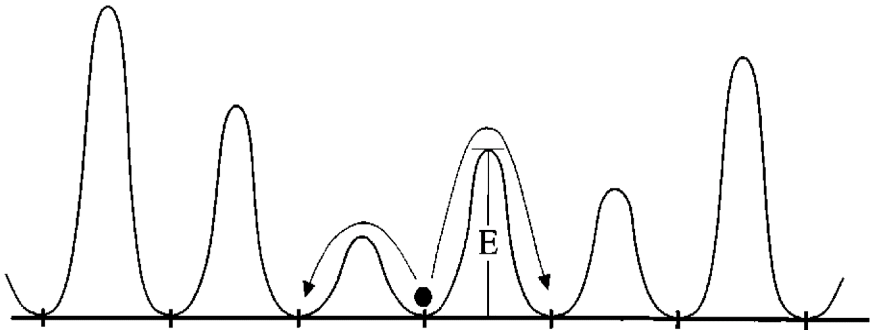
J. Liouville
(1809-1882)

Frigyes Riesz
(1880-1956)

$$\frac{\partial^q}{\partial |x|^q} n(x,t) = \frac{\cos^{-1}(\pi q/2)}{2\Gamma(p-q)} \times$$

$$\times \left[\frac{\partial^p}{\partial x^p} \int_{-\infty}^{x} dx' \frac{n(x',t)}{(x-x')^{q-p+1}} - \frac{\partial^p}{\partial (-x)^p} \int_{x}^{+\infty} dx' \frac{n(x',t)}{(x'-x)^{q-p+1}} \right]$$

Revisiting the realm of Brownian diffusion…



$\phi(\Delta t)$ – waiting time pdf

$\psi(\Delta x)$ – step-size pdf

$$\phi(\Delta t) = \frac{1}{\tau}\exp\left(-\frac{\Delta t}{\tau}\right)$$

$\xleftarrow{\text{Random process}}$

S. Poisson
(1781-1840)

K.F. Gauss
(1777-1855)

CLT: $\longrightarrow$

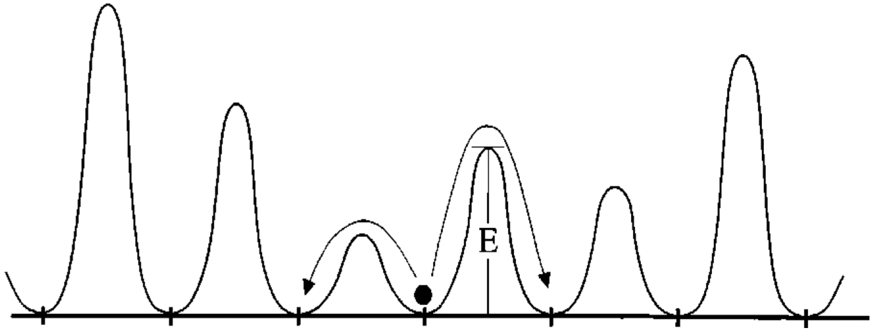
$$\psi(\Delta x) = \frac{1}{2a\sqrt{\pi}}\exp\left(-\frac{(\Delta x)^2}{4a^2}\right)$$

2^{nd} moment $\psi(\Delta x)$

$$\boxed{\frac{\partial n(x,t)}{\partial t} = \frac{\partial^2}{\partial x^2}\left[D n(x,t)\right]} \longleftarrow D = a^2 / \tau$$

1^{st} moment $\phi(\Delta t)$

Generalization I: Heavy-tailed waiting time pdf



$\phi(\Delta t)$ – waiting time pdf

$\psi(\Delta x)$ – step-size pdf

$$\boxed{\phi(\Delta t) \propto \frac{1}{(1+\Delta t/\tau)^{1+\beta}}}$$

$(0 < \beta < 1)$

CTRW:

1st moment diverges

E.W. Montroll (1916-1983)

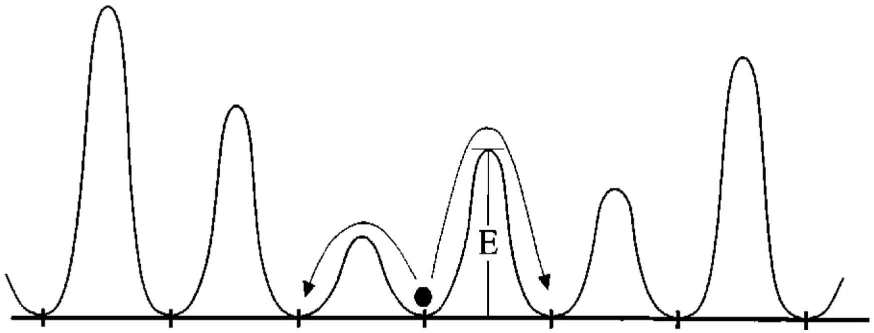
$$\psi(\Delta x) = \frac{1}{2a\sqrt{\pi}} \exp\left(-\frac{(\Delta x)^2}{4a^2}\right)$$

CLT:

$$\boxed{\frac{\partial n(x,t)}{\partial t} = \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \frac{\partial^2}{\partial x^2}\left[D n(x,t)\right]}$$

$$\boxed{\left\langle x^2(t) \right\rangle \propto t^{\beta}}$$

Generalization II: Heavy-tailed step-size pdf



$\phi(\Delta t)$ – waiting time pdf

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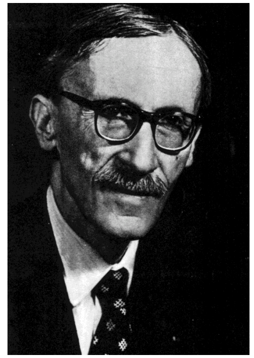
Poisson distribution



P. Levy (1886-1971)

$$\psi(\Delta x) = F^{-1}\exp(-a^q|k|^q)$$

$$\psi(\Delta x) \propto |\Delta x|^{-(1+q)}$$

$(0 < q \leq 2)$

G-CLT

2nd moment diverges

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial^q}{\partial|x|^q}\left[D_q n(x,t)\right]$$

$$D_q = \boxed{a^q} / \boxed{\tau}$$

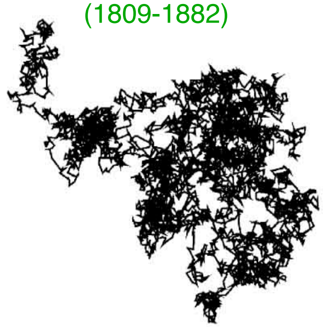
q-moment $\psi(\Delta x)$

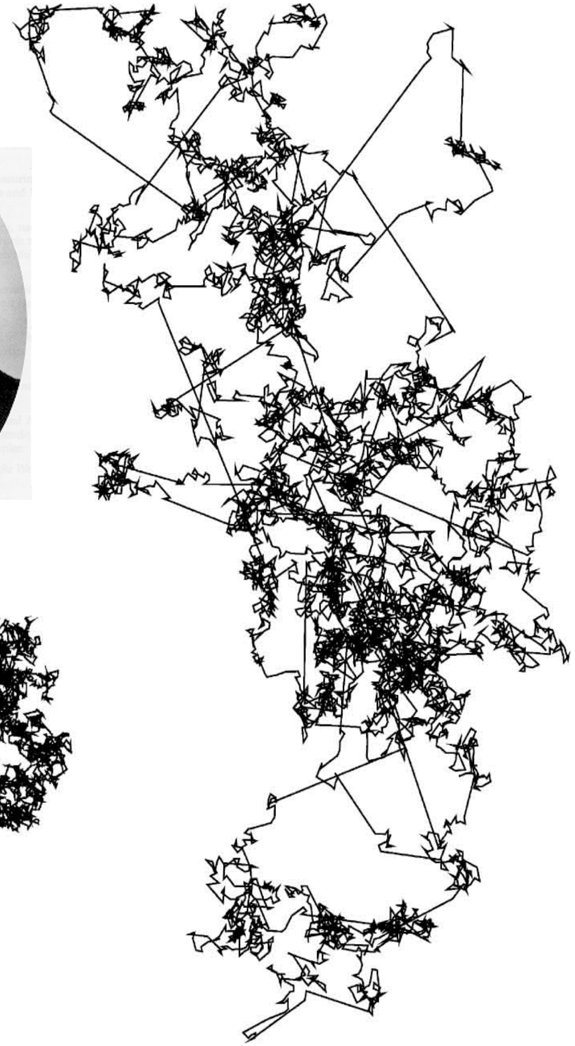
1st moment $\phi(\Delta t)$

Bi-fractional diffusion equation

$$\boxed{\frac{\partial^{\beta}}{\partial t^{\beta}} n(x,t) = \frac{\partial^{q}}{\partial |x|^{q}} \left[D_{q,\beta} n(x,t) \right]}$$

