Multi-fluid magnetohydrodynamics with neutral hydrogen and helium atoms

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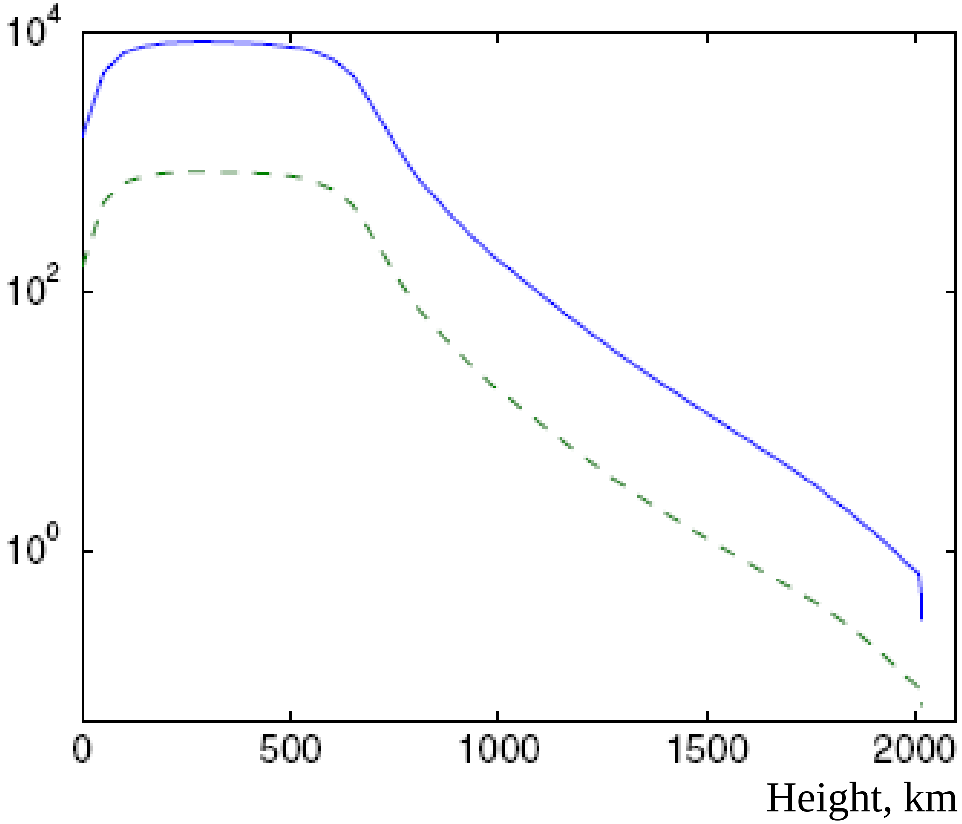
Neutral atoms in the solar atmosphere



FAL93-3 model (Fontenla et al. 1993)

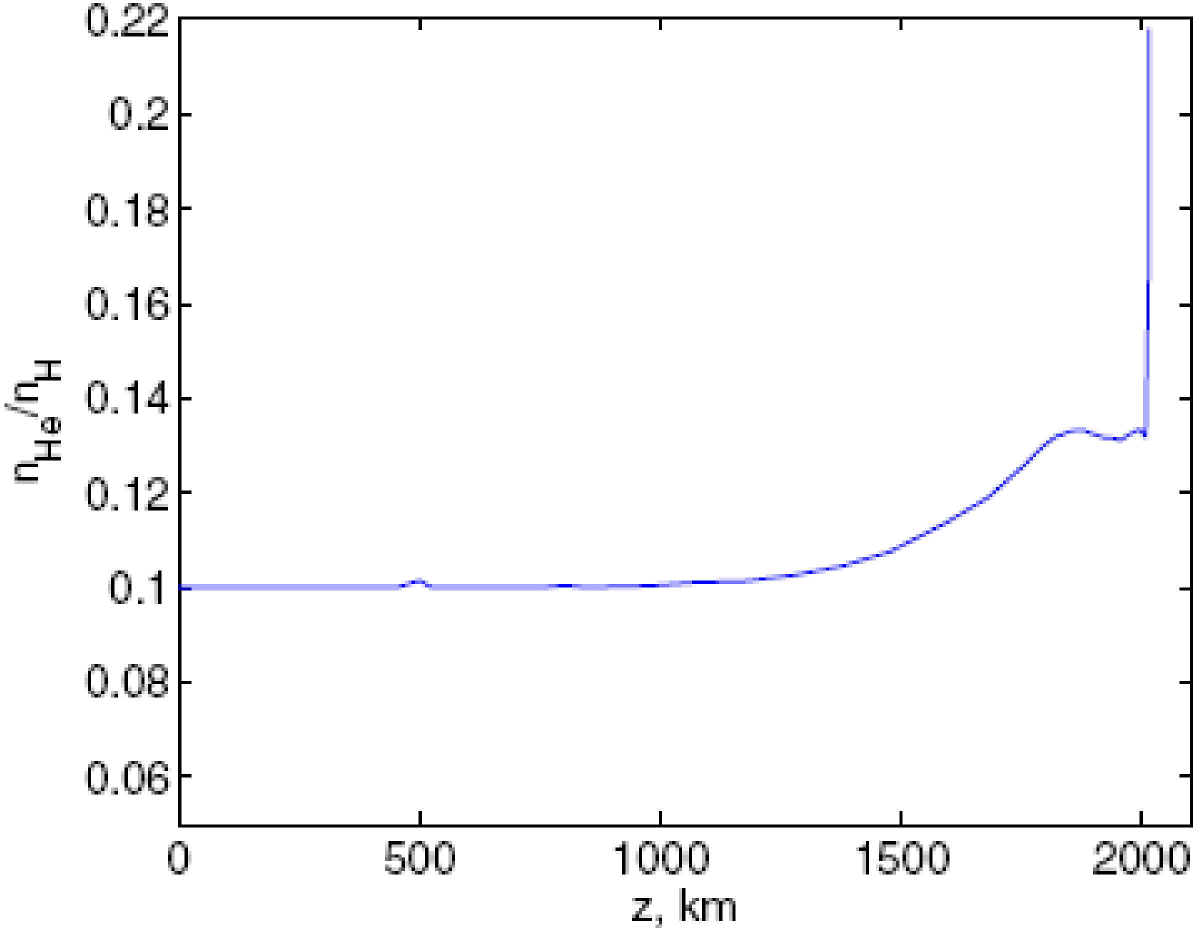
Blue solid line: ratio of neutral hydrogen and electron number densities.

