Theory and Model for the New Generation of the Lunar Laser Ranging Data

Sergei Kopeikin - Team Leader/Coordinator
Department of Physics and Astronomy, University of Missouri-Columbia, MO 65211, USA
kopeikins@missouri.edu/Fax:+1-573-882-4195

Alberto Escapa, Juan Getino, Alexander Gusev, Sergei Klioner, Jürgen Müller, Tomas Murphy, Jürgen Oberst, Erricos Pavlis, Despina Pavlis, Natalia Petrova, Michael Soffel

Abstract. Lunar Laser Ranging (LLR) measurements are crucial for advanced exploration of the evolutionary history of the lunar orbit, the laws of fundamental gravitational physics, selenophysics and geophysics as well as for future human missions to the Moon. Current LLR technique measures distance to the corner cube reflector (CCR) on the Moon with a precision approaching one millimeter that strongly demands further significant improvement of the theoretical model of the orbital and rotational dynamics of the Earth-Moon system. This model should inevitably be based on the theory of general relativity, fully incorporate the relevant geophysical/selenophysical processes and rely upon the most recent IAU standards. We propose new methods and approaches in developing such a mathematical model. The model takes into account all classic and relativistic effects in the orbital and rotational motion of the Moon and Earth at the millimeter-range level. It utilizes the IAU 2000 resolutions on reference frames and demonstrates how to eliminate from the data analysis all spurious (coordinate-dependent) relativistic effects playing no role in selenophysics/geophysics. The new model is based on both the locally-inertial and barycentric coordinates and elaborates on the currently used LLR code to take advantage of one millimeter accuracy for computation and evaluation of a more complete and detailed set of solution parameters. We explore the new opportunities of the one-millimeter LLR to perform the most precise fundamental test of general relativity in the solar system in robust and physically-adequate way.

Scientific Rationale

LLR technique is currently the most effective way to study the interior of the Moon and dynamics of the Moon-Earth system. The most important contributions from LLR include: detection of a molten lunar core and indication to the presence of a solid inner core [1, 2], detection of lunar free libration [3], an accurate test of the strong principle of equivalence for massive bodies [4], and setting of a stringent limit on time variability of the universal gravitational constant [5]. LLR has also given access to more subtle tests of relativity [6–8], measurements of the Moons tidal acceleration [9] and geodetic precession of the lunar orbit [10, 11], and has provided orders-of-magnitude improvements in the accuracy of the lunar orbit [12–15] and its rotation [16, 17]. On the geodesy front, LLR contributes to the determination of the Earth orientation parameters, such as nutation, precession, polar motion, UT0, and to their long-term variation [18, 19]. LLR also contributes to the realization of both the terrestrial and selenodesic reference frames [20]. The laser ranging realization of a dynamically-defined inertial reference frame in contrast to the kinematically-realized frame of VLBI offers new possibilities for mutual cross-checking and confirmation [19, 21].

Over the years, LLR has benefited from a number of improvements both in observing technology and data modeling [22]. Recently, a one millimeter precision in determining range distances between a laser on the Earth and a retro-reflector on the Moon has been achieved [23, 24]. With the precision of one millimeter, accumulation of more accurate LLR data will lead to new, fascinating discoveries in fundamental gravitational theory, geophysics, and physics of lunar interior [25] whose unique interpretation will intimately rely upon our ability to develop a systematic theoretical approach to analyze the new generation of the LLR data [14, 26].

Nowadays, the theory of the lunar motion should incorporate not only the numerous Newtonian perturbations but has to deal with much more subtle relativistic phenomena being currently incorporated to the LLR codes [27–29]. A theoretical approach, used for construction of the ephemerides, accepts that the post-Newtonian description of the planetary motions can be achieved with the Einstein-Infeld-Hoffmann (EIH) equations of motion of point-like masses [30], which are valid in the barycentric frame of the solar system with time coordinate, t, and spatial coordinates, $x^i \equiv \boldsymbol{x}$.