J. Liouville
(1809-1882)





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$$0 < \beta \leq 1$$

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$$\times \left[\frac{\partial^{p}}{\partial x^{p}} \int_{-\infty}^{x} dx' \frac{n(x',t)}{(x-x')^{q-p+1}} - \frac{\partial^{p}}{\partial (-x)^{p}} \int_{x}^{+\infty} dx' \frac{n(x',t)}{(x'-x)^{q-p+1}} \right]$$

Sub-, super-, and diffusive transport

q-moment $\psi(\Delta x)$

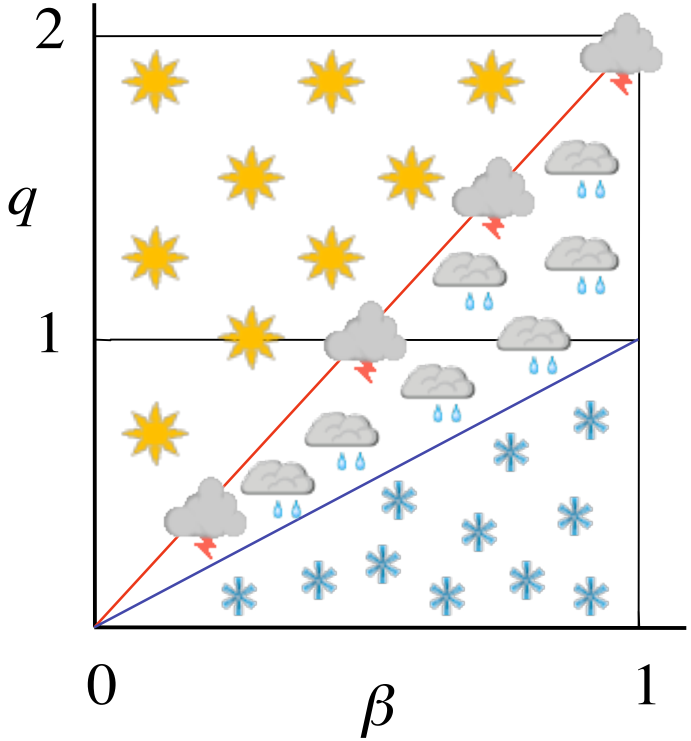
$$D_{q,\beta} = a^q / \tau^\beta$$

β-moment $\phi(\Delta t)$

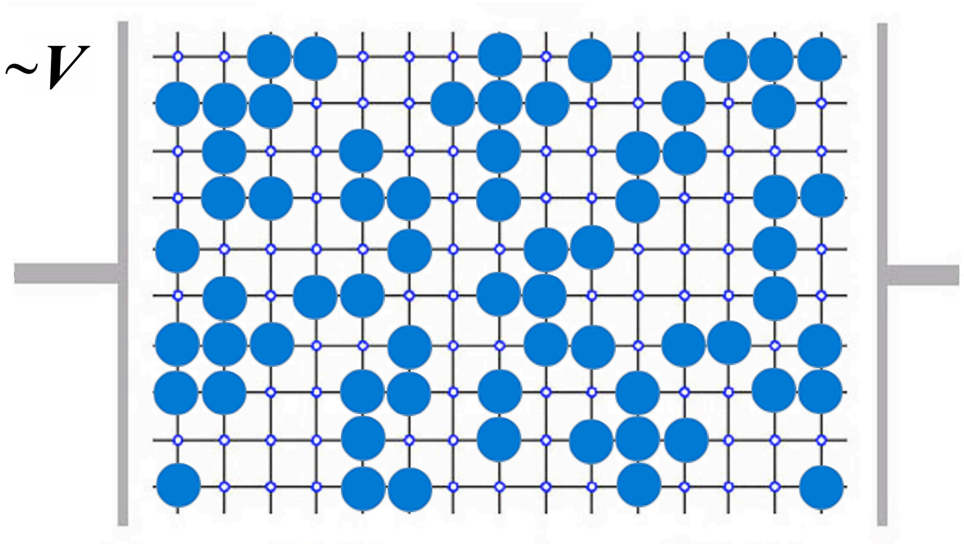
$$\frac{\partial^\beta}{\partial t^\beta} n(x,t) = \frac{\partial^q}{\partial |x|^q}\left[D_{q,\beta}\, n(x,t)\right]$$

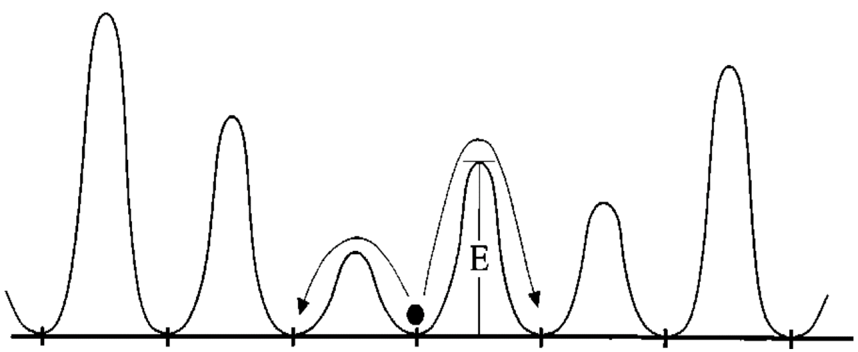
$$\left\langle \left| x(t) - x(t_0) \right|^q \right\rangle^{1/q} \propto t^{\beta/q} \equiv t^H$$

	Sub-diffusion	$0 < H < 0.5$
	Diffusion	$H = 0.5$
	Super-diffusion	$0.5 < H < 1$
	Super-ballistic	$H > 1$