Green dashed line: ratio of neutral helium and electron number densities.

Plasma is only weekly ionized in the photosphere, but becomes almost fully ionized in the transition region and corona.

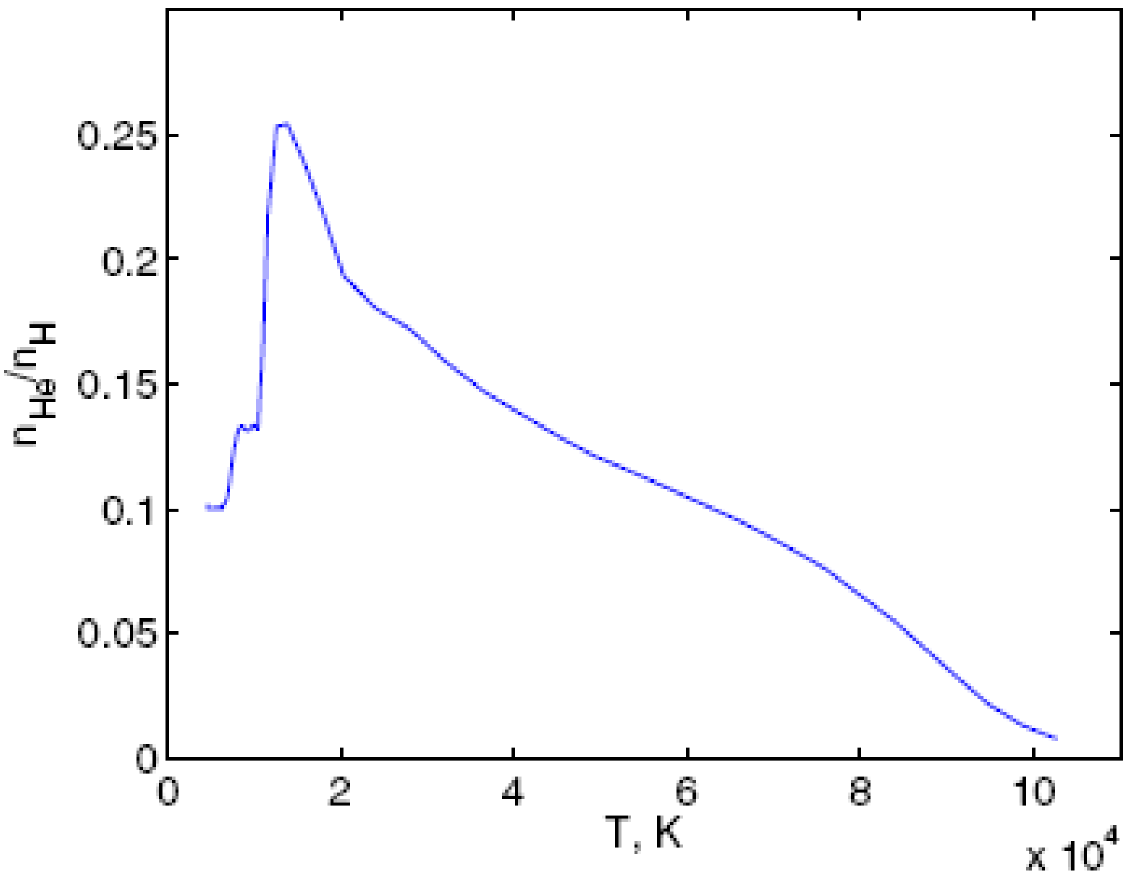
Neutral helium in the solar atmosphere

The ratio of neutral helium and neutral hydrogen is around 0.1 in the lower heights. But it increases up to 0.22 near chromosphere/corona transition region i.e. at 2000 km.



FAL93-3 model (Fontenla et al. 1993)

The ratio of neutral helium and neutral hydrogen number densities is increased in the temperature interval 10000-40000 K peaking at 20000 K.



FAL93-3 model (Fontenla et al. 1993)

Multi-fluid equations

We consider partially ionized incompressible plasma which consists of electrons, protons, singly ionized helium, neutral hydrogen and neutral helium atoms.

We neglect the viscosity, the heat flux, and the heat production due to collision between particles. Then the governing equations are:

$$\nabla \cdot \vec{V}_a = 0,$$

$$m_a n_a \left(\frac{\partial \vec{V}_a}{\partial t} + \left(\vec{V}_a \cdot \nabla \right) \vec{V}_a \right) = -\nabla p_a - e_a n_a \left(\vec{E} + \frac{1}{c} \vec{V}_a \times \vec{B} \right) + \vec{R}_a,$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},$$