Due to the covariant nature of general theory of relativity the barycentric coordinates are not unique and are defined up to the space-time transformation [31, 32]

$$t \mapsto t - \frac{1}{c^2}\sum_B \nu_B \frac{G\mathrm{M}_B}{R_B}\left(\boldsymbol{R}_B \cdot \boldsymbol{v}_B\right) \ , \qquad \boldsymbol{x} \ \mapsto \ \boldsymbol{x} - \frac{1}{c^2}\sum_B \lambda_B \frac{G\mathrm{M}_B}{R_B}\boldsymbol{R}_B \ , \tag{1}$$

where summation goes over all the massive bodies of the solar system ($B = 1, 2, ..., N$); G is the universal gravitational constant; c is the speed of light; $(\boldsymbol{R}_B \cdot \boldsymbol{v}_B)$ denotes an Euclidean dot product of two vectors $\boldsymbol{R}_B$ and $\boldsymbol{v}_B$; M_B is mass of a body B; $\boldsymbol{x}_B = \boldsymbol{x}_B(t)$ and $\boldsymbol{v}_B = \boldsymbol{v}_B(t)$ are coordinates and velocity of the body B; $\boldsymbol{R}_B = \boldsymbol{x} - \boldsymbol{x}_B$; ν_B and λ_B are constant, but otherwise free parameters being responsible for a particular choice of the coordinates. These parameters can be chosen arbitrarily for each body B of the solar system. The barycentric coordinates are global coordinates covering the entire solar system. Therefore, they are of little help for efficient physical decoupling of the post-Newtonian effects existing in the orbital and rotational motions of the Moon and Earth and for the description of motion of satellites around these bodies.

Furthermore, there exists another problem stemming from the gauge freedom of the general theory of relativity. Part of this freedom is associated with the choice of time t and spatial coordinates $\boldsymbol{x}$ through parameters ν_B, λ_B in equation (1). Each term in LLR code depending implicitly on these parameters has no direct physical meaning as it can be eliminated after making a specific choice of the coordinates. Current LLR code uses $\nu_B = \lambda_B = 0$, which corresponds to working in harmonic coordinates. It simplifies equations to large extent but one has to keep in mind that these coordinates have no physical privilege and that only coordinate-independent effects can be measured.

It was noticed [33, 34] that the post-Newtonian EIH force in the lunar equations of motion admits essentially larger freedom of transformations than is given in equation (1). This is because the Earth-Moon system resides in the tidal gravitational field of the Sun and other planets making the local background space-time curved. This allows to introduce the local coordinates attached to the Earth-Moon system in infinite number of ways that significantly complicates physical interpretation of the LLR data as it reveals that many harmonics in the orbital and/or rotational motion of the Moon are very sensitive to the choice of the local coordinates. This dependence should be clarified, otherwise it can lead to misinterpretation of various aspects of gravitational physics of the Earth-

Moon system [35, 36], thus, degrading the value of extremely accurate LLR measurements for deeper exploration of the lunar interior [14].

The gauge freedom of the three body problem was studied in papers [33, 34, 37]. It was found that the post-Newtonian equations of motion of a test body orbiting Earth, can be significantly simplified by making a space-time transformation from the barycentric coordinates $x^\alpha = (ct, \boldsymbol{x})$, to geocentric coordinates $X^\alpha = (cT, \boldsymbol{X})$

$$T = t + \frac{A(t, \boldsymbol{r}_E)}{c^2} + \frac{B(t, \boldsymbol{r}_E)}{c^4} + O\left(\frac{1}{c^5}\right) , \qquad X^i = x^i - x^i_E(t) + \frac{C^i(t, \boldsymbol{r}_E)}{c^2} + O\left(\frac{1}{c^4}\right) , \tag{2}$$

