Spatial scales of the thermal mode in the coronal plasma

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Some facts

- In ideal MHD, the entropy mode is a solution of the linearized MHD equations. It represents nonpropagating perturbations of density that remain static, neither decaying nor growing.
  (e.g., Goedbloed & Poedts 2004)
- When nonadiabatic mechanisms are taken into account (e.g., radiative cooling, thermal conduction, heating), the entropy mode is then called thermal mode and can be unstable.
  (e.g., Parker 1953; Field 1965; Heyvaerts 1974)
- Thermal instability occurs due to imbalance between temperature-independent energy gains (heating) and temperature-dependent radiative losses. The amplitude of the thermal mode grows in time and can lead to catastrophic cooling.
- This mechanism allows the formation of cool condensations in the high-temperature coronal medium (e.g., prominences, coronal rain).

Are the observed spatial scales of coronal rain blobs related with the expected length scales of thermal modes in the corona?

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Nonadiabatic MHD equations

$$
\begin{aligned}
\frac{\mathrm{D}\rho}{\mathrm{D}t} &= -\rho\nabla\cdot\mathbf{v} \\
\rho\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} &= -\nabla p + \frac{1}{\mu}(\nabla\times\mathbf{B})\times\mathbf{B} \\
\frac{\partial\mathbf{B}}{\partial t} &= \nabla\times(\mathbf{v}\times\mathbf{B}) \\
\frac{\mathrm{D}p}{\mathrm{D}t} - \frac{\gamma p}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t} &= -(\gamma-1)\left[L(T,\rho) - \nabla\cdot(\kappa\cdot\nabla T) - H\right] \\
p &= \rho R\frac{T}{\tilde{\mu}}
\end{aligned}
$$

- Radiative losses: $L(T,\rho)$
- Thermal flux: $-\nabla\cdot(\kappa\cdot\nabla T)$
- Arbitrary heating: H

Thermal conduction

- Highly anisotropic in the magnetized coronal plasma
- Parallel conduction caused by electrons
- Perpendicular conduction caused by ions

$$\kappa = \kappa_{\parallel} \hat{e}_B \hat{e}_B + \kappa_{\perp} \left(\hat{\mathbf{I}} - \hat{e}_B \hat{e}_B \right)$$

$$\begin{aligned} \kappa_{\parallel} &= 1.8 \times 10^{-10} \frac{T^{5/2}}{\ln \Lambda} \\ \kappa_{\perp} &= 5.78 \times 10^{11} \frac{\ln \Lambda \, \rho^2}{|\mathbf{B}|^2 T^{1/2}} \end{aligned}$$

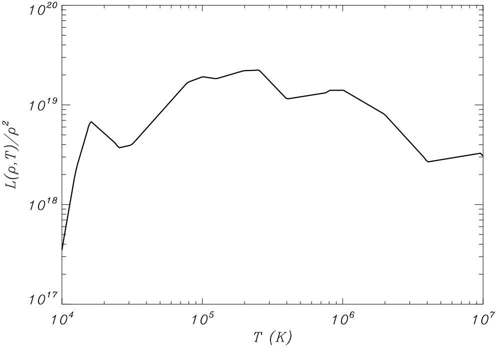
In coronal conditions

$$\kappa_{\perp} \ll \kappa_{\parallel}$$

Radiative losses

- Semi-empirical approximation for optically thin plasma
- Piecewise function of temperature
- Losses computed from the CHIANTI database assuming coronal abundances

$$L(\rho, T) = \rho^2 \chi^* T^\alpha$$



Adapted from Soler, Ballester, & Parenti (2012)

Linear perturbations

- Simplest scenario: homogeneous coronal plasma with a straight and constant magnetic field
- We linearize the nonadiabatic MHD equations
- Temporal dependence: $\exp(st)$, with s the growth rate
- Fourier modes in space: $k_\parallel$ and $k_\perp$ with respect to the magnetic field

Dispersion relation of compressible modes

$$s^4 + \left(v_\mathrm{A}^2 + \tilde{v}_s^2\right)\left(k_\parallel^2 + k_\perp^2\right) s^2 + \left(k_\parallel^2 + k_\perp^2\right) k_\parallel^2 v_\mathrm{A}^2 \tilde{v}_s^2 = 0$$

