Similarity and scaling- what the principle of similitude can tell us about turbulence, SOC, and ecosystems

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- *Order and control parameters*
- *Formal dimensional analysis (Buckingham's Pi theorem) an introduction*
- *Some examples, flocking 'birds', turbulence- finding order and control parameters*
- *Implications for SOC*
- *Macroecological patterns- from Pi theorem*
- *A 'Reynolds number' for life?*

➢*more details in SCC et al, POP 2009, SCC et al PPCF 2009, Wicks, SCC et al PRE 2007 (method), SCC et al arXiv:1108.4802 (ecology)*

Universality-
the details are irrelevant, only need *relevant* parameters

Pendulum

$$F = mg, F_t = mg\sin\theta, a_t = l\frac{d^2\theta}{dt^2}$$

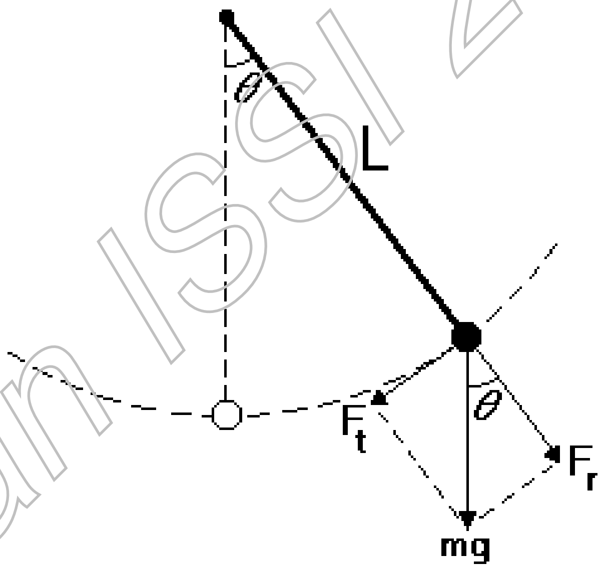
$$F_t = ma_t; \frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta = -\omega^2\frac{\partial V}{\partial\theta}$$

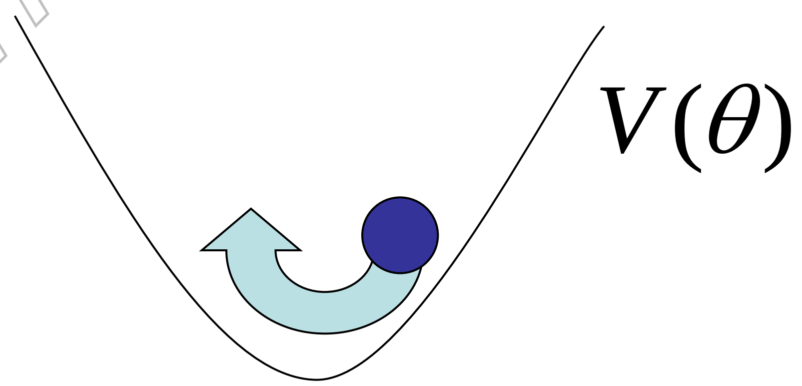
$$V(\theta) = 1 - \cos(\theta) \sim \frac{\theta^2}{2} + \ldots$$

same behaviour at

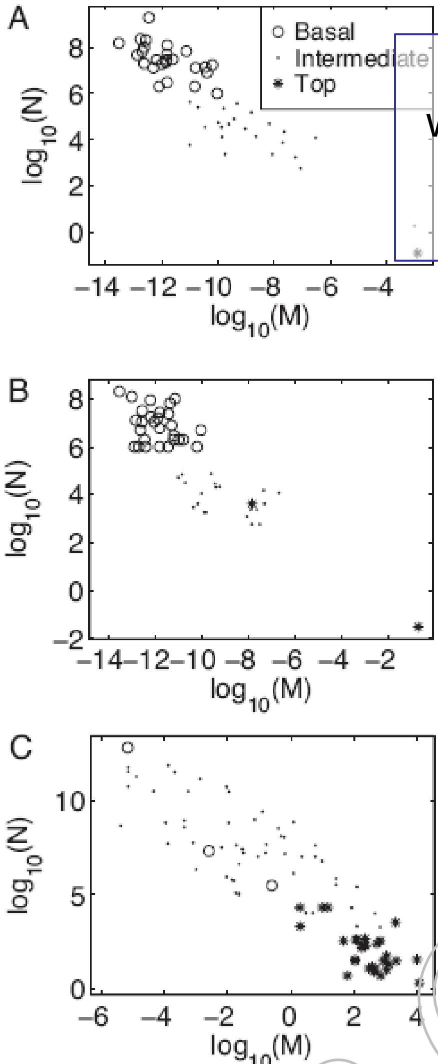
any local minimum in $V(\theta)$

(insensetive to details)



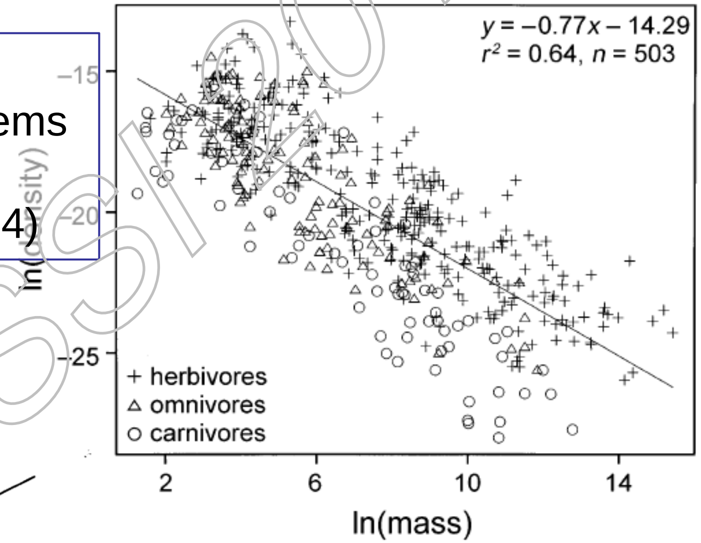


Macroecological patters…plotting the wrong variables?

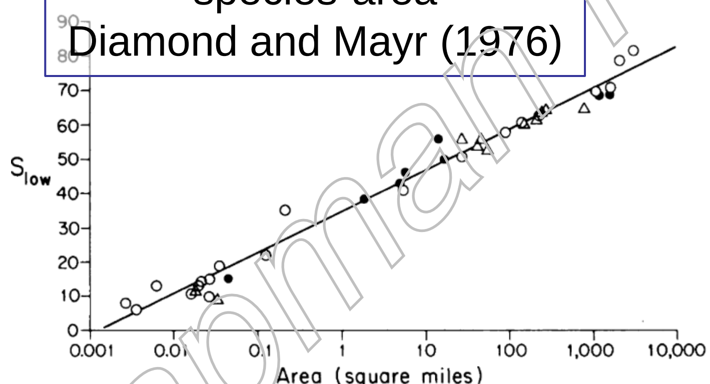


density-body size
within an ecosystem, across ecosystems

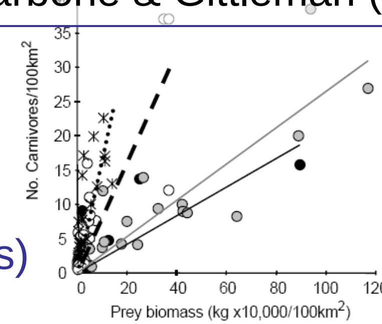
Cohen et al (2009) Brown et al (2004)



species-area
Diamond and Mayr (1976)

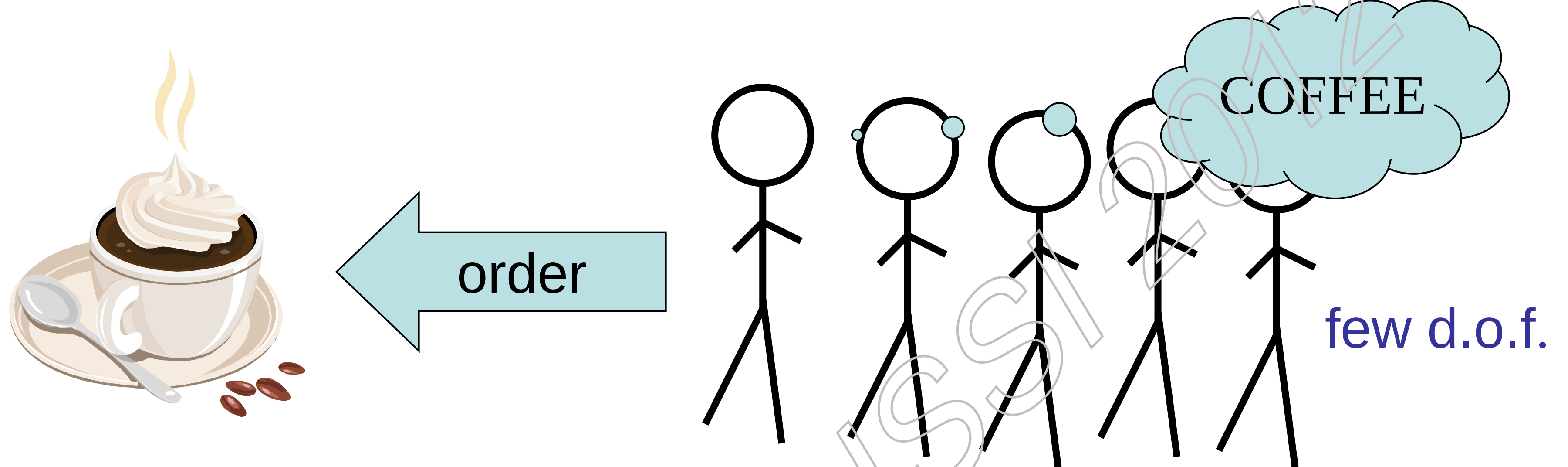


Predator density-prey biomass
Carbone & Gittleman (2002)