where $A(t, \boldsymbol{x})$, $B(t, \boldsymbol{x})$, $C^i(t, \boldsymbol{x})$ are polynomials of distance $\boldsymbol{r}_E = \boldsymbol{x} - \boldsymbol{x}_E(t)$ of the field point $\boldsymbol{x}$ from geocenter, $\boldsymbol{x}_E(t)$. The polynomial coefficients are functions of the time t and are determined by solving a system of ordinary differential equations, which follows from the gravity field equations and the tensor law of transformation of the metric tensor. Contrary to the test particle, the Moon is a massive body, which makes the exploration of the gauge freedom of the lunar motion more involved. This requires introduction of one global (SSB) frame and three local reference frames associated with the Earth-Moon barycenter (EMB), the geocenter, and the center of mass of the Moon (selenocenter). Any local coordinates can be used for processing and interpretation of LLR data but only those post-Newtonian effects, which do not depend on the transformations (2) can have direct physical meaning. This point was understood to some extent [38] but the problem of observable quantities in the one-millimeter LLR code must be discussed at a deeper theoretical level.

Scientific Objectives

Existing computer-based theories of the lunar ephemeris [27–29] consist of three major blocks:

- the barycentric EIH equations of orbital motion of the Moon, Earth, Sun, and other planets of the solar system in harmonic coordinates;
- the Newtonian rotational equations of motion of the Moon and Earth;
- the barycentric post-Newtonian equations of light rays propagating from laser to CCR on the Moon and back.

As we have noticed, the disadvantage of the barycentric approach is that it mixes up the post-Newtonian effects associated with the orbital motion of the Earth-Moon barycenter around the Sun with those, which are attributed exclusively to the relative motion of the Moon around Earth. Therefore, analytic decoupling of the orbital motion of the Earth-Moon barycenter from the relative motion of the Moon around Earth with identification of the gauge-dependent terms in LLR code is the primary goal of our research. It leads us to the necessity of re-formulation of the LLR data processing algorithm in terms of few other reference frames besides the barycentric one. The origin of these frames should be fixed at the Earth-Moon barycenter, at Earth's geocenter, and at the lunar center of mass (selenocenter). We distinguish the Earth-Moon barycenter from the geocenter because the Moon is not a test particle, thus, making the Earth-Moon barycenter displaced from the geocenter ~1710 km below the surface of the Earth. Mathematical design of each frame is another goal of our study that is reduced to finding a metric tensor by means of solution of the gravity field equations with an appropriate boundary condition [26]. We shall explore, then, the gauge freedom of the coordinates in the Earth-Moon system by means of matching the metric tensors defined in each reference frame in the overlapping domains of their applicability.

The multi-frame post-Newtonian theory of the lunar ephemeris will revamp the LLR data processing software in order to suppress the spurious gauge-dependent solutions, which overwhelm the barycentric LLR code at the one-millimeter accuracy of LLR measurements, thus, hindering the interpretation of selenophysics, geophysics and fundamental gravitational physics. Careful mathematical construction of the local frames will allow us to pin down and correctly interpret all physical effects having classical (lunar interior, Earth geophysics, tides, asteroids, etc.) and relativistic nature.

The theory of the rotation of Earth and Moon is an important component of our study. The one-millimeter LLR data suggest that Earth and Moon should be considered as multi-layered systems. We propose to use the Hamiltonian mechanics of heavenly bodies in order to analyze their movements in response to the gravitational Newtonian and post-Newtonian torques. This approach extends to the Moon a mathematical model that had been previously developed in order to explain the small changes in Earths rotational axis. This model has being awarded the 2003 European Unions Descartes Prize for Research.

The advanced post-Newtonian dynamics of the Sun-Earth-Moon system are to include the following structural elements:
1) construction of a set of astronomical reference frames decoupling orbital dynamics of the Earth-Moon system from the rotational motion of Earth and Moon with the full account of the post-Newtonian corrections and elimination of the gauge modes;
2) relativistic definition of the integral parameters like mass, the center of mass, the multipole moments of the gravitating bodies;
3) derivation of the relativistic equations of orbital motion of the Earth-Moon center of mass with respect to the barycentric reference frame of the solar system;
4) derivation of the relativistic orbital equations of motion of Earth and Moon with respect to the reference frame of the Earth-Moon system;
5) derivation of the Hamiltonian equations of rotational motion of the multi-layer Earth and Moon;
6) derivation of the relativistic equations of motion of CCR on the lunar surface with respect to the selenocentric reference frame;
7) derivation of the relativistic equations of motion of a laser station with respect to the geocentric reference frame.

