Interrogating Field-Aligned Solar Flare Models: Comparing, Contrasting and Improving

Dr. Graham S. Kerr & Dr. Vanessa Polito

1. Abstract

Understanding the physical mechanisms responsible for, and at play during, solar flares still remains one of the most important open issues in astrophysics. These energetic events release a tremendous amount of magnetic energy, $> 10^{32}$ ergs, resulting in efficient particle acceleration and are often associated with the ejection of coronal material, strongly influencing space weather. Recent high-resolution observations from space and ground based instruments, such as the Interface Region Imaging Spectrograph (IRIS), the Swedish 1-m Solar Telescope (SST), and the Dunn Solar Telescope (DST) have provided an unprecedented view into the complex and dynamic plasma environment during solar flares. In particular, the observation of the chromosphere, which is the site of the bulk of energy deposition, is key to understanding the physical mechanisms at play during flares. The imaging and spectroscopic observations are typically complemented by state-of-the-art solar flare and radiation transfer numerical models as a means to both (1) assist in the interpretation of the observables through forward modelling and (2) assess the validity of models of flare energy transport. This first point is particularly true in the case of optically thick, chromospheric emission lines, which are non-trivial to interpret due to their complex formation properties and due to the presence of non-thermal and non-equilibrium processes. While several magnetic field-aligned flare loop models with 1D radiative transfer have been developed and are extensively used, there has been little concerted effort to critically compare, contrast, and validate, the output of these models to each other. This is despite the fact that these models are critical as a means to interpret the observations.

Our ISSI team aims to bring together, for the first time, the three most widely used field-aligned loop models that simulate the atmospheric response to flare energy injection: `RADYN`, `HYDRAD` & `FLARIX`. We propose to assess how differences in numerical techniques and code features (single fluid vs multi fluid, boundary conditions, treatment of energy transport and deposition, numerical grid size and resolution, treatment of radiative processes, treatment of thermal conduction etc.,) affect model-model and model-data comparison. We will also compare the results of several post-processing codes, which are often used in conjunction with the (radiation-) hydrodynamic models to obtain the accurate synthesis of spectral lines not included in the original codes: `RH`, `MALI` & `MULTI`. Our approach will be to select a set of well-observed flares with multi-wavelength coverage. From this canonical dataset, we will agree upon a set of observables, use the forward modelling to predict those observables, and also use these as a basis for comparing, contrasting and calibrating the relative performance of the models. As the quality of solar flare observations continues to increase (e.g. the upcoming Daniel K. Inouye Solar Telescope, DKIST, and Solar Orbiter), and as the community is reliant on flare loop models to aid in the analysis and interpretation of these observations, it is crucial to have validated models, a solid understanding of the advantages and disadvantages of each code, and to lay the foundations for moving towards model improvements.

2. Scientific rationale and proposed work

In the standard flare model, energy is transported from the coronal release site via directed beams of relativistic non-thermal electrons and deposited in the lower atmosphere. The existence of accelerated electrons is unambiguous, known from observations of hard X-ray (HXR) sources. HXR data are then used to determine the electron beam parameters that drive flare models. Recently, however, additional energy transport mechanisms have been proposed, namely high-frequency Alfvénic waves that act either instead of, or in tandem with, electron beams (e.g. Fletcher & Hudson 2008). To determine if our models of electron beam energy transport in flares are correct, and if additional heating mechanisms are required, we need both high resolution observations and advanced numerical simulations. The latter is the focus of this proposal.

