Partial ionization effects on resonant absorption and Kelvin-Helmholtz instability

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Outline

Transverse oscillations in the solar corona

- First observed with *TRACE* in 1999
  Nakariakov et al. (1999); Aschwanden et al. (1999)
- After an energetic disturbance (flare), the whole loop displays a damped transverse oscillation $\sim \cos\left(2\pi t/P + \phi\right) \exp\left(-t/\tau\right)$

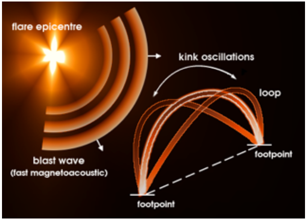
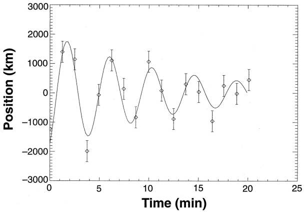


Image credit: E. Verwichte



Nakariakov et al. (1999)

- Physical interpretation: Global kink MHD mode
  see, e.g., Edwin & Roberts (1983)
- Rapid attenuation consistent with damping by resonant absorption
  see, e.g., Ruderman & Roberts (2002); Goossens et al. (2002)

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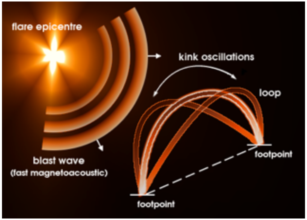

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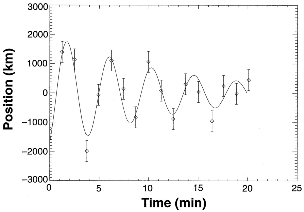

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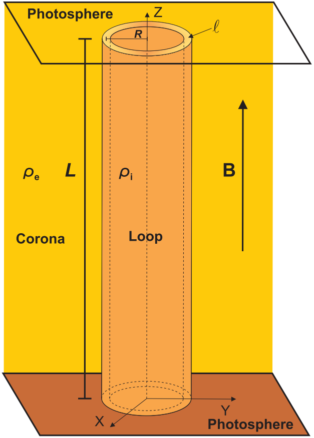
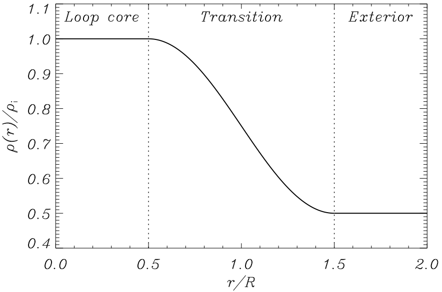
Simple model: straight magnetic cylinder





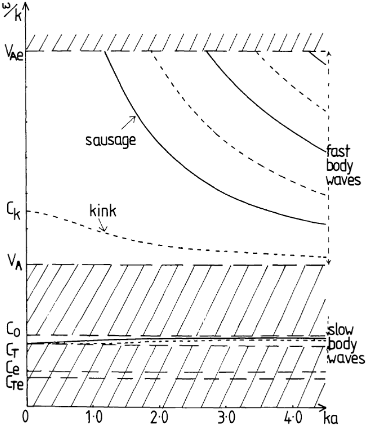
- $l = 0 \rightarrow$ Abrupt density jump
- $l = 2R \rightarrow$ Fully nonuniform tube

Fully ionized plasma

Normal modes for $l = 0$

Edwin & Roberts (1983)

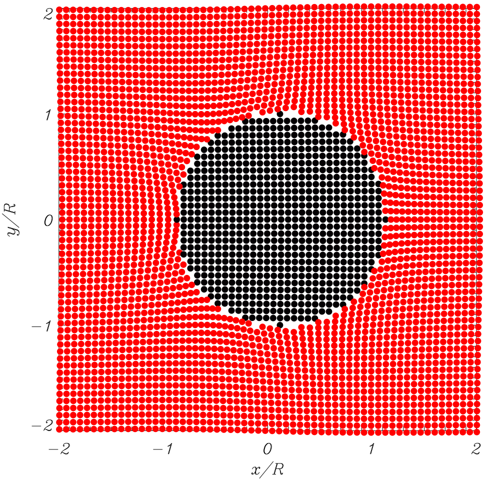
- Linear ideal MHD equations
- Perturbations: $f(r)\exp(im\varphi + ik_z z - i\omega t)$
- $f(r) \rightarrow$ Bessel functions (J_m inside, K_m outside)
- $k_z = \frac{\pi}{L} \rightarrow$ Fundamental mode



