Clock networks for future height systems: concepts and realizations

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-- Spacetime Metrology, Clocks and Relativistic Geodesy --
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Motivation

Heights and ways to determine heights

Definition of the IHRS

Clock networks for the realization of IHRS;

Clock networks for the height system unification;
Motivation
The precise height information is necessary for:

- global sea level rise;
- tunnel and bridge that connect different countries;
- floods and droughts;
- natural disasters, e.g., earthquake, landslide and sedimentation.
The geodetic reference frame is required with:

- one order of accuracy higher than the magnitude of the effects to be observed;
- consistency and reliability worldwide;
- long-term stability (the same accuracy at any time).

The geodetic reference frame can be split into:

- terrestrial reference frame;
- height reference frame;
- gravity reference frame;
- ...

Motivation
The International Terrestrial Reference Frame (ITRF):

- geometric coordinates \((X, \dot{X})\) consistent globally;
- accuracy at \textbf{mm to cm} level.
The problems for the existing height systems:

- more than 100 realizations worldwide;
- discrepancies of dm to m level (different vertical datums);
- different types of physical heights (missing standardization);
- 1 - 2 orders of accuracy less than ITRF.

To realize an **international height reference system (IHRS)** which is **highly-accurate, consistent and stable** in a global scale is now one of the main goals of the International Association of Geodesy (IAG).
Different types of heights and ways to determine heights
Height: vertical distance from the point to a reference surface.

Different types of height:

- ellipsoidal height $h$;
Height: vertical distance from the point to a reference surface.

Different types of height:

- ellipsoidal height $h$;
- orthometric and normal height $H$;
Different types of height

Height: vertical distance from the point to a reference surface.

Different types of height:
- ellipsoidal height $h$;
- orthometric and normal height $H$;
- geopotential number $C = W_0 - W_P$;

Relationships:
- $H = h - N$;
- $H = \frac{C}{\{\bar{\varepsilon}, \bar{\gamma}\}}$;
## Different types of height

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ellipsoidal height</strong></td>
<td><strong>orthometric height</strong></td>
</tr>
<tr>
<td>convenient, efficient and economically obtained;</td>
<td>both geometric and physical meaning;</td>
</tr>
<tr>
<td>no physical meaning;</td>
<td>the mean gravity is not obtained directly;</td>
</tr>
</tbody>
</table>
Ways to determine heights

- Geodetic(spirit) levelling: \( H_P = H_0 + \delta h_1 + \delta h_2 + \cdots + \delta h_n \)

- time, money and labor consuming;
- error accumulated over long distance;
- benchmark destroyed by natural disasters;
- offsets between different local height systems.
Ways to determine heights

- Geodetic(spirit) levelling: $H_P = H_0 + \delta h_1 + \delta h_2 + \cdots + \delta h_n$
- GNSS + geoid: $H_P = h_P - N_P$

- efficient, economical and high accuracy over long distance;
- fewer framework points are required;
- challenge in determining a high-accuracy and high-resolution geoid.
Ways to determine heights

- Geodetic(spirit) levelling: \( H_P = H_0 + \delta h_1 + \delta h_2 + \cdots + \delta h_n \)
- GNSS + geoid: \( H_P = h_P - N_P \)
- Chronometric levelling: \( \frac{\Delta f}{f} = \frac{W_A - W_B}{c^2} = \frac{\Delta W}{c^2} = -\frac{\Delta C}{c^2} \)

- efficient and high accuracy over large distance;
- direct measurement on potential difference resp. height differences;
Errors analysis of these methods

Geodetic levelling \( H_P = H_O + \delta h_1 + \delta h_2 + \cdots + \delta h_n \)
- random errors: mm - cm;
- systematic errors:
  - accumulated errors: 1 - 3 cm/100km;
  - offsets of datum: dm;

GNSS + geoid \( H = h - N \)
- ellipsoid height \( (h) \) error: 1 - 3 cm;
- geoid height \( (N) \) error:
  - regional: 2 - 5 cm;
  - global: dm;