Over the past few years, the advent of high-resolution imaging and spectroscopic instruments from both ground and space-based observatories (such as IRIS, SST, DST) has significantly improved our understanding of solar flares. Routine observations of the flaring transition region (TR) and chromosphere at unprecedented spatial ($\sim 0.3 - 0.4''$) and temporal (down to a few seconds) resolution with the IRIS satellite have provided crucial new insights into flare dynamics and the plasma response to flare heating. A notable example is the capability of IRIS to resolve the site of high-temperature upflows in the Fe XXI line ($> 10^7$ K) at the flare footpoints (*chromospheric evaporation*,e.g. Young et al. 2015; Polito et al. 2015, 2016; Graham & Cauzzi 2015, to name a few). The magnitude and duration of the Fe XXI upflows and the simultaneous downflows observed in cooler lines (such as Hα & Mg II) *chromospheric condensation*) have also proven to hold important information on the duration of the heating and its partition into unresolved flare loop strands (Reep et al. 2018a). Further, observations of the chromospheric emission lines (such as C II & Mg II lines with IRIS, and Hα & Ca II with SST), have provided important diagnostics of the properties and dynamics of the flaring atmosphere, which are key to distinguishing between competing models of flares (for example, by determining the importance of electron beams vs Alfvén heating, e.g. Kerr et al. 2016).

Such major observational advancements can only be fully exploited if there is a parallel development and improvement of the theoretical models which are used to interpret those observations. This is particularly true when modelling the optically thick emission from the lower atmosphere, requiring advanced radiative transfer calculations, as well the treatment of non-equilibrium conditions in the tenuous optically thin corona. Field-aligned hydrodynamic models are typically used to study the atmospheric plasma response to the heating in an individual flare loop. The advantage of such models is that they allow us to simulate the plasma dynamics at very small spatial scales (down to metre scales is required at times). Achieving the required temporal and spatial resolution for a flare simulation in a 2D or 3D model that includes a chromosphere would be very computationally demanding. The 1D assumption is justified by the fact that in the low plasma β regime of the solar atmosphere, mass and energy transport across the magnetic field is highly inhibited, and it is therefore appropriate to treat each flare strand as an isolated plasma loop. Further, it is essential that we understand the complex physics involved in a field-aligned model before progressing to 3D.

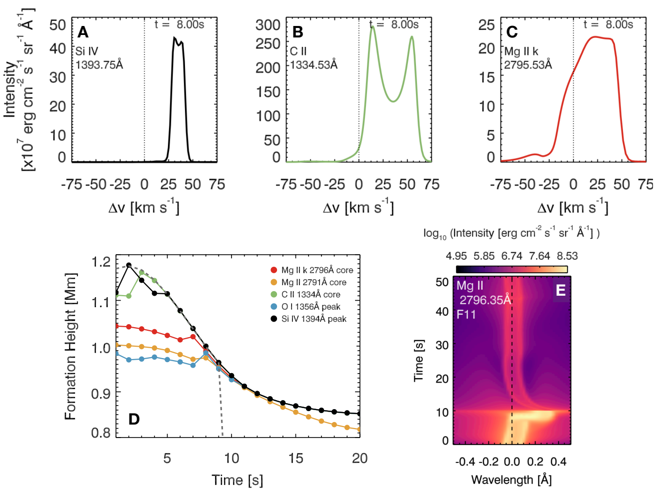

Figure 1: *Illustrating spectral line modelling using* `RADYN`*,* `MS_RADYN`*, and* `RH`*. Panels (A-C) show synthetic Si* IV*, C* II*, and Mg* II *resonance lines during a flare. Panel (D) shows the formation heights of these and other lines. Panel (E) shows the temporal evolution of the Mg* II *k line. Such products, and other simulation details, permit us to understand flaring line formation, and ultimately extract physical properties of the plasma. They also facilitate model-model and model-data comparison.*

The field-aligned codes `HYDRAD` (Bradshaw & Mason 2003a; Bradshaw & Cargill 2013; Reep et al. 2019), `RADYN` (Carlsson & Stein 1995, 1997; Allred et al. 2015) and `FLARIX` (Varady et al. 2010; Heinzel et al. 2016) are now well established and widely used in the solar flare community. These codes solve the equations describing the conservation of mass, momentum, charge, and energy in a single field-aligned magnetic strand rooted in the photosphere and stretching out to include the chromosphere, TR and corona. `HYDRAD` and `RADYN` use an adaptive grid where the size of the grid cells can vary to allow shocks and steep gradients in the atmosphere to be resolved as required (`HYDRAD` can also vary the number of grid cells), while `FLARIX` uses a fixed, but optimized, grid with ~ 2000 points. The codes have various similarities and differences as regards treatment of radiation and flare energy transport.