- Transverse (fast) modes
  - Sausage ($m = 0$)
  - Kink ($m = 1$)
  - Fluting ($m \geq 2$)
- Longitudinal (slow) modes
- Torsional/Rotational (Alfvén) modes

The (boring) kink mode when $l = 0$

- Global transverse (kink) motion of the flux tube
- No damping, no change of polarisation

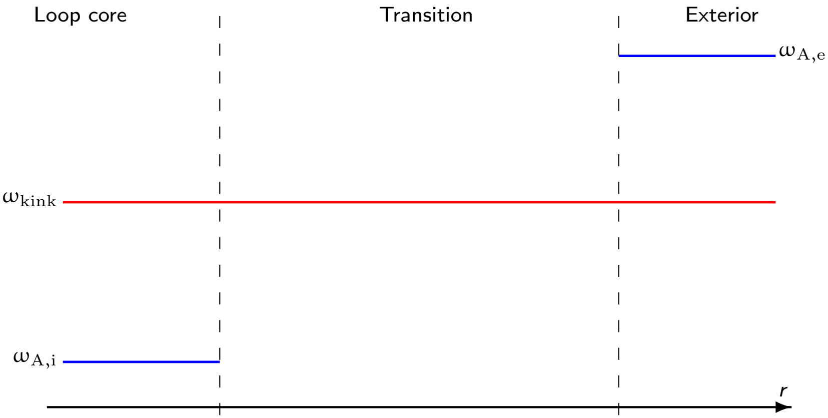


- Thin tube ($L/R \gg 1$) approximation:

$$P = \frac{L}{v_{\mathrm{A,i}}}\sqrt{\frac{2\left(\rho_{\mathrm{i}} + \rho_{\mathrm{e}}\right)}{\rho_{\mathrm{i}}}}$$

The kink mode when $l \neq 0$: Resonant absorption

- When $l \neq 0$ the kink mode is resonantly coupled to Alfvén waves



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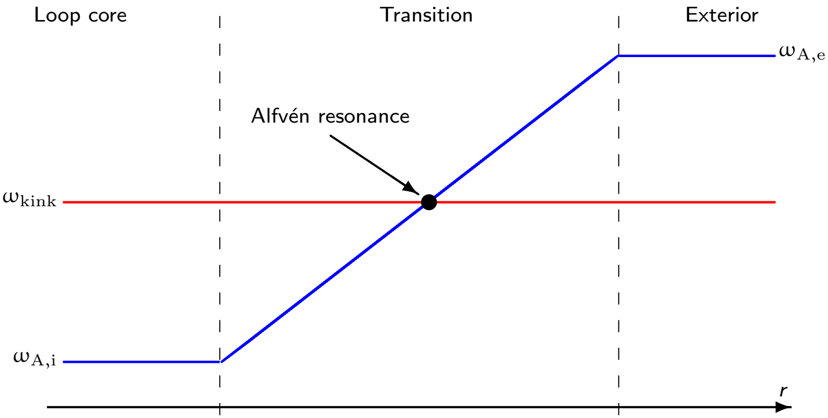
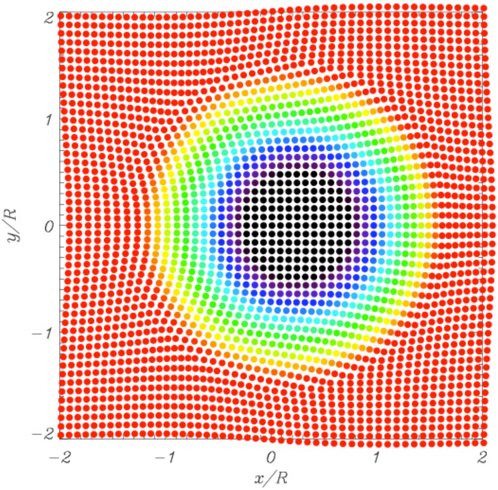


From transverse motions to rotational motions

Soler & Terradas (2015)

- Damping of the global transverse (kink) motion
- Motions become rotational (Alfvénic) in the nonuniform layer



- Thin tube ($L/R \gg 1$), thin boundary ($l/R \ll 1$) approximations:

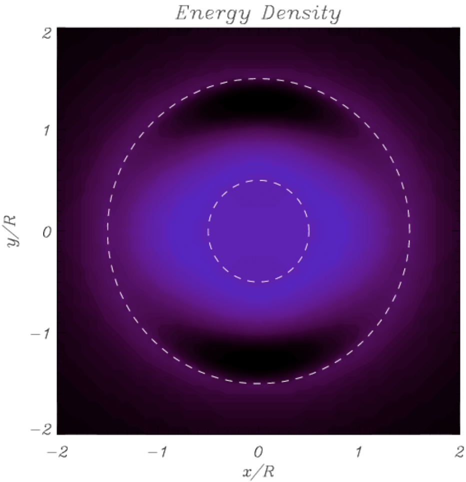
$$\tau_{\rm RA} = F \frac{R}{l} \frac{\rho_{\rm i} + \rho_{\rm e}}{\rho_{\rm i} - \rho_{\rm e}} P$$

(Ruderman & Roberts 2002, Goossens et al. 2002)

Flux of energy to the nonuniform boundary

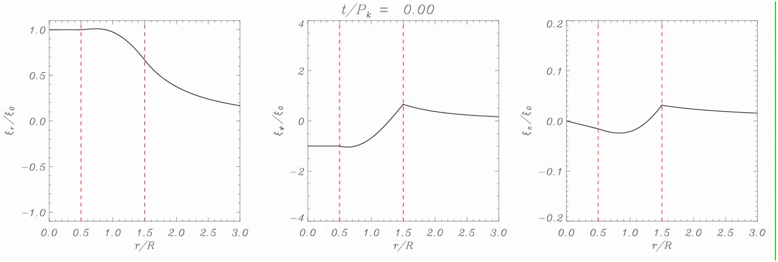
Soler & Terradas (2015)

- Wave energy becomes localized at the boundary of the flux tube
- Potential locations for plasma heating!



Phase mixing
Soler & Terradas (2015)

- Small length scales are needed for resistive/viscous heating.
- Oscillations are phase-mixed with time in the nonuniform boundary
- Smaller and smaller spatial scales are generated as time increases



- Phase-mixing length scale (Mann et al. 1995):

$$L_{\mathrm{ph}}(r=R) = \frac{2\pi}{\left|\partial \omega_{\mathrm{A}}/\partial r\right|_{r=R} t} = G \frac{L}{v_{\mathrm{A,i}}} \frac{\rho_{\mathrm{i}}+\rho_{\mathrm{e}}}{\rho_{\mathrm{i}}-\rho_{\mathrm{e}}} \frac{l}{t}$$

Resistive heating time scale

- The efficiency of resistive dissipation depends on R_m

$$R_m = \frac{L_0 v_0}{\eta}$$

- $L_0 \rightarrow$ Length scale, $L_0 \sim L_{\rm ph}(t) \sim t^{-1}$
- $v_0 \rightarrow$ Characteristic velocity, $v_0 \sim v_{\rm A,i} \approx 10^6$ m/s
- $\eta \rightarrow$ Resistivity (very small in the corona), $\eta \approx 10^9 T^{-3/2} \approx 1$ m^2/s

- Efficient dissipation/heating when $R_m \sim 1$

$$\tau_{\rm RES} \sim \frac{l\, v_{\rm A,i}}{\eta} P$$

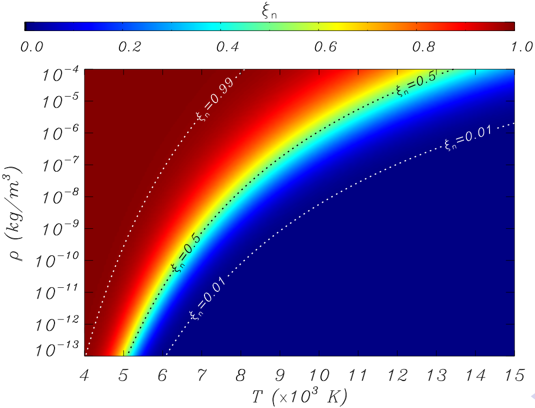
No heating during the damping of the global kink motion!

$$\tau_{\rm RES} \gg \tau_{\rm RA} > P$$

What about partial ionization?