- Alfvén and nonadiabatic sound velocities: $v_\mathrm{A} = \frac{B}{\sqrt{\mu\rho}}$, $\tilde{v}_s = \sqrt{\frac{\tilde{\gamma} p}{\rho}}$
- Effective adiabatic index (Soler et al. 2008):

$$\tilde{\gamma} = \frac{(\gamma-1)\left(\frac{T}{p}\kappa_\parallel k_\parallel^2 + \frac{T}{p}\kappa_\perp k_\perp^2 + \frac{T}{p}\left(\frac{\partial L}{\partial T}\right)_\rho - \frac{\rho}{p}\left(\frac{\partial L}{\partial \rho}\right)_T\right) + \gamma s}{(\gamma-1)\left(\frac{T}{p}\kappa_\parallel k_\parallel^2 + \frac{T}{p}\kappa_\perp k_\perp^2 + \frac{T}{p}\left(\frac{\partial L}{\partial T}\right)_\rho\right) + s}$$

Magnetoacoustic waves (complex s) and the thermal mode (real s) are coupled even in a homogeneous plasma

Thermal mode

- Approximations to decouple the thermal mode:
  1. Low-β plasma: fast waves decouple from slow + thermal modes
  2. Nonadiabatic terms small compared to ideal terms: slow waves decouple from the thermal mode

Approximate thermal mode growth rate

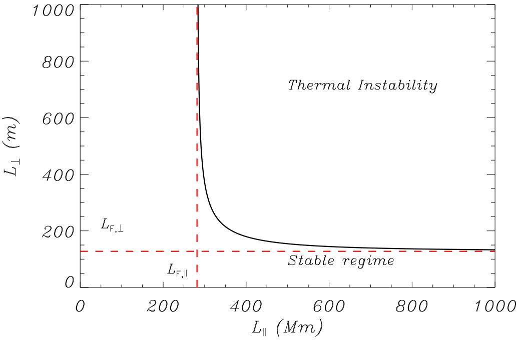
$$s \approx -\frac{\gamma - 1}{\gamma}\left(\frac{T}{p}\kappa_{\parallel}k_{\parallel}^2 + \frac{T}{p}\kappa_{\perp}k_{\perp}^2 + \frac{T}{p}\left(\frac{\partial L}{\partial T}\right)_{\rho} - \frac{\rho}{p}\left(\frac{\partial L}{\partial \rho}\right)_{T}\right)$$

- **Instability criterion:** $s > 0$

- Thermal conduction has a stabilizing effect for short length scales
- Critical lengths or Field's lengths: longest lengths that are stable, $L_{\mathrm{F},\parallel}$ and $L_{\mathrm{F},\perp}$
- In the corona, $L_{\mathrm{F},\parallel} \gg L_{\mathrm{F},\perp}$ due to the highly anisotropic nature of thermal conduction

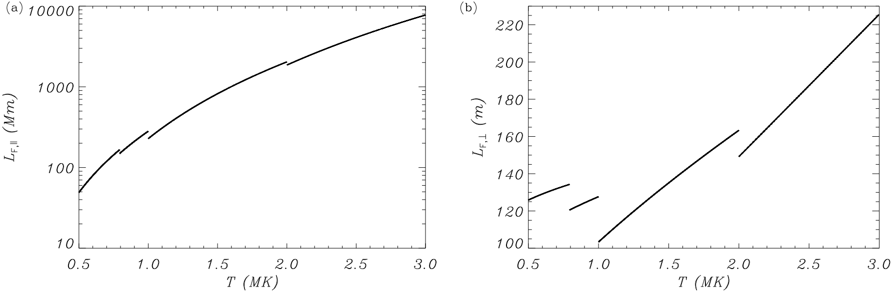
Field's lengths in the corona

$$L_{\mathrm{F},\parallel} = 2\pi\sqrt{\frac{\kappa_\parallel}{\frac{\rho}{T}\left(\frac{\partial L}{\partial \rho}\right)_T - \left(\frac{\partial L}{\partial T}\right)_\rho}}, \qquad L_{\mathrm{F},\perp} = 2\pi\sqrt{\frac{\kappa_\perp}{\frac{\rho}{T}\left(\frac{\partial L}{\partial \rho}\right)_T - \left(\frac{\partial L}{\partial T}\right)_\rho}}$$



Coronal conditions: $T = 10^6$ K, $\rho = 4 \times 10^{-13}$ kg m^{-3}, and $B = 10$ G.