All three simulate the response of the atmosphere to injection of energy, typically via a beam of non-thermal electrons (but flare-accelerated ions can be included too). The treatment of the transport of the electron beam differs between the codes. `RADYN` uses a Fokker-Plank treatment to model the evolution of the non-thermal electron distribution as a function of time (including return current effects), `HYDRAD` uses the treatment of Emslie (1978), and `FLARIX` uses a test-particle module that provides the time-dependent beam propagation including scattering terms. Thermal conduction is treated as a modified form of Spitzer conduction in each code. Dissipation of Alfvénic waves has also been recently implemented in both `HYDRAD` and `RADYN` (Reep & Russell 2016; Reep et al. 2018b; Kerr et al. 2016).

Each code has been conceived and developed to focus in more details on a specific plasma physics problem. `RADYN` and `FLARIX` are radiation hydrodynamic codes which couple the hydrodynamic equations to the non-LTE (NLTE) 1D radiative transfer and time-dependent non-equilibrium atomic level population equations, for elements important for chromospheric energy balance. `RADYN` considers H, He & Ca, with Mg also sometimes included, whereas `FLARIX` considers H, Ca, and Mg (with plans to update the code to include He). Continua from other species are treated in LTE as background metal opacities. Optically thin losses are included by summing all transitions from the `CHIANTI` atomic database (Dere et al. 1997; Del Zanna et al. 2015) apart from those transitions solved in detail. In `RADYN` additional backwarming and photoionisations by soft X-ray, extreme ultraviolet, and ultraviolet radiation is included. Both `RADYN` and `FLARIX` currently employ the assumption of complete redistribution (CRD) when solving the radiation transport problem. In `RADYN` and `FLARIX` the loop is modelled as one leg of a symmetric flare loop. It should also be noted that `RADYN` also allows to calculate "a-posteriori" (i.e. with no feedback on the plasma equations of mass, momentum, and energy) the time-dependent non-equilibrium populations and radiation transport of a desired ion via the minority species version of that code, `MS_RADYN` (such as Si IV, see Kerr et al. 2019).

`HYDRAD` does not solve the detailed optically thick radiation transport and atomic level population equations, instead employing approximations of chromospheric radiation losses. Losses from H, Ca and Mg are included via the approach of Carlsson & Leenaarts (2012). The code has also recently adopted a more accurate method for computing NLTE H populations following the prescription of Sollum (1999) which approximates the radiation field in the chromosphere (Reep et al. 2019). Ion population equations, however, are solved self-consistently in full non-equilibrium ionization (NEI) for any desired element, returning a more accurate calculation of the optically thin radiative losses and spectral synthesis of optically thin lines using those ion fractions. While the treatment of optically thick radiation is less accurate than in `RADYN` or `FLARIX`, `HYDRAD` has the advantage of being significantly less computationally demanding. Other important differences are that `HYDRAD` features a multi-fluid plasma that treats the electron and hydrogen temperatures separately, it solves a full length flux tube (foot-point to foot-point) of arbitrary geometry (e.g. based on a magnetic field extrapolation) and includes effects due to cross-sectional area expansion (varying inversely with the magnetic field strength), which has been shown to play an important role in dynamics.

Ideally, both the accurate treatment of NEI as well as the inclusion of full NLTE radiative transfer are needed to ensure the correct interpretation of all the available plasma observables during flares. For instance, considerable differences are found in the synthetic spectra of coronal and TR ions between equilibrium and non-equilibrium simulations (e.g. Bradshaw & Mason 2003b; Bradshaw & Raymond 2013; Doyle et al. 2013; Bradshaw & Testa 2019; Kerr et al. 2019), suggesting that an invalid assumption of equilibrium ion populations can lead to incorrect conclusions about the properties of the plasma and therefore erroneous physical constraints for the numerical models of flare heating.

