Long-range Correlations and Extreme Events: Dynamics and Predictions

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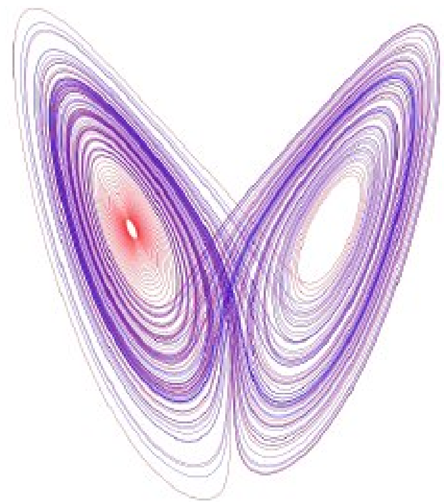
Nonlinear Dynamics and Complexity

Dynamics (Lorenz, 1963)



- Deterministic dynamics
- Chaos
- Quantitative results
- Weak connection with data

Geometry / Structure (Mandelbrot, 1977)

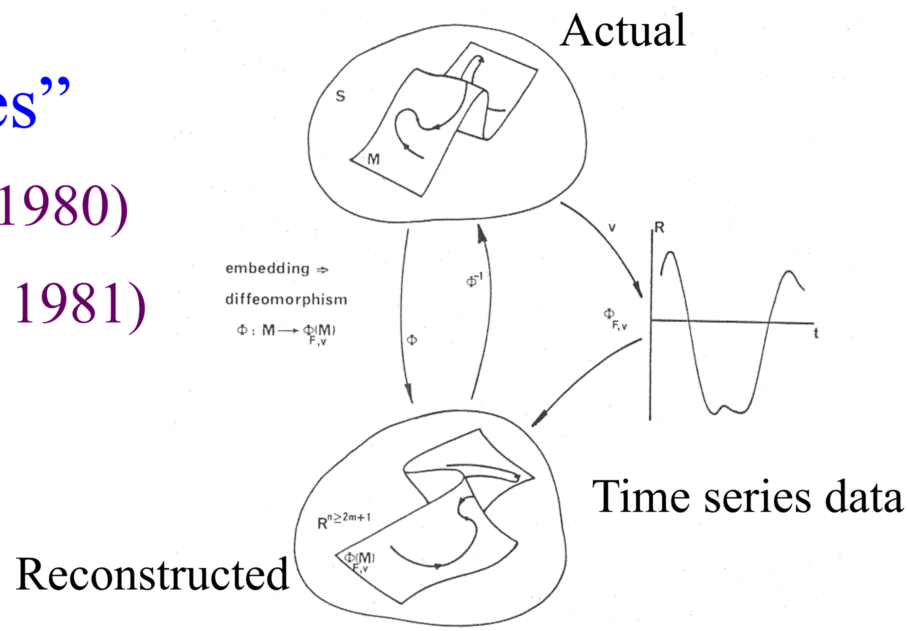
- Real objects in nature
  - (Trees, clouds, coastline, etc.)
- Fractals, Multifractals

Dynamics + Structure

Reconstruction of Dynamics

"Geometry from a time series"

(Packard et al., PRL, 1980)



Embedding theorem (Takens, 1981)

Time series data: $x(t)$

Time-delay embedding:

$$x_k(t_i) = x(t_i + (k-1)\tau)$$

Reconstructed space:

$$X_i = \{x_1(t_i), x_2(t_i), x_3(t_i), ..\}$$

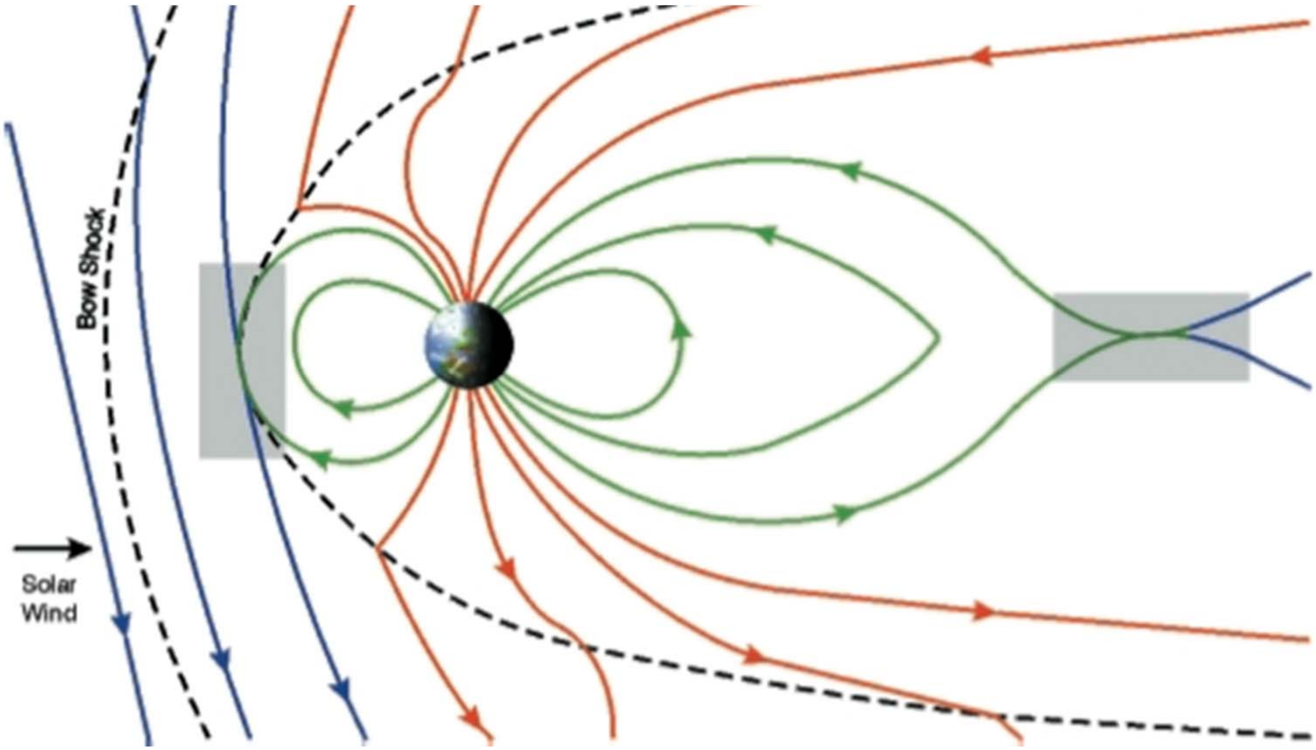
(Broomhead and King, Phys. A, 1986)

Correlation Dimension

$$C(r) = \sum \Theta(r - |X_i - X_j|) \sim r^{\nu}$$

(Grassberger & Procaccia, PRL, 1983)

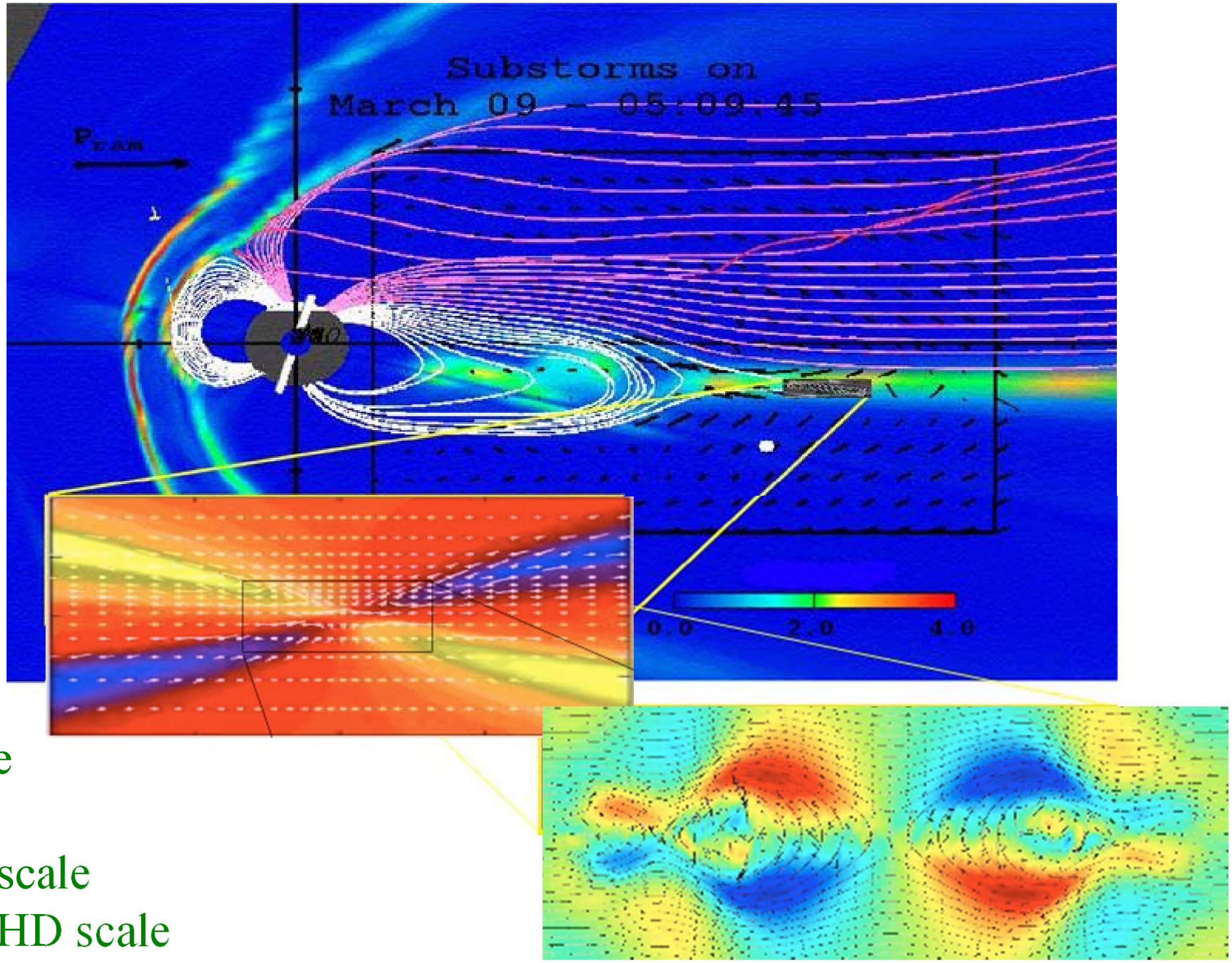
Earth’s Magnetosphere



Episodic nature of the magnetosphere:

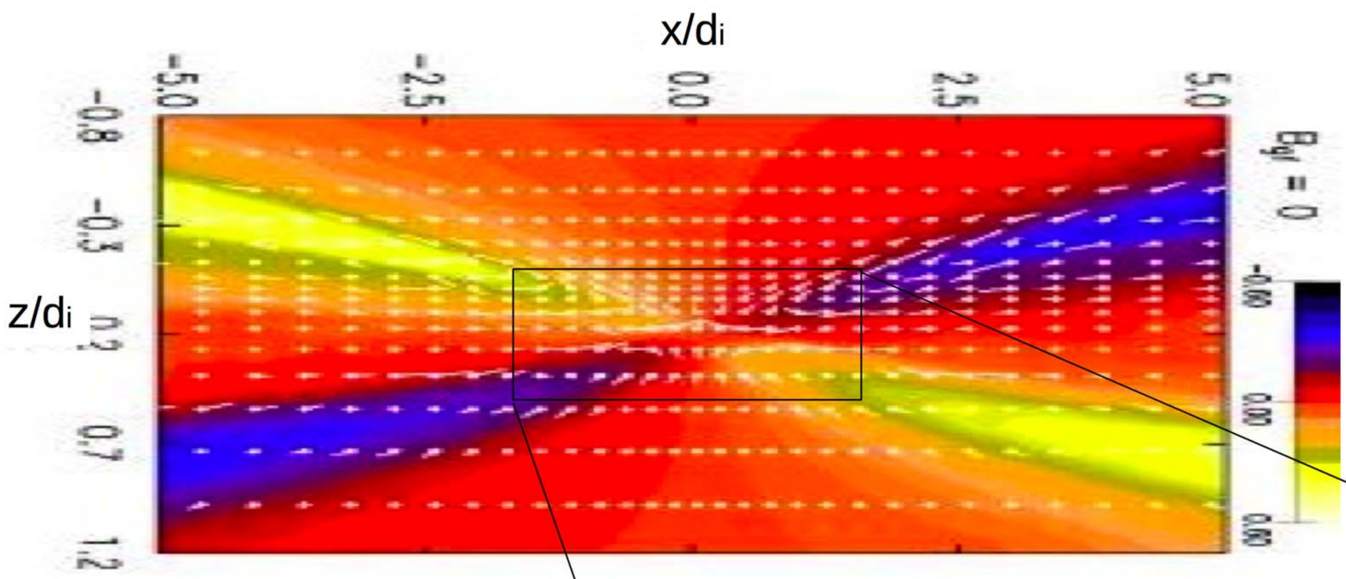
Storms and substorms driven by the solar wind