Random walks on percolation cluster





$$\sigma_{ac}(\omega) \propto \omega^{\eta}$$

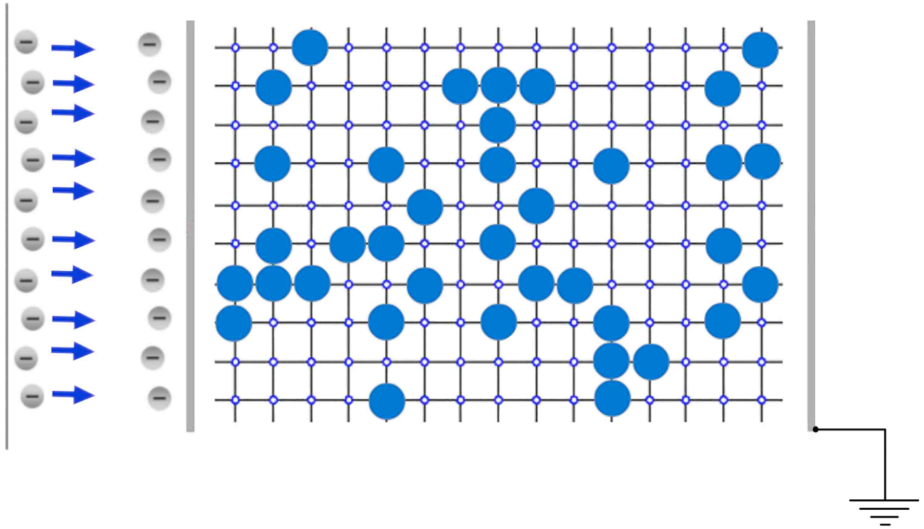
$$\eta = \mu/(2\nu + \mu - \beta)$$

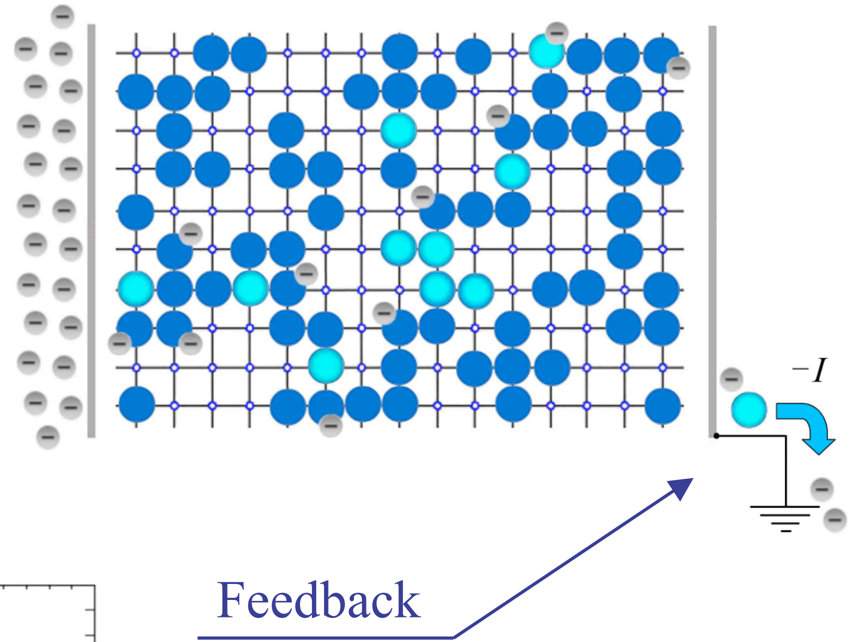
$$\sigma_{ac}(\omega) = \frac{ne^2}{k_B T} D(\omega)$$

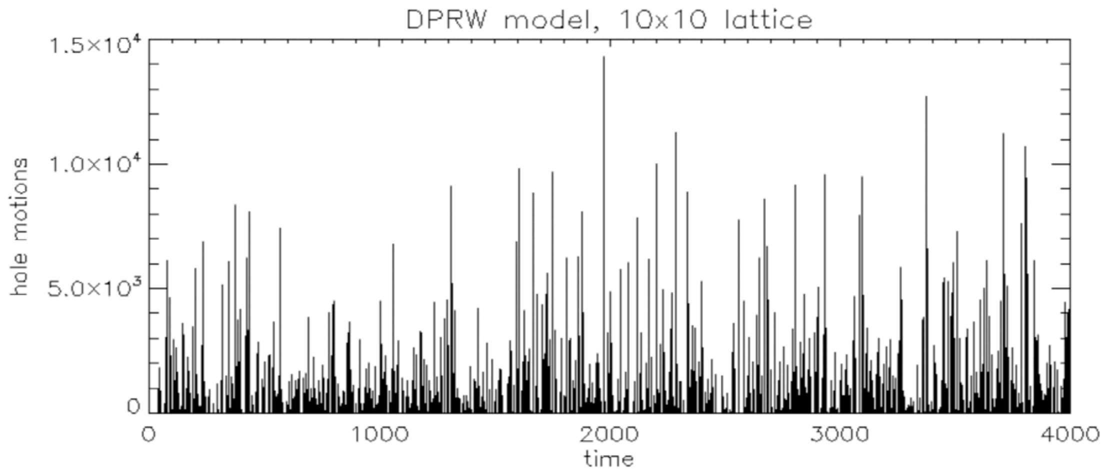
$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial^{\eta}}{\partial t^{\eta}} \frac{\partial^2}{\partial x^2}\left[Dn(x,t)\right]$$

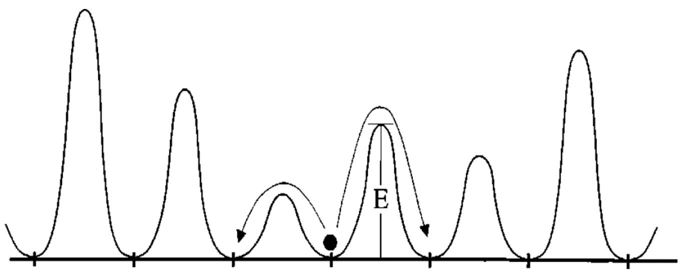
1. Y. Gefen, A. Aharony, and S. Alexander, Phys. Rev. Lett. **50**, 77 (1983).
2. A. V. Milovanov, Phys. Rev. E **79**, 046403 (2009).

Dynamic Polarization Random Walk model









The state of critical percolation occurs dynamically via random walks on a self-adjusting random cluster

Dielectric-relaxation formalism

Polarization response:

$$\delta P(t,\vec{r}) = \int_{-\infty}^{+\infty} \chi(t-t')\delta E(t',\vec{r})dt'$$

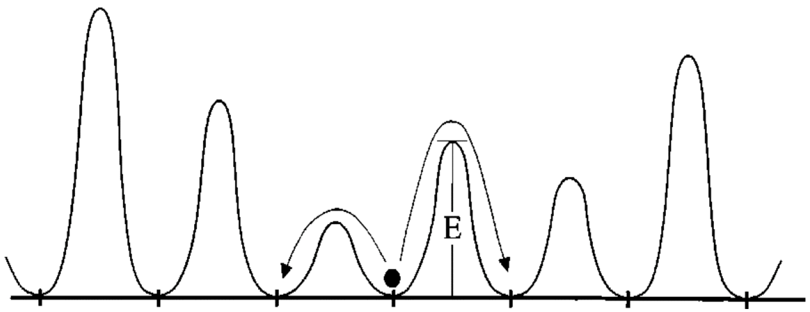
$$\delta P(\omega,\vec{k}) = \chi(\omega)\delta E(\omega,\vec{k})$$

Kramers-Kronig relation:

$$\chi(\omega) \propto \mathrm{P}\int \frac{\sigma_{ac}(\omega')}{\omega'(\omega'-\omega)} d\omega'$$

Linear-response theory for the frequency-dependent conductivity specialized to hopping conduction:

$$\sigma_{ac}(\omega) = \frac{ne^2}{k_B T} D(\omega) \quad \Leftarrow \quad \frac{1}{n_d} D(\omega) = \lim_{\varepsilon \to 0+}\left[(i\omega)^2 \int_0^{\infty} e^{-i\omega t} e^{-\varepsilon t} \left\langle r^2(t) \right\rangle dt\right]$$



Hopping on percolation geometry:

$$\sigma_{ac}(\omega) \propto \omega^{\eta} \quad \Rightarrow \quad \chi(\omega) \propto \omega^{\eta-1}$$

Self-consistent model of dielectric relaxation

Continuity implies that:

$$\frac{\partial}{\partial t}\delta\rho(t,\vec{r})+\nabla\cdot\frac{\partial}{\partial t}\delta P(t,\vec{r})=\frac{\partial}{\partial t}\delta\rho(t,\vec{r})+\nabla\cdot\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\chi(t-t')\delta E(t',\vec{r})dt'=0$$

Applying Maxwell's equation we have:

$$\nabla\cdot\delta E(t,\vec{r})=4\pi\delta\rho(t,\vec{r})\;\Rightarrow\;\frac{\partial}{\partial t}\left[\delta\rho(t,\vec{r})+4\pi\int_{-\infty}^{+\infty}\chi(t-t')\delta\rho(t',\vec{r})dt'\right]=0$$

In the frequency domain:

$$\delta\rho(\omega,\vec{k})+4\pi\chi(\omega)\delta\rho(\omega,\vec{k})=\phi(\vec{k})/\omega$$

$$\delta\rho(\omega,\vec{k})=\frac{1}{\omega+T^{-\gamma}\omega^{1-\gamma}}\phi(\vec{k})$$