Post-processing codes, such as `RH` (Uitenbroek 2001; Pereira & Uitenbroek 2015), `MULTI` (Carlsson 1986) and `MALI` (Heinzel 1995), are also typically used in conjunction with snapshots of the flare atmospheres from the (radiation-) hydrodynamic codes to synthesize spectral lines not included in the original codes. This also allows more advanced synthesis of certain spectral lines which require additional physics, such as the need for partial redistribution (PRD) when computing the Mg II h & k lines, as observed by IRIS. Figure 1 shows spectral lines produced using `RADYN` and `RH`. It is possible to determine formation heights as a function of time, and other formation properties, which can then be related to plasma properties. A caveat with this approach is that these codes use statistical equilibrium to obtain the atomic level populations, so time dependent effects may be lost. A comparison of the results of post-processing via `RH` and `MALI` will feature as part of our project. These codes are based on the same techniques (multilevel accelerated lambda iterations, MALI, according to Rybicki & Hummer 1991) but we will ensure that they give consistent results, as well as assessing any differences resulting from the atom files used. Further, the method of use will be investigated (for example, should the NEI population levels from hydrogen be prescribed in these post-processing codes when computing other species, is full angle-dependent PRD required).

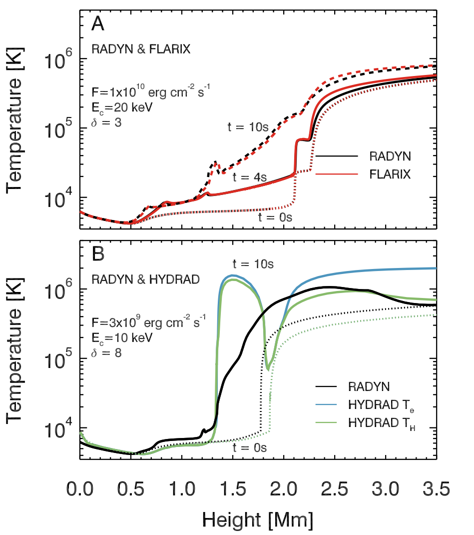

Figure 2: *Examples of comparing flare temperature stratification in RADYN to FLARIX (A), and to HYDRAD (B). Dotted lines are t = 0. For the RADYN/FLARIX comparison the codes used similar initial conditions and physical processes. For the RADYN/HYDRAD comparison the non-thermal electron beam inputs were the same, but initial conditions and code features were not prescribed to be the same.*

These codes are valuable resources to the flare community, and have been used variously to (1) understand the complex formation properties of radiation produced in the flaring chromosphere/TR, including demonstrating that Doppler flows can be easily misinterpreted in the case of optically thick emission (e.g Kuridze et al. 2015; Kerr et al. 2016; Kerr 2017; Rubio da Costa & Kleint 2017; Kerr et al. 2019), (2) investigate the formation of continua during flares, important for determining if models are sufficiently heating the atmosphere in certain locations (e.g. Heinzel et al. 2017; Kowalski et al. 2017; Simões et al. 2017), (3) perform model-data comparisons that aim to both diagnose the flaring plasma and to test models of energy transport (e.g. Kerr et al. 2016; Reep et al. 2016; Kerr 2017; Capparelli et al. 2017; Polito et al. 2018; Brown et al. 2018; Reep et al. 2018a), and (4) explore fundamental processes during solar flares (e.g. Abbett & Hawley 1999; Allred et al. 2005, 2015; Kašparová et al. 2009; Reep et al. 2015; Reep & Russell 2016; Kerr et al. 2016; Reep et al. 2018b). Without these codes, understanding flare dynamics and the resulting radiation from both space-based and ground-based observatories would be significantly more difficult. Indeed there are still many open observations yet to be explained, including (but certainly not limited to!): what the Mg II profiles observed at the leading edge of flare ribbons (Panos et al. 2018) tell us about energy deposition, if observations support the model finding that Si IV becomes optically thick (Kerr et al. 2019) (if not, then this points to a problem with the model),what are the effects of including ion acceleration in the simulations and the observable signatures of ion heating, and can models explain observations of dimming of the Helium 10830Å and D3 lines (Xu et al. 2016) or the large broadening of Mg II lines (e.g. Carlsson et al. 2015). Finally, to what extent do these numerical models compare with flare atmospheres derived from spectroscopic/spectro-polarimetric inversions (e.g. Libbrecht et al. 2019). This latter point is particularly exciting as it opens an additional avenue for model-data comparisons.