Multiscale Magnetosphere

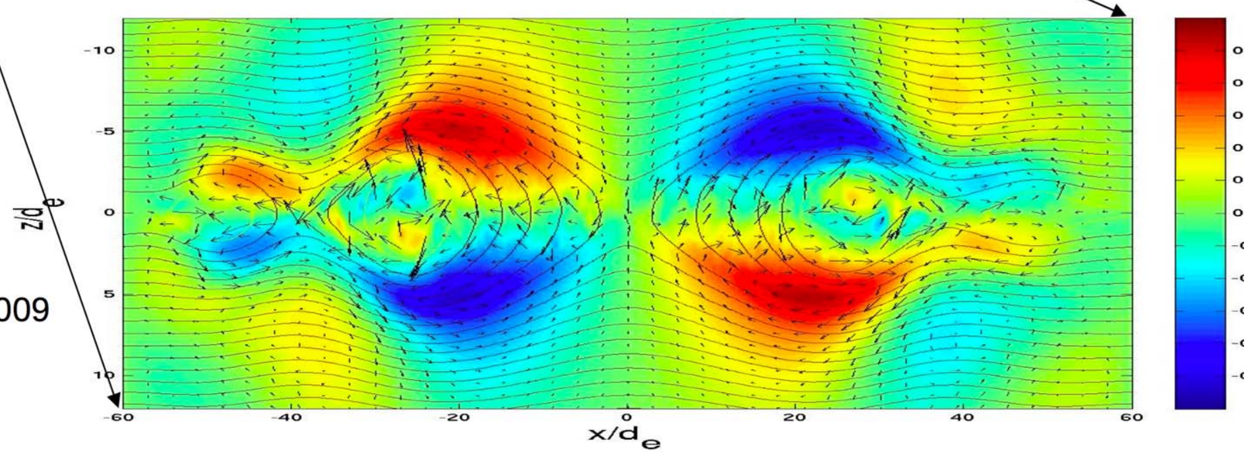


Global scale
MHD scale
Hall-MHD scale
Electron-MHD scale

Reconnection at Electron and Ion Scales



Hall MHD simulations
Huba, Phys. Plasmas, 2005



EMHD simulations
Jain and Sharma, Phys. Plasmas, 2009

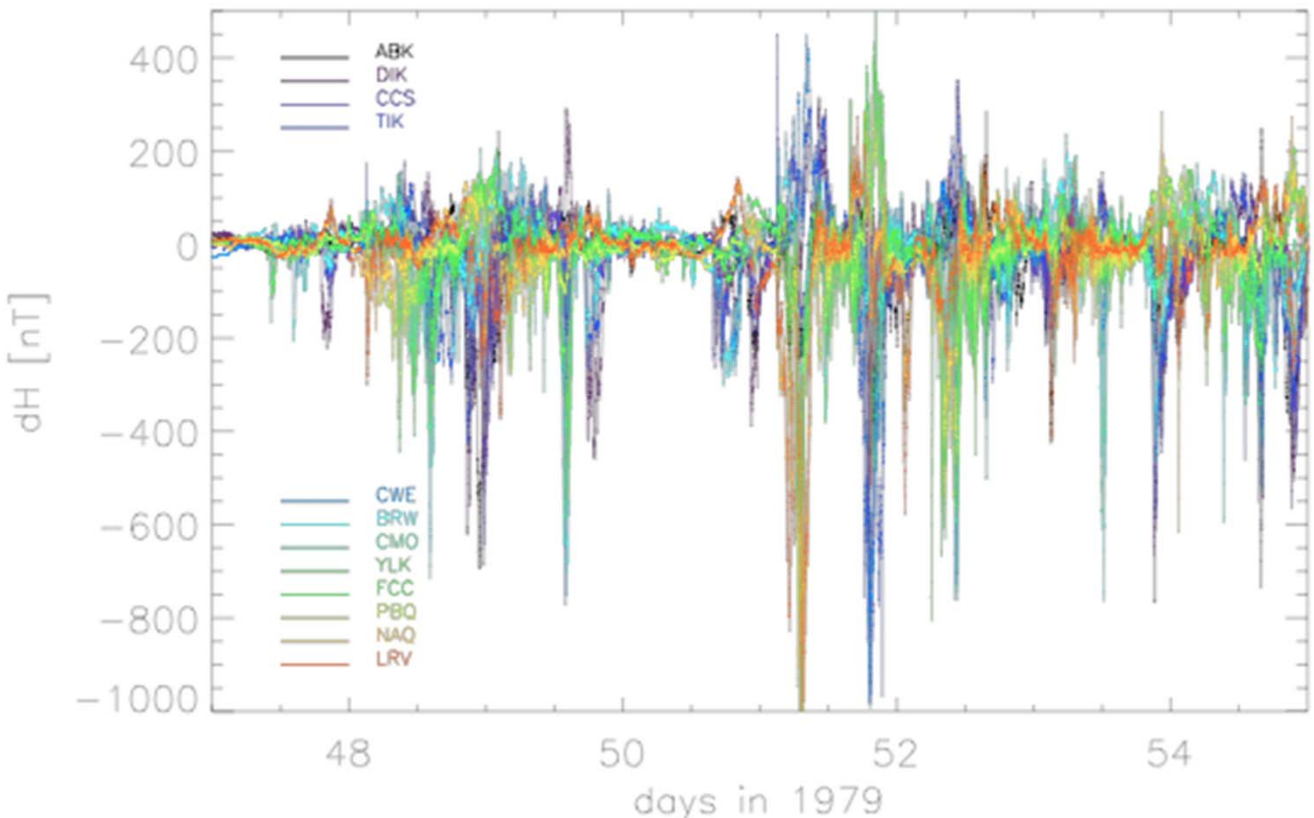
!

Auroral Electrojet Indices

Envelopes:

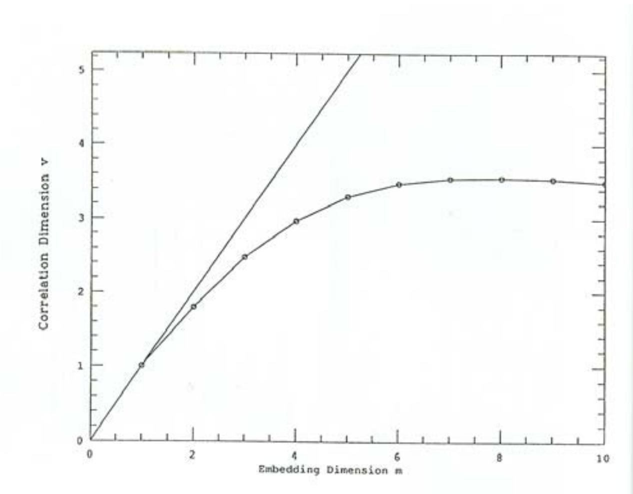
upper AU

lower AL



Reconstruction of Phase Space:

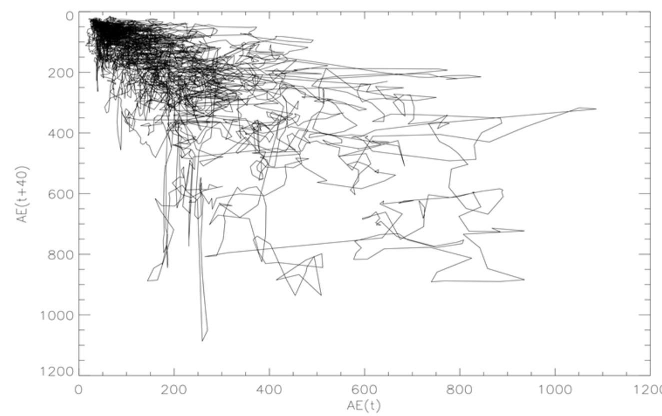
Low Dimensional Dynamics

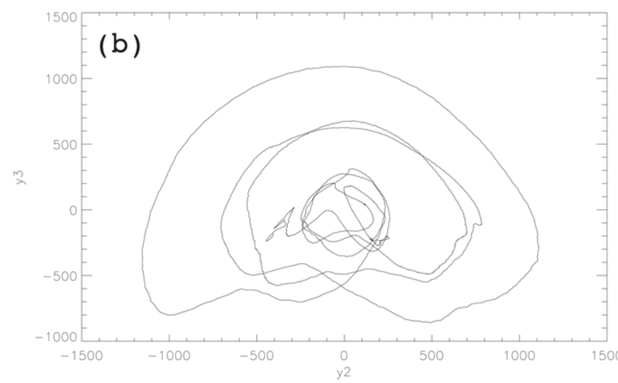


Correlation dimension

Convergence to a low value

Vassiliadis et al., GRL, 1990





Note:

Uses Magnetospherric data alone:

Autonomous system

Complex Driven System:

Input-output model

Ortho-normal coordinates

- Singular spectrum analysis

Sharma et al., GRL, 1993

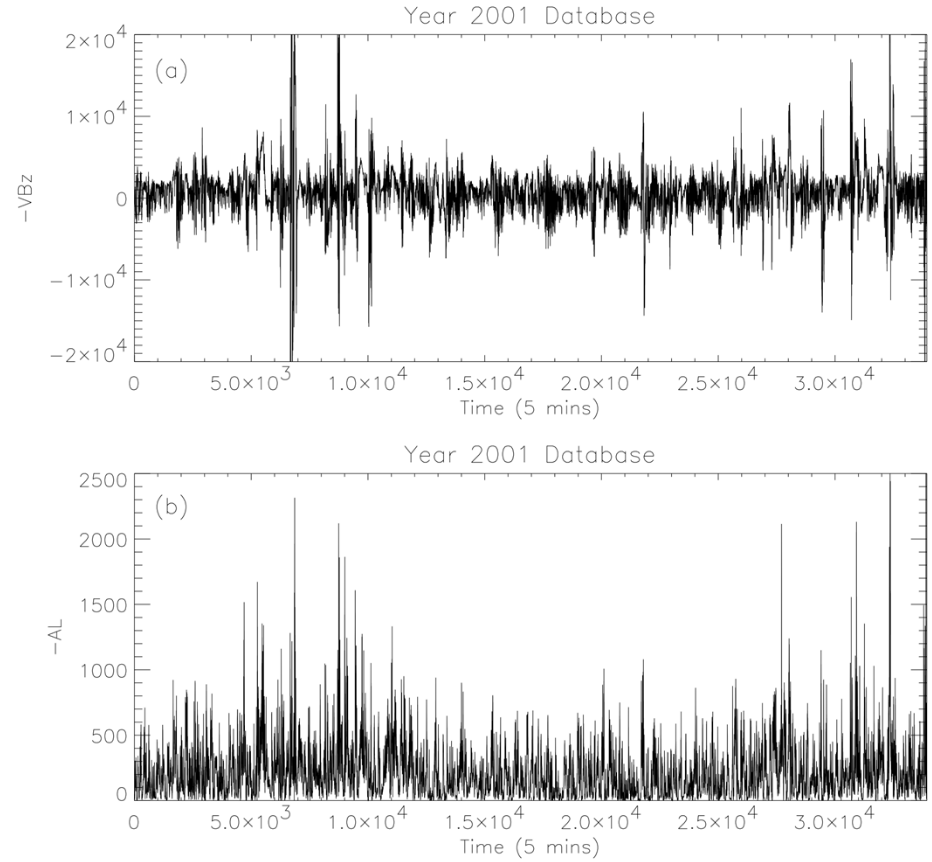
Strongly Driven Magnetosphere:

Substorms during 81 intense storms in 2001

Solar wind data
(ACE satellite)

Magnetospheric response
(AL index)

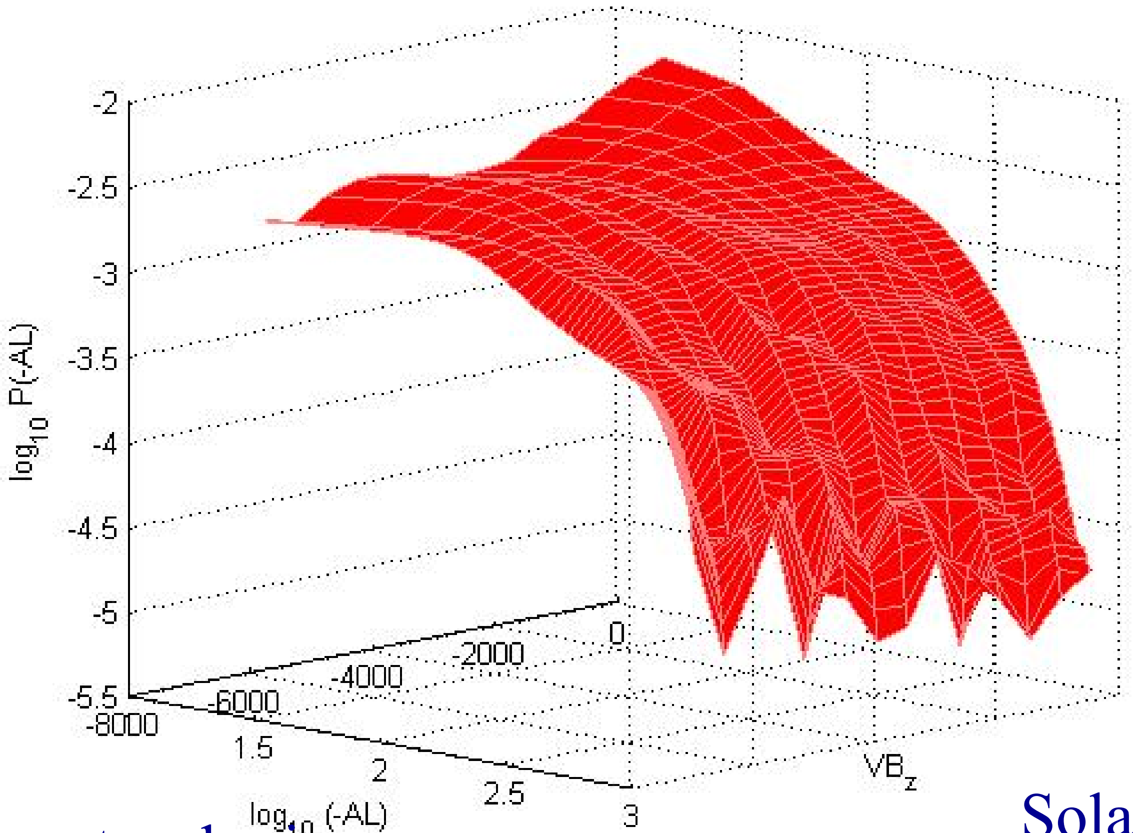
1 min res.,
0.5 x 10^6 data pts.