$$\nabla \times \vec{B} = -\frac{4\pi}{c} \vec{j},$$

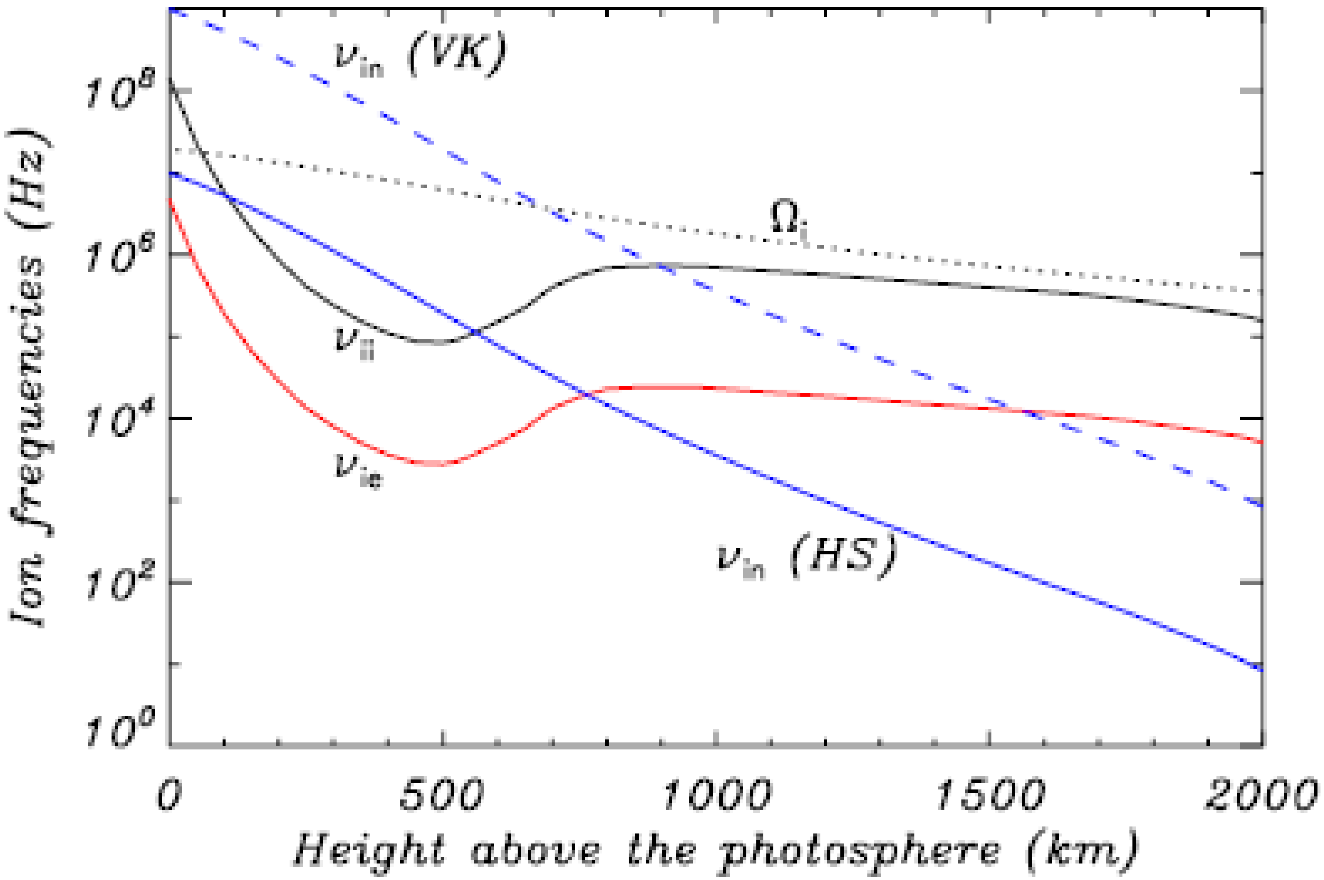
$$\vec{j} = -e \left(n_e \vec{V}_e - n_{H^+} \vec{V}_{H^+} - n_{He^+} \vec{V}_{He^+} \right).$$

Three-fluid equations

- For time scales longer than ion-electron collision time, the electron and ion gases can be considered as a single fluid.

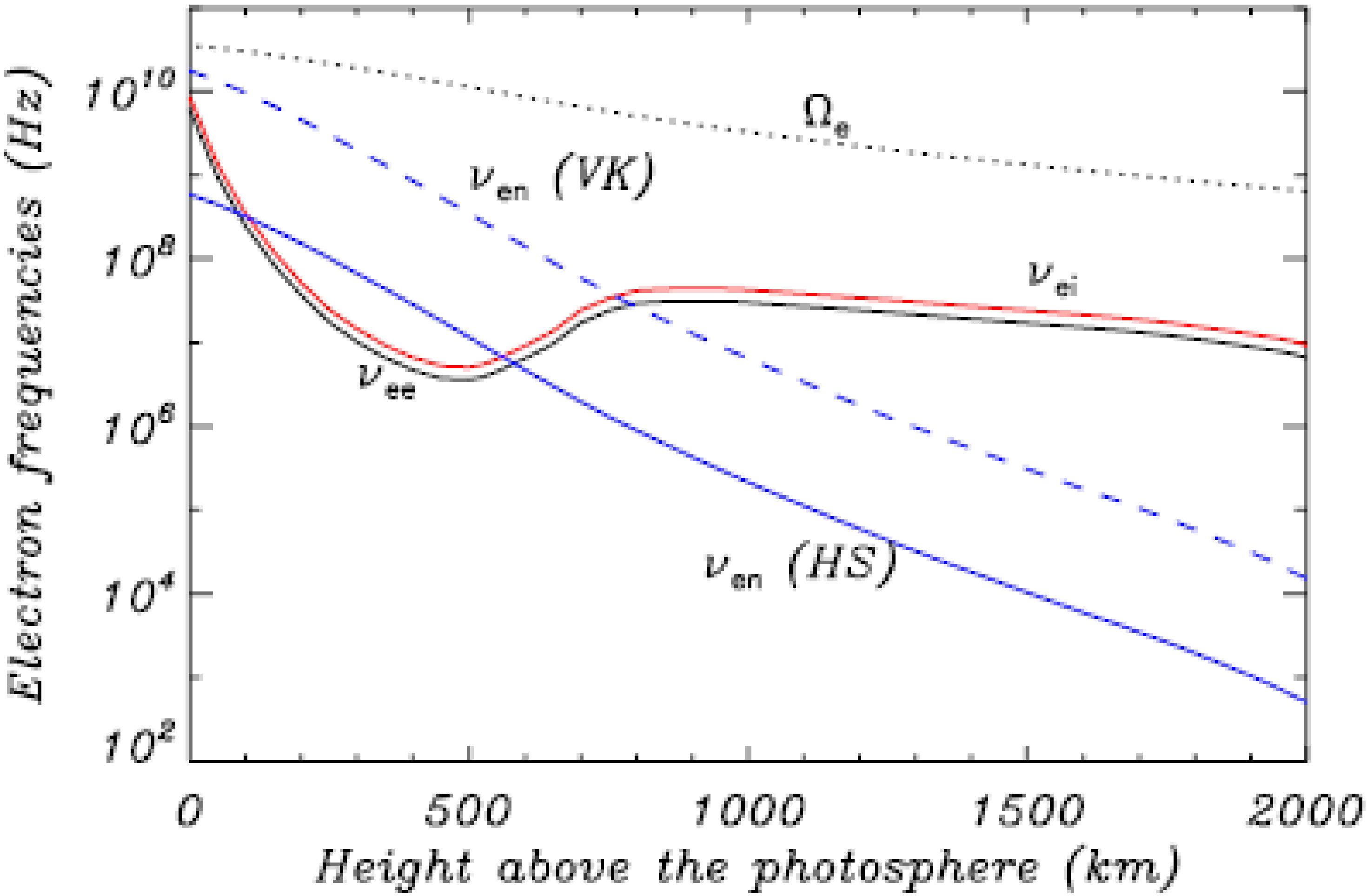
- Then the five-fluid description can be changed by three-fluid description, where one component is the charged fluid (electron+protons+singly ionized helium) and other two components are the gases of neutral hydrogen and neutral helium gases.