To date, a *comprehensive* comparison of these available field-aligned codes and post-processing codes, has not been performed. **We propose to critically compare and contrast these vital resources, to validate model output, determine the essential physics that must be included in such codes, and to lay groundwork for model improvements in preparation for future high-resolution observations.** This is essential now that these codes are so widely used.

There have been initial efforts to compare `RADYN` and `FLARIX`, focusing on the atmospheric stratification from identical beam injections (Kašparová et al. 2019). In this case the codes used as similar initial conditions as possible and considered the same basic set of physical processes (so that the influence of numerical approaches was assessed). Results are shown in Figure 2(A), which shows a remarkably close temperature stratification in both models (differences are presently being investigated). This is highly encouraging, suggesting that differences in numerical techniques do not lead to divergent results. Figure 2(B) shows a comparison between `RADYN` and `HYDRAD`, where the identical beam parameters were used, but the codes were not set up to be closely aligned. Here the differences are somewhat more striking. Finally, we emphasize that the F-CHROMA project created a publicly available database of solar flare RADYN models and observations (http://www.fchroma.org), but a systematic study comparing those observations with the synthetic observables is still lacking.

3. Goals and expected output

We will model a set of well-observed, multi-wavelength, flares observations, making a concerted effort to understand what drives the flares at each stage, how energy is transported, how the emission is formed, what clues to the underlying processes are contained in the emission, and how should we interpret the signatures. Our team will, for the first time, perform a detailed and comprehensive analysis of the similarities and differences of flare model predictions from a collection of codes. Guided by where the models disagree we will explore the differences in numerical methods and what physics is included to determine the cause of these differences. This will help us identify what the crucial physical components are, where future modelling efforts should be spent, and to develop a consistent physics-based understanding of the observed flares. Some specific tasks will include the following:

(1) perform a systematic set of basic experiments designed to investigate the effects of including or neglecting certain features in each of the field-aligned codes. This will include the post-processing codes, and will build upon the efforts of Kašparová et al. (2019).

(2) select at least two well observed flares (including IRIS and HXR data that can be used to determine the electron beam parameters), and task each code with simulating the atmospheric evolution and resulting radiative output. We will agree upon a canonical set of observations to predict and form the basis of the model comparisons. From these models we will use the predictions from each code to provide physical insight into the processes underlying the observed emission, as well as to what extent the models are able to match observations. We will use knowledge gained in task **(1)** as guides for determining the sources of differences in the codes to isolate the most important physics.

(3) discuss model improvements and what is required for future development, using what we have learned is the crucial physics in tasks **(1)** and **(2)**. Further, observations from SST, including spectral inversions, are set to provide observations that demand explanation and can be used to critically interrogate our models of energy transport. We will discuss how to best model such observations and what models may be presently lacking (such as non-thermal processes involving helium and other species).

Expected Output: Through these team meetings we will start new collaborations that will lead to novel work which will be presented at international conferences as well as published in refereed journals. One publication will focus on the detailed comparison between the field-aligned `RADYN`, `FLARIX` and `HYDRAD` flare codes with contribution from all the team members. This will explain the physical differences between the codes, advise potential users of what code is most appropriate for their intended use, and provide guidance on how to interpret code results. The results of **(2)** will be of particular use to the community as a means to interpret flare observations from DKIST, and we will publish a paper discussing the physical interpretation of the flare observations, from our selected dataset, through the lens of our codes whose differences will at that point be well understood. Finally, a review paper that discusses recent flare modelling, as well as laying out future modelling challenges, priorities and directions will be prepared.