- The plasma in the chromosphere, prominences, coronal rain. . . is only partially ionized!
- The Saha equation for a hydrogen plasma:

$$\frac{n_{\rm i}^2}{n_{\rm n}} = \left(\frac{2\pi m_{\rm e} k_{\rm B} T}{h^2}\right)^{3/2} \exp\left(-U_{\rm i}/k_{\rm B} T\right)$$



Neutral fraction

$$\xi_{\rm n} = \frac{n_{\rm n}}{n_{\rm i} + n_{\rm n}}$$

What about partial ionization?

New physics

- Collisions between ions and neutrals are an important physical process in partially ionized plasmas

Remarks

- Ion-neutral collisions are a dissipative mechanism whose efficiency is frequency-dependent (the larger the frequency, the stronger the dissipation)
- The role of ion-neutral collisions is independent of the spatial scale (unlike resistivity)
- Ion-neutral collisions do not need small scales to heat the plasma

Possible effects

- Ion-neutral collisions introduce a new dissipation time scale
- Ion-neutral collisions may affect the resonant absorption rate

Partial ionization effects on resonant absorption

- Partially ionized Hydrogen + Helium plasma
- Strongly coupled ions and neutrals
- Single-fluid approximation (e.g., Braginskii 1965)
- Ideal MHD equations + generalized induction equation with ambipolar term

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \{[\eta_{\rm A} (\nabla \times \mathbf{B}) \times \mathbf{B}] \times \mathbf{B}\}$$

- Ambipolar Diffusion $\longrightarrow$ ion-neutral collisions

Only Hydrogen (Braginskii 1965):

$$\eta_{\rm A} = \frac{\xi_{\rm n}^2}{\mu \alpha_{\rm in}}, \qquad \alpha_{\rm in} = n_{\rm i} n_{\rm n} \frac{m_{\rm i} m_{\rm n}}{m_{\rm i} + m_{\rm n}} \sqrt{\frac{m_{\rm i} + m_{\rm n}}{m_{\rm i} m_{\rm n}} \frac{8 k_{\rm B} T}{\pi}} \sigma_{\rm in}$$

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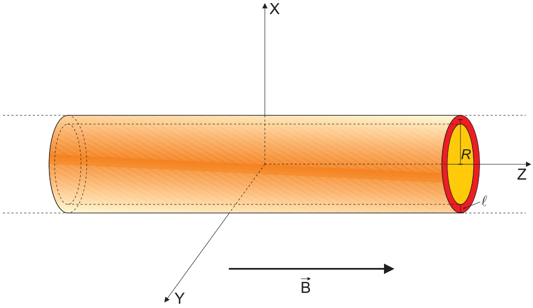
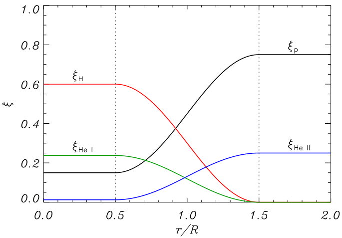
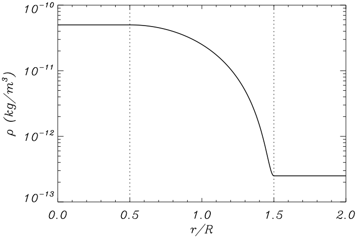
Hydrogen + Helium (Zaqarashvili et al. 2013):

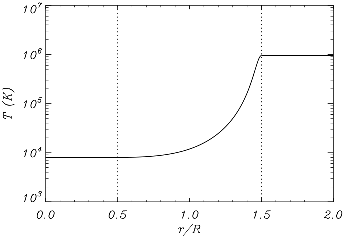
$$\eta_{\rm A} = \frac{\xi_{\rm H}^2 \alpha_{\rm He\,I} + \xi_{\rm He\,I}^2 \alpha_{\rm H} + 2\xi_{\rm H}\xi_{\rm He\,I}\alpha_{\rm H\,He\,I}}{\mu \left(\alpha_{\rm H}\alpha_{\rm He\,I} - \alpha_{\rm H\,He\,I}^2\right)}$$