These equations are to be implemented in the LLR data processing software operating with the round-trip times of the laser pulse between observer on Earth and CCR array on the Moon. The computational advantage of the new approach in the lunar code is that it unambiguously separates physical effects from the choice of coordinates - allowing us, thus, to get robust and unbiased measurement of the true physical parameters of Earth and Moon. The new approach is particularly useful for comparing different models of the multi-layer lunar interior and for making fundamental tests of general theory of relativity in the solar system.

[1] J. G. Williams, D. H. Boggs, J. T. Ratcliff, C. F. Yoder, and J. O. Dickey, in *Lunar and Planetary Institute Conference Abstracts* (2001), vol. 32 of *Lunar and Planetary Inst. Technical Report*, p. 2028.
[2] J. G. Williams, *Geophys. Res. Lett.* **34**, 3202 (2007).
[3] J. G. Williams, X. X. Newhall, C. F. Yoder, and J. O. Dickey, in *Lunar and Planetary Institute Conference Abstracts* (1996), vol. 27 of *Lunar and Planetary Inst. Technical Report*, p. 1439.
[4] J. Müller and K. Nordtvedt, *Phys. Rev. D* **58**, 062001 (1998).
[5] K. Nordtvedt, in *Gyros, Clocks, Interferometers ...: Testing Relativistic Gravity in Space*, edited by C. Lämmerzahl, C. W. F. Everitt, and F. W. Hehl (2001), vol. 562 of *Lecture Notes in Physics, Berlin Springer Verlag*, pp. 317–329.
[6] J. Müller, K. Nordtvedt, and D. Vokrouhlický, *Phys. Rev. D* **54**, 5927 (1996).
[7] J. G. Williams, S. G. Turyshev, and T. W. Murphy, International Journal of Modern Physics D **13**, 567 (2004), arXiv:gr-qc/0311021.
[8] M. Soffel, S. Klioner, J. Müller, and L. Biskupek, *Phys. Rev. D* **78**, 024033 (2008).
[9] J. Chapront, M. Chapront-Touzé, and G. Francou, *Astron. Astrophys.* **387**, 700 (2002).
[10] B. Bertotti, I. Ciufolini, and P. L. Bender, Physical Review Letters **58**, 1062 (1987).
[11] J. O. Dickey, X. X. Newhall, and J. G. Williams, Advances in Space Research **9**, 75 (1989).
[12] X. X. Newhall, E. M. Standish, Jr., and J. G. Williams, in *Dynamics, Ephemerides, and Astrometry of the Solar System*, edited by S. Ferraz-Mello, B. Morando, and J.-E. Arlot (1996), vol. 172 of *IAU Symposium*, pp. 37–44.
[13] S. M. Kudryavtsev, Highlights of Astronomy **14**, 472 (2007).
[14] S. M. Kopeikin, E. Pavlis, D. Pavlis, V. A. Brumberg, A. Escapa, J. Getino, A. Gusev, J. Müller, W.-T. Ni, and N. Petrova, Advances in Space Research **42**, 1378 (2008), 0710.1450.
[15] E. M. Standish, in *Recent Developments in Gravitation and Cosmology*, edited by A. Macias, C. Lämmerzahl, and A. Camacho (2008), vol. 977 of *American Institute of Physics Conference Series*, pp. 254–263.
[16] J. Chapront, M. Chapront-Touzé, and G. Francou, *Astron. Astrophys.* **343**, 624 (1999).
[17] J. G. Williams, D. H. Boggs, J. T. Ratcliff, and J. O. Dickey, in *Lunar and Planetary Institute Conference Abstracts*, edited by S. Mackwell and E. Stansbery (2003), vol. 34 of *Lunar and Planetary Inst. Technical Report*, p. 1161.
[18] J. Müller, J. G. Williams, and S. G. Turyshev, in *Lasers, Clocks and Drag-Free Control: Exploration of Relativistic Gravity in Space*, edited by H. Dittus, C. Lämmerzahl, and S. G. Turyshev (2008), vol. 349 of *Astrophysics and Space Science Library*, pp. 457–472.
[19] J. Müller, M. Soffel, and S. A. Klioner, Journal of Geodesy **82**, 133 (2008).