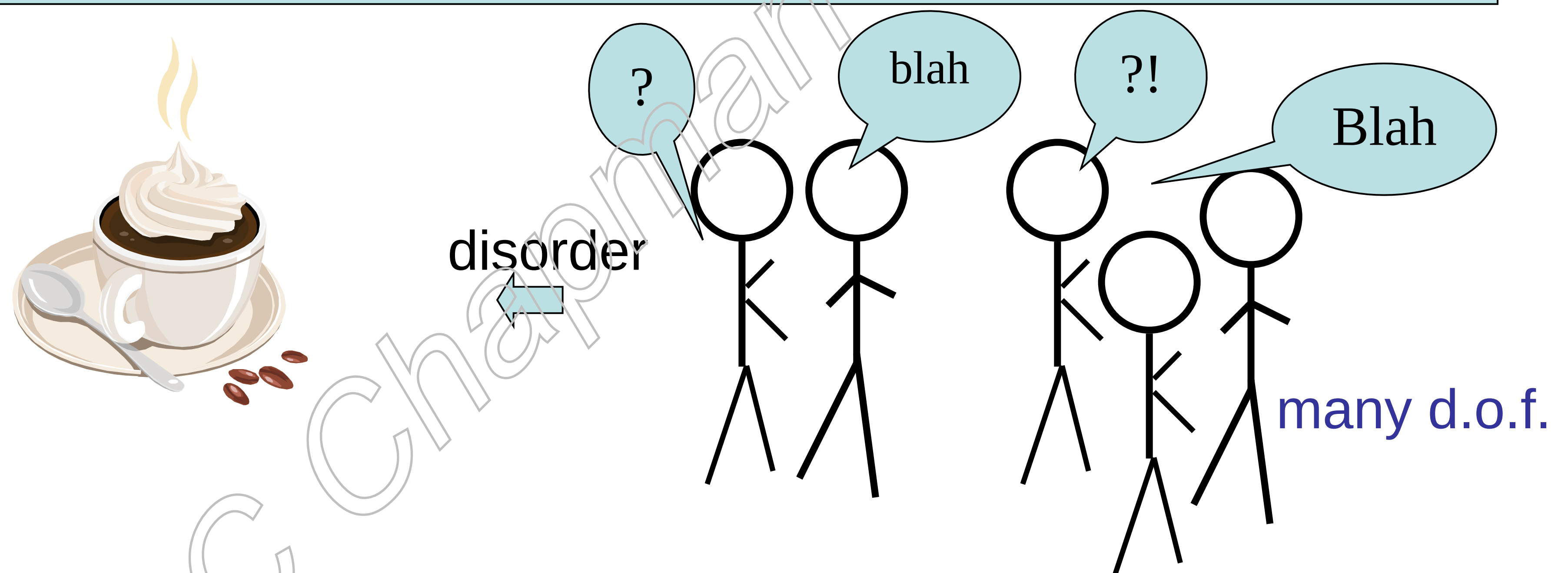


Macroecological patterns for:
diversity (richness, no. of species)
density (abundance, no. of individuals/species)
metabolic rate
with: size(mass), area, trophic level, productivity, latitude…
(and exceptions, complications, caveats..)

Order and control parameters



Control parameter=(desire for knowledge)/(desire for coffee)



Competition between order and disorder

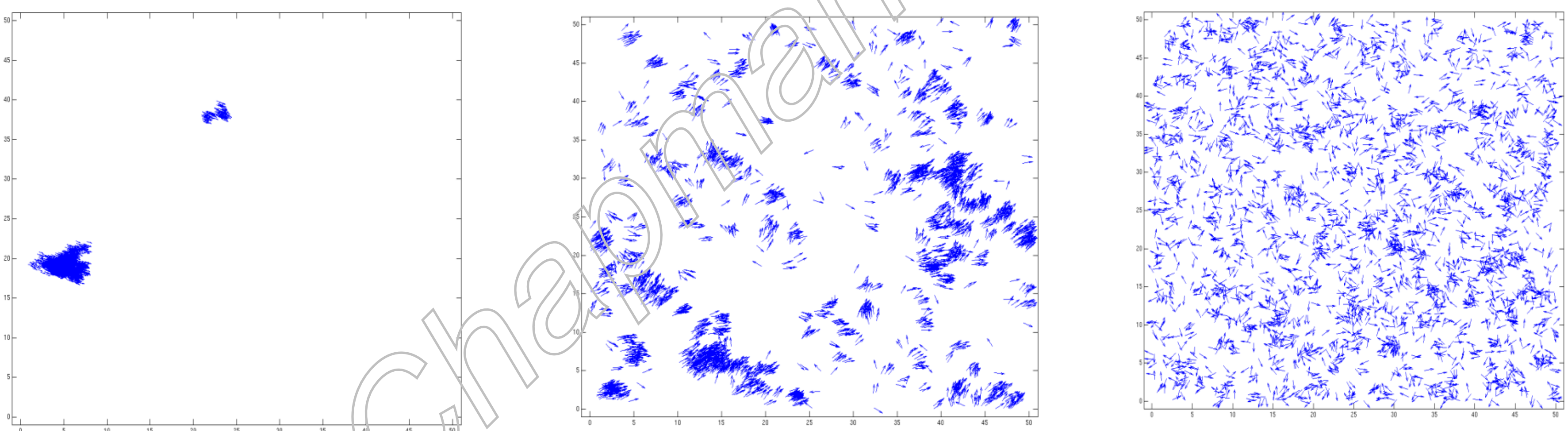
Rules: random fluctuation plus 'following the neighbours'

$$\mathbf{x}_{n+1}^{k} = \mathbf{x}_{n}^{k} + \mathbf{v}_{n}^{k} dt, \quad \left|\mathbf{v}_{n}^{k}\right| \text{ constant}$$

$$\theta_{n+1}^{k} = \left\langle \theta_{n}^{k} \right\rangle_{k \cap R} + \delta\theta, \;\; \delta\theta = \left[-\eta, \eta\right] \text{ iid random variable}$$

order parameter: total speed $\frac{1}{N}\left|\sum_{i=1}^{N} \mathbf{v}_i\right|$

Control parameter depends on noise

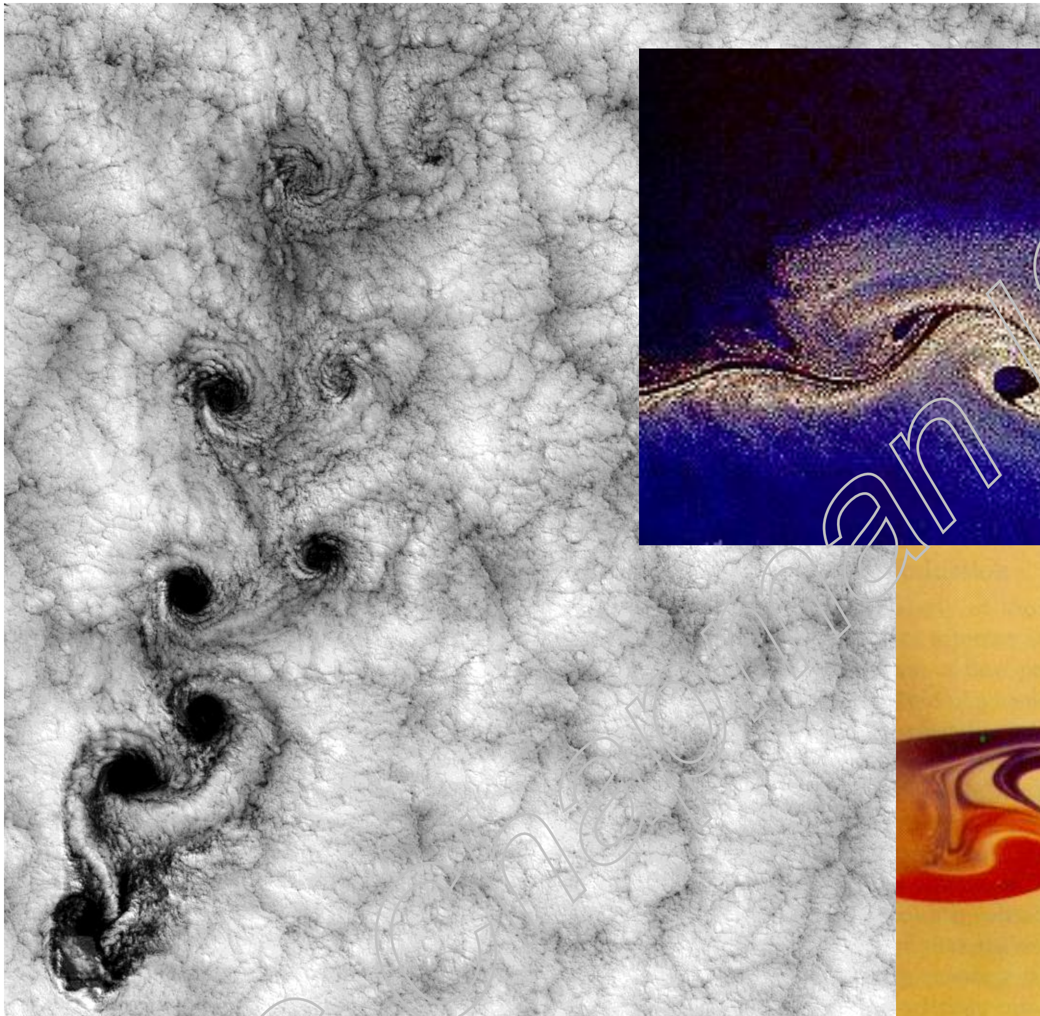


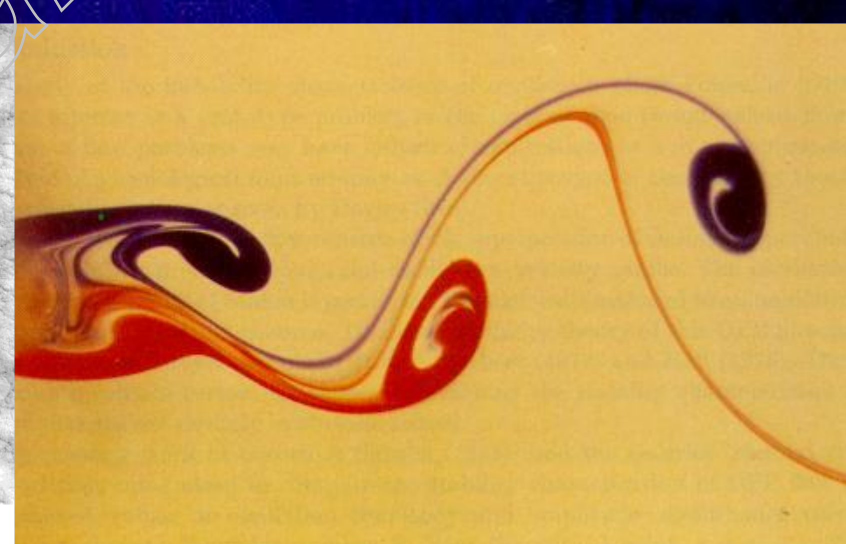
Vicsek PRL 1995 ‘bird’ model

Using MI: March, SCC et al, Physica D 2005, Wicks, SCC et al PRE 2007

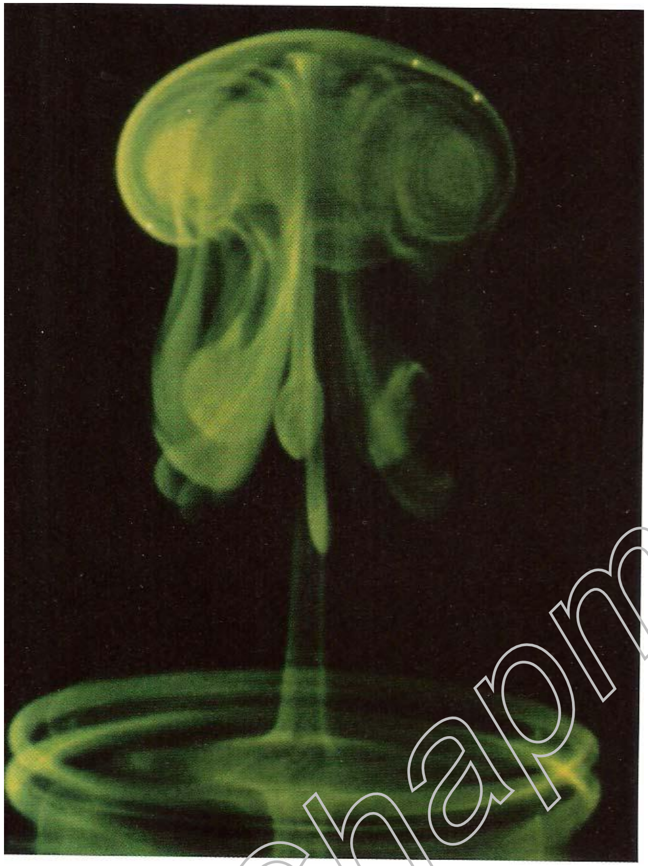
Similarity analysis-
a method to obtain parameters