Collision frequencies



Soler et al. 2015

Collision frequencies



Soler et al. 2015

Three-fluid equations

We use the definition of total density of charged fluid

$$\rho_0 = \rho_{H^+} + \rho_{He^+}$$

and the total velocity of charged fluid as

$$\vec{V} = \frac{\rho_{H^+}\vec{V}_{H^+} + \rho_{He^+}\vec{V}_{He^+}}{\rho_{H^+} + \rho_{He^+}}.$$

The sum of momentum equations for electrons, protons and singly ionized helium is

$$\rho_0 \frac{d\vec{V}}{dt} + \rho_0 \xi_{H^+} \xi_{He^+} (\vec{w}\cdot\nabla)\vec{w} = -\nabla p + \frac{1}{c}\vec{j}\times\vec{B} + \vec{F}_t,$$

where $\vec{w} = \vec{V}_{H^+} - \vec{V}_{He^+}$ is the relative velocity of protons and helium ions.

Three–fluid equations

It can be shown that $|\vec{w}| << |\vec{V}|$ for the time scales longer than ion gyro period.

Then we obtain the three-fluid equations as

$$\rho_0 \frac{d\vec{V}}{dt} = -\nabla p + \frac{1}{4\pi}\left(\nabla \times \vec{B}\right)\times \vec{B} + \vec{F}_i,$$

$$\rho_H \frac{d\vec{V}_H}{dt} = -\nabla p_H + \vec{F}_H,$$

$$\rho_{He} \frac{d\vec{V}_{He}}{dt} = -\nabla p_{He} + \vec{F}_{He},$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{B}\right).$$

where

$$\vec{F}_i = -\left(\alpha_{H^+H} + \alpha_{H^+He} + \alpha_{He^+H} + \alpha_{He^+He}\right)\vec{V}_i + \left(\alpha_{H^+H} + \alpha_{He^+H}\right)\vec{V}_H + \left(\alpha_{H^+He} + \alpha_{He^+He}\right)\vec{V}_{He},$$

$$\vec{F}_H = -\left(\alpha_{H^+H} + \alpha_{He^+H} + \alpha_{HeH}\right)\vec{V}_H + \left(\alpha_{H^+H} + \alpha_{He^+H}\right)\vec{V}_i + \alpha_{HeH}\vec{V}_{He},$$

$$\vec{F}_{He} = -\left(\alpha_{H^+He} + \alpha_{He^+He} + \alpha_{HeH}\right)\vec{V}_{He} + \left(\alpha_{H^+He} + \alpha_{He^+He}\right)\vec{V}_i + \alpha_{HeH}\vec{V}_H.$$

Three-fluid linear equations

We use a vertical magnetic flux tube embedded in the stratified solar atmosphere. Magnetic field of the tube is axis-symmetric i.e. $B_{0\theta}=0$.

We consider the linear Alfvén waves polarized in the θ direction i.e. only non-vanishing perturbations are θ-components of velocity and magnetic field.

$$\frac{\partial}{\partial t}(rv_\theta)=\frac{\vec{B}_0\cdot\nabla}{4\pi\rho_i}(rb_\theta)-\frac{\alpha_H+\alpha_{He}}{\rho_i}(rv_\theta)+\frac{\alpha_H}{\rho_i}(rv_{H\theta})+\frac{\alpha_{He}}{\rho_i}(rv_{He\theta}),$$

$$\frac{\partial}{\partial t}(rv_{H\theta})=\frac{\alpha_H}{\rho_H}(rv_\theta)-\frac{\alpha_H+\alpha_{HeH}}{\rho_H}(rv_{H\theta})+\frac{\alpha_{HeH}}{\rho_H}(rv_{He\theta}),$$

$$\frac{\partial}{\partial t}(rv_{He\theta})=\frac{\alpha_{He}}{\rho_{He}}(rv_\theta)-\frac{\alpha_H+\alpha_{HeH}}{\rho_{He}}(rv_{He\theta})+\frac{\alpha_{HeH}}{\rho_{He}}(rv_{H\theta}),$$

$$\frac{\partial b_\theta}{\partial t}=r\left(\vec{B}_0\cdot\nabla\right)\left(\frac{v_\theta}{r}\right).$$

Dispersion relation in three-fluid plasma

We consider a homogeneous plasma and after Fourier transform derive the dispersion relation of Alfvén waves in the three-fluid plasma

$$\xi_H \xi_{He} a_H a_{He} \varpi^4 + i\left[\xi_H a_H (1+\xi_{He}) + \xi_{He} a_{He} (1+\xi_H)\right]\varpi^3 -$$
$$-\left[1+\xi_H + \xi_{He} + \xi_H \xi_{He} a_H a_{He}\right]\varpi^2 - i\left[\xi_H a_H + \xi_{He} a_{He}\right]\varpi + 1 = 0,$$