Chen, Ph. D. Dissertation, 2007

Conditional Probability Distribution Functions

pdf



Solar Wind E

Magnetospheric
response

Multiscale phenomenon

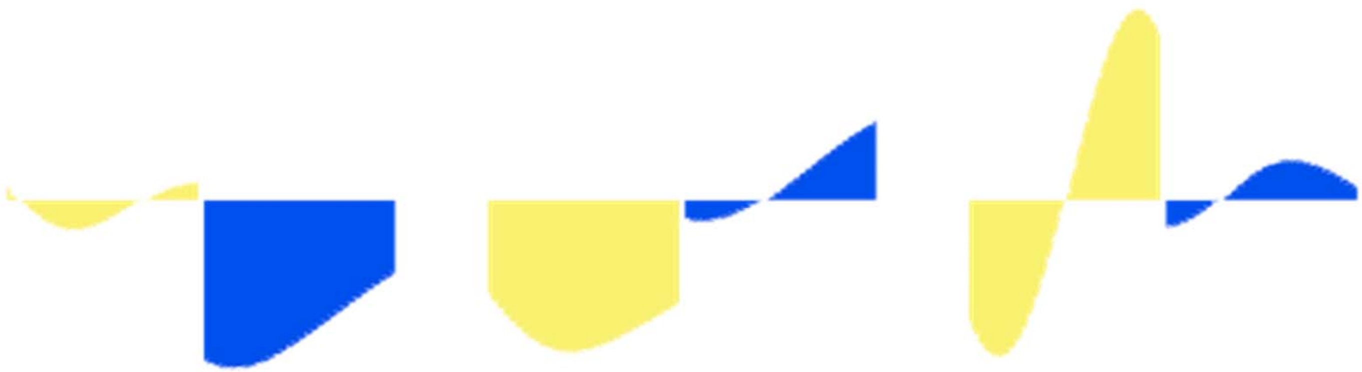
Ukhorskiy et al., 2004
Veeramani et al., 2007
Sharma et al., 2008

Reconstruction of Dynamics

- Time-delay embedding
- Singular spectrum analysis
- Input-output analysis

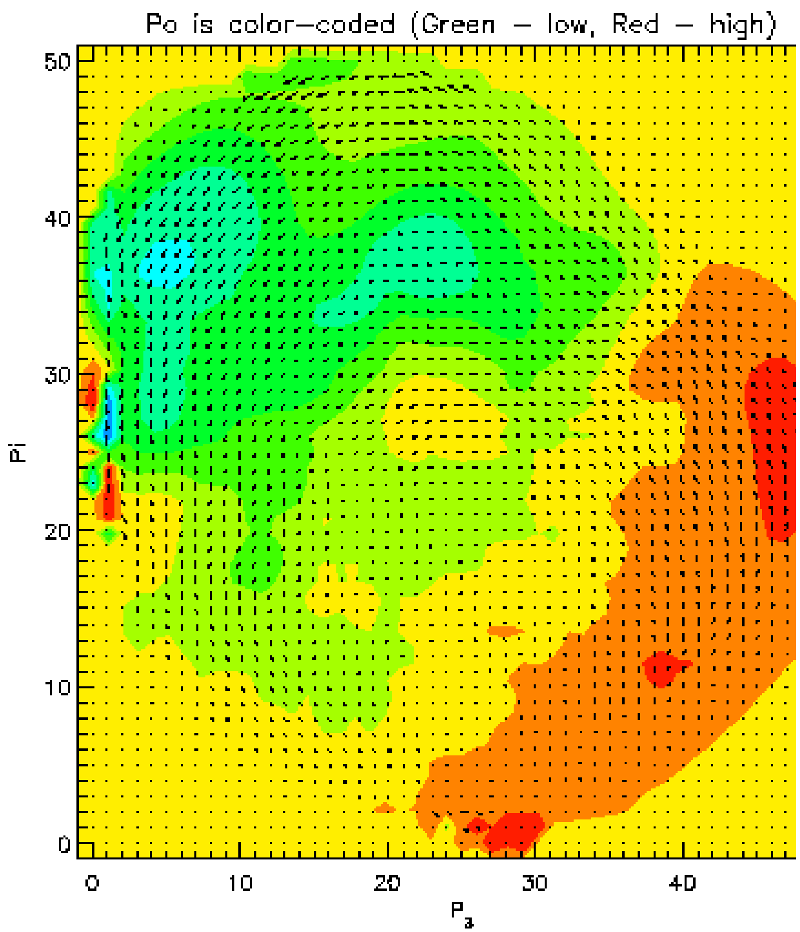
Input : solar wind induced E field

Output: auroral electrojet index AL

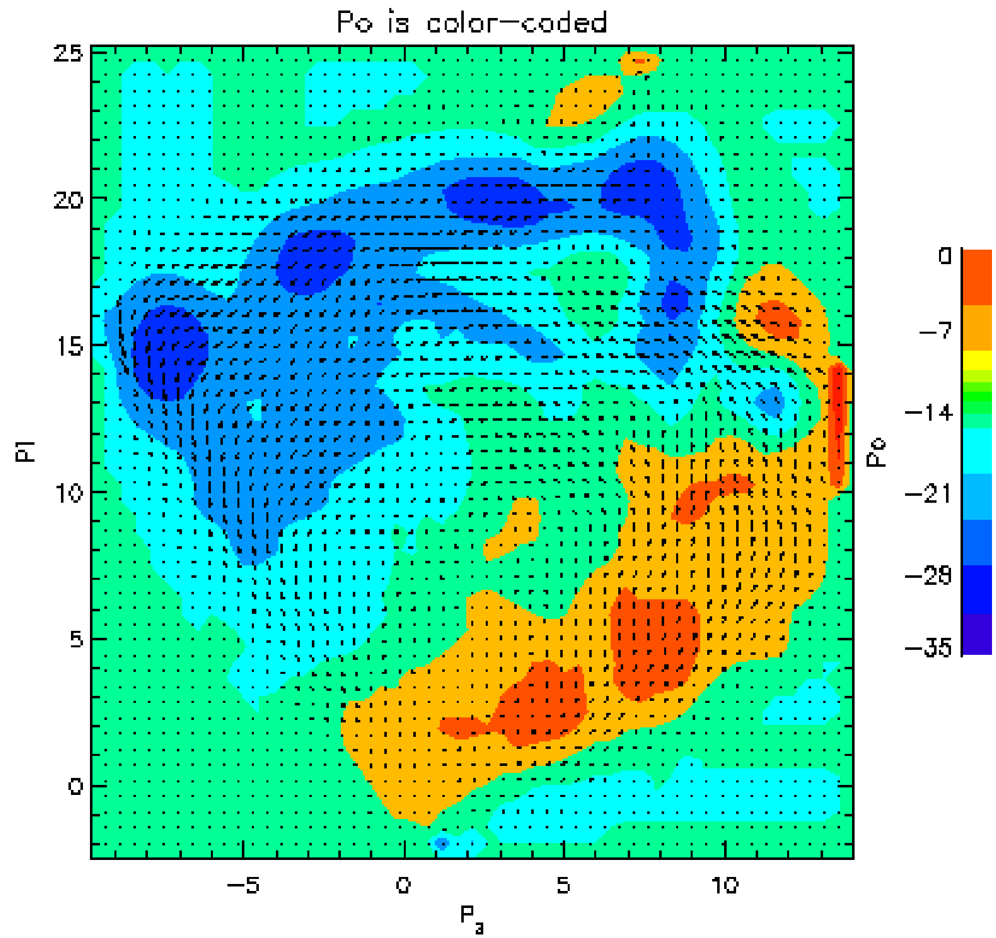


Leading components of the reconstructed space

Manifold of Magnetospheric Dynamics:
- Phase Transition-like Behavior



From vBs-AL Index Data
Sitnov et al., JGR, 2000; Phys. Rev. E, 2001.



From vBs-**Pseudo** AL Index: Global MHD simulations
Shao et al., JGR, 2003

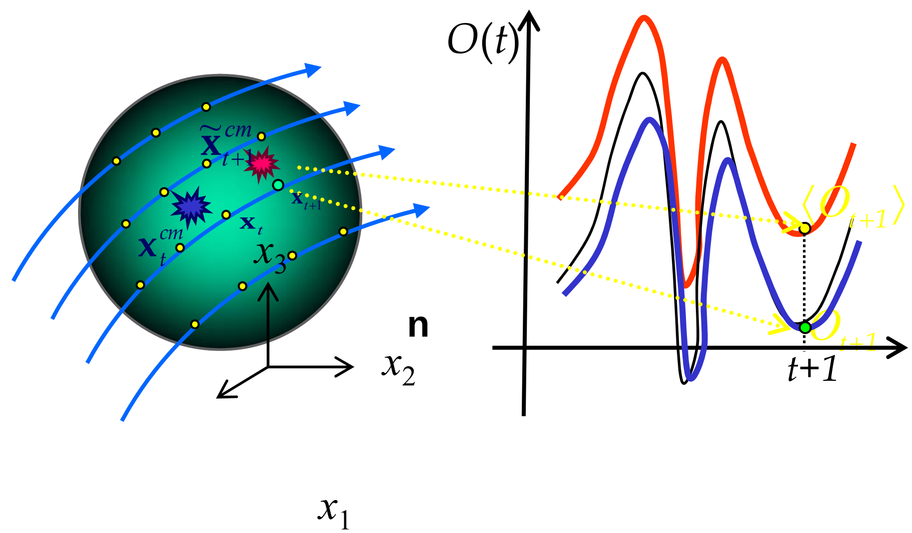
Mean-Field Model and Prediction

Nearest neighbors of current state x_t

Center of mass x_t^{cm}

- mean state for given input I_t

Mean state for $t+1$ represented by x_{t+1}^{cm}



Predicted output:

$$O_{t+1} = \mathcal{F}(\mathbf{x}_t^{cm}) = \langle O_{k+1} \rangle, \qquad \mathbf{x}_t \to \mathbf{x}_t^{cm}$$