The principle of similitude and parameters…





Similarity in action…



Peck and Sigurdson, A Gallery of Fluid Motion, CUP(2003)
See also G. I. Taylor, Proc. Roy. Soc., (1950)

Buckingham, Phys. Rev., 1914

Buckingham π theorem

System described by $F(Q_1 ... Q_p)$ where $Q_{1..p}$ are the relevant macroscopic variables

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.)

there are $M = P - R$ distinct dimensionless groups.

Then $F(\pi_{1..M}) = C$ is the general solution for this universality class.

To proceed further we need to make some intelligent guesses for $F(\pi_{1..M})$

See e.g. *Barenblatt, Scaling, self - similarity and intermediate asymptotics, CUP, [1996]*

also *Longair, Theoretical concepts in physics, Chap 8, CUP [2003]*

Example: simple (nonlinear) pendulum

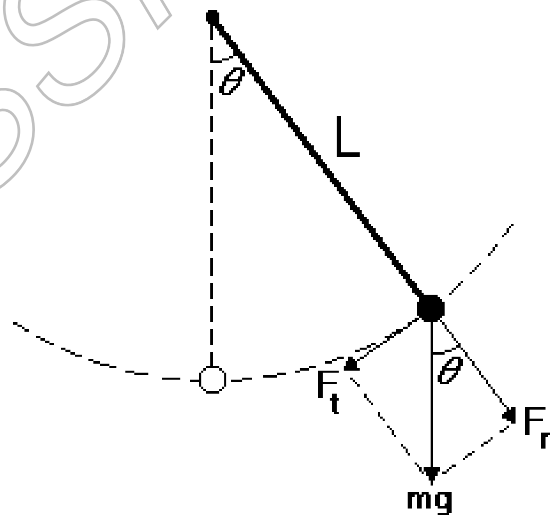
System described by $F(Q_1 ... Q_p)$ where Q_k is a macroscopic variable

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.) there are $M = P - R$ dimensionless groups

Step 1: write down the relevant macroscopic variables:

variable	dimension	description
θ_0	$-$	angle of release
m	$[M]$	mass of bob
τ	$[T]$	period of pendulum
g	$[L][T]^{-2}$	gravitational acceleration
l	$[L]$	length of pendulum



Step 2: form dimensionless groups: $P = 5, R = 3$ so $M = 2$

$\pi_1 = \theta_0, \pi_2 = \dfrac{l}{g\tau^2}$ and no dimensionless group can contain m

then solution is $F(\theta_0, {l}/{g\tau^2}) = C$

Step 3: make some simplifying assumption: $f(\pi_1) = \pi_2$ then the period: $\tau = f(\theta_0)\sqrt{{l}/{g}}$

NB $f(\theta_0)$ is universal ie same for all pendula-

we can find it knowing some other property eg conservation of energy..

Similarity analysis Vicsek flocking model

quantity	dimension	description
η	$-$	noise
u_0	$[L][T]^{-1}$	'bird' speed
R	$[L]$	interaction radius
δt	$[T]$	timestep
ρ	$[L]^{-2}$	av. 'bird' density

$P = 5, R = 2, M = P - R = 3$ dimensionless groups:

$\Pi_1 = \eta$ - random fluctuations

$\Pi_2 = \dfrac{u_0 \delta t}{R}$ - dist. per timestep/interaction radius

$\Pi_3 = \pi R^2 \rho$ - av. no. of birds in interaction radius

true for system size $\rightarrow \infty$ with R, ρ finite

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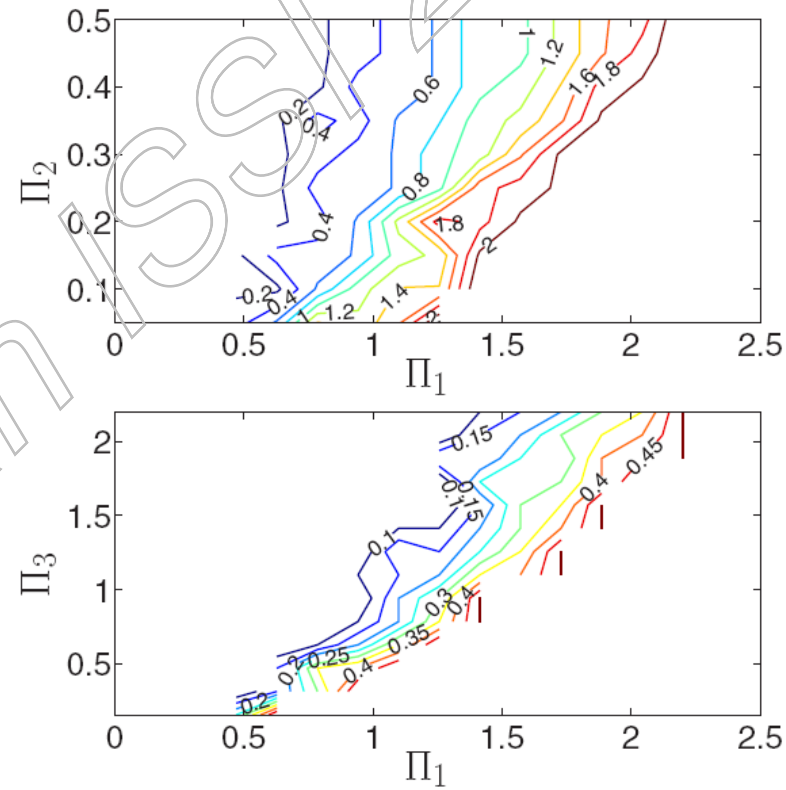


Position of phase transition

Wicks, SCC et al, PRE (2007)

Example: fluid turbulence, the Kolmogorov '5/3 power spectrum'

System described by $F(Q_1 \ldots Q_p)$ where Q_k is a macroscopic variable

F must be a function of dimensionless groups $\pi_{1..M}(Q_{1..p})$

if there are R physical dimensions (mass, length, time etc.) there are $M = P - R$ dimensionless groups

Step 1: write down the relevant variables (incompressible so energy/mass):

variable	dimension	description
$E(k)$	$[L]^3[T]^{-2}$	energy/unit wave no.
ε_0	$[L]^2[T]^{-3}$	rate of energy input
k	$[L]^{-1}$	wavenumber

Step 2: form dimensionless groups: $P = 3, R = 2,$ so $M = 1$

$$\pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}$$

Step 3: make some simplifying assumption:

$F(\pi_1) = \pi_1 = C$ where C is a non universal constant, then: $E(k) \sim \varepsilon_0^{2/3} k^{-5/3}$

call this the '*inertial range*'

Buckingham π theorem (similarity analysis)
universal scaling, anomalous scaling

Turbulence:

variable	dimension	description
$E(k)$	L^3T^{-2}	energy/unit wave no.
ε_0	L^2T^{-3}	rate of energy input
k	L^{-1}	wavenumber

$$M=1, \pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}, E(k) \sim \varepsilon_0^{2/3} k^{-5/3}$$

introduce another characteristic speed....

variable	dimension	*description*
$E(k)$	L^3T^{-2}	energy/unit wave no.
ε_0	L^2T^{-3}	rate of energy input
k	L^{-1}	wavenumber
v	LT^{-1}	characteristic speed

$$M=2, \pi_1 = \frac{E^3(k)k^5}{\varepsilon_0^2}, \pi_2 = \frac{v^2}{Ek}$$

$$\text{let } \pi_1 \sim \pi_2^{\alpha}, E(k) \sim k^{-(5+\alpha)/(3+\alpha)}$$

Homogeneous Isotropic Turbulence and Reynolds Number

Step 1: write down the relevant variables:

variable	dimension	description
L_0	$[L]$	driving scale
η	$[L]$	dissipation scale
U	$[L][T]^{-1}$	bulk (driving) flow speed
ν	$[L]^2[T]^{-1}$	viscosity

Step 2: form dimensionless groups: $P = 4, R = 2,$ so $M = 2$

$\pi_1 = \frac{UL_0}{\nu} = R_E, \pi_2 = \frac{L_0}{\eta}$ and importantly $\frac{L_0}{\eta} = f(N)$, where N is no. of d.o.f

Step 3: d.o.f from scaling ie $f(N) \sim N^{\alpha}$ here $\frac{L_0}{\eta} \sim N^3$,or $N^{3\beta}$ or $\frac{L_0}{\eta} \sim \lambda^{N/3}$ or ...

Step 4: assume steady state and conservation of the dynamical quantity, here energy...

transfer rate $\varepsilon_r \sim \frac{u_r^3}{r}$, injection rate $\varepsilon_{inj} \sim \frac{U^3}{L_0}$, dissipation rate $\varepsilon_{diss} \sim \frac{\nu^3}{\eta^4}$ - gives $\varepsilon_{inj} \sim \varepsilon_r \sim \varepsilon_{diss}$

this relates π_1 to π_2 giving: $R_E = \frac{UL_0}{\nu} \sim \left(\frac{L_0}{\eta}\right)^{4/3} \sim N^{\alpha}, \alpha > 0$ thus N grows with R_E

Avalanche model (a la BTW 1987 SOC)

Step 1:

variable	dimension	description
L_0	$[L]$	system size
δl	$[L]$	grid size
h	$[S][T]^{-1}$	average driving rate per node
ε	$[S][T]^{-1}$	system average dissipation/loss

Step 2: form dimensionless groups: $P = 4, R = 2,$ so $M = 2$

$$\pi_1 = \frac{h}{\varepsilon} = R_A, \pi_2 = \frac{L_0}{\delta l} = f(N)$$ where N is no. of d.o.f.