[20] C. Huang, W. Jin, and H. Xu, Journal of Geodesy **73**, 125 (1999).
[21] D. J. Pearlman, M.R. and J. Bosworth, Advances in Space Research **30**, 135 (2002).
[22] F. Meyer, F. Seitz, and J. Mueller, in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, edited by U. Schreiber, C. Werner, G. W. Kamerman, and U. N. Singh (2002), vol. 4546 of *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, pp. 154–159.
[23] J. Battat, T. Murphy, E. Adelberger, C. D. Hoyle, R. McMillan, E. Michelsen, K. Nordtvedt, A. Orin, C. Stubbs, and H. E. Swanson, APS Meeting Abstracts p. 12.003 (2007).
[24] T. W. Murphy, E. G. Adelberger, J. B. R. Battat, L. N. Carey, C. D. Hoyle, P. Leblanc, E. L. Michelsen, K. Nordtvedt, A. E. Orin, J. D. Strasburg, et al., *Publ. Astron. Soc. Pacific* **120**, 20 (2008), 0710.0890.
[25] T. W. Murphy, E. L. Michelson, A. E. Orin, E. G. Adelberger, C. D. Hoyle, H. E. Swanson, C. W. Stubbs, and J. B. Battat, International Journal of Modern Physics D **16**, 2127 (2007).
[26] S. Kopeikin and Y. Xie, ArXiv e-prints (2009), 0902.2416.
[27] M. Chapront-Touze and J. Chapront, *Astron. Astrophys.* **124**, 50 (1983).
[28] E. M. Standish, JPL IOM 312.F-98-048, Jet Propulsion Laboratory (1998).
[29] E. V. Pitjeva, *Sol. Sys. Res.* **39**, 176 (2005).
[30] A. Einstein, L. Infeld, and B. Hoffmann, The Annals of Mathematics **39**, 65 (1938).
[31] V. A. Brumberg, *Essential Relativistic Celestial Mechanics* (New York: Adam Hilger, 1991).
[32] M. H. Soffel, *Relativity in Astrometry, Celestial Mechanics and Geodesy* (Berlin: Springer, 1989).
[33] V. A. Brumberg and S. M. Kopejkin, Nuovo Cimento B Serie **103**, 63 (1989).
[34] T. Damour, M. Soffel, and C. Xu, *Phys. Rev. D* **49**, 618 (1994).
[35] S. Kopeikin, ArXiv e-prints (2008), 0809.3392.
[36] S. M. Kopeikin, Physical Review Letters **98**, 229001 (2007), arXiv:gr-qc/0702120.
[37] S. A. Klioner and A. V. Voinov, *Phys. Rev. D* **48**, 1451 (1993).
[38] J. Mueller, M. Schneider, M. Soffel, and H. Ruder, *Astrophys. J. Lett.* **382**, L101 (1991).

Timeline of the Project

Construction of the post-Newtonian reference frames and transformations: **October 30, 2009**
Derivation of the post-Newtonian equations of motion of the Moon and the Earth: **May 30, 2010**
Hamiltonian theory of rotation of the multi-layer Moon and Earth: **March 1, 2011**
Analytic formulation of the one-millimeter LLR code: **June 30, 2011**

Schedule of the Project

First meeting: **November 1-6, 2009**
Second meeting: **June 6-11, 2010**
Third meeting: **November 14-19, 2010**
Forth meeting: **June 5-10, 2011**

The Expected Output

Four papers submitted to peer review journals. Technical report of the formulation of the one-millimeter LLR code.