4. The international team

Our 12 person team comprises the lead developers and users of the most advanced field-aligned flare models, and are experts in the fields of radiation transfer and radiation hydrodynamics, magnetohydrodynamics, and spectroscopic inversions, as applied to flares. Many team members also have extensive experience with the analysis of solar flare observations, and work at the forefront of model-data comparisons. In addition to `RADYN`, `HYDRAD`, and `FLARIX`, our team includes members with experience of an MHD code that has been applied to flares (we can discuss the effect of including/neglecting the magnetic field), and spectroscopic inversion codes (we can address if inversions of observations and numerical modelling lead to similar atmospheric stratification in flares). As well as representing six nations, our team comprises members who are at various stages of their career. Note that USA based team members represent several geographically dispersed institutions, and the team leaders (Kerr, Polito) are USA-based post-docs but originally from Europe (UK, Italy). Table 1 lists the team members and their institutions. We have considered two potential Young Scientist team members, both current PhD students, who we will approach if our proposal is selected: Akiko Tei (Japan; `FLARIX`), and Christopher Osborne (UK; `RADYN`).

5. Schedule and funding

We propose to have two 5-day long meetings, to be held at ISSI-Bern, and request lodging and per diem for 12 team members plus up to two young scientists. During the interval between meetings the team will actively work on projects initiated in Meeting One, with regular communication coordinated by the TLs. Before Meeting One we will select the flare datasets to model, and begin that modelling. Meeting One would be held late 2019, and focus on identifying important differences between the codes, performing baseline comparisons, and discussing the forward modelling of the dataset. Meeting Two would be held approximately 12-18 months later, focusing on the results of simulating the flares selected in Meeting One, identifying key areas of future model improvements, and identifying how we help the community prepare for the oncoming solar maximum with new observatories (e.g. DKIST).

Name	Affiliation	Country	Relevant Expertise
Graham S. Kerr (TL)	NASA/GSFC	USA	RADYN, RH, Obs
Vanessa Polito (TL)	Harvard-Smithsonian Center for Astrophysics	USA	RADYN, RH, Obs
Mats Carlsson	University of Oslo/Rosseland Centre for Solar Physics	Norway	RADYN, RH, MULTI
Joel C. Allred	NASA/GSFC	USA	RADYN, FP, RH
Adam F. Kowalski	University of Colorado / National Solar Observatory	USA	RADYN, RH, Obs
Jeffrey W. Reep	Naval Research Laboratory	USA	HYDRAD, RH, Obs
Stephen J. Bradshaw	Rice University	USA	HYDRAD, RH, Obs
Petr Heinzel	Astronomical Institute, Czech Academy of Sciences	Czech Republic	FLARIX, MALI, Obs
Jana Kašparová	Astronomical Institute, Czech Academy of Sciences	Czech Republic	FLARIX, Obs, NC Cols
Paulo Simões	Universidade Presbiteriana Mackenzie	Brazil	RADYN, Obs
Tine Libbrecht	Stockholm University	Sweden	MULTI3D, Obs, HAZEL
Craig D. Johnston	University of St. Andrews	UK	HYDRAD, Lare3D

Table 1: TL = Team Leader; FP = Fokker-Planck solver used in RADYN; NC Cols = Non-thermal collisional rates; Obs = observational analysis; HAZEL = Spectro-polarimetric Inversion code; Lare3D = a 3D MHD code

6. Value added by ISSI

The ISSI meetings provide a unique setting for extended discussions on a well-focused topic, that would be difficult to carry out during conferences or large workshops. ISSI will allow us to gather a team of experts on the most widely used field-aligned models in the flare community from 10 institutions of 6 different countries, that would not otherwise have the opportunity to collaborate so closely and participate in face-to-face discussions. Several members of our team (including the TLs) have taken part in prior ISSI International Teams (in both Bern and Beijing) and found that these meetings are extremely helpful, leading to significant progress. The format of the meetings (two five-day meetings) is also very suited to our project, which requires lengthy discussions, and sufficient time to work closely to compare the capabilities of each code.