Step 3: d.o.f from scaling ie $f(N) \sim N^{\alpha}, N \sim \left(\frac{L_0}{\delta l}\right)^{\alpha}$ with Euclidean dimension $D \geq \alpha > 0$

Step 4: assume steady state and conservation of the dynamical quantity, here sand...S

conservation of flux of sand gives $h \times$(no of nodes) $\sim \varepsilon$

so $h\left(\frac{L_0}{\delta l}\right)^{D} \sim \varepsilon$ this relates π_1 to π_2 giving $R_A = \frac{h}{\varepsilon} \sim \left(\frac{\delta l}{L_0}\right)^{D} \sim N^{-\frac{D}{\alpha}}$

this is in the opposite sense to fluid turbulence, N is maximal when $R_A \to 0$

How is SOC different to turbulence? consider...

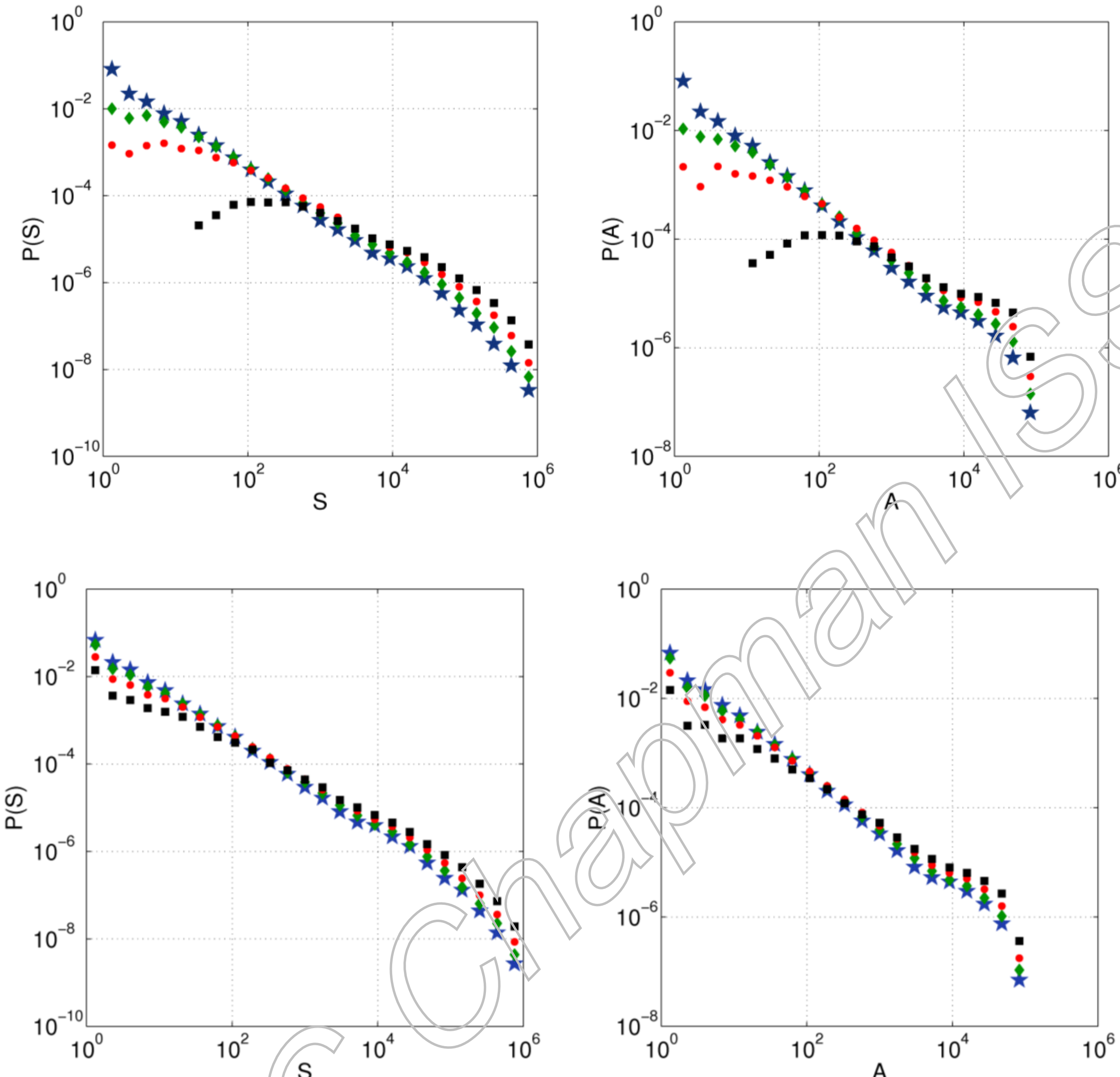
Intermediate driving (or what happens as we change $R_A \sim h/\varepsilon$):

If $L_0 \gg \delta l$ we can consider intermediate behaviour $gL_0 \gg h\delta t > g\delta l$ where the smallest avalanches are swamped, but large avalanches persist.

Corresponds to:

reducing the available d.o.f. by increasing h, and hence R_A

- *SCC et al, Phys. Plasmas 2009,*
- *SCC et al Plas. Phys. Cont. Fusion 2009*



Centre driven BTW sandpile

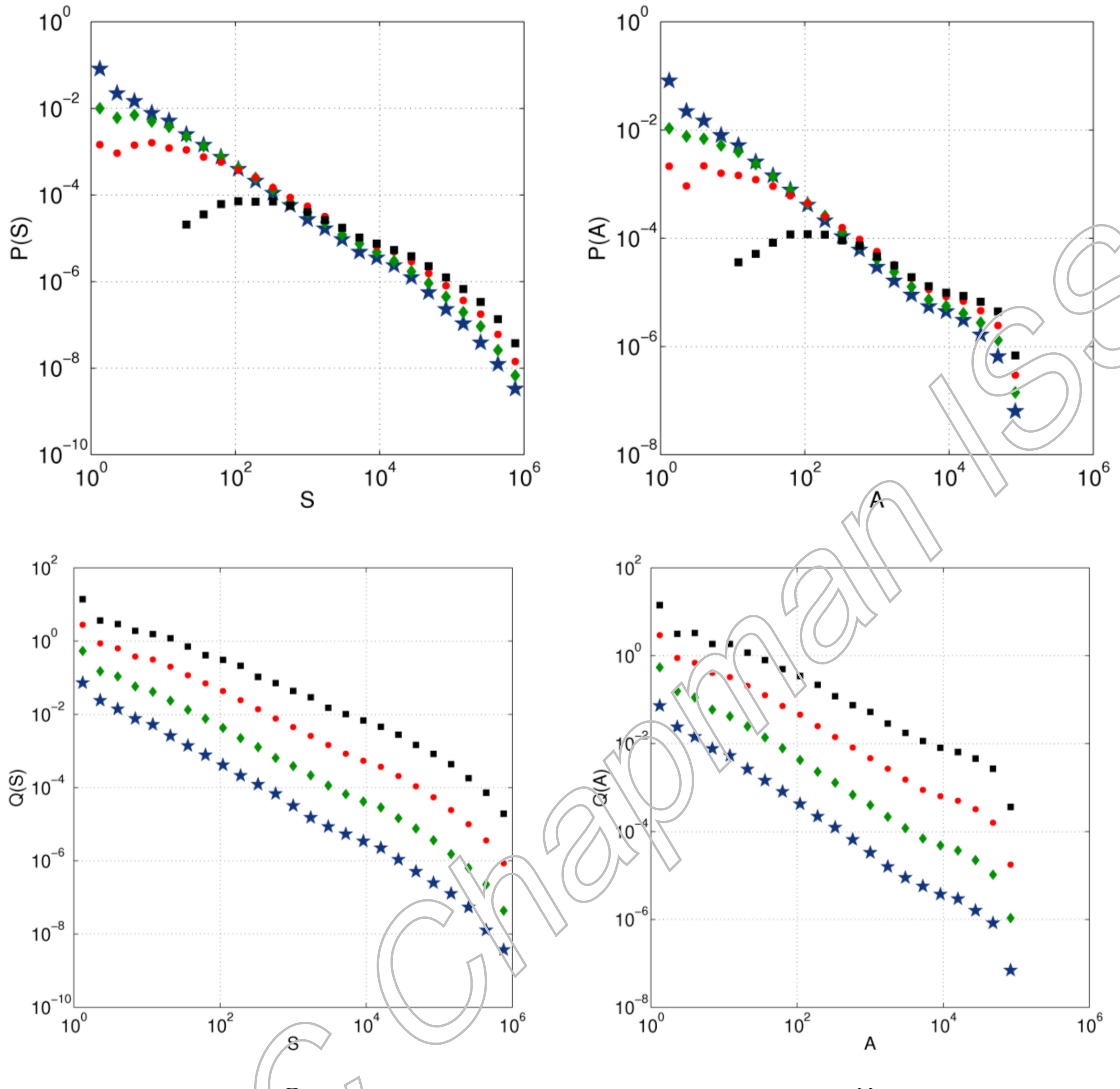
box *400×400*

h=1,4,8,16

[* ◆ ● ■]

Top- constant drive

Bottom- broadband white noise drive



Centre driven BTW sandpile

box 400×400

$h=1,4,8,16$

[∗ ◆ ● ■]

Top- constant drive

Bottom- broadband white noise drive

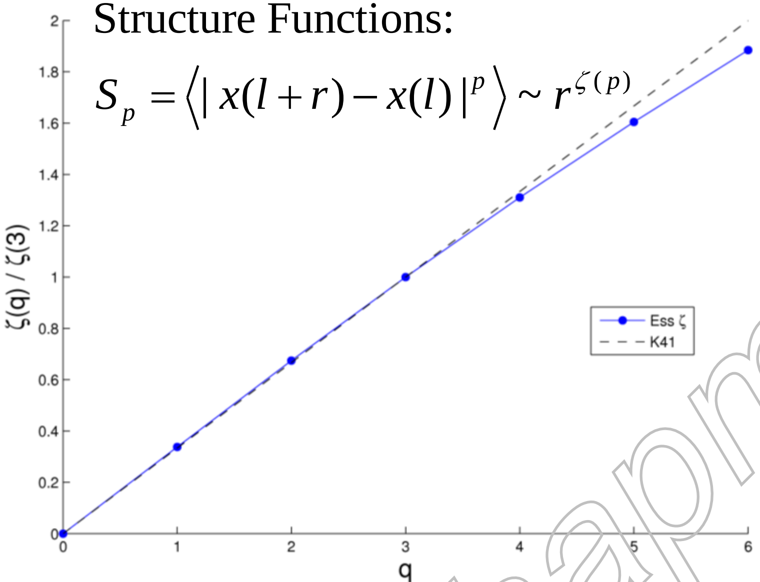
(curves displaced for clarity)