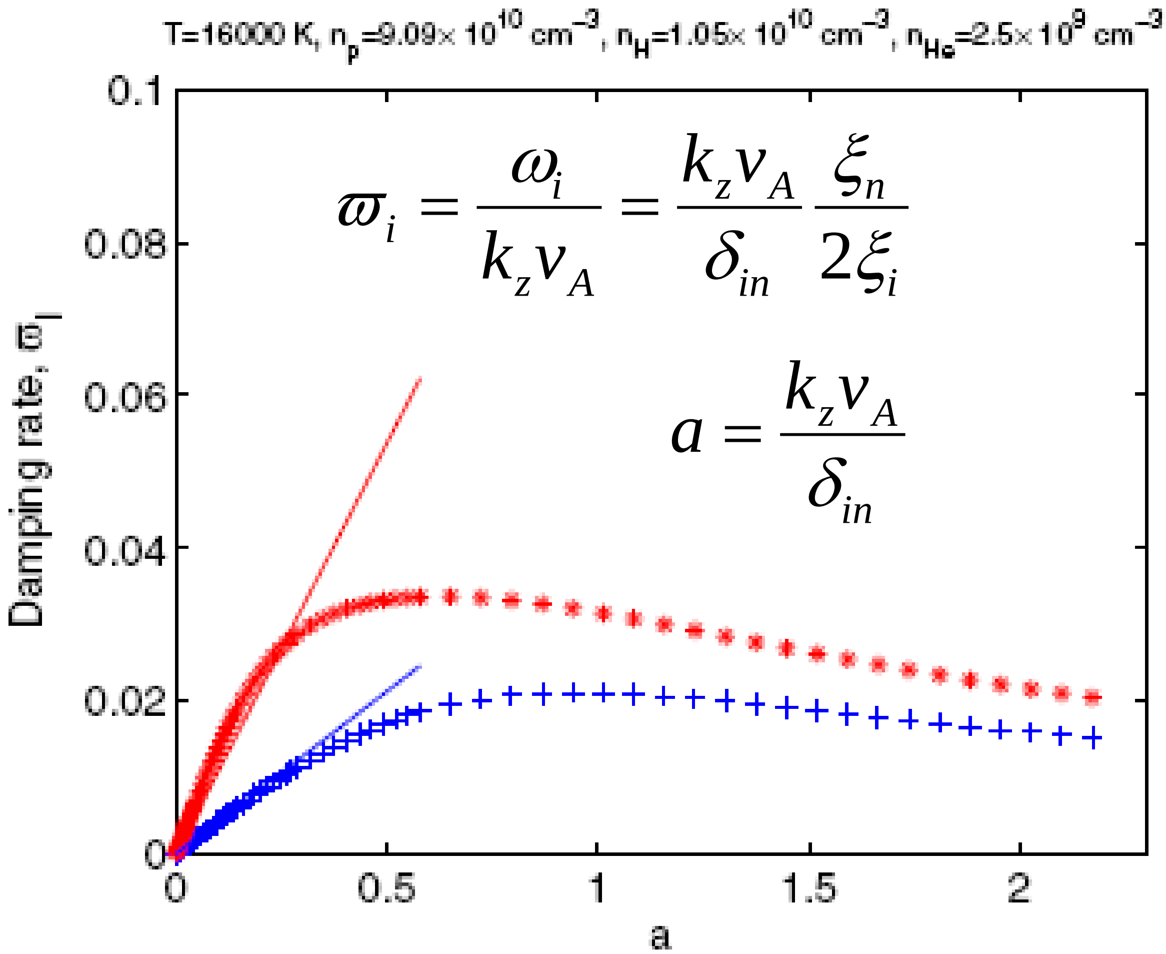
where

$$\varpi = \frac{\omega}{k_z v_A}, a_H = \frac{k_z v_A \rho_0}{\alpha_H}, a_{He} = \frac{k_z v_A \rho_0}{\alpha_{He}}, \xi_H = \frac{\rho_H}{\rho_0}, \xi_{He} = \frac{\rho_{He}}{\rho_0}.$$

The dispersion relation has four different roots: the two complex solutions, which correspond to Alfvén waves damped by ion-neutral collision and two purely imaginary solutions, which correspond to damped vortex solutions of neutral hydrogen and neutral helium fluids.

Alfven waves: upper chromosphere

Chromosphere: 2015 km height above the photosphere.

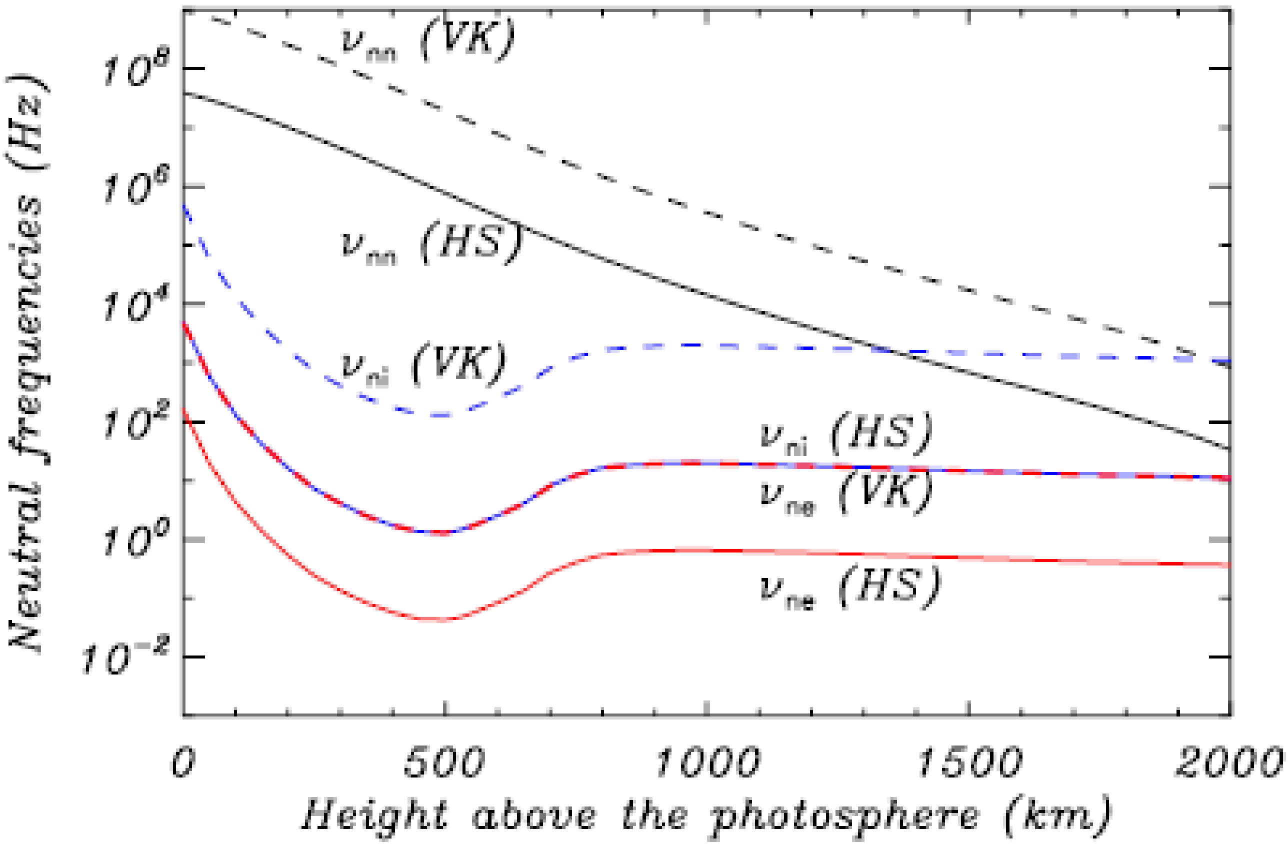


$$\varpi_i = \frac{\omega_i}{k_z v_A} = \frac{k_z v_A}{\delta_{in}} \frac{\xi_n}{2\xi_i}$$