Prediction procedure: search of mean response

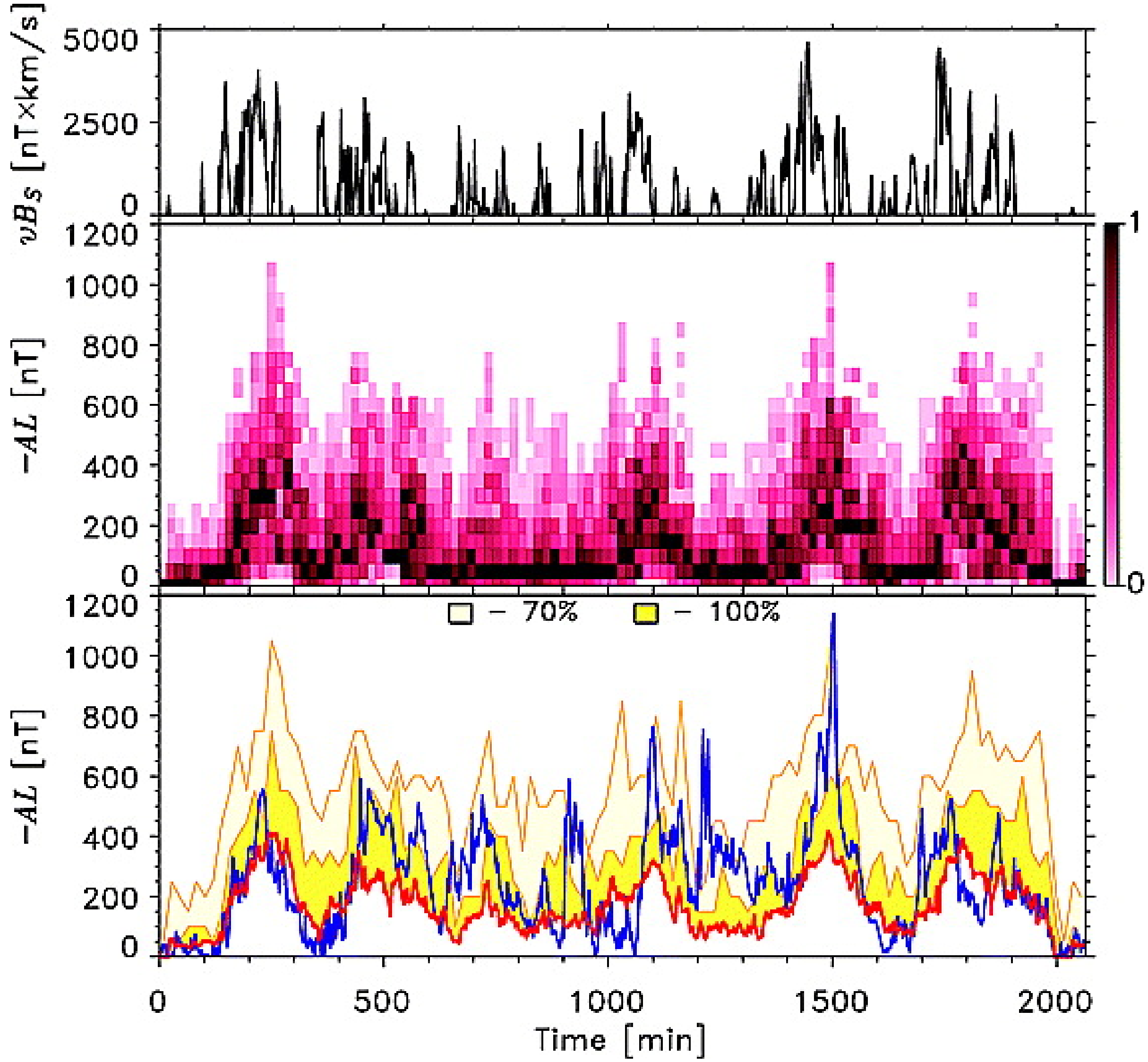
Ukhorskiy et al., *GRL*, 2002

Prediction with Mean-field Model

Solar wind conditions

Distribution of past events - Conditional probabilities

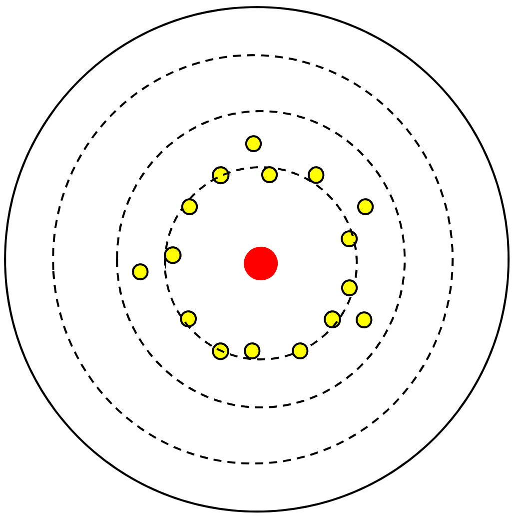
Predicted and actual AL



Ukhorskiy et al., GRL, 2004

Mean-Field & Weighted Mean-Field Models

Mean-field model



Average over all nearest neighbors

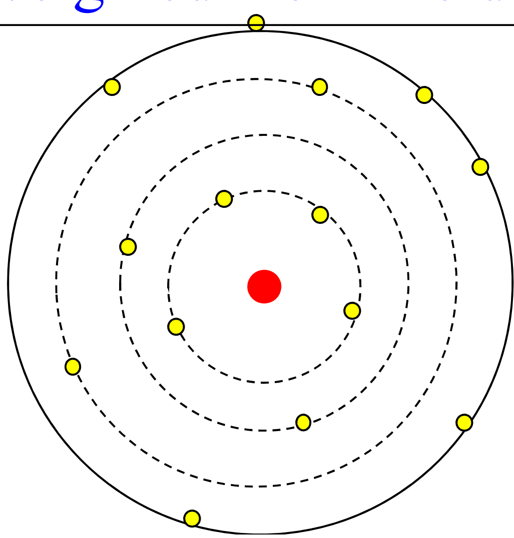
Ukhorskiy et al., *GRL, 2002, 2004*

Closer nearest neighbors should contribute more

$$O_{n+1} = \frac{1}{NN}\sum_{k=1}^{NN} X_k \bullet g_k$$

$$g_k = \frac{1}{d_k^2} \bigg/ \sum_{i=1}^{NN} \frac{1}{d_i^2}$$

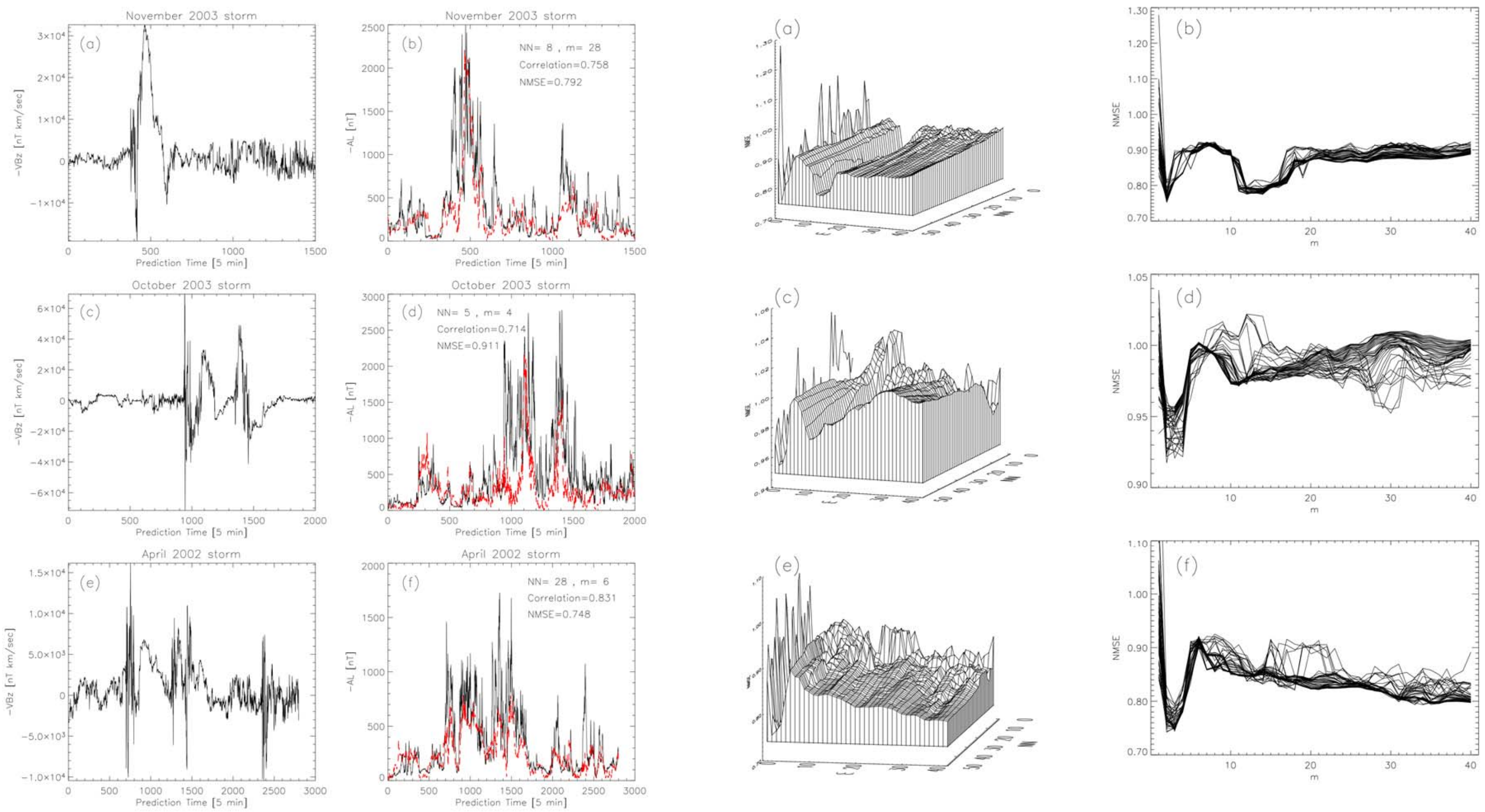
Weighted mean-field model



d_k – distance from center of mass

Chen and Sharma, *JGR, 2006,2007*

Predictions for strongly driven magnetosphere



Intense storms of 2003 (Oct 26 – Nov 3) and 2002 April

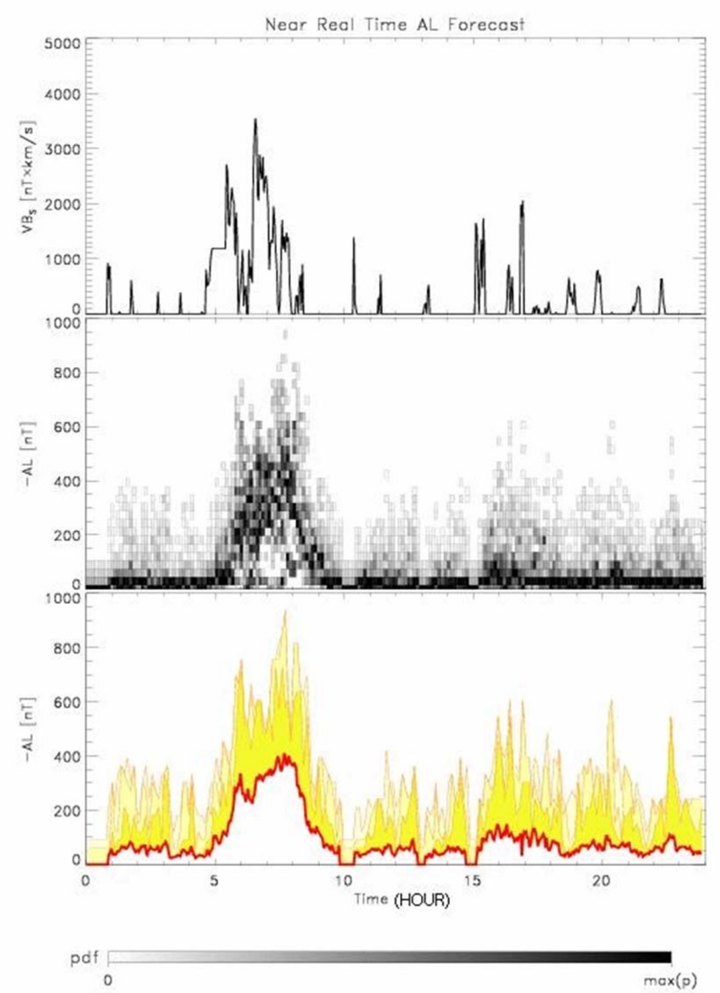
Chen et al., JGR, 2006

Forecasting Space Weather

Near Real-time data of the solar wind from ACE satellite

Forecasts of AL and Dst using the weighted mean-field model

Pdf's provide measures of predictability, and can used to estimate associated risk
Forecast using the weighted mean-field model



http://www.spp.astro.umd.edu/spaceweather

Spatio-Temporal Properties:
Information Theoretic Approach

Shannon, 1948,
Wiener, 1949
Jaynes, 1983

- Average mutual information (AMI):

$$I(x,\tau)=\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}(x(t),x(t-\tau))\log_2\left[\frac{p_{ij}(x(t),x(t-\tau))}{p_i(x(t))p_j(x(t-\tau))}\right]$$

- AMI represents the expectation of the average degree of independence incorporating all higher orders

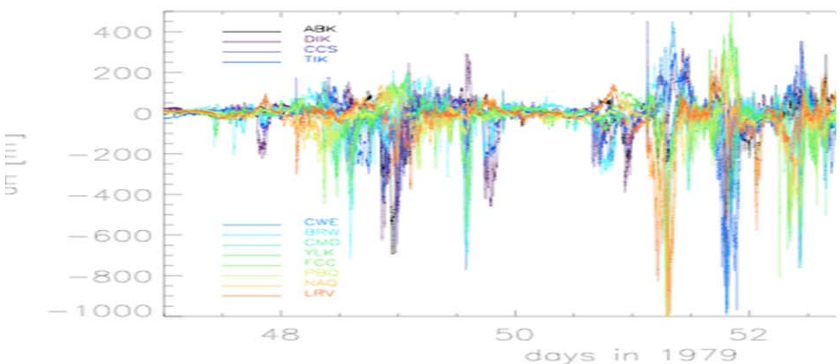
- AMIs of 12 spatially distributed magnetometer time series