Partially ionized flux tube









Time scales

- Linear eigenvalue problem $\longrightarrow$ Global kink mode $\longrightarrow$ Damping
- Thin Tube Approximation, $L/R \gg 1$
- Thin Boundary Approximation, $l/R \ll 1$

$$\frac{1}{\tau} \approx \frac{1}{\tau_{\rm RA}} + \frac{1}{\tau_{\rm AD}}$$

- Resonant Absorption
- Ambipolar Diffusion

$$\tau_{\rm RA} = F \frac{R}{l} \frac{\rho_{\rm i} + \rho_{\rm e}}{\rho_{\rm i} - \rho_{\rm e}} P$$

$$\tau_{\rm AD} = \frac{1}{\pi^2} \frac{1}{\mu \rho_{\rm i} \eta_{\rm A,i}} P^2$$

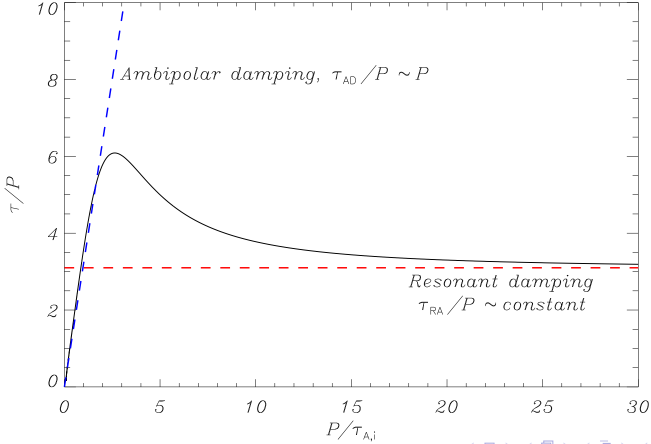
$$P = 5 \text{ min}, \quad \frac{l}{R} = 0.4 \quad \longrightarrow \quad \tau_{\rm RA} \approx 8 \text{ min}, \quad \tau_{\rm AD} \approx 85 \text{ min}$$

Critical Period $\longrightarrow$ $P_{\rm crit} = \pi^2 F \frac{R}{l} \frac{\rho_{\rm i}+\rho_{\rm e}}{\rho_{\rm i}-\rho_{\rm e}} \mu \rho_{\rm i} \eta_{\rm A,i} \approx 1$ s !!!

Numerical eigenvalue solution

Soler et al. (2009)

- Global kink mode eigenvalue problem numerically solved with PDE2D (Sewell 2005)
- Agreement with the approximate analytic results



Time scales (again)

- Ordering of time scales in the partially ionized case:

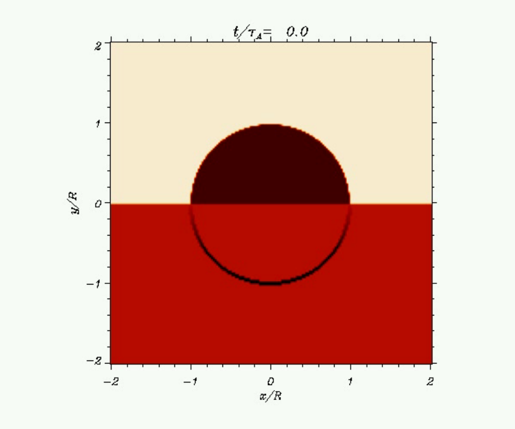
$$\tau_{\rm RES} \gg \tau_{\rm AD} > \tau_{\rm RA} > P$$

- $t < \tau_{\rm RA}$ $\longrightarrow$ Damped global kink motion (but no heating!)
- $\tau_{\rm RA} < t < \tau_{\rm AD}$ $\longrightarrow$ Phase mixing evolves (but still no heating!)
- $t > \tau_{\rm AD}$ $\longrightarrow$ Dissipation (heating) by ion-neutral collisions

- Transverse wave heating in partially ionized plasmas happens earlier than in fully ionized plasmas ($\tau_{\rm RES} \gg \tau_{\rm AD}$)
- Still, no heating during the observable kink oscillation ($\tau_{\rm AD} > \tau_{\rm RA}$)
- Ambipolar heating $\longrightarrow$ increase of T $\longrightarrow$ ionization degree increases $\longrightarrow$ efficiency of ambipolar diffusion decreases $\longrightarrow$ less heating!
- Approximate time scales should be checked using self-consistent nonlinear simulations (including ionization/recombination)

Nonlinear Kelvin-Hemlholtz instability

- Strong shear at the boundary $\longrightarrow$ Kelvin-Hemlholtz instability (Terradas et al. 2008, Antolin et al. 2014)