Chronometric levelling \( h = \frac{\Delta W}{\bar{g}} = \frac{\Delta f}{f} \times \frac{c^2}{\bar{g}} \)
\[
\frac{\Delta f}{f} (10^{-18}) \longrightarrow \Delta W (0.1 \text{ m}^2/\text{s}^2) \longrightarrow h (1.0 \text{ cm})
\]
## Comparison of the three methods

<table>
<thead>
<tr>
<th></th>
<th>Geodetic levelling</th>
<th>GNSS + geoid</th>
<th>Chronometric levelling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pros</strong></td>
<td>• high accuracy over short distances; easy operating.</td>
<td>• high accuracy over long distances; convenient and economic; fewer framework benchmarks.</td>
<td>• high accuracy over long distances; direct measurement of potential differences; all-weather, all-day; efficient and consistent.</td>
</tr>
<tr>
<td><strong>Cons</strong></td>
<td>• high cost in time, money and labor; weather and topography depend; costly to maintain the frame; systematic errors; impossible to realize a global consistent height system.</td>
<td>• GNSS height errors; determination of a high-accuracy and high-resolution geoid is challenging; resolution is a limit.</td>
<td>• instrument is large; instrument is expensive.</td>
</tr>
</tbody>
</table>
Definition of the IHRS
IAG Resolution No.1, 2015 define the IHRS as:

- vertical coordinates are geopotential numbers $C_P$ ($C_P = W_0 - W_P$, $W_0 = 62,636,853.4$ m$^2$/s$^2$);
- the position $P$ is given by the coordinate vector $X_P$ ($X_P, Y_P, Z_P$) in the ITRF, i.e., $W(P) = W(X(P))$;
- the estimation of $X(P)$ and $W(P)$ includes their changes over time $\dot{X}(P), \dot{W}(P)$.
- the unit of length is the meter and the unit of time is the second;
- coordinates are given in the mean-tide system.
The IHRF is established with:

- a global network of reference stations:
  - a core network (perdurable, long-term stability);
  - regional and national densifications (local accessibility).
- high-precise primary coordinates ($X_p, \dot{X}_p, W_p, \dot{W}_p$) of these reference stations.

The reference stations of IHRF shall be collocated with:

- fundamental geodetic observatories (VLBI, GNSS, ...);
- reference tide gauges and national vertical networks;
- gravity reference stations.
Three approaches to determine $W_P$ or $C_P$:

1. Levelling + Gravimetry: $W_P = W_0 - C_P, C_P = \int g dh$

2. GBVP: $W_P = U_P + T_P, T_P$ (determination of geoid)

3. Global Gravity Model: $W_P = f(X, C_{nm}, S_{nm})$

Drawbacks:

1. Levelling + Gravimetry: local vertical datums, different gravity reductions, systematic levelling errors, etc.

2. GBVP: different standards, restricted accessibility to gravity data, etc.

3. GGM: limited spatial resolution, different standards, inconsistency between approaches to determine GGMs.
The classical geodetic methods and observing tools show their **limits in realizing the IHRS**, i.e., the estimation of $W_p$ for the reference points.
Clock networks for the realization of IHRS
Clock networks for IHRS

datum clock $W_0$

core clock $W_P$ or $C_P$

national clock $W_P$ or $C_P$

transportable clock
Clock networks for IHRS

- spaced clock $W_S$
- datum clock $W_0$
- core clock $W_P$ or $C_P$
- national clock $W_P$ or $C_P$
- transportable clock
The gravity potential field at different altitude:

- Earth surface: 0 km
- LEO: 450 km;
- MEO: 20000 km;
- Geostationary orbit: 35786 km.
The gravity potential field at different altitude:

- Earth surface: 0 km
- LEO: 450 km;
- MEO: 20000 km;
- Geostationary orbit: 35786 km.

The gravity potential field is more regular in a higher orbit, e.g., the geostationary orbit. Such a orbit is a good choice for the space-based clocks.
Clock networks for the unification of local height systems
To realize the IHRS, one approach is to unify local height systems.