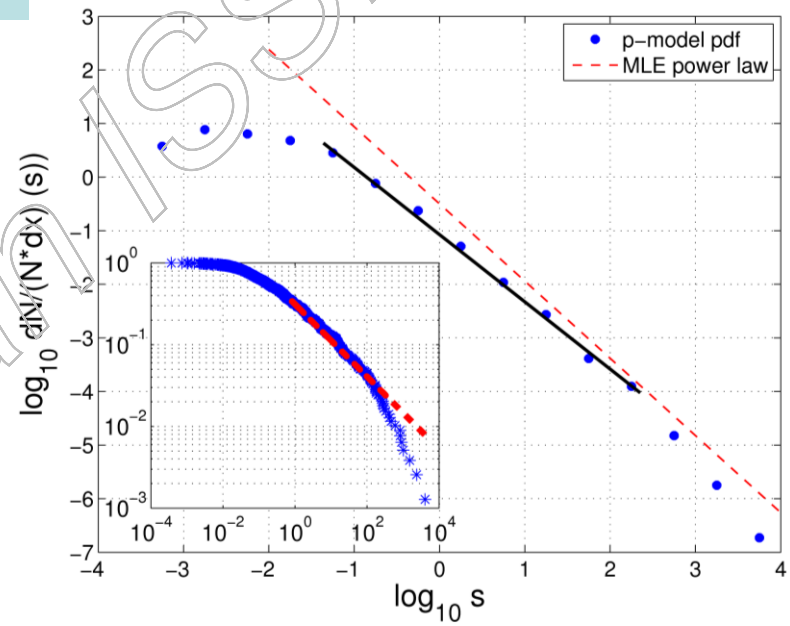
A cautionary tale..
p-model for intermittent turbulence- shows finite range power law avalanches

p-model timeseries shows multifractal behaviour in structure functions as expected

Structure Functions:

$$S_p = \langle | x(l+r) - x(l) |^p \rangle \sim r^{\zeta(p)}$$





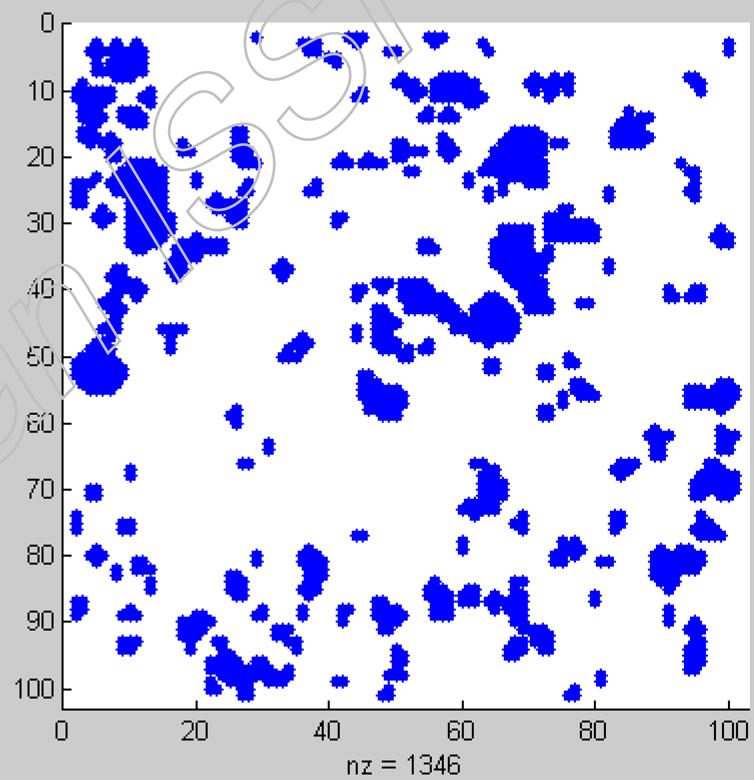
Thresholding the timeseries to form an avalanche distribution- finite range power law
Watkins, SCC et al, PRL, 2009

Fractal dynamics- Edwards Wilkinson

A *linear* model
Shown: 100^2 grid D=0.3
Solves:

$$\frac{\partial \bar{h}}{\partial t} = D\nabla^2 \bar{h} + \eta$$

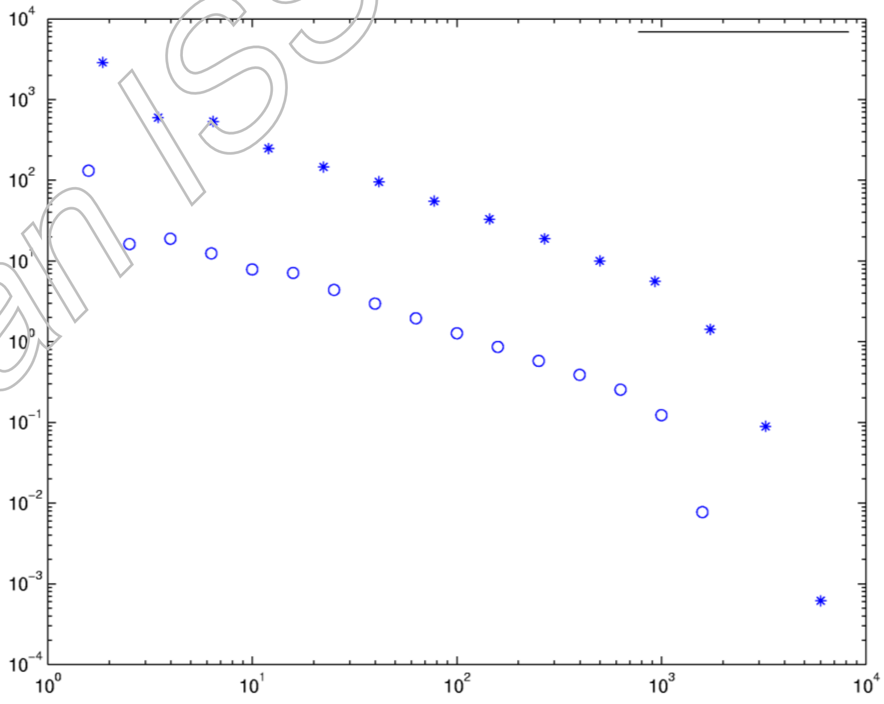
where η is iid 'white'
random source of grains
'height' $\bar{h} = h - \langle h \rangle$
blue patches are $\bar{h} > h_0$



Edwards Wilkinson- statistics

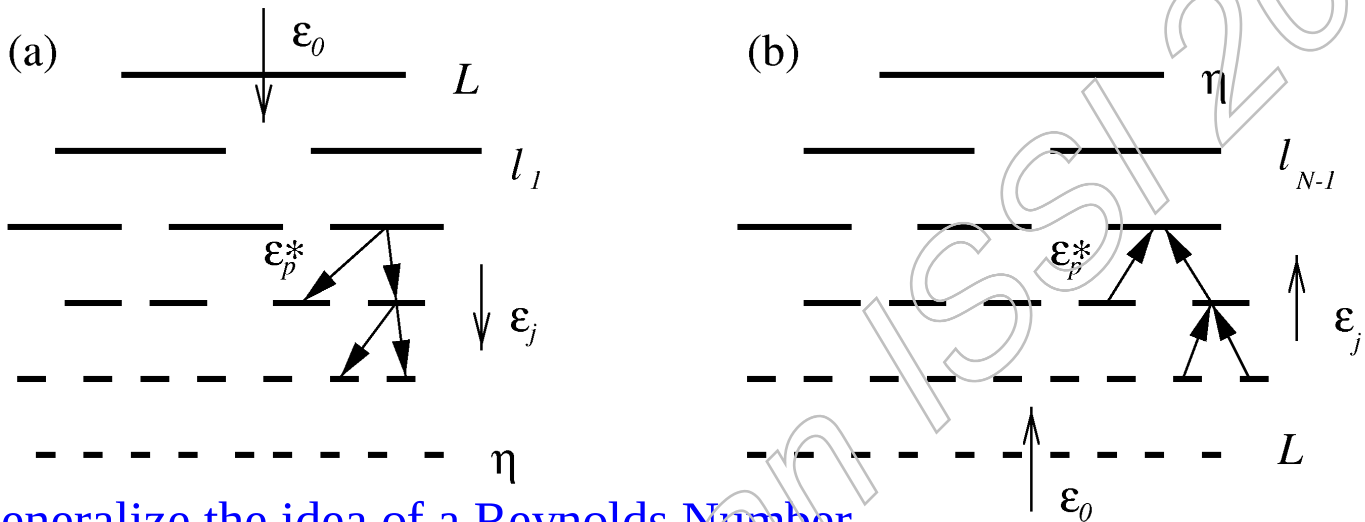
Statistics of instantaneous patch size are power law

Linear model- driver (random rain of particles) has inherent fractal scaling (Brownian surface) +selfsimilar diffusion which preserves scaling



SCC et al, PPCF (2004)

Cascade can be 'anything':Turbulence, food web...



Generalize the idea of a Reynolds Number...

a control parameter for the onset of disorder (turbulence, burstiness)

Cascade - forward or inverse- with:

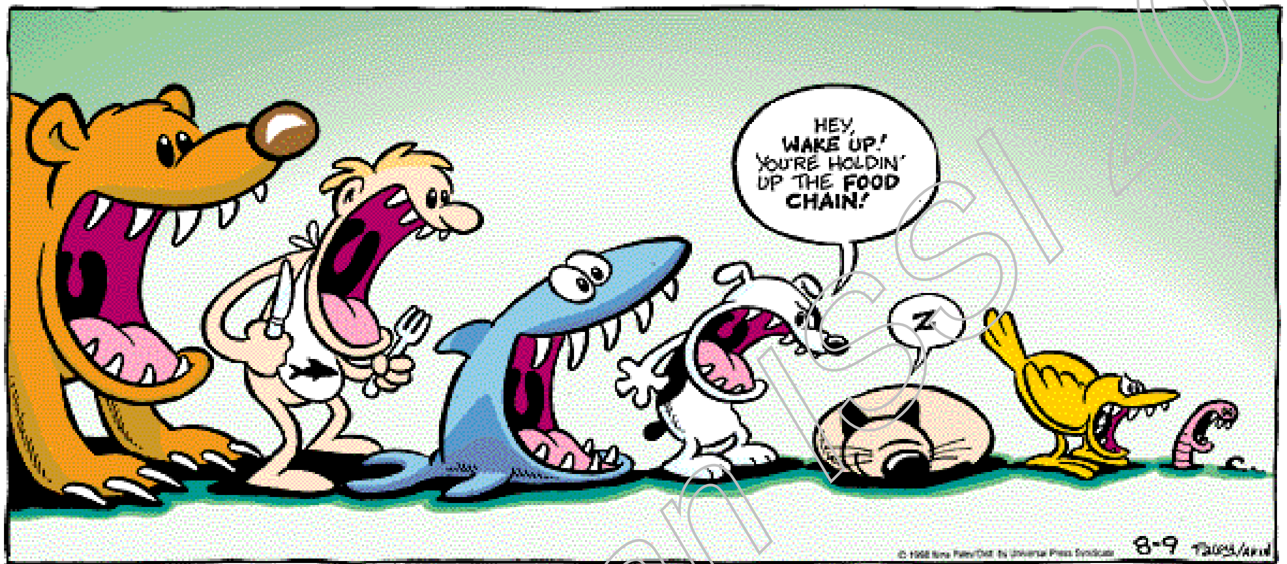
$\pi_1 = R_E$ the Reynolds Number,

at least one other, $\pi_k = f(N)$ where N is the number of degrees of freedom

flux of some dynamical quantity is conserved- steady state

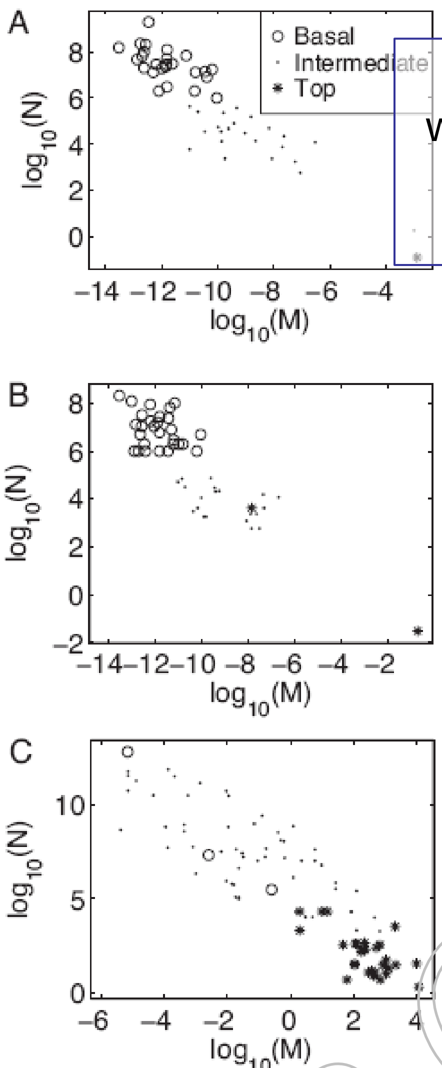
Order and control parameters- in macroecology

A Reynolds number for ecosystems?

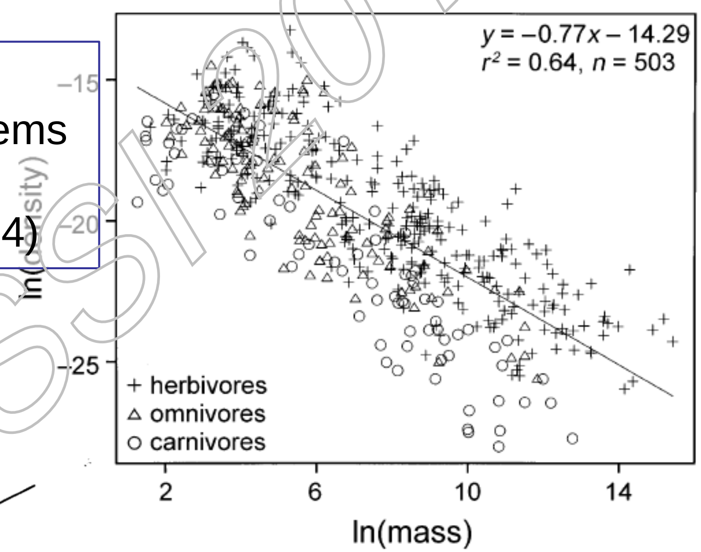


- (interchangeable) categories occupy a particular niche in the web
- caregories all linked by predation/consumption which processes some resource (energy, biomass..)
- System driven by primary producers introducing energy/biomass and all categories removing it
- It does not matter what the resource is as long as we can conserve flux
- still ok if there are losses i.e. a fraction is passed from one category to the next, or if there is recycling (bottom species feeding off dead top predators)- we will sum over the ecosystem
- Steady state: timescale over which we change *R* is slow compared to timescale to propagate the resource through the web (recycling time)

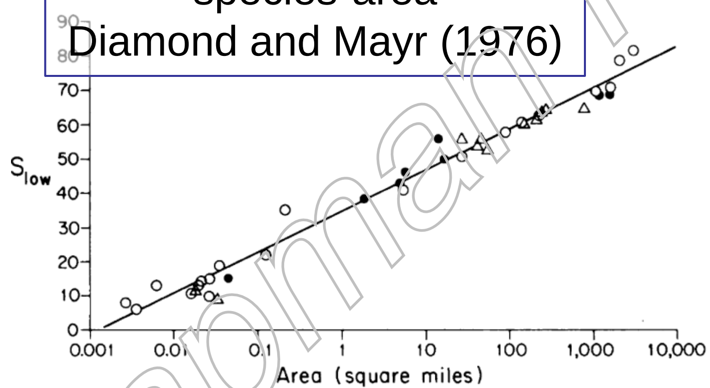
Macroecological patterns, some examples…



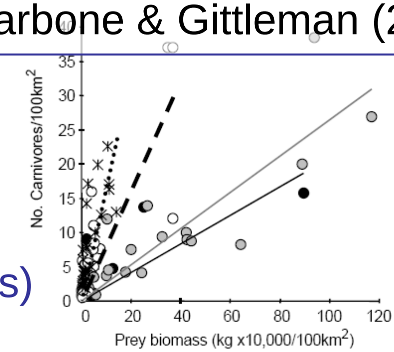
density-body size
within an ecosystem, across ecosystems
Cohen et al (2009) Brown et al (2004)



species-area
Diamond and Mayr (1976)



Predator density-prey biomass
Carbone & Gittleman (2002)



Macroecological patterns for:
diversity (richness, no. of species)
density (abundance, no. of individuals/species)
metabolic rate
with: size(mass), area, trophic level, productivity, latitude…
(and exceptions, complications, caveats..)

Π theorem – find relevant ecosystem variables..

1. Statistical properties of p^{th} category
2. Resource flow into ecosystem
3. Characteristic lengthscales

➢ We assume that the ecosystem is in a

dynamically balanced steady state

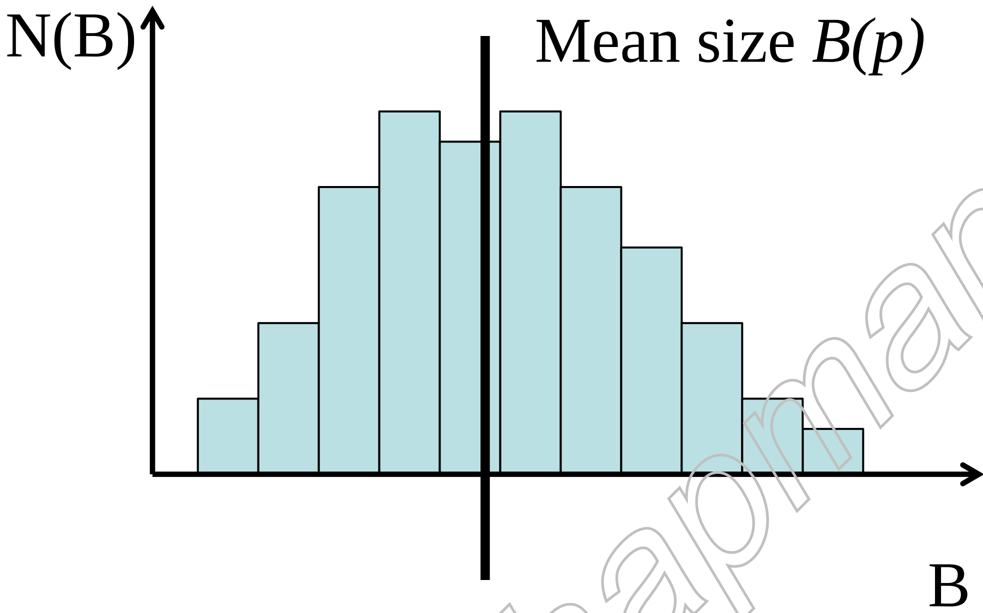
implies a separation of timescales-
ecosystem can adapt [quickly enough] to [external changes] to maintain a balance between
the rate of uptake and utilization of resource

[cf 'homeostatis' White et al (2004), Ernest and Brown (2001)]

1) Statistical properties of p^{th} category-

- all species in the category have similar average size, metabolic rate
- all connected into same resource flow

Observed histograms of body size of category found to be peaked



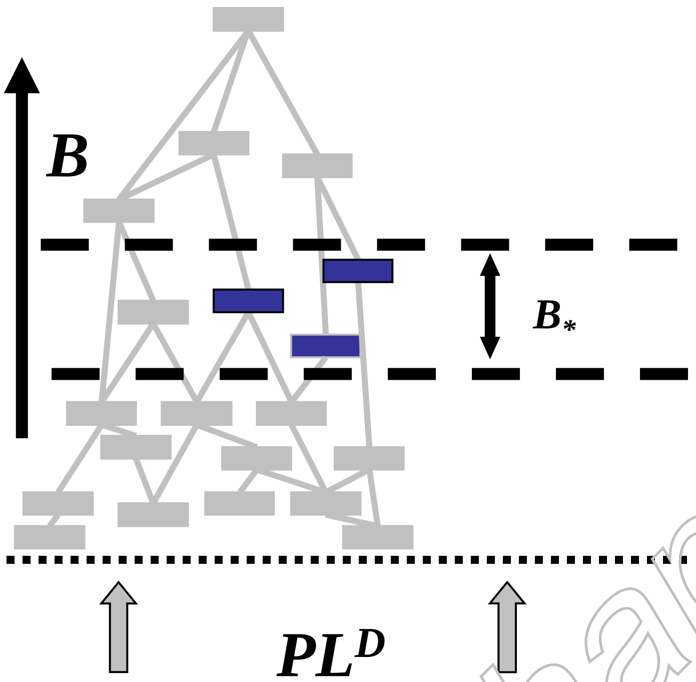
Observe characteristic:

- Size (no of cells) $B(p)$
- Abundance $n(p)$
- Richness $S(p)$
- Metabolic rate R *per cell*

p^{th} category members all connected to the ecosystem by resource flow- Size, abundance, richness, metabolic rate all depend on available resource