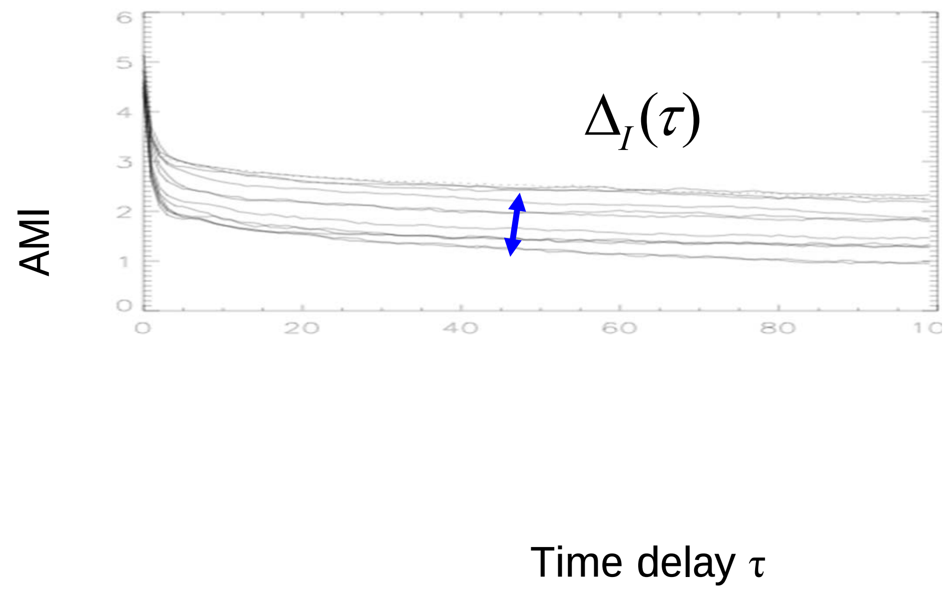
- Spread of Localized Integrated Mutual Information (SLIMI):

$$\Delta_I(\tau)=\underset{i}{Max}\{I(\tau,x_i0\}-\underset{i}{Min}\{I(\tau,x_i)\}$$

Edwards et al., AGU Spring Mtg., 2000
Chen et al., JGR, 2008

Average Mutual Information: Data from 12 Auroral stations





- AMI represents the average degree of independence or correlation time
- Spread among the AMIs of 12 spatially distributed magnetometers:

$$\Delta_I(\tau) = \underset{i}{Max}\{I(\tau, x_i 0\} - \underset{i}{Min}\{I(\tau, x_i)\}$$

Long-range Correlations:
Detrended Fluctuation Analysis (DFA)

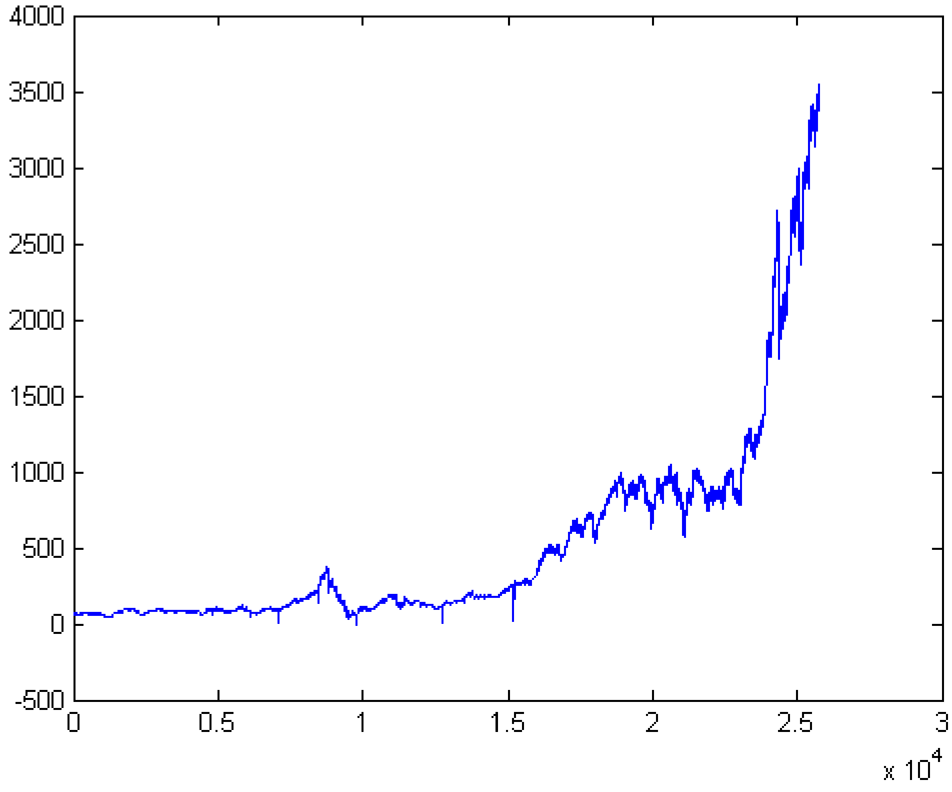
- Detection of long-range correlations in time series data
- Based on auto-correlation function C(s)
- Removes effects due to non-stationarities (?)
- Yields a scaling exponent:

$$F(s) \sim s^{\alpha}$$

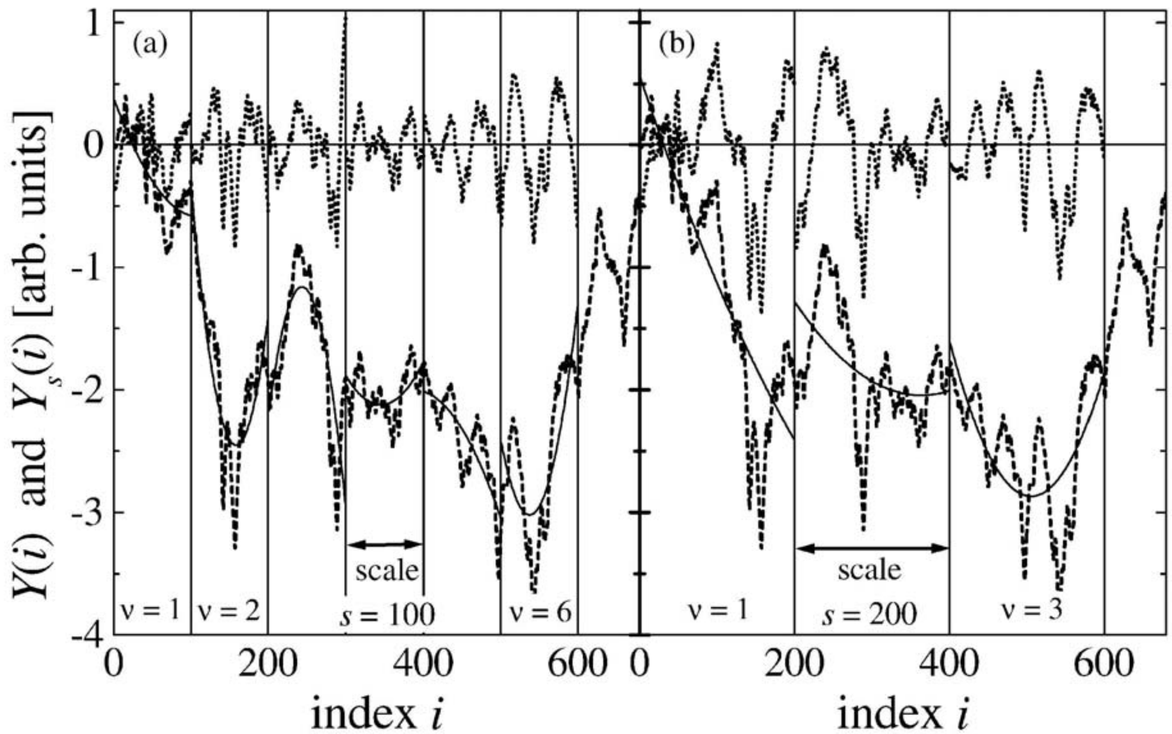
$\alpha = 0.5$ - uncorrelated (diffusion)

> 0.5 - long-range correlation

Why detrend data?