Distribution of relaxation times

From Mittag-Leffler relaxation function to the Kohlrausch-Williams-Watts stretched-exponential decay function:

$$\boxed{E_\gamma\left[-(t/T)^\gamma\right] \cong \exp\left[-(t/T)^\gamma / \Gamma(1+\gamma)\right]}$$

This stretched-exponential relaxation function can conveniently be considered as a weighted average of single-exponential relaxation functions - each associated with a single relaxation event in the system:

$$\boxed{\exp\left[-(t/T)^\gamma / \Gamma(1+\gamma)\right] \cong \int_0^\infty d\theta\, w_\gamma(\theta) \exp\left[-(t/\theta)\right]}$$

where the weighting function is expressible in terms of Levy (stable) distribution

$$\boxed{w_\gamma(\theta) = (T/\theta^2) L_{\gamma,-1}(T/\theta) \propto \theta^{-2}\theta^{1+\gamma} \propto \theta^{-\eta}} \longleftarrow \text{Heavy-tailed!}$$

Fractional diffusion equation

The Mittag-Leffler function describes the relaxation dynamics of particles governed by the fractional diffusion equation

$$\boxed{\frac{\partial}{\partial t}\delta\rho(t,\vec{r}) = {}_0D_t^{1-\gamma}\kappa_\gamma\nabla^2\delta\rho(t,\vec{r})}$$

where

$$\boxed{{}_0D_t^{1-\gamma}\delta\rho(t,\vec{r}) = \frac{1}{\Gamma(\gamma)}\frac{\partial}{\partial t}\int_0^t dt'\,\frac{\delta\rho(t',\vec{r})}{(t-t')^{1-\gamma}}}$$

is the Riemann-Liouville fractional derivative.

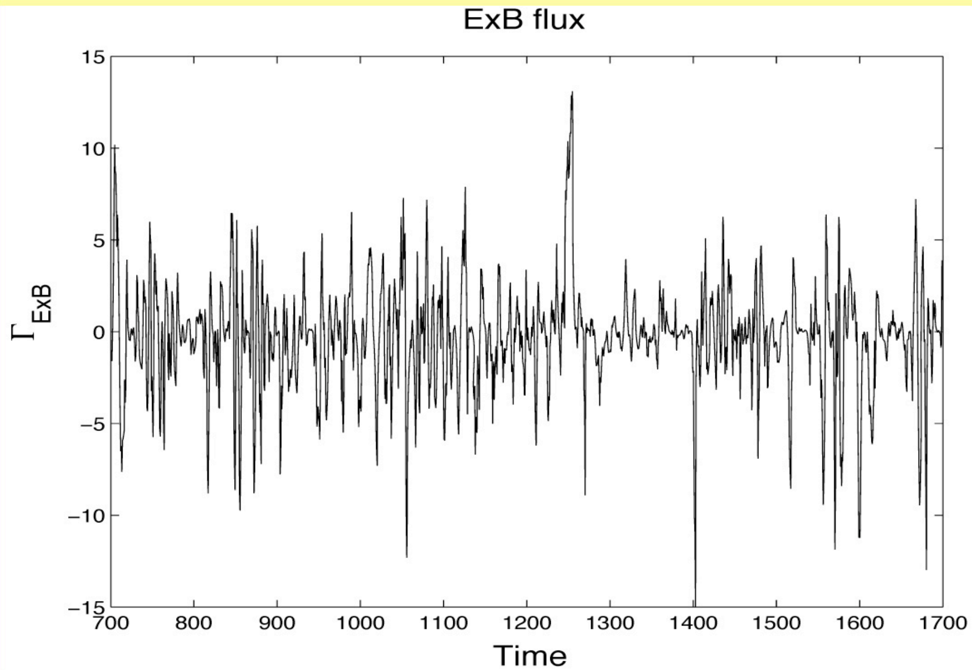
This equation is a hallmark of systems with a broad distribution of relaxation times (e.g., the SOC systems)

Critical exponents of the DPRW model

Exponent	Expression	$d=1$	$d=2$	$d=3$	$d=\infty$
η	$\mu/(2\nu+\mu-\beta)$	0	0.34	0.6	1
z	$1+\eta$	1	1.34	1.6	2
γ	$1-\eta$	1	0.66	0.4	0
α	$2-2\eta$	2	1.3	0.8	0
H	$1/z$	1	0.75	0.6	1/2
τ	$3-\alpha z/d_f$	1	2.1	2.5	3

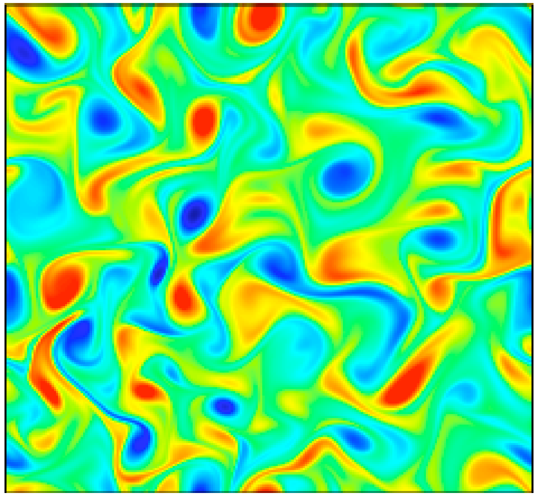
SANDPILE: PROTOTYPE MODEL OF SOC (BTW 1987)

- By examining Table 1 one can see that the DPRW model reproduces with remarkable accuracy the critical exponents from major sand-pile models: Zhang (1989); Tang & Bak (1988).

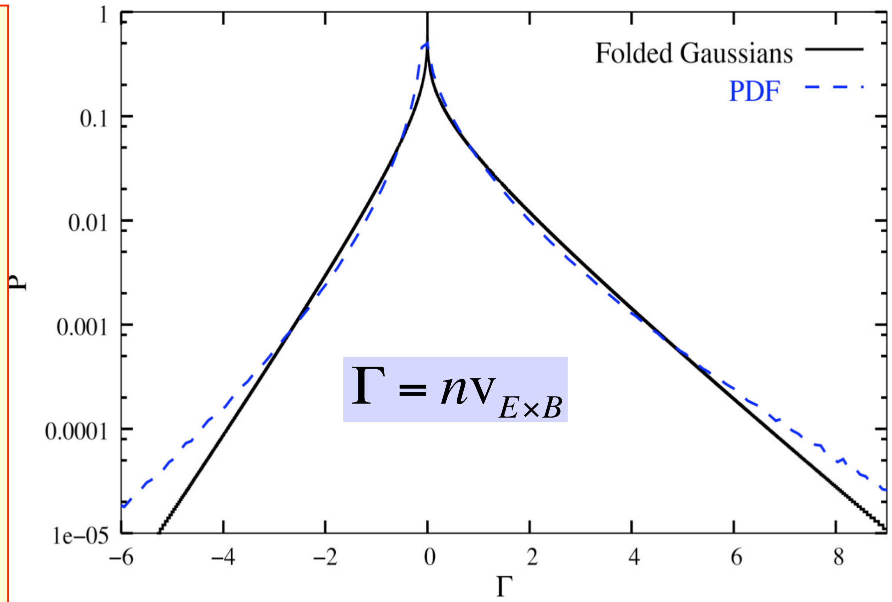


2D drift-wave turbulence

Bursting flux!