2) Resource flow into ecosystem

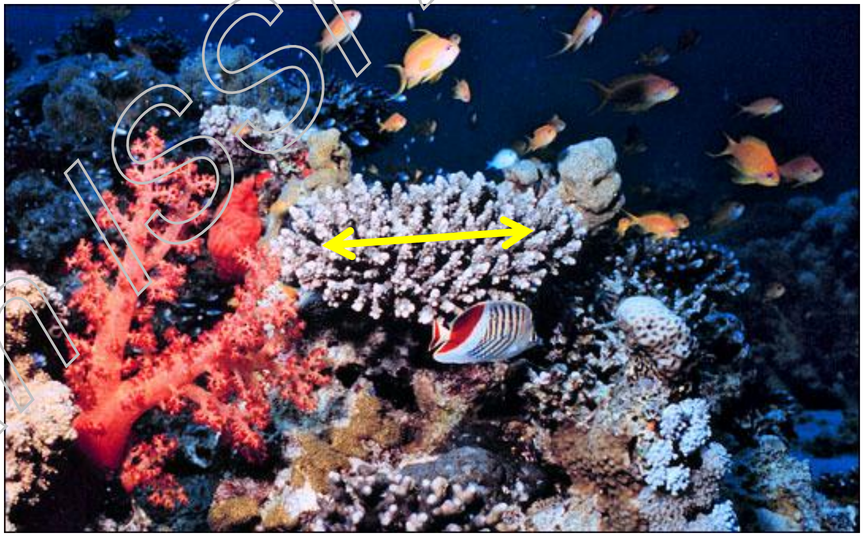
p^{th} category members all connected to the ecosystem by resource flow-
Size, abundance, richness, metabolic rate all depend on available resource



- Position in foodweb (and available resource) is relative to other categories- we need to know at least one, B_*
- Primary producers nett productivity P

[L^D is habitat size, so resource rate of supply is PL^D]

3) Characteristic length-scales



Habitat size L^D so rate of supply of resource is PL^D

category characteristic length-scale r

[reflects how individuals are dispersed]

Ecosystem macroscopic variables...

$$P = 8, R = 3,$$
$$M = P - R = 5 \text{ dimensionless groups}$$

$$\Pi_1 = \frac{\alpha P L^2}{R}, \Pi_2 = n_* L^D, \Pi_3 = S_*, \Pi_4 = B_*, \Pi_5 = \frac{r_*}{L}$$

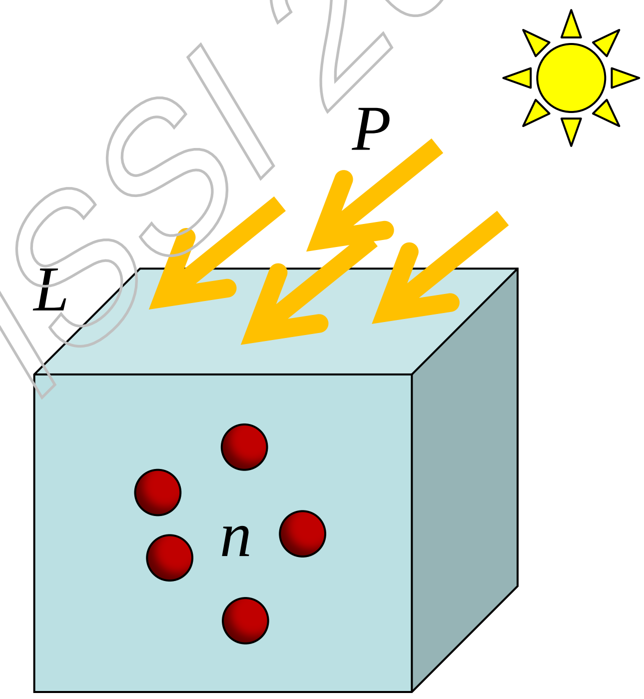
variable	dimension	description
B_*	$[-]$	mean body size p^{th} category
n_*	$[L]^{-D}$	mean density/abundance of p^{th} category (no of individuals/unit area/vol.)
S_*	$[-]$	richness/no of species in p^{th} category
R	$[\varepsilon][T]^{-1}$	mean metabolic rate per cell
P	$[\varepsilon][T]^{-1}[L]^{-2}$	rate of energy supply/unit area
α	$[-]$	primary producer efficiency
r_*	$[L]$	characteristic length-scale, p^{th} category
L	$[L]$	habitat size

dimensions of energy $\varepsilon = [M][L]^2[T]^{-2}$

Go through this slowly… bottom up ecosystem
Single cell organisms- all identical ‘type’ or species

$$\boxed{\Pi_1 = \frac{\alpha P L^2}{R}, \Pi_2 = n L^D}$$

$$\boxed{\frac{\alpha P L^2}{R} = n L^D}, \text{ or } \Pi_1 = \Pi_2$$



threshold for life:

$$\Pi_1 = 1$$

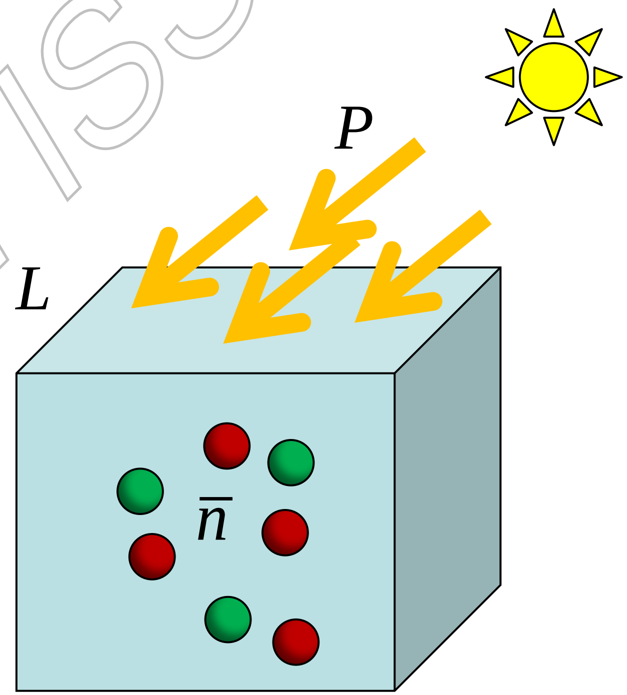
$\boxed{\textit{one} \text{ single cell in } L^D \text{ with metabolic rate } R}$

l.h.s is control parameter, r.h.s. are order parameters

Single cell organisms- more than one 'type' or species

S types (species) and there is a species label *k=1..S*,
the density of the k^{th} species is *n(k)*
Average density per species:

$$\bar{n} = < n(k) >_k = \frac{1}{S}\sum_{k=1}^{S} n(k)$$



$$\boxed{\Pi_1 = \frac{\alpha P L^2}{R}, \Pi_2 = \bar{n} L^D, \Pi_3 = S}$$

$$\boxed{\frac{\alpha P L^2}{R} = \bar{n} S L^D} = \sum_{k=1}^{S} n(k) L^D$$

Multi-cell organisms- more than one 'type' or species

now possible to distinguish types of organism- label the different types or categories distinguished in this way with index *p*.

p^{th} category clustered around an average body size,
on average composed of *B(p)* cells with metabolic rate *RB(p)*
Within each *p* there are *k=1..S(p)* differentiable species
each with density *n(k,p)*, average body size *B(p)*

average density:

$$\bar{n}(p) = \langle n(k,p) \rangle_k = \frac{1}{S(p)} \sum_{k=1}^{S(p)} n(k,p)$$

now observe a given category p_*, $n_* = \bar{n}(p_*), S_* = S(p_*)$

with characteristic average size $B_* = B(p_*)$

then

$$\boxed{\Pi_1 = \frac{\alpha P L^2}{R}, \Pi_2 = n_* L^D, \Pi_3 = S_*, \Pi_4 = B_*}$$

now

$$\frac{\alpha P L^2}{R} = \sum_p \sum_{k=1}^{S(p)} n(k,p) B(p) L^D = \sum_p \bar{n}(p) S(p) B(p) L^D$$

$$\frac{\alpha P L^2}{R} = n_* S_* B_* L^D \sum_p \frac{\bar{n}(p) S(p) B(p)}{\bar{n}(p_*) S(p_*) B(p_*)}$$

$$\boxed{\frac{\alpha P L^2}{R} = n_* S_* B_* L^D \Psi(p_*)}$$

$\Psi(p_*)$ is dimensionless;

$1/(\Psi(p_*))$ fraction of total rate of resource

supplied to the ecosystem utilized by p_*^{th} category organisms not uniformly distributed in space so density depends on length scale r over which it is observed, $n=n(r,k,p)$, efficiency of primary producers $\alpha=\alpha(r)$

$$n(r,k,p)=n(L,k,p)/g(k,p,r/L)$$
$$\alpha(r)=\alpha(L)/g_{\alpha}(r/L)$$

$$\boxed{\Pi_1=\frac{\alpha_* PL^2}{R},\Pi_2=n_*L^D,\Pi_3=S_*,\Pi_4=B_*,\Pi_5=\frac{r_*}{L}}$$

$$\bar{n}(r,p)=<n(r,k,p)>_k=\frac{1}{S(p)}\sum_{k=1}^{S(p)}n(r,k,p)=\frac{1}{S(p)}\sum_{k=1}^{S(p)}\frac{n(L,k,p)}{g(k,p,r/L)}=\bar{n}(L,p)/\bar{g}(p,r/L)$$

with $g_*=\bar{g}(p_*)$

$$\frac{\alpha(L)PL^2}{R}=\sum_p \bar{n}(L,p)S(p)B(p)L^D=\sum_p \bar{n}(r,p)\bar{g}(p,r/L)S(p)B(p)L^D$$

$$=n_*S_*B_*g_*(\frac{r_*}{L})L^D\sum_p\frac{\bar{n}(p)\bar{g}(p,r/L)S(p)B(p)}{\bar{n}(p_*)\bar{g}(p_*,r_*/L)S(p_*)B(p_*)}$$

or with $G(r/L)=\bar{g}(r/L)/g_{\alpha}(r/L)$

$$\boxed{\frac{\alpha_* PL^2}{R}=n_*S_*B_*G(\frac{r_*}{L})L^D\Psi(p_*)}$$

which is $\Pi_1=\Pi_2\Pi_3\Pi_4G(\Pi_5)\Psi(p_*)$

So it follows that… for any ecosystem (as specified) there will be the following macroscopic patterns or trends

$$\boxed{\frac{\alpha_* P L^2}{R} = n_* S_* B_* G\left(\frac{r_*}{L}\right) L^D \Psi(p_*)}$$

Species **diversity/richness** S_*
$S_* \sim PL^2$ -**increases** with **total nett productivity summed over habitat**
Wrights rule (Wright 1983)
$S_* \sim 1/R$ -**decreases with typical metabolic rate**

abundance (density of individuals) n_* –ditto

cf latitudinal gradient rule- dependence on temperature, sunlight etc…

Range of body size relates to **number of trophic levels** –ditto
π theorem has given these trends/patterns without knowing Ψ