For the unification, we have to estimate:

- offsets between height datums;
- tilts along levelling lines.

Existing approaches for the unification:

- Levelling + gravimetry;
- Oceanographic modelling;
- Modified GBVP;
We proposed to use clock networks for the height system unification, with the end-to-end simulator as:

Questions to be answered:
- how well can clock networks unify local height systems;
- how many clocks are required;
- how should the clocks be distributed;
- what is the effect of clocks’ accuracy?
The a priori height system: EUVN/2000
The a priori height system: EUVN/2000

Four height systems:
- **G1**: Marseille (France)
- **G2**: Newlyn (UK);
- **G3**: Amsterdam (Netherlands);
- **G4**: Genova (Italy).

Number of points per group:

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>29</td>
<td>16</td>
<td>73</td>
<td>84</td>
<td>202</td>
</tr>
</tbody>
</table>
The introduced errors:

- **levelling heights:**
  - random errors: 1.0 cm;
  - offsets;
  - tilts;
- **clocks:** random errors \((1.0 \times 10^{-18})\)

The four local height systems with introduced offsets (unit: cm) and tilts (unit: cm/100km)

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
</tr>
</thead>
<tbody>
<tr>
<td>offsets</td>
<td>-18.0</td>
<td>25.0</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>tilt in lat.</td>
<td>3.0</td>
<td>-2.0</td>
<td>1.5</td>
<td>-3.0</td>
</tr>
<tr>
<td>tilt in lon.</td>
<td>2.0</td>
<td>3.0</td>
<td>-1.5</td>
<td>-2.0</td>
</tr>
</tbody>
</table>
Height system unification

Four identical clocks are used in each local height system.

RMS of errors, unit in cm

<table>
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<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
</tr>
</thead>
<tbody>
<tr>
<td>true errors</td>
<td>22.81</td>
<td>62.29</td>
<td>15.36</td>
<td>53.65</td>
</tr>
<tr>
<td>adjusted errors</td>
<td>0.84</td>
<td>1.19</td>
<td>1.29</td>
<td>1.58</td>
</tr>
</tbody>
</table>

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The number of clocks: \((2, 2, 3, 3)\)

- true error
- adj. error (G1)
- adj. error (G2)
- adj. error (G3)
- adj. error (G4)
The number of clocks: \((3, 3, 3, 3)\)
Height system unification

The number of clocks: (4, 4, 4, 4)

[Graph and map showing errors and height distribution]
Height system unification

The number of clocks: \((5, 5, 5, 5)\)
Height system unification

Clock’s accuracy: $5.0 \times 10^{-18}$
Height system unification

Clock’s accuracy: $20.0 \times 10^{-18}$
Height system unification

Distribution of clocks:

- offsets and tilts along the latitudinal direction;
- number of clocks: (2, 2, 3, 2);

Two clocks
Distribution of clocks:

- offsets and tilts along the latitudinal direction;
- number of clocks: (2, 2, 3, 2);

**diagonal distribution**
Distribution of clocks:
- offsets and tilts along the latitudinal direction;
- number of clocks: (2, 2, 3, 2);

(latitudinal distribution)
Distribution of clocks:
- offsets and tilts along the latitudinal direction;
- number of clocks: (2, 2, 3, 2);

longitudinal distribution
Summary
A thorough overview of heights, measuring concepts, error sources and the IHRS was given;

A hybrid clock network which is composed of the space-based, earth-bound and transportable clocks is designed to realize a consistent, accurate and stable IHRS in the future;

- several clocks in space, e.g., in the geostationary orbit, enable the links between continental clocks or sever as the space reference;
- datum clock is the absolute origin;
- core and national clocks are the backbone of the IHRF;
- transportable clocks can be used for the densification in local areas;
The clock network can potentially be used to connect different regional height systems:

- clock networks can estimate the offsets between different height systems and the tilts along levelling lines
  - a few clocks in each region are sufficient;
  - clocks should be properly distributed so that the tilts can be detected;
  - clocks nowadays lack of accuracies for the unification, but they still show potential for unification.
Thanks for your attention!

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