Flux PDF



V. Naulin et al., Phys. Lett. A 321, 355 (2004)

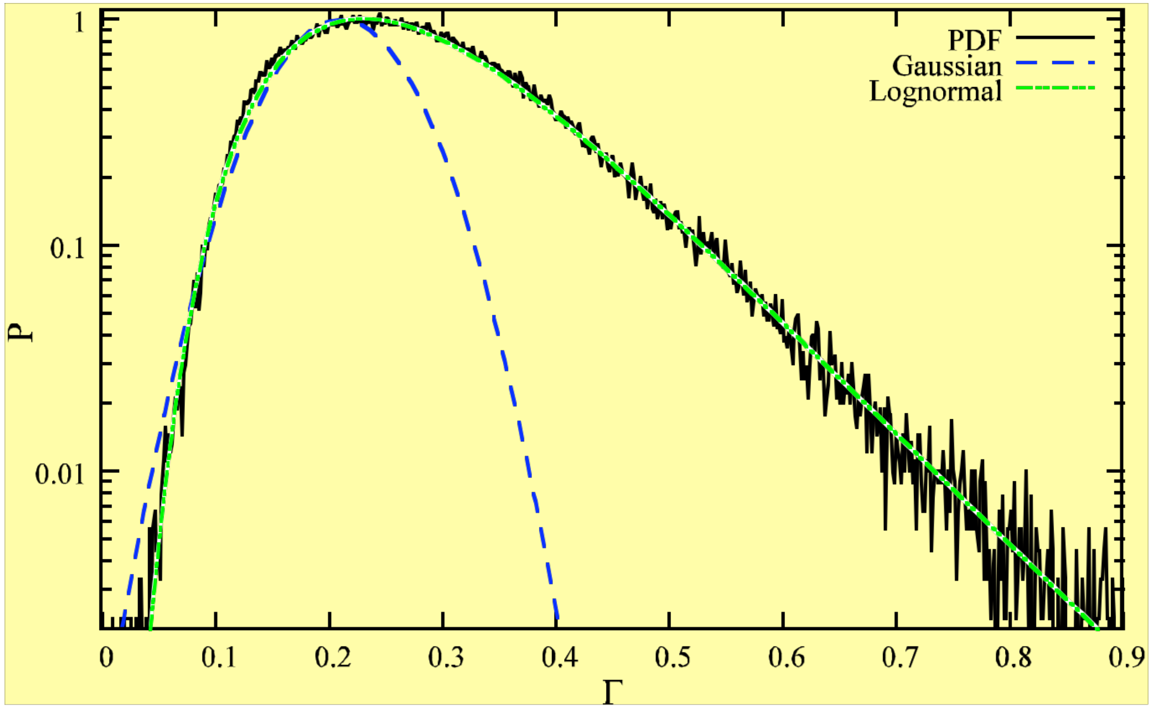
Properties:

- **Plasma turbulence** and transport strongly intermittent
- **No** easy parametrization
- **Not** simple diffusion in Fickian sense
- **PDF's** of radial displacements of tracer particles are non-Gaussian with exponential tails and the moments indicating **superdiffusive transport**

Turbulent particle density flux

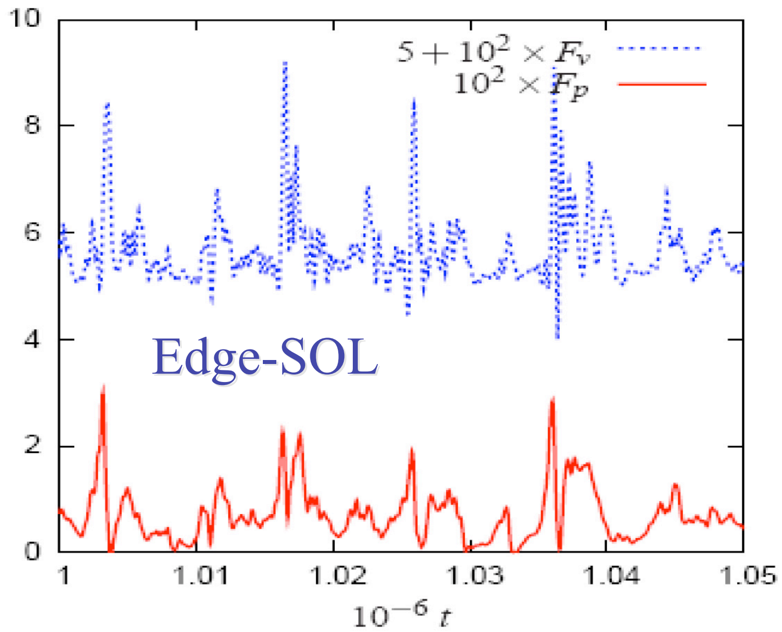
Particle density flux:
flux surface averaged

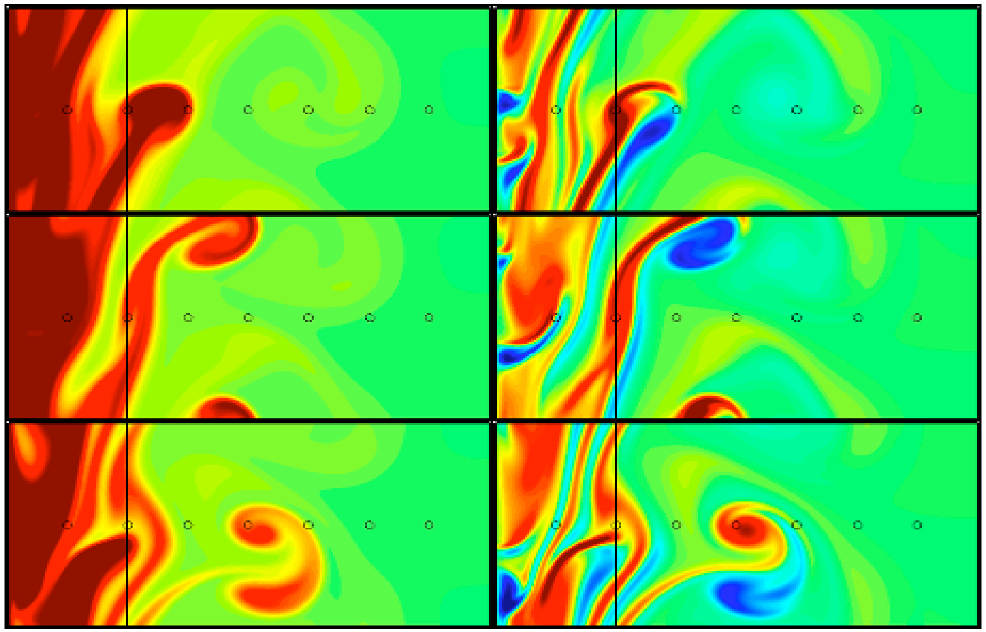
The log-normal distribution as well as the EVD yields a good approximation of the flux-surface averaged plasma transport

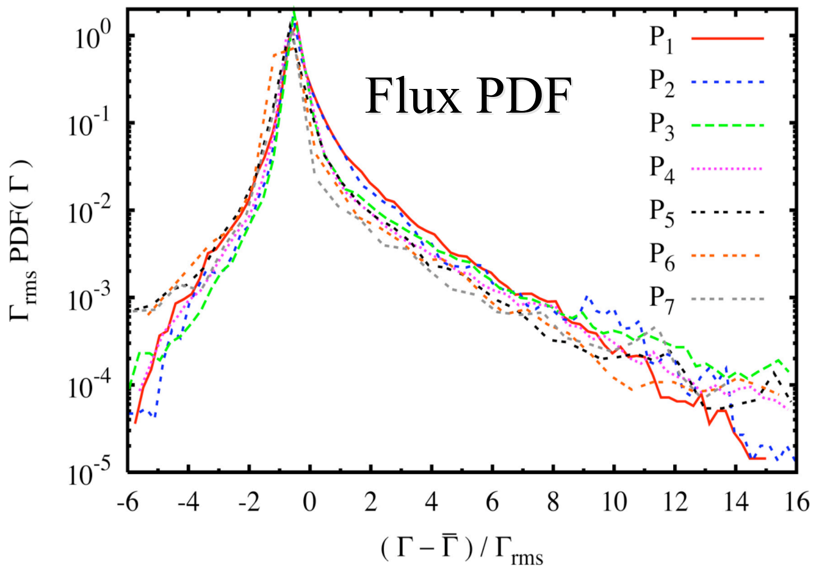


The probability distribution function for the plasma flux across the magnetic field is strongly ***non-Gaussian*** ***with exponential tails!***

Global model for interchange driven turbulence (Edge-SOL)