Spatial dependence gives the species area rule

species-area rule

$$\frac{\alpha_* P L^2}{R} = n_* S_* B_* G\left(\frac{r_*}{L}\right) L^D \Psi(p_*)$$

$$S_* \sim \frac{L^{2-D}}{G\left(r_*/L\right)}$$

[actually tells us that S_* must relate to how habitat, dispersal grow with scale]

power law SAR:

$S_* \sim L^z$ observed $z \sim [0.15 - 0.4]$

$G\left(r_*/L\right) \sim \left(r_*/L\right)^\gamma$

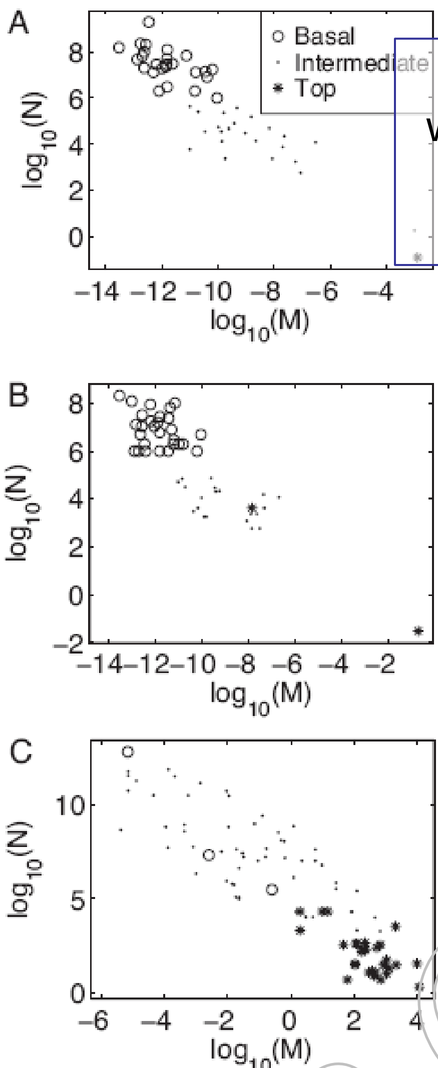
depends on both primary producers and observed category,

available surface area (fractally rough terrain)

also habitat ie trees, coral, foraging/dispersal

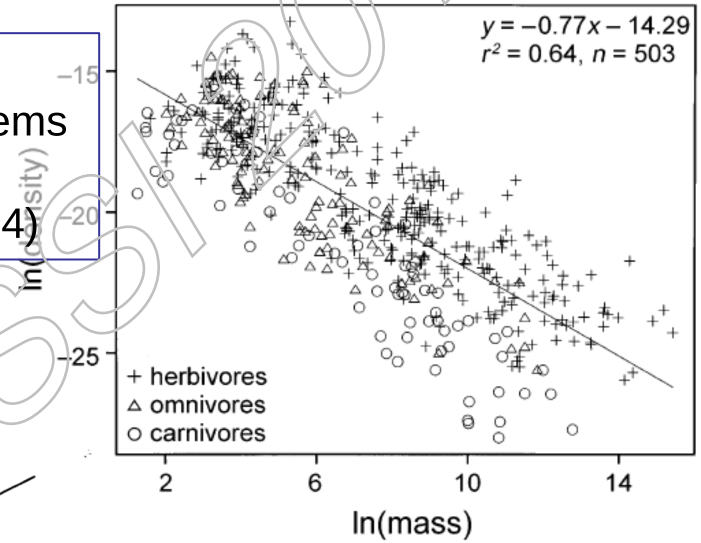
See eg Haskell et al (2002), Ritchie and Olff (1999),Palmer (2007), Milne (1992), Kunin (1992)

Macroecological patterns, trends and scatter…

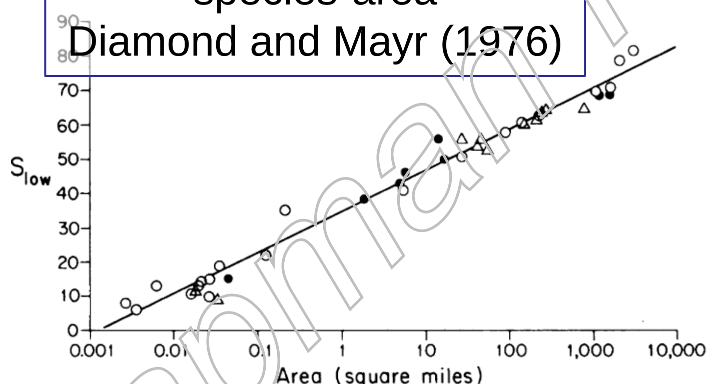


density-body size
within an ecosystem, across ecosystems

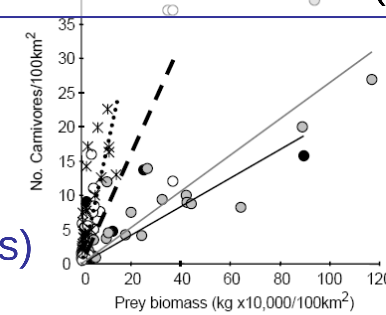
Cohen et al (2009) Brown et al (2004)



species-area
Diamond and Mayr (1976)



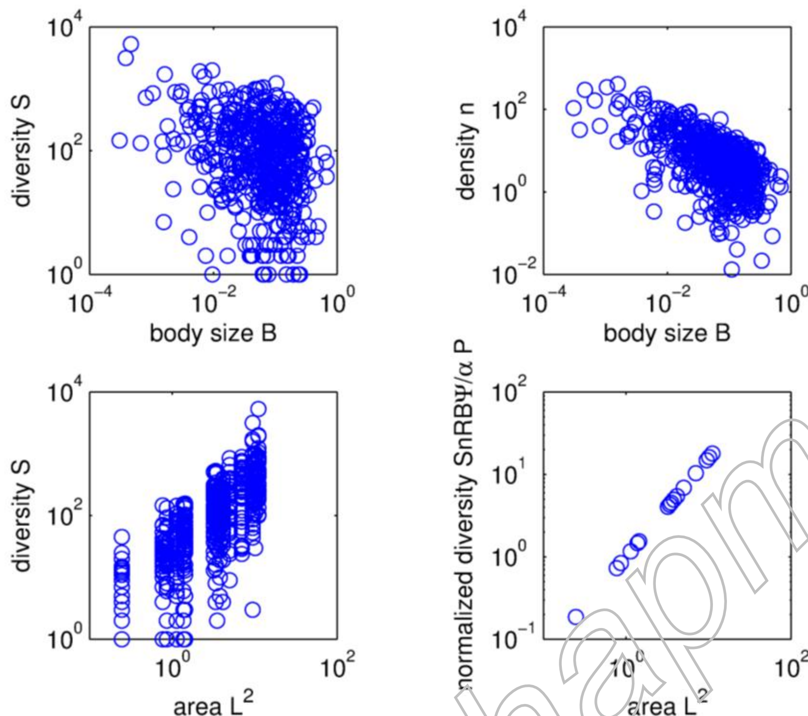
Predator density-prey biomass
Carbone & Gittleman (2002)



Macroecological patterns for:
diversity (richness, no. of species)
density (abundance, no. of individuals/species)
metabolic rate
with: size(mass), area, trophic level, productivity, latitude…
(and exceptions, complications, caveats..)

Testing for species-area rule .. synthetic data

$$\boxed{\frac{\alpha_* P L^2}{R} = n_* S_* B_* G\left(\frac{r_*}{L}\right) L^D \Psi(p_*)}$$

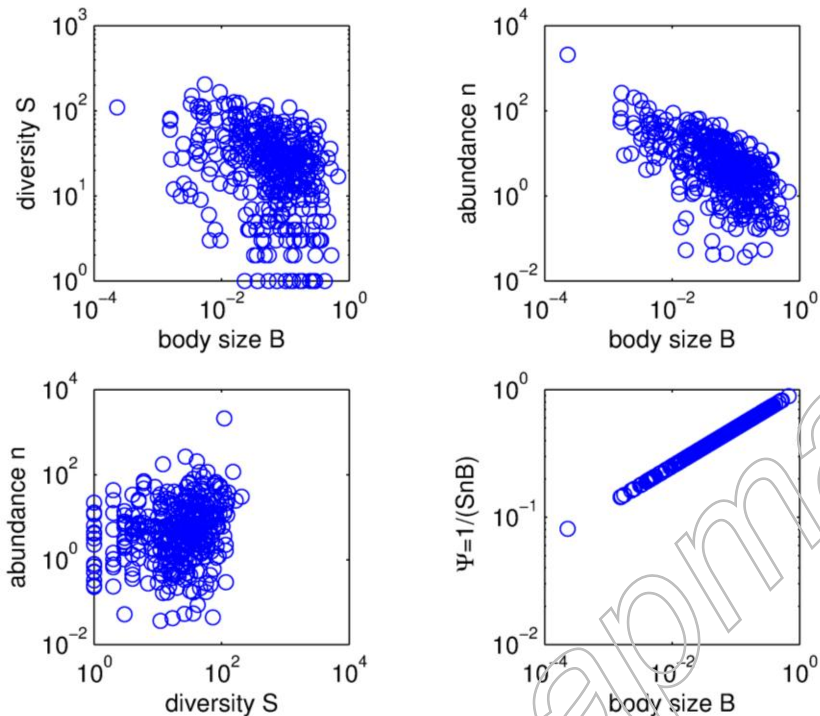


So species- area is actually:

$$\bar{S} = \frac{S_* n_* R B_* \Psi(p_*)}{\alpha_* P} = \frac{L^{2-D}}{G(r_*, L)}$$

‘normalized’ or dimensionless diversity

Extracting Ψ from single ecosystem data… synthetic data shown



$$\boxed{\frac{\alpha_* P L^2}{R} = n_* S_* B_* G\left(\frac{r_*}{L}\right) L^D \Psi(p_*)}$$

$$\Psi(n_* S_* B_*) = \frac{n' S' B'}{n_* S_* B_*} \Psi(n' S' B')$$

Ψ is ‘ecosystem function’ captures level of complexity of the ecosystem and/or how observed species/guilds are categorized

Summary

- Dimensional analysis+ conservation/ dynamical steady state →control / order parameters
- Distinguishing SOC/turbulence
- Framework for systems where there is a flow of something…
- leads to many of the observed macroecological ***patterns*** in ecology- is this why they are ubiquitous?
- **Where these patterns fail- may imply fast change ie ecological collapse**
- ecosystems data normalization to isolate trends/refine patterns, Ecosystem ‘classes’ in the ‘complexity’ function Ψ - is it universal?
- Thresholds for life?