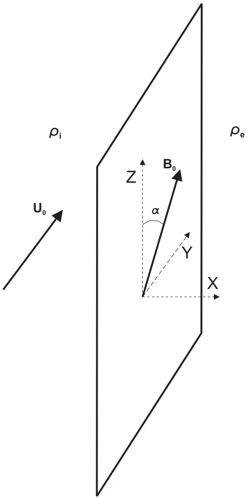
(From Terradas et al. 2008)

- Onset of the KHI ($\Delta v \perp \mathbf{B}$):

$$\frac{\Delta v}{v_{\mathrm{A,i}}} > \pi \frac{R}{L} \sqrt{\frac{2\left(\rho_{\mathrm{i}} + \rho_{\mathrm{e}}\right)}{\rho_{\mathrm{e}}}}$$

Simple interface model

Soler et al. (2010)



- Fully ionized plasma, linear analysis
- $k_y \gg k_z$ $(L/R \gg \pi)$
- Most unstable case $(\mathbf{U}_0 \perp \mathbf{B}_0,\ \alpha = 0)$:

$$\frac{U_0}{v_{\mathrm{A,i}}} > \pi \frac{R}{L} \sqrt{\frac{2\left(\rho_{\mathrm{i}} + \rho_{\mathrm{e}}\right)}{\rho_{\mathrm{e}}}}$$

- Stabilization by "magnetic twist" $(\mathbf{U}_0 \angle \mathbf{B}_0)$:

$$\sin\alpha > \frac{U_0}{v_{\mathrm{A,i}}} \sqrt{\frac{\rho_{\mathrm{e}}}{\rho_{\mathrm{i}} + \rho_{\mathrm{e}}}}$$

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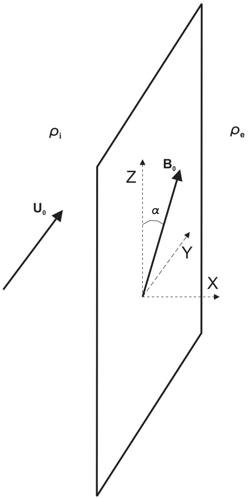

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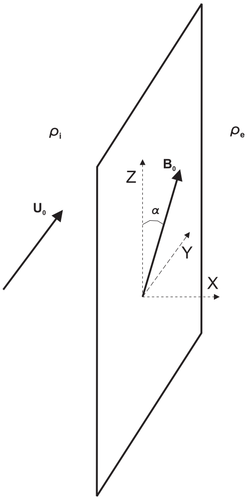

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Including partial ionization

- Linearized two-fluid equations: ions-electrons + neutrals
- We retain the distinct behavior of ions and neutrals

$$\begin{aligned}
\rho_{\mathrm{ion}} \left(\frac{\partial}{\partial t} + \mathbf{U}_0 \cdot \nabla \right) \mathbf{v}_{\mathrm{i}} &= -\nabla p_{\mathrm{ie}} + \frac{1}{\mu} \left(\nabla \times \mathbf{b} \right) \times \mathbf{B}_0 - \rho_{\mathrm{n}} \nu_{\mathrm{ni}} \left(\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{n}} \right) \\
\rho_{\mathrm{n}} \left(\frac{\partial}{\partial t} + \mathbf{U}_0 \cdot \nabla \right) \mathbf{v}_{\mathrm{n}} &= -\nabla p_{\mathrm{n}} - \rho_{\mathrm{n}} \nu_{\mathrm{ni}} \left(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{i}} \right) \\
\frac{\partial \mathbf{b}}{\partial t} &= \nabla \times \left(\mathbf{U}_0 \times \mathbf{b} \right) + \nabla \times \left(\mathbf{v}_{\mathrm{i}} \times \mathbf{B}_0 \right) \\
\nabla \cdot \mathbf{v}_{\mathrm{i}} &= \nabla \cdot \mathbf{v}_{\mathrm{n}} = 0
\end{aligned}$$

- ν_{ni} = Neutral-ion collision frequency
- Perfectly coupled fluids, $\nu_{\mathrm{ni}} \to \infty$ (ideal MHD)
- Partial ionization effects are present when ν_{ni} in finite

Approximate KHI growth rates

Soler et al. (2012) + Martínez-Gómez et al. (2015)