$$a = \frac{k_z v_A}{\delta_{in}}$$

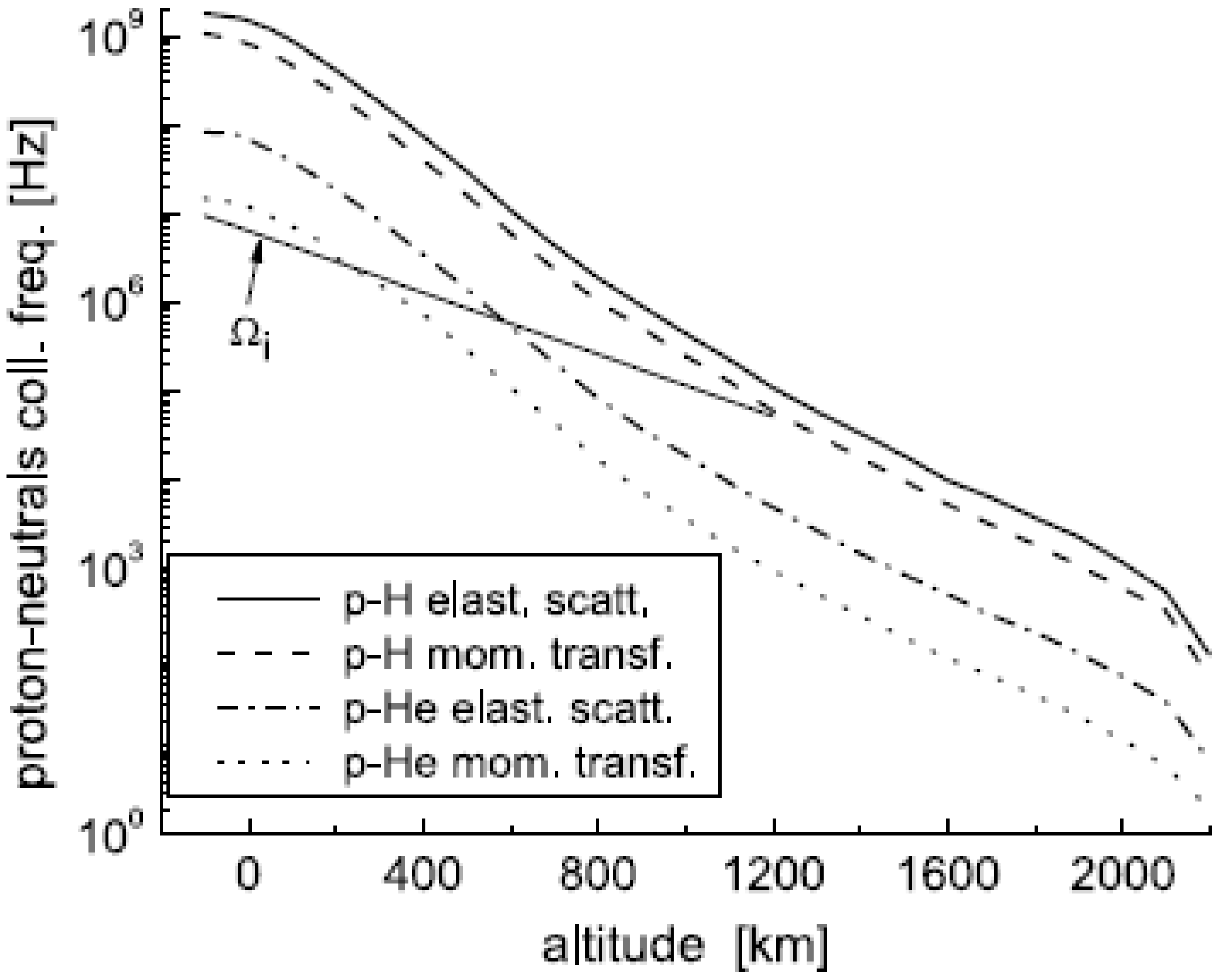
Zaqarashvili et al. 2011

Collision frequencies



Soler et al. 2015

Collision frequencies



Vranjes and Krstic (2013)

Single-fluid MHD

We consider the total density

$$\rho = \rho_0 + \rho_H + \rho_{He},$$

total velocity

$$V_\theta = \frac{\rho_0 u_\theta + \rho_H u_{H\theta} + \rho_{He} u_{He\theta}}{\rho_0 + \rho_H + \rho_{He}},$$

relative velocity between ions and neutral hydrogen

$$w_{H\theta} = u_\theta - u_{H\theta}$$

and relative velocity between ions and neutral helium

$$w_{He\theta} = u_\theta - u_{He\theta}.$$

Then we find that

$$u_\theta = V_\theta + \xi_H w_{H\theta} + \xi_{He} w_{He\theta}.$$

Single–fluid MHD

Consecutive subtractions of multi-fluid equations and neglect of inertial terms leads to the equations

$$rw_{H\theta}=\left[\frac{\alpha_{He}}{\alpha}\xi_H+\frac{\alpha_{HeH}}{\alpha}(\xi_H+\xi_{He})\right]\frac{\vec{B}_0\cdot\nabla}{4\pi}(rb_\theta),$$

$$rw_{He\theta}=\frac{B_z}{4\pi}\left[\frac{\alpha_H}{\alpha}\xi_{He}+\frac{\alpha_{HeH}}{\alpha}(\xi_H+\xi_{He})\right]\frac{\vec{B}_0\cdot\nabla}{4\pi}(rb_\theta),$$

where $\alpha=\alpha_H\alpha_{He}+\alpha_H\alpha_{HeH}+\alpha_{He}\alpha_{HeH}.$

Then the sum of multi-fluid equations leads to the single-fluid equations

$$\frac{\partial}{\partial t}(rV_\theta)=\frac{\vec{B}_0\cdot\nabla}{4\pi\rho(z)}(rb_\theta),$$

$$\frac{\partial b_\theta}{\partial t}=r\left(\vec{B}_0\cdot\nabla\right)\left(\frac{V_\theta}{r}\right)+r\left(\vec{B}_0\cdot\nabla\right)\left[\frac{\eta_c(z)}{B_0^2}\frac{\vec{B}_0\cdot\nabla}{r^2}(rb_\theta)\right],$$

where

$$\eta_c=\frac{B_0^2}{4\pi}\left(\frac{\alpha_{He}\xi_H^2+\alpha_H\xi_{He}^2+\alpha_{HeH}\left(\xi_H+\xi_{He}\right)^2}{\alpha_H\alpha_{He}+\alpha_H\alpha_{HeH}+\alpha_{He}\alpha_{HeH}}\right)$$

is the coefficient of Cowling diffusion.