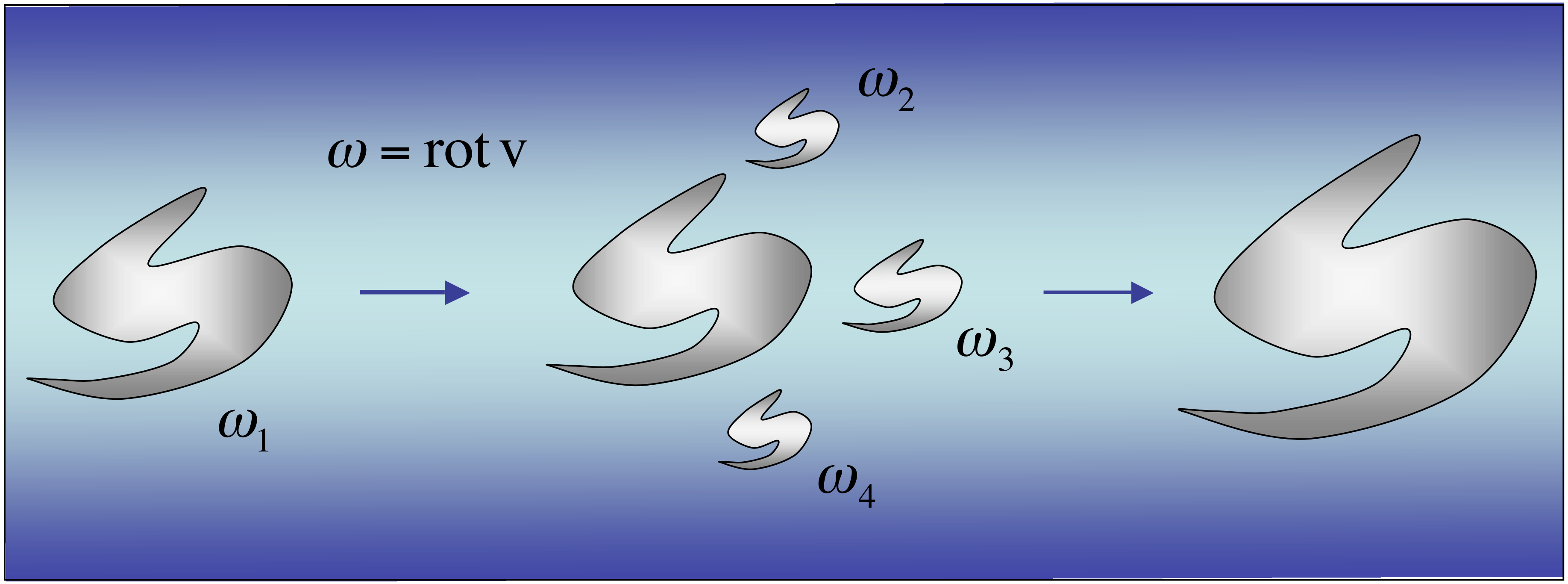






Particle density (left) and vorticity (right) during a burst. In the edge/scrape-off layer (SOL) region transport is strongly intermittent and characterized by large-amplitude, radially propagating blob-like structures of particles and heat, generated close to the last closed flux surface. Garcia et al., PPCF **48**, L1 (2006)

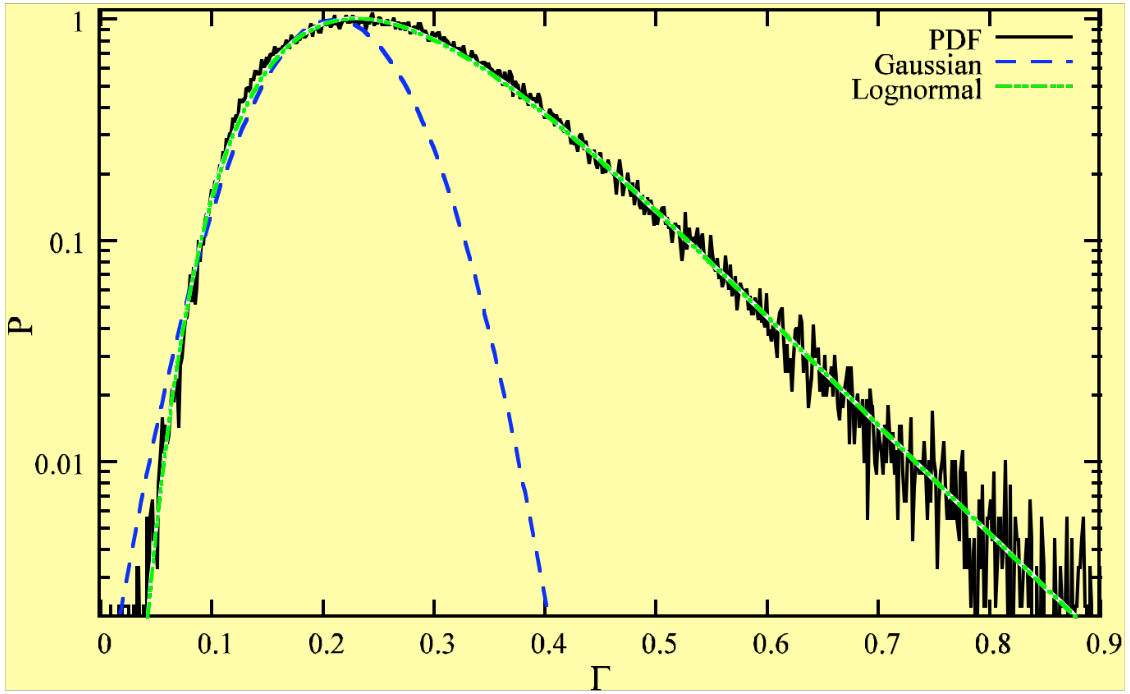
Stimulated vortex formation + merging:



Turbulent amplification:

$$\frac{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \ldots}{N} \rightarrow \text{Gaussian} \quad \Rightarrow \quad \sqrt{\langle \mathrm{v}^2 \rangle} \propto \sqrt{N}$$

Amplification of the range of log-normal behavior:



Levy!

Montroll & Shlesinger, 1982

Space-fractional diffusion equation:

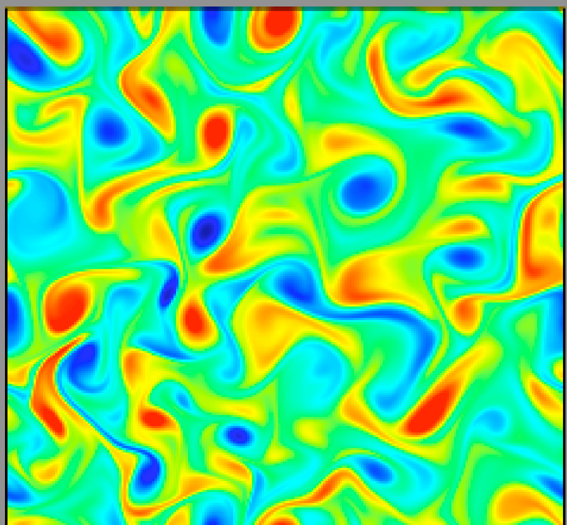
$$\boxed{\frac{\partial n(x,t)}{\partial t} = \frac{\partial^q}{\partial |x|^q}\left[D_q n(x,t)\right]}$$

Time-scale separation:

In drift-wave turbulence, the so-called Rhines length designates the spatial scale separating vortex motion from drift wave-like motion

$$\lambda_{\mathrm{Rh}} \propto \sqrt{\mathrm{u}_{E\times B}}$$

$$\tau_{\mathrm{turn}} \propto \lambda_{\mathrm{Rh}} / \mathrm{u}_{E\times B} \propto 1/\sqrt{\mathrm{u}_{E\times B}}$$