Dependence on temperature

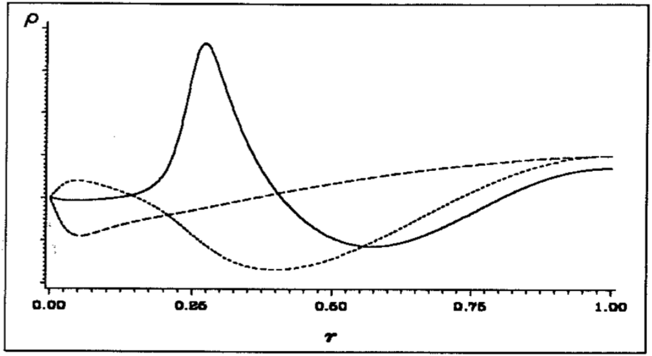


- $L_{\mathrm{F},\parallel}$ is strongly dependent on temperature
- $L_{\mathrm{F},\perp}$ is very weakly dependent on temperature
- Jumps are artificial and are caused by the piecewise form of the radiative loss function, L

The values of $L_{\mathrm{F},\perp}$ seem to small to explain the observed transverse size of coronal rain blobs

Additional ingredient: magnetic twist

- In a twisted loop, a new family of thermal modes can appear
- They are called "discrete thermal modes"
  Van der Linden & Goossens (1991); Van der Linden (1991, PhD Thesis)
- The transverse length scale of these modes is not related to the Field's length and can be much larger

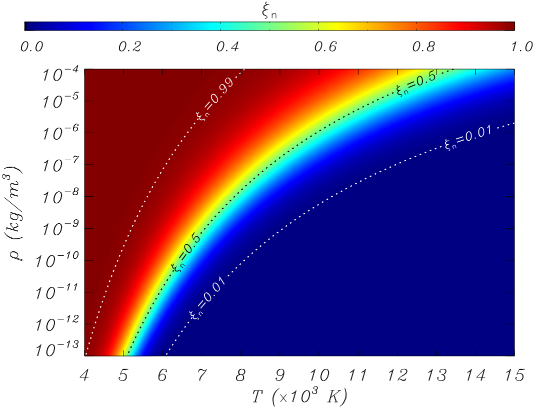


Density perturbations of three "discrete thermal modes" in a twisted flux tube
From Van der Linden (1991, PhD Thesis)

Additional ingredient: partial ionization

- The plasma in cool coronal rain blobs is only partially ionized
- The Saha equation for a hydrogen plasma:

$$\frac{n_\mathrm{i}^2}{n_\mathrm{n}} = \left(\frac{2\pi m_\mathrm{e} k_\mathrm{B} T}{h^2}\right)^{3/2} \exp\left(-U_\mathrm{i}/k_\mathrm{B} T\right)$$



Neutral fraction

$$\xi_\mathrm{n} = \frac{n_\mathrm{n}}{n_\mathrm{i} + n_\mathrm{n}}$$

Neutral thermal conduction

- In a partially ionized plasma, thermal conduction by neutrals can be very efficient
- In the first approximation, conductivity by neutrals can be taken as isotropic

$$\kappa_n = 2.24 \times 10^{-2} \xi_n T^{1/2}$$

- Effective conductivities in a partially ionized plasma should be modified as follows:

$$\kappa_\parallel \to \kappa_\parallel + \kappa_n$$

$$\kappa_\perp \to \kappa_\perp + \kappa_n \approx \kappa_n$$

- Perpendicular conduction governed by neutrals
- **The perpendicular Field's length may increase as the plasma gets partially ionized!**

Summary

- Thermal modes in the solar corona can be unstable and may lead to catastrophic cooling (seed for condensations)
- Very different parallel and perpendicular Field's lengths due to the highly anisotropic nature of thermal conduction
- Effects as, e.g., partial ionization and magnetic twist may be necessary to explain the observed scales of coronal rain blobs

Work for the future

- Role of neutral thermal conduction on the dynamics of partially ionized condensations
- Role of magnetic twist on the perpendicular scales associated with "discrete thermal modes"

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