Main equation

We consider new coordinate s, which is distance along unperturbed magnetic field (Hollweg 1984) , then the two equations can be combined into the single equation

$$\frac{\partial^2}{\partial t^2}\left(\frac{V_\theta}{r}\right)=\frac{B_{0s}}{4\pi\rho r^2}\frac{\partial}{\partial s}\left[r^2 B_{0s}\frac{\partial}{\partial s}\left(\frac{V_\theta}{r}\right)\right]+\frac{B_{0s}}{4\pi\rho r^2}\frac{\partial}{\partial s}\left[r^2 B_{0s}\frac{\partial}{\partial s}\left(\frac{4\pi\rho\eta_c}{B_{0s}^2}\frac{\partial}{\partial t}\left(\frac{V_\theta}{r}\right)\right)\right].$$

This equation is simplified near the axis of symmetry, where (Hollweg 1984)

$$B_{0s}r^2 \approx const.$$

Then the equation is rewritten as

$$\frac{\partial^2}{\partial t^2}\left(\frac{V_\theta}{r}\right)=V_A^2(s)\frac{\partial^2}{\partial s^2}\left[\left(1+\frac{\eta_c(s)}{V_A^2(s)}\right)\left(\frac{V_\theta}{r}\right)\right].$$

Fourier analyses with time gives

$$V_A^2(s)\frac{\partial^2}{\partial s^2}\left[\left(1-i\frac{\omega\eta_c(s)}{V_A^2(s)}\right)\left(\frac{V_\theta}{r}\right)\right]+\omega^2\frac{V_\theta}{r}=0.$$

Thin flux tube: Dispersion relation

In the lower region we consider isothermal atmosphere, where the thin magnetic flux tube is embedded. Then the plasma β is constant with height in the case of same temperature inside and outside the tube (Roberts 2004).

Then, the Alfvén speed is also constant with height, which leads to the equation with constant coefficients.

Consequently, Fourier analyses with s gives the dispersion relation

$$\omega^2 + i\eta_c k_s^2 \omega - k_s^2 V_A^2 = 0,$$

which has two complex solutions

$$\omega = \pm k_s V_A \sqrt{1 - \frac{\eta_c^2 k_s^2}{4V_A^2}} - i\frac{\eta_c k_s^2}{2}.$$

Real part of the complex frequency gives cut-off wave number

$$k_s = \pm \frac{2V_A}{\eta_c}.$$

Thin flux tube: Damping rate

Normalized damping rate is

$$|\tilde{\omega}_i| = \left|\frac{\omega_i}{k_s V_A}\right| = \frac{1}{2}\frac{k_s V_A}{\rho}\frac{\alpha_{He}\rho_H^2 + \alpha_H \rho_{He}^2 + \alpha_{HeH}(\rho_H + \rho_{He})^2}{\alpha_H \alpha_{He} + \alpha_H \alpha_{HeH} + \alpha_{He}\alpha_{HeH}}.$$

In the low chromosphere, where plasma is weakly ionized, we have $\alpha_{HeH} >> \alpha_H, \alpha_{He}$ therefore

$$|\tilde{\omega}_i| = \frac{1}{2}\frac{k_s V_A}{\rho}\frac{(\rho_H + \rho_{He})^2}{\alpha_H + \alpha_{He}}.$$

This expression was used by De Pontieu et al. (2001) and Soler et al. (2010).

On the other hand, in higher regions of the chromosphere, where $\alpha_{HeH} << \alpha_H, \alpha_{He}$ we have

$$|\tilde{\omega}_i| = \frac{1}{2}\frac{k_s V_A}{\rho}\left[\frac{\rho_H^2}{\alpha_H} + \frac{\rho_{He}^2}{\alpha_{He}}\right].$$

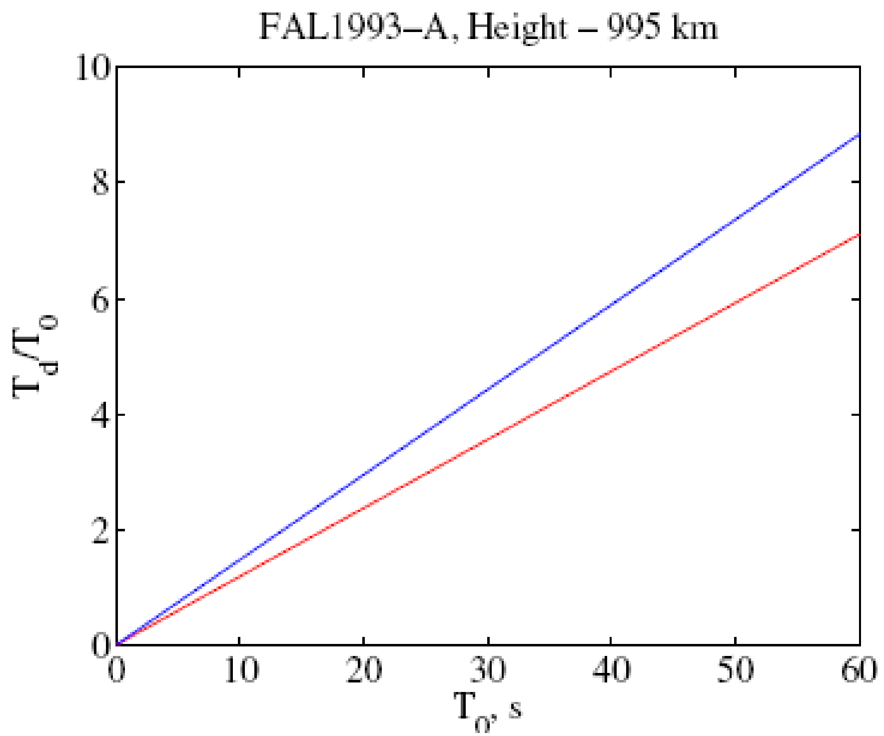
In the middle chromosphere, spicules and prominences the general expression should be used.