References

Abbett, W. P. & Hawley, S. L. 1999, ApJ, 521, 906

Allred, J. C., et al. 2005, ApJ, 630, 573

Allred, J. C., et al. 2015, ApJ, 809, 104

Bradshaw, S. J. & Cargill, P. J. 2013, ApJ, 770, 12

Bradshaw, S. J. & Mason, H. E. 2003a, A&A, 401, 699

Bradshaw, S. J. & Mason, H. E. 2003b, A&A, 407, 1127

Bradshaw, S. J. & Raymond, J. 2013, SSR, 178, 271

Bradshaw, S. J. & Testa, P. 2019, ApJ, 872, 123

Brown, S. A., et al. 2018, ApJ, 862, 59

Capparelli, V., et al. 2017, ApJ, 850, 36

Carlsson, M. 1986, Uppsala Astronomical Observatory Reports, 33

Carlsson, M. & Leenaarts, J. 2012, A&A, 539, A39

Carlsson, M., et al. 2015, ApJL, 809, L30

Carlsson, M. & Stein, R. F. 1995, ApJL, 440, L29

Carlsson, M. & Stein, R. F. 1997, ApJ, 481, 500

Del Zanna, G., et al. 2015, A&A, 582, A56

Dere, K. P., et al. 1997, A&AS, 125

Doyle, J. G., et al. 2013, A&A, 557, L9

Emslie, A. G. 1978, ApJ, 224, 241

Fletcher, L. & Hudson, H. S. 2008, ApJ, 675, 1645

Graham, D. R. & Cauzzi, G. 2015, ApJL, 807, L22

Heinzel, P. 1995, A&A, 299, 563

Heinzel, P., et al. 2016, in IAU Symposium, Vol. 320, Solar and Stellar Flares and their Effects on Planets, ed. A. G. Kosovichev, S. L. Hawley, & P. Heinzel, 233–238

Heinzel, P., et al. 2017, ApJ, 847, 48

Kašparová, J., et al. 2019, submitted to proceedings of Astronomical Society of the Pacific Conference Series, “Radiative signatures from the cosmos: A conference in honor of Ivan Hubeny”

Kašparová, J., et al. 2009, A&A, 499, 923

Kerr, G. S. 2017, PhD Thesis, Univ. Glasgow

Kerr, G. S., et al. 2019, ApJ, 871, 23

Kerr, G. S., et al. 2016, ApJ, 827, 101

Kowalski, A. F., et al. 2017, ApJ, 836, 12

Kuridze, D., et al. 2015, ApJ, 813, 125

Libbrecht, T., et al. 2019, A&A, 621, A35

Panos, B., et al. 2018, ApJ, 861, 62

Pereira, T. M. D. & Uitenbroek, H. 2015, A&A, 574, A3

Polito, V., et al. 2016, ApJ, 816, 89

Polito, V., et al. 2015, ApJ, 803, 84

Polito, V., et al. 2018, ApJ, 856, 178

Reep, J. W., et al. 2015, ApJ, 808, 177

Reep, J. W., et al. 2019, ApJ, 871, 18

Reep, J. W., et al. 2016, ApJ, 818, 44

Reep, J. W., et al. 2018a, ApJ, 856, 149

Reep, J. W. & Russell, A. J. B. 2016, ApJL, 818, L20

Reep, J. W., et al. 2018b, ApJ, 853, 101

Rubio da Costa, F. & Kleint, L. 2017, ApJ, 842, 82

Rybicki, G. B. & Hummer, D. G. 1991, A&A, 245, 171

Simões, P. J. A., et al. 2017, A&A, 605, A125

Sollum, E. 1999, Masters Thesis, University of Oslo

Uitenbroek, H. 2001, ApJ, 557, 389

Varady, M., et al. 2010, IEEE Transactions on Plasma Science, 38, 2249

Xu, Y., et al. 2016, ApJ, 819, 89

Young, P. R., et al. 2015, ApJ, 799, 218