- We consider the interface with $\mathbf{U}_0 \perp \mathbf{B}_0$: $\mathbf{U}_0 = U_0 \hat{e}_y$, $\mathbf{B}_0 = B_0 \hat{e}_z$
- Perturbations, $\exp\left(ik_y y + ik_z z - i\omega t\right)$, $k_z = \frac{\pi}{L}$, $k_y = \frac{m}{R} = \frac{1}{R}$

Uncoupled case ($\nu_{\rm ni} = 0$)

- Ions-electrons KH unstable when $\frac{U_0}{v_{\rm A,i}} > \pi \frac{R}{L} \sqrt{\frac{2(\rho_{\rm ion,i} + \rho_{\rm ion,e})}{\rho_{\rm ion,e}}}$
- Neutrals are always KH unstable (no velocity threshold)

$$\gamma_{\rm KHI} \approx \frac{\sqrt{\rho_{\rm n,i}\rho_{\rm n,e}}}{\rho_{\rm n,i} + \rho_{\rm n,e}} \frac{U_0}{R}$$

Realistic strongly coupled case ($\nu_{\rm ni} \gg \omega$)

- Neutrals remain KH unstable for slow flows when $\nu_{\rm ni}$ is finite
- Ion-neutral collisions cannot stabilize the neutral fluid

$$\gamma_{\rm KHI} \approx \frac{2\rho_{\rm n,i}\rho_{\rm n,e}}{\left(\rho_{\rm n,i} + \rho_{\rm n,e}\right)^2} \frac{U_0^2/R^2}{\nu_{\rm ni}}$$

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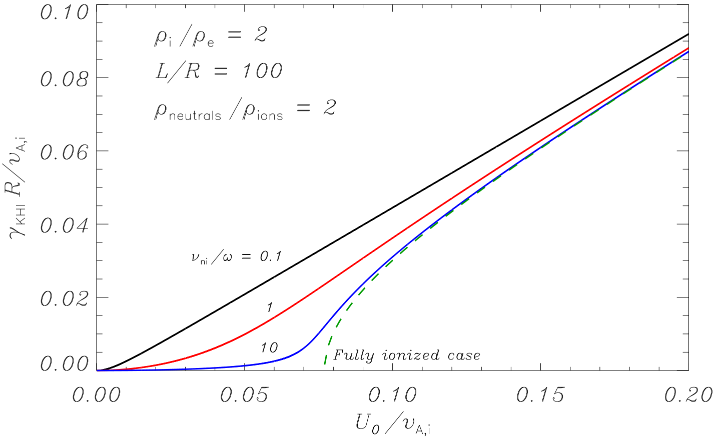
Full solution

Soler et al. (2012) + Martínez-Gómez et al. (2015)

- Numerical solution of the dispersion relation



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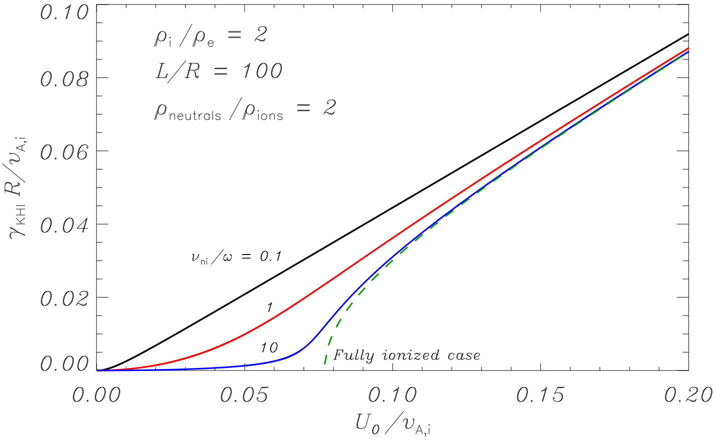

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Conclusions

Effects on resonant absorption and associated heating

- Partial ionization introduces a new time scale, τ_{AD}, for the dissipation of wave energy by ambipolar diffusion
- Heating by ambipolar diffusion in partially ionized plasmas happens earlier than heating by resistivity in fully ionized plasmas, $\tau_{\mathrm{AD}} \ll \tau_{\mathrm{RES}}$
- No heating during the observable global kink motion, $\tau_{\mathrm{RA}} < \tau_{\mathrm{AD}}$

Effects on the KHI onset

- Partially ionized tubes are more unstable than fully ionized tubes
- No velocity threshold and no effect of magnetic twist on neutrals
- Neutrals can trigger the KHI even in twisted flux tubes

Partial ionization effects are not negligible