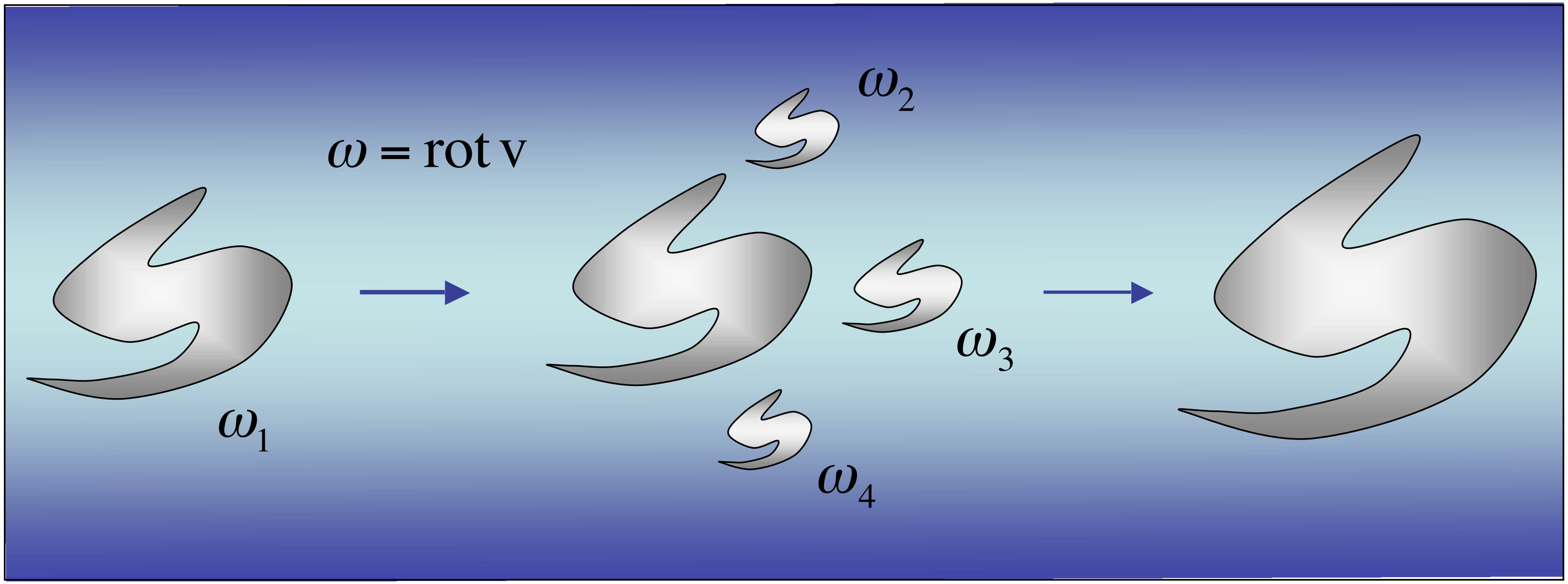
The nonadiabaticity parameter, δ , characterizes deviation between the potential and the density fluctuations in the Hasegawa-Wakatani (HW) model

$$\gamma_{\mathrm{L}} \approx \delta/8$$

Avalanching dynamics:

$$\tau_{\mathrm{turn}} << \gamma_{\mathrm{L}}^{-1}$$

Vortex avalanches: Propagating drift-wave turbulence



Self-Organized Critical Directed Percolation:

$$t \propto |x|^q \quad \Rightarrow \quad q \approx 1.7 \;\; (= 1.73 \pm 0.05)$$

Maslov & Zhang, 1996

Stimulated galaxy formation:

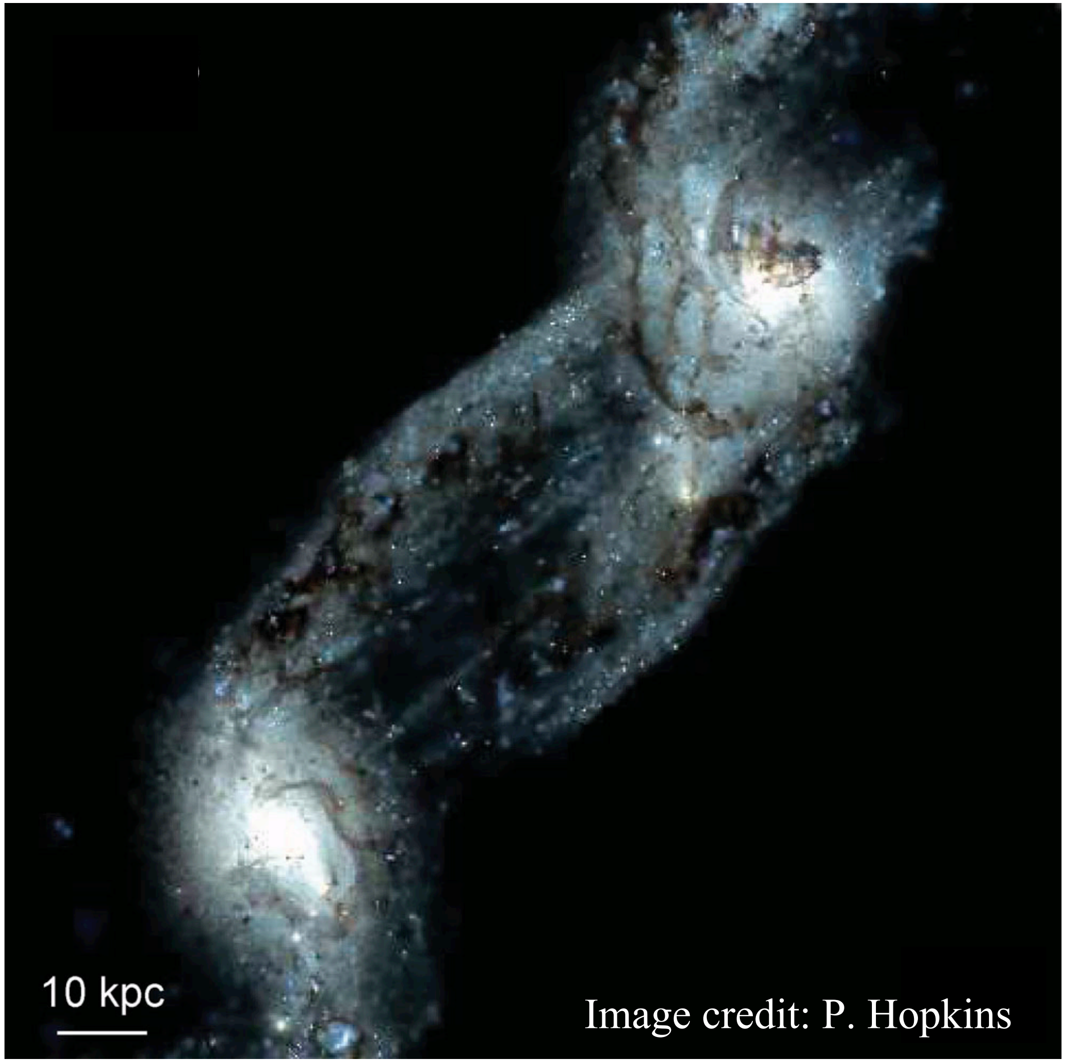
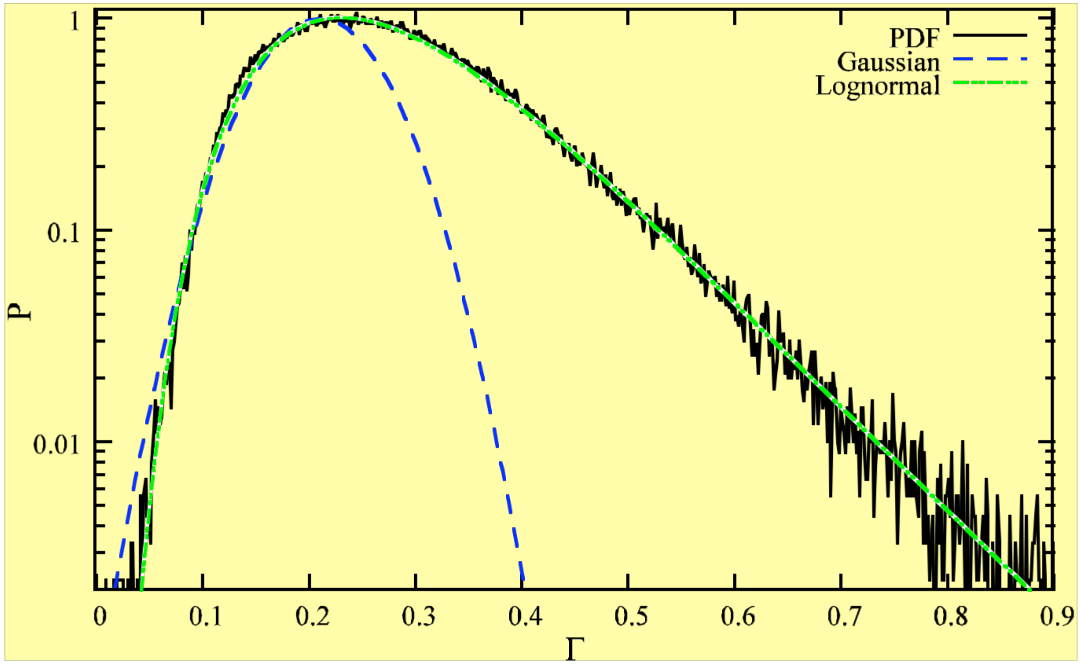