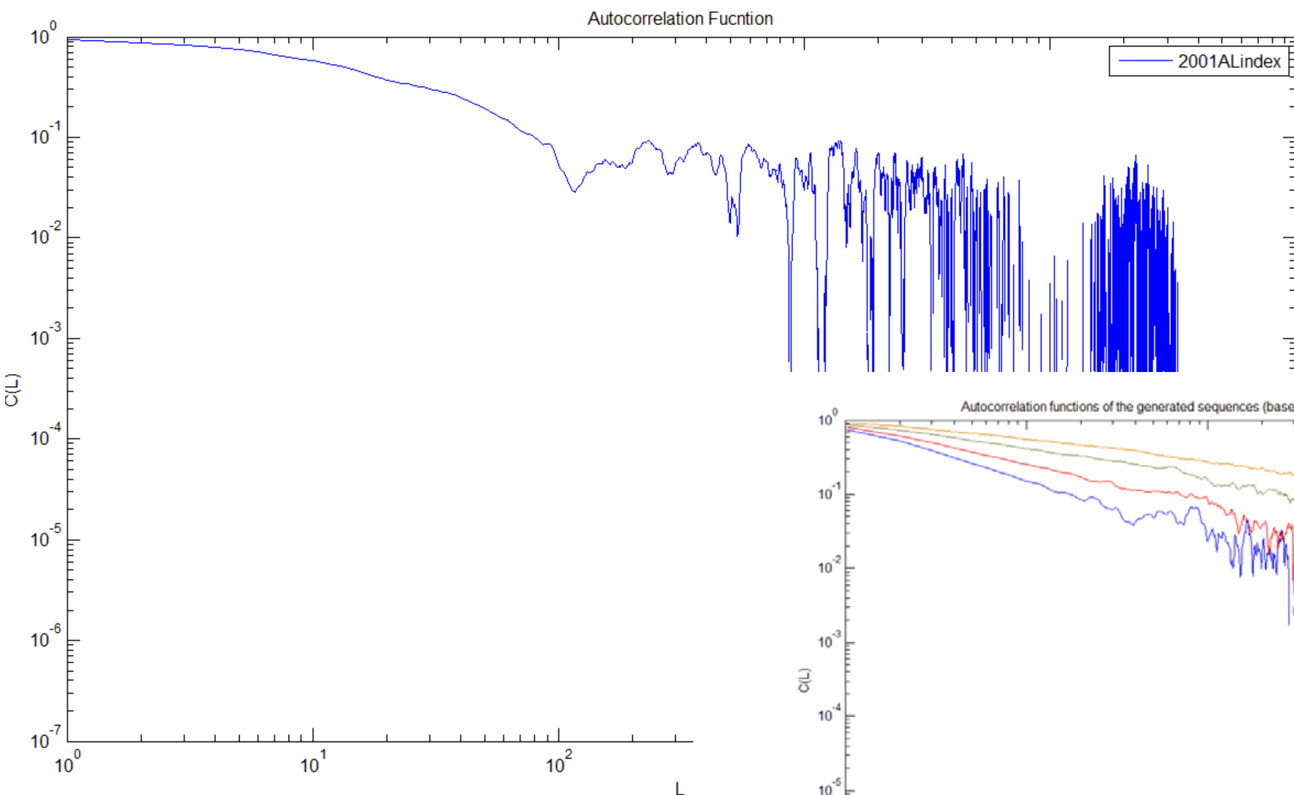
Detrended Fluctuation Analysis

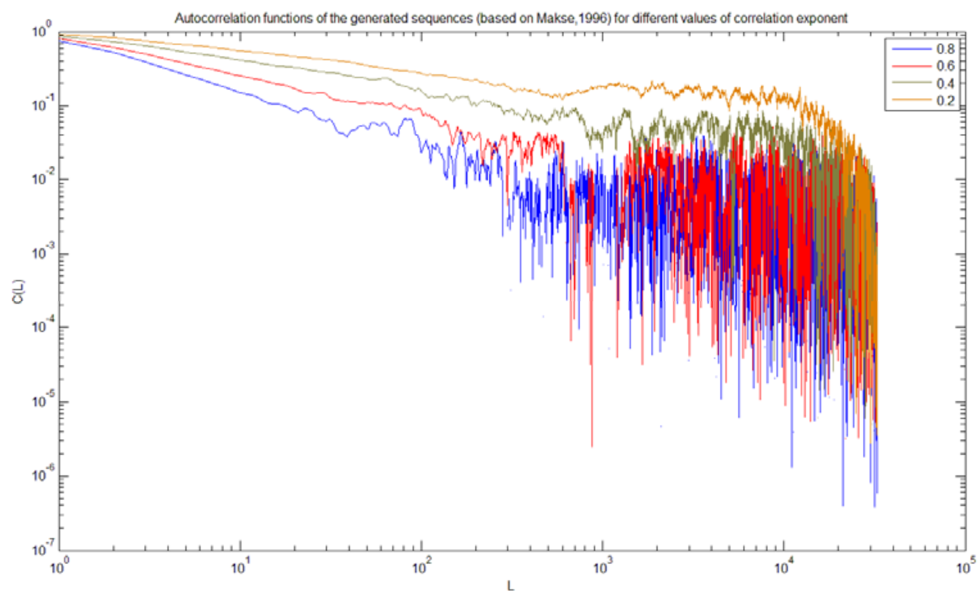


Kantelhardt et al. Physica A, 2001

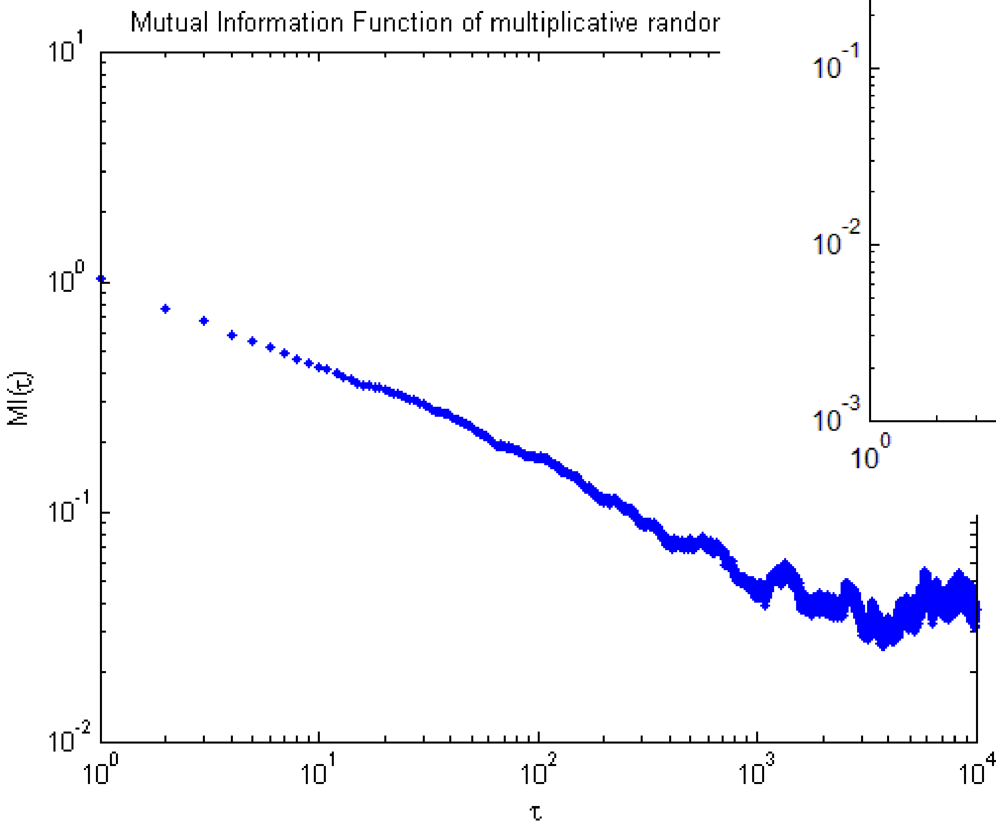
Bryce and Sprangue, Sci. Rep. 2:315, 2012

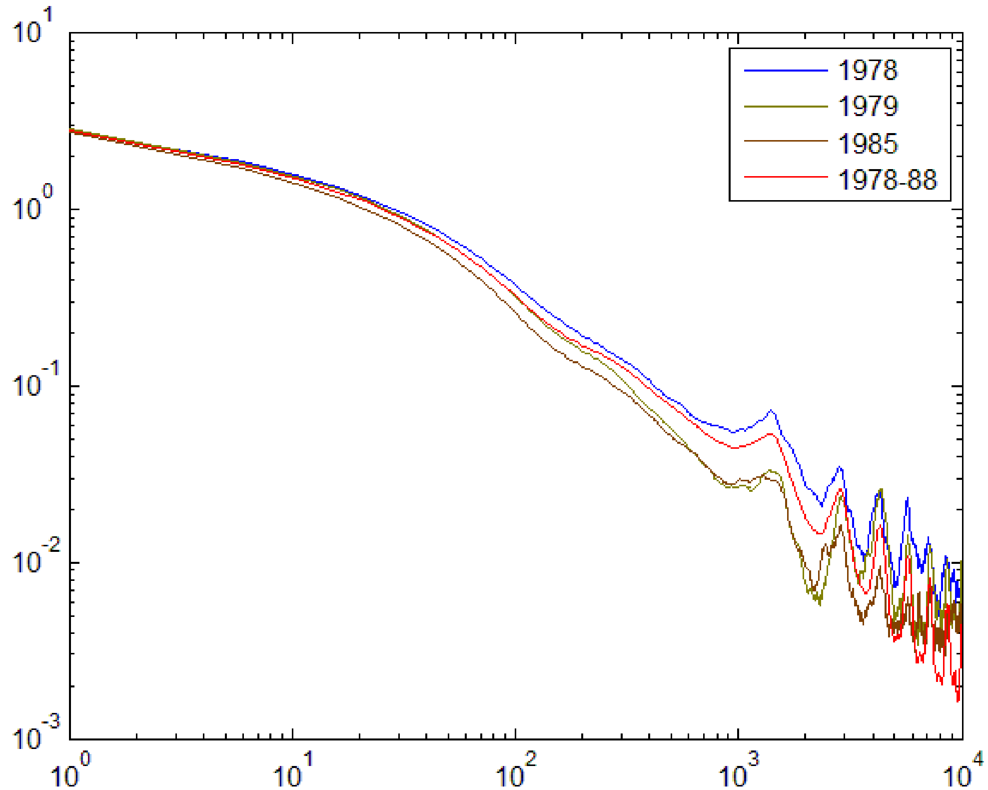
Auto-correlation Function of AL index



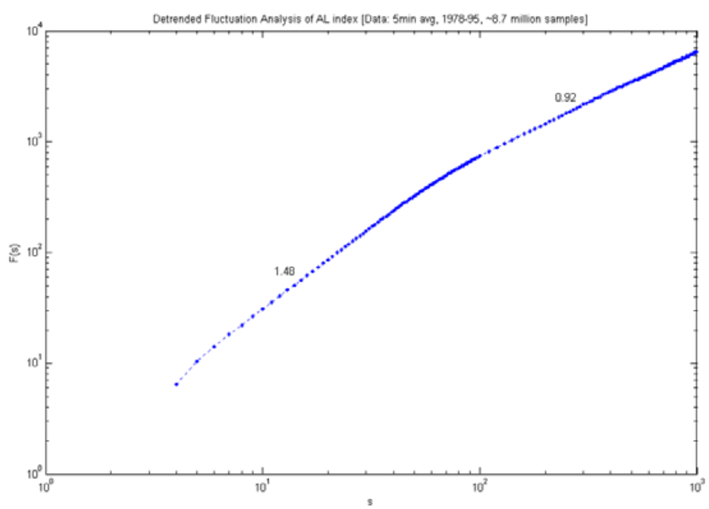


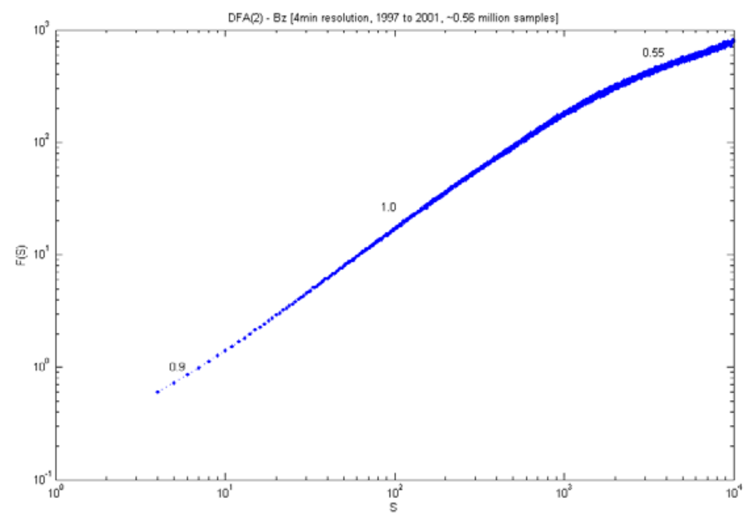
Mutual Information Function – AL index





DFA analysis of AL and Solar Wind





- Uncorrelated beyond ~ 4-5 hrs

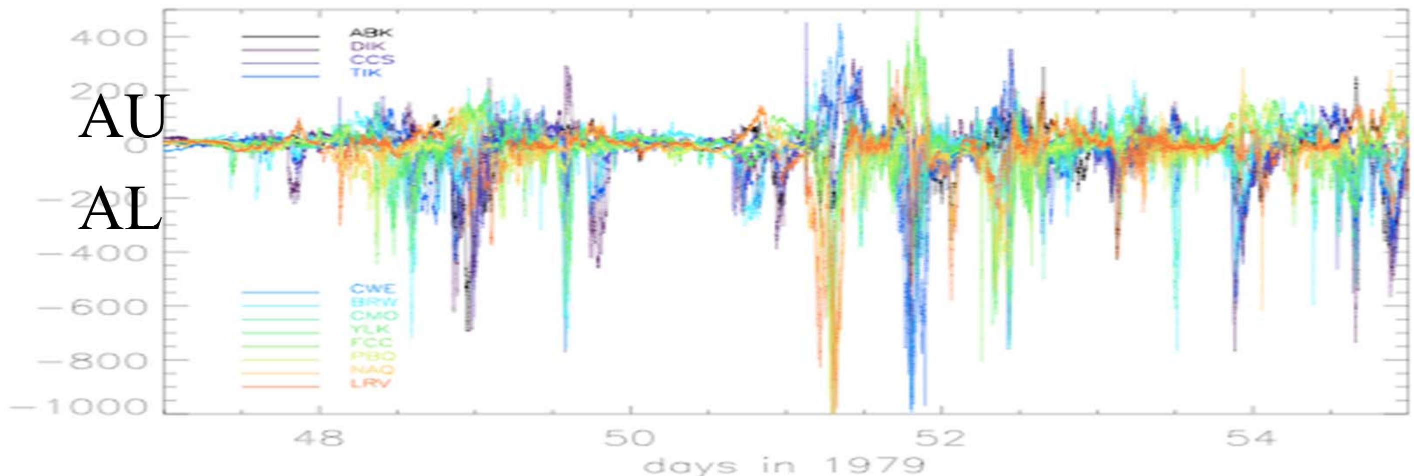
Veeramani et al., 2008

Correlation Time of 4 – 5 hrs

Envelopes:

upper AU

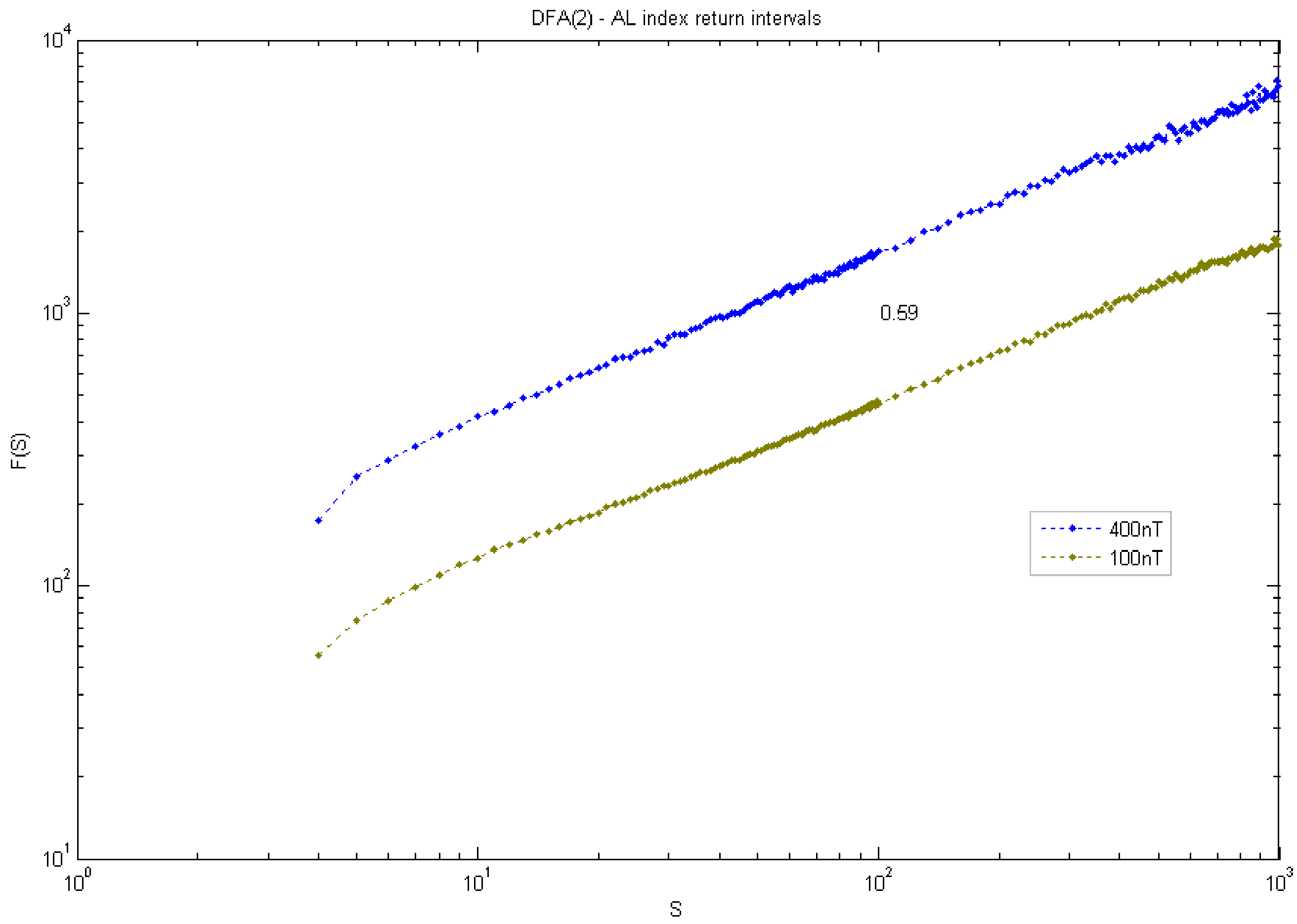
lower AL



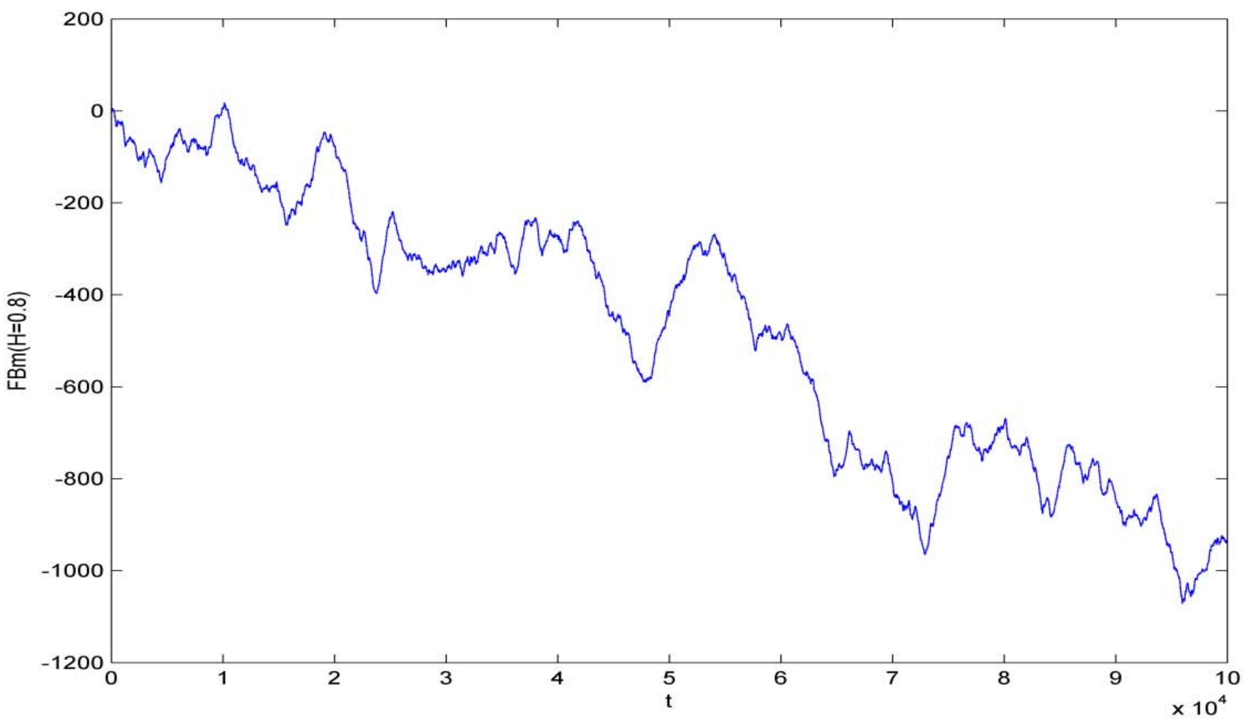
Randomness from the definition of indices

Artificial stochasticity - Anthropogenic

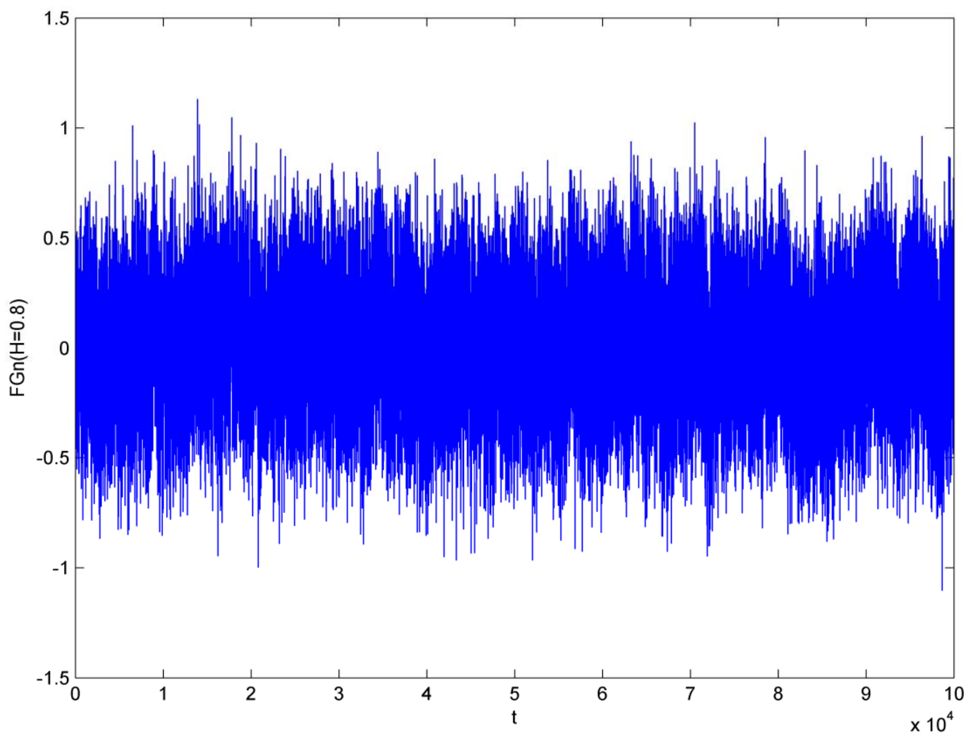
DFA Analysis – Return Intervals



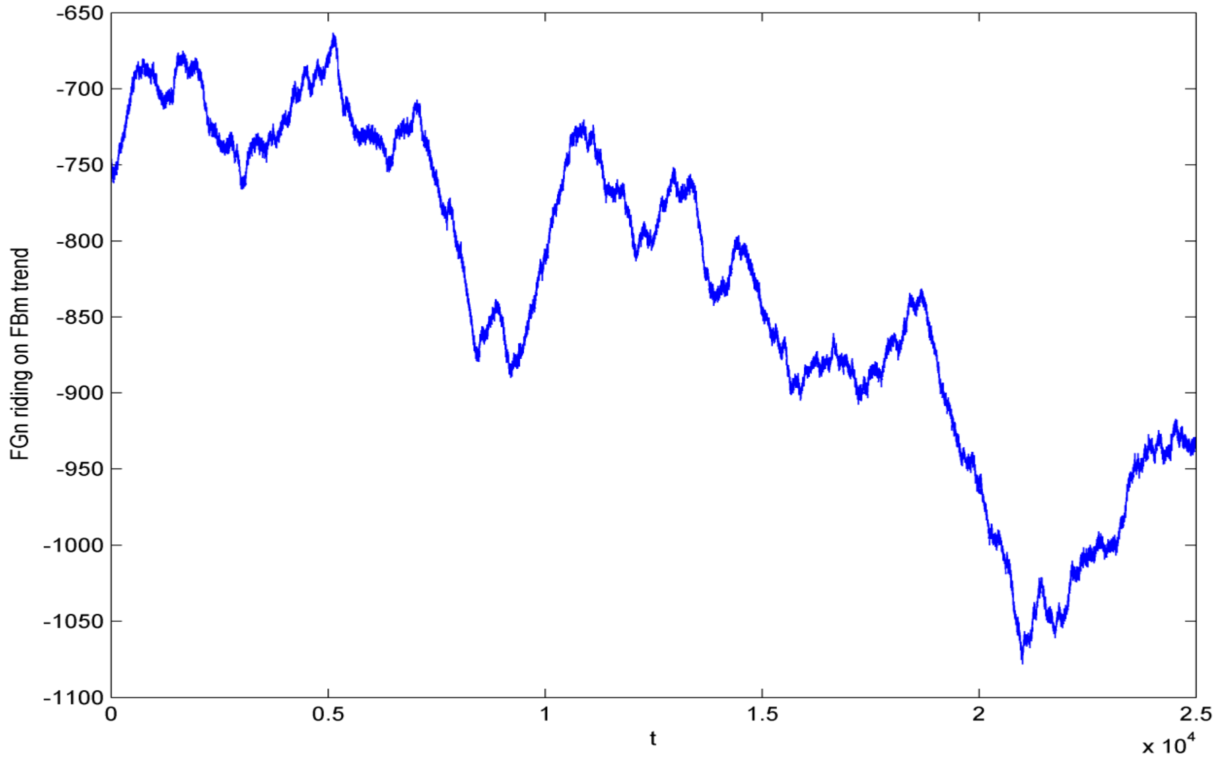
Fractional Brownian motion with Hurst exponent of 0.8



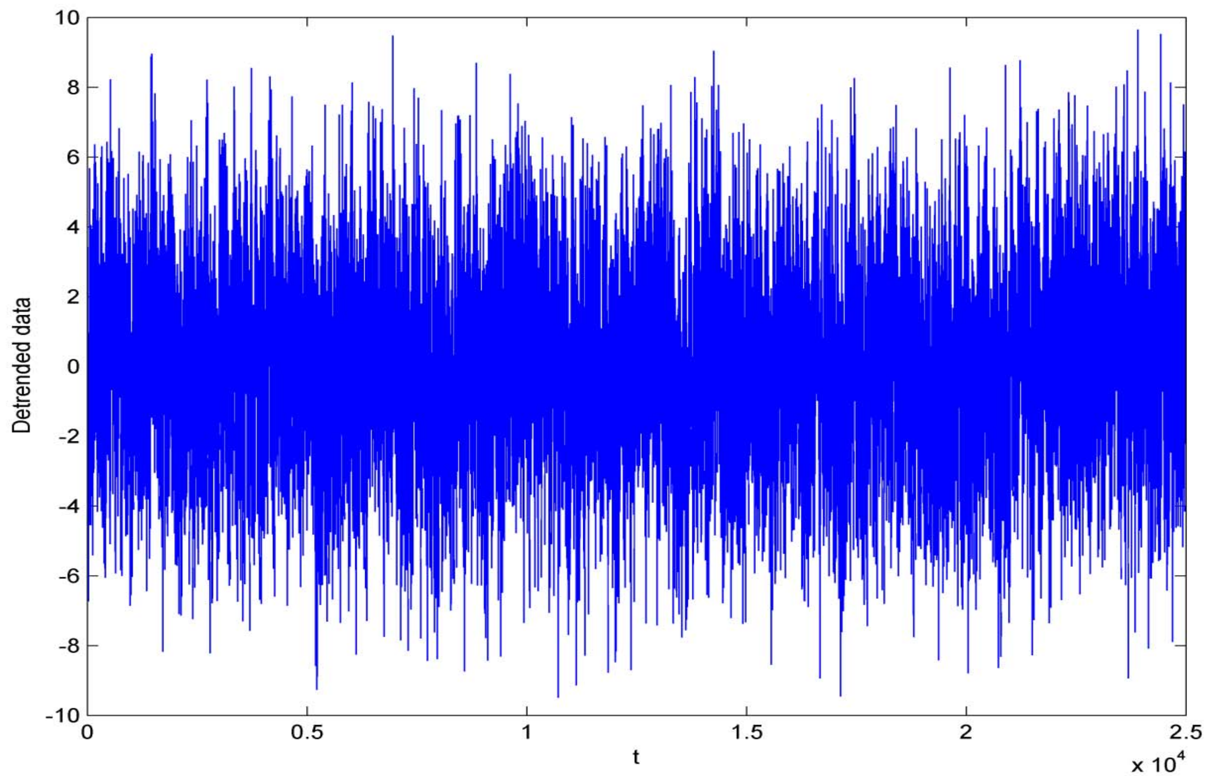
Fractional Gaussian noise with Hurst exponent of 0.8