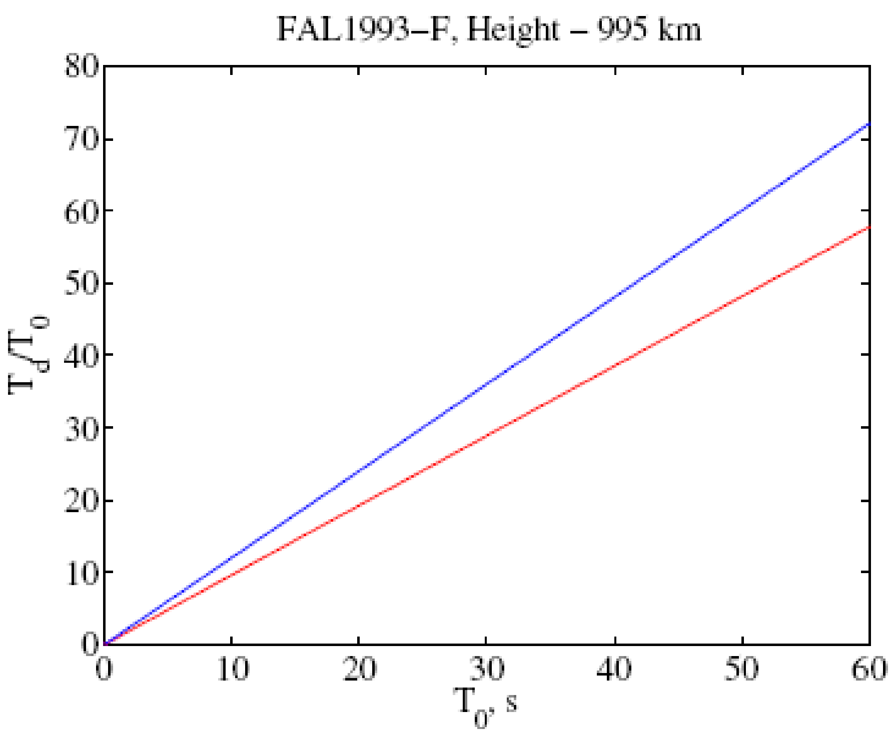
Faint cell center area (FAL93-A)

$n_i = 3.26\ 10^{10}\ cm^{-3}$,
$n_H = 3.01\ 10^{13}\ cm^{-3}$
$n_{He} = 3.01\ 10^{12}\ cm^{-3}$

Bright network (FAL93-F)

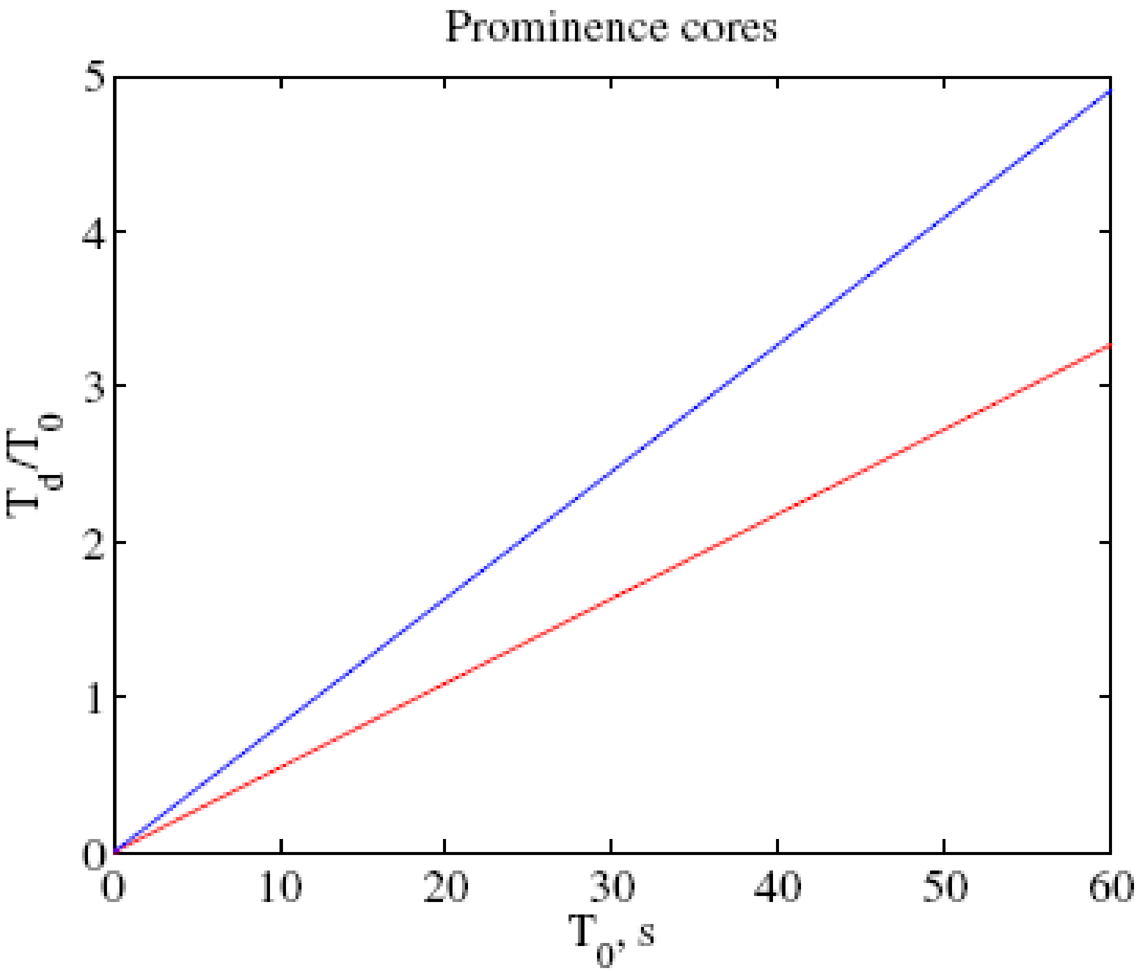
$n_i = 2.49\ 10^{11}\ cm^{-3}$
$n_H = 5.24\ 10^{13}\ cm^{-3}$
$n_{He} = 5.24\ 10^{12}\ cm^{-3}$





Prominences

$n_i = 10^{10}\ cm^{-3}$
$n_H = 2\ 10^{10}\ cm^{-3}$
$n_{He} = 2\ 10^{9}\ cm^{-3}$



Damped vortex modes

- Multi-fluid approach allows the existence of damped vortex mode solutions, which are absent in the single-fluid approach.
- They are connected to neutral fluids of hydrogen and helium.
- The solution connected with neutral hydrogen damps very quickly.
- The solution connected with neutral helium damps less quickly.
- Do these vortex modes have something with coronal rain blobs?

Conclusions

- Neutral helium atoms may enhance the damping of high-frequency Alfvén waves in solar plasma for T=8000-40000 K.
- The damping rate is maximal near ion-neutral collision frequency in the multi-fluid approach.
- The single-fluid approach is valid for time scales of > 1 s.
- Are damped Alfvén waves important for coronal rain?
- Damped vortex solutions?