The Added Value Provided by ISSI to the Project

The ISSI environment and services will bring together scientists from various disciplines that otherwise rarely meet at one location and help to create a fruitful scientific enviroment to push forward the international study of the Moon and Earth with the method of Lunar Laser Ranging. Regular meetings of the team members hosted by ISSI will facilitate the approbation of the new theories of the Moon's orbital and rotational movements, which will lead to the revamping of the LLR data processing package while keeping it harmonized with the current barycentric LLR code.

List of Confirmed Members

1. Prof. Michael Soffel
   Lohrmann Observatory, Dresden Technical University, Mommsenstrae, 13, 01062 Dresden, Germany
   Tel: +49 351 4633 4200
   Fax: +49 351 4633 7019
   Email: soffel@rcs.urz.tu-dresden.de

2. Prof. Juergen Mueller
   Institute of Geodesy, Leibniz University of Hannover, Germany
   Tel: +49 511 762 3362
   Fax: +49 511 762 4006
   Email: mueller@ife.uni-hannover.de

3. Prof. Juergen Oberst
   Institut fr Planetenforschung, Deutsches Zentrum fur Luft- und Raumfahrt Rutherfordstr, 2, 12489 Berlin
   Tel: +49 306 705 5336
   Fax: +49 ????????????
   Email: juergen.oberst@dlr.de

4. Prof. Sergei Klioner
   Lohrmann Observatory, Dresden Technical University, Mommsenstrae, 13, 01062 Dresden, Germany
   Tel: +49-351-463-32821

Fax: +49-351-463-37019
e-mail: Sergei.Klioner@tu-dresden.de

5. Prof. Juan Getino
   Departamento de Matemtica Aplicada, Universidad de Valladolid, Prado de la Magdalena s/n, E-47005 Valladolid, Spain
   Tel: ???????????????
   Fax: ???????????????
   E-mail: getino@maf.uva.es
6. Prof. Alberto Escapa
   Departamento de Matemtica Aplicada, Escuela Politécnica Superior, Universidad de Alicante, Apartado de correos 99, 03080-Alicante,Spain
   Tel:+34 96 590 9762
   Fax:+34 96 590 9707
   E-mail: Alberto.Escapa@ua.es
7. Prof. Alexander Gusev
   Kazan State University, Russia
   Tel: +7 843 521 65 48
   Fax: +7 843 264 56 04
   E-mail: Alexander.Gusev@ksu.ru
8. Dr. Natalia Petrova
   Kazan State University, Russia
   Tel: ????????????????????
   Fax:?????????????????????
   E-mail: nk_petrova@mail.ru
9. Prof. Erricos Pavlis
   Joint Center for Earth Systems Technology (JCET/UMBC), University of Maryland Baltimore County & NASA Goddard Space Flight Center, Academic IV Wing A, Suite 114E, 1000 Hilltop Circle, Baltimore, Maryland 21250 , U S A
   Tel: +1 410 455-5832
   Fax: +1 410 455-5868
   E-mail: epavlis@umbc.edu
10. Dr. Despina Pavlis
    STG Inc, 7701 Greenbelt Road, Suite 400, Greenbelt, Maryland 20770, USA
    Tel: +1 301 486-3126
    Fax: +1 301 614-8601
    E-mail: dpavlis@sgt-inc.com
11. Prof. Tom Murphy
    Department of Physics, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093, USA
    Tel: +1 858 534-3290
    Fax: +1 858 534-0262
    E-mail: tmurphy@physics.ucsd.edu

Proposed Experts

1. Prof. Victor A. Brumberg
Institute of Applied Astronomy,
Russian Academy of Sciences,
10 Kutuzova Emb.,
191187 St Petersburg, Russian Federation
Tel: +7 617 267-2520
Fax; +7 617 275-1119
Email: vabrumberg@yahoo.com

Current address: 100 Norway Street, Apartment 5C, Boston, MA 02115-3426, USA.

Facilities Required

Financial Support Requested of ISSI

Team leader requests the travel support for reimbursement of the round trip between Columbia, Missouri, USA and Bern, Switzerland (approximately in the range 1500 - 2000 US dollars). Per diem coverage of the living expenses of team members while residing in Bern.