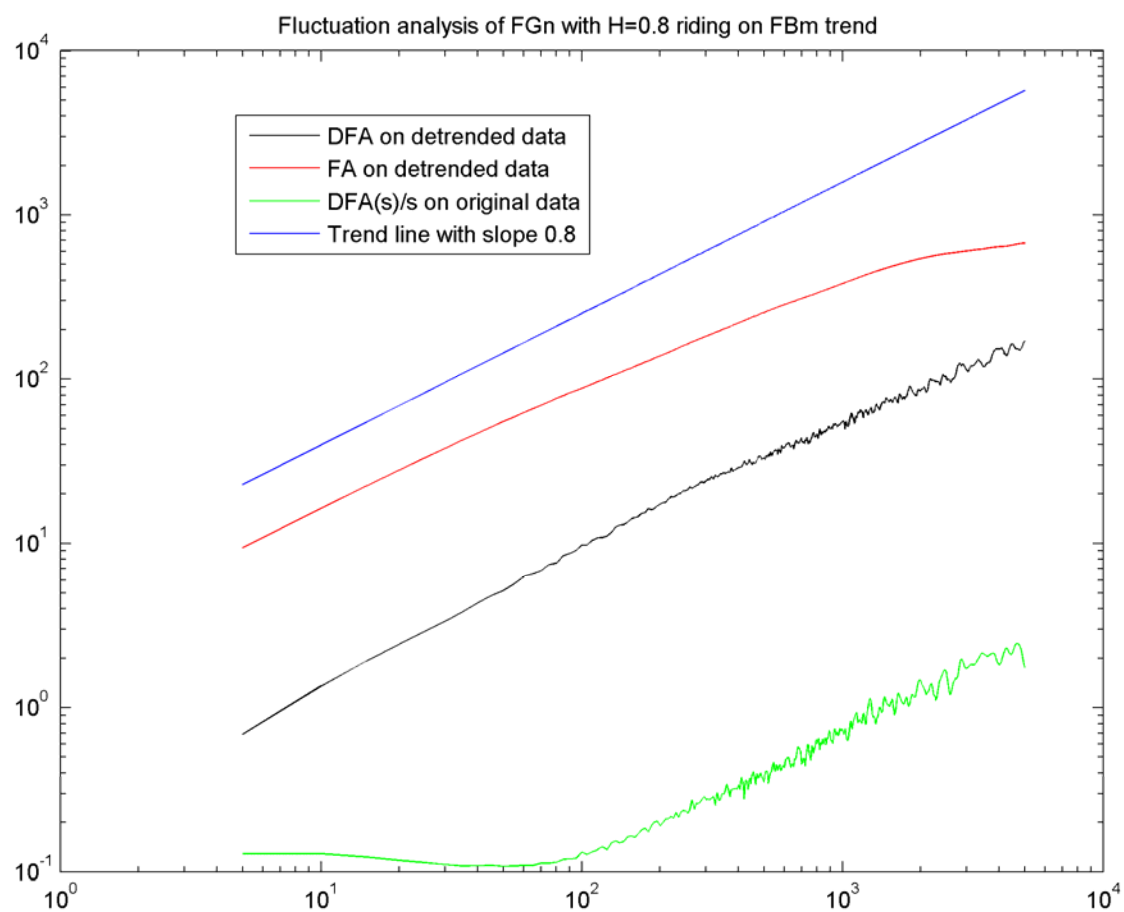
FGn data riding on FBm trend (FBm + σ.FGn)



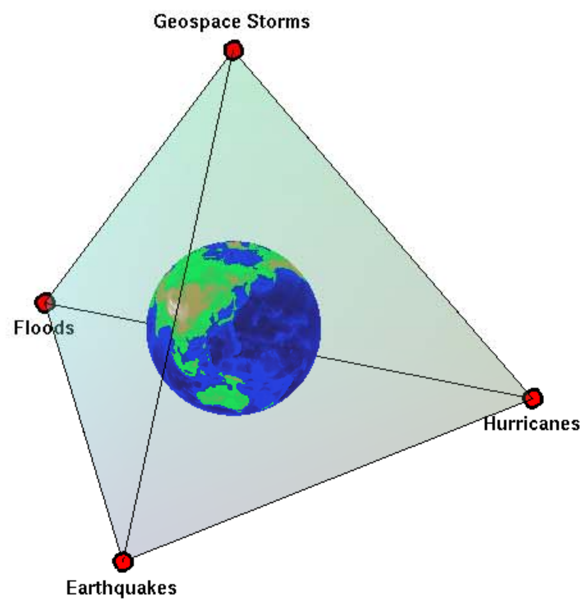
Detrended data (σ.FGn)



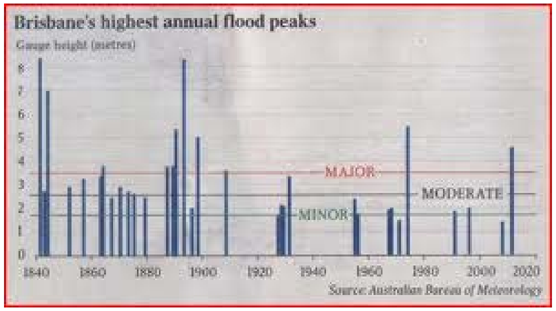
Fluctuation analysis: Actual and Detrended data

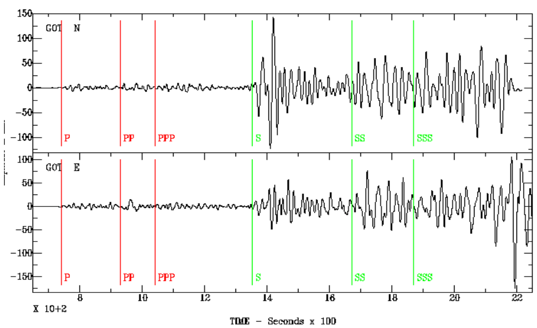


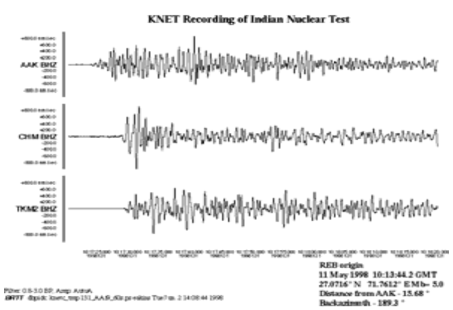
Extreme Events

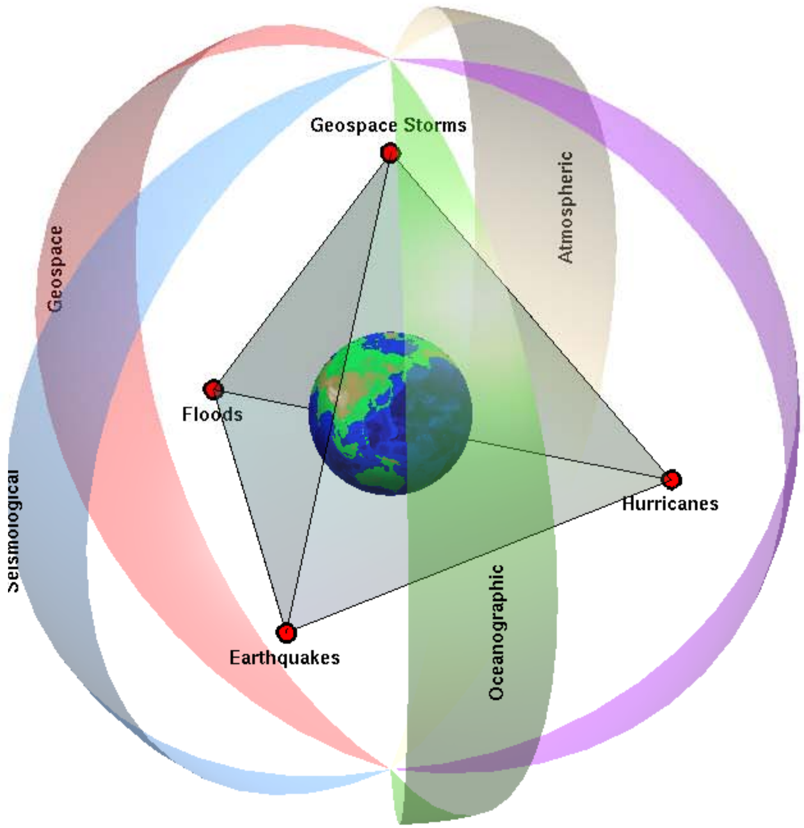


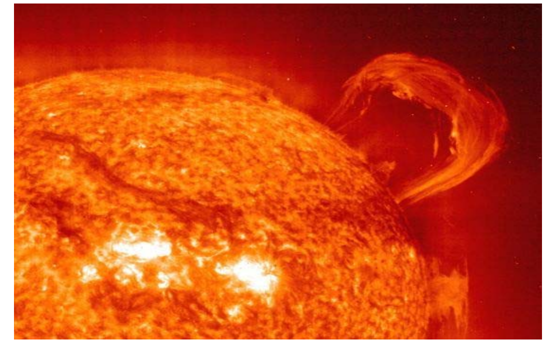
Extreme Events:
Data-enabled Modeling Framework

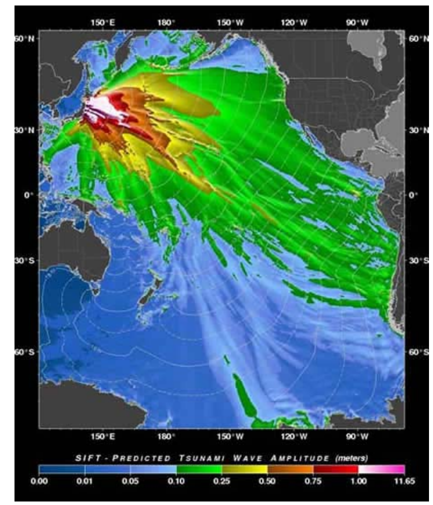












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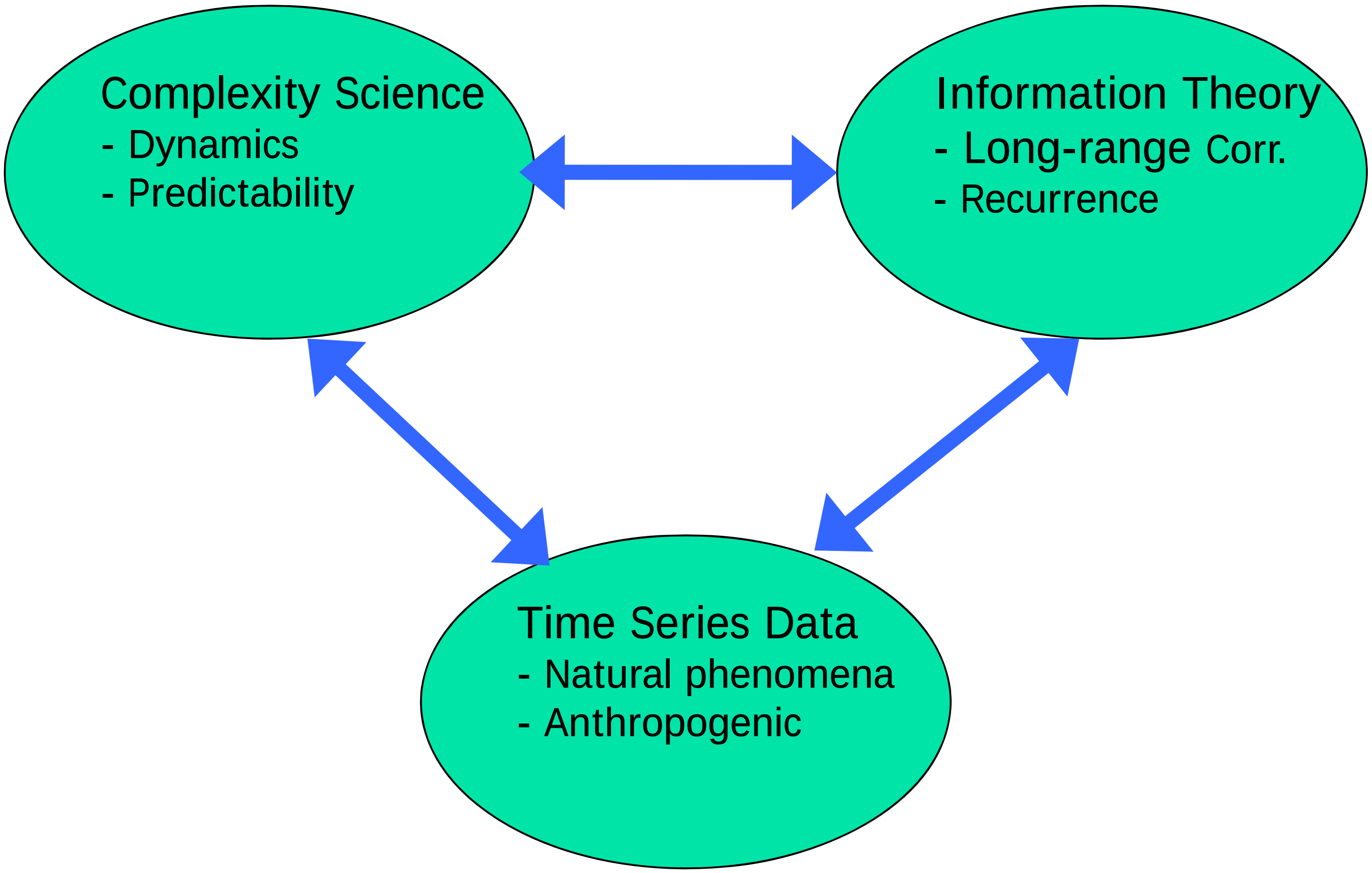

“Simple Lessons from Complexity”

Goldenfeld and Kadanoff, Science 1999

As science turns to complexity, one must realize that complexity demands attitudes quite different from those heretofore common in physics. Up to now, physicists looked for fundamental laws true for all times and all places. But each complex system is different; apparently there are no general laws of complexity. Instead, one must reach for “lessons” that might, with insight and understanding, be learned in one system and applied to another.

Criticality and Long-range correlation

Unique relationship in Second order phase transition

Not all transitions in systems with long-range correlations are critical

"Self-organized complexity"

Don Turcotte

Dynamics of systems with long-range correlations