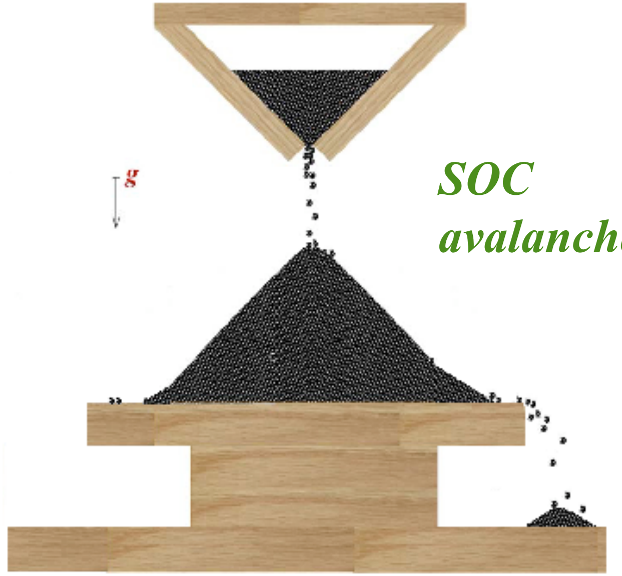
Image credit: P. Hopkins

If the original process of galaxy formation occurs through the stimulated birth of one galaxy due to a nearby recently formed galaxy, and if this process occurs near its percolation threshold, then a hierarchical structure with power-law correlations arises at the time of galaxy formation.

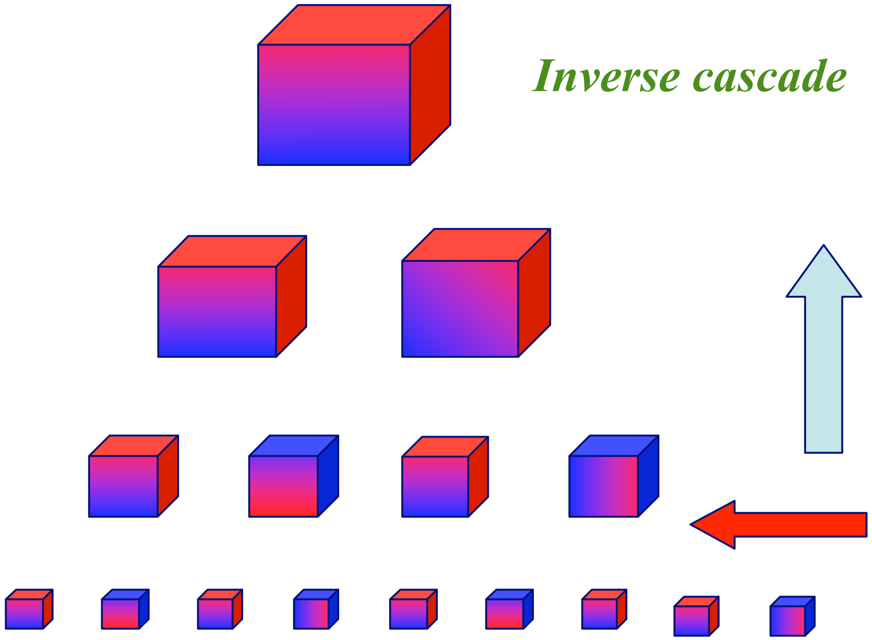
Schulman & Seiden, 1986

→ Amplification mechanism provided by galaxy merging

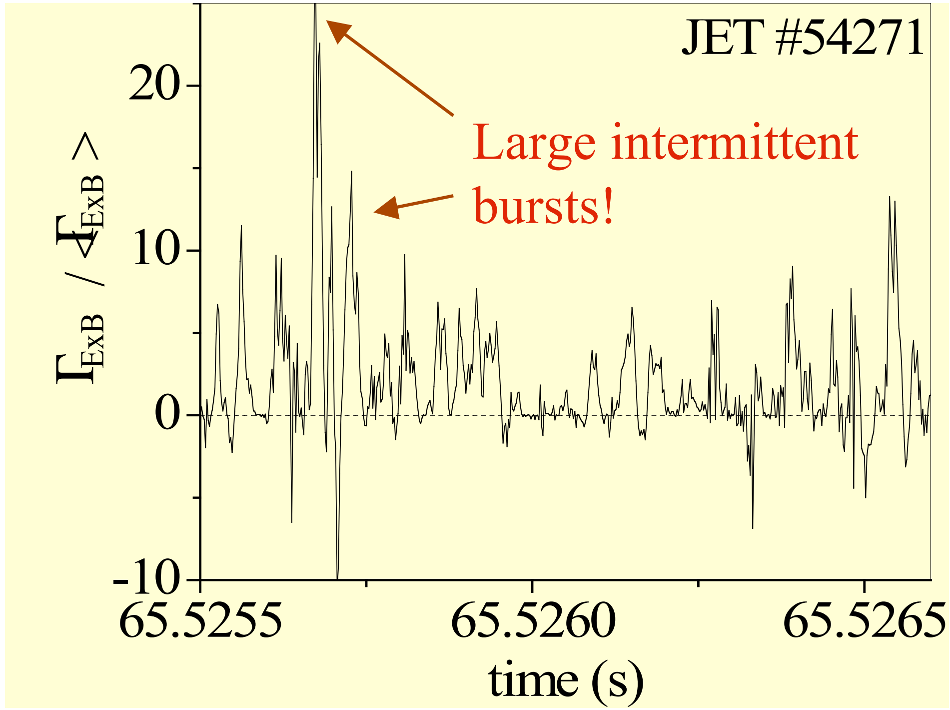




SOC avalanches



Inverse cascade



Conclusions

- An amplification mechanism is needed to introduce nonlocality into the dynamics
- Amplification of a log-normal distribution generates a new distribution of the Levy type
- Synergetic coupling between SOC and turbulence: Amplification of SOC-associated avalanches via the inverse turbulence cascade produces large-amplitude bursts of transport
- Consistent derivation of a Levy fractional diffusion equation: A random walker driven by a Levy white noise
- Nonlocal diffusion explains the behavior of cold pulses

epl draft

A mixed SOC-turbulence model for nonlocal transport and space-fractional Fokker-Planck equation

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PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion
PACS 05.40.Fb – Random walks and Lévy flights
PACS 52.25.Fi – Transport properties

Abstract. - The phenomena of nonlocal transport in magnetically conned plasma are theoretically analyzed. A hybrid model is proposed, which brings together the notion of inverse energy cascade, typical of drift-wave and two-dimensional fluid turbulence, and the ideas of avalanching behavior, associable with self-organized critical (SOC) behavior. Using statistical arguments, it is shown that an amplification mechanism is needed to introduce nonlocality into dynamics. We obtain a consistent derivation of nonlocal Fokker-Planck equation with space-fractional derivatives from a stochastic Markovian process with the transition probabilities defined in reciprocal space.