ISSI, March 2017

Seismology of propagating kink MHD waves in the presence of flow.

ISSI Workshop
Towards dynamic solar atmospheric magneto-seismology.
27 - 31 March 2017
Bern, Switzerland

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IAP P7/08 Charm
KU Leuven GOA/2015-014
FWO Vlaanderen

1. Flows and MHD waves

- **Standing transverse MHD waves in coronal loops (Schrijver et al. (1999); Aschwanden et al. (1999)); rare events.**
- **Propagating transverse MHD waves (COMP (Tomczyk et al. 2007), SDO/AIA (McIntosh et al. 2011)); are everywhere.**
- **Transverse waves = kink waves $(m = 1)$ (Nakariakov et al. (1999)).**
- **Fundamental radial mode of kink waves $\cong$ surface Alfvén wave / Alfvénic wave. (Wentzel 1979, Goossens et al. 2009, 2012).**
- **Propagating waves and flows: Okamoto and De Pontieu (2011) : upward and downward propagating and standing waves along spicules. Phase speeds $\cong$ 100 - 200 km/sec ; expansion velocity of the spicule $\cong$ 50 - 100 km/sec.**
- **Propagating waves and flows: Morton (2014): waves and damping of waves in spicules.**
- **"An advanced theory of wave propagation with magneto-seismological tools that incorporates the influence of flows will be required to confirm the validity of this assumption."**

- **Propagating waves and flows: Bo Li et al. (2014): theoretical analysis of standing axisymmetric or sausage waves in the presence of a field aligned flow.**

- **Propagating waves and flows: Morton et al. (2015): Alfvénic wave propagation in coronal open field regions. Primary interest is in counter-streaming waves.**

- **Phase velocities of the outward propagating and inward propagating Alfvénic waves: $\cong 442 \pm 2$ km/sec and 365 ± 76 km/sec. Avarage speed of the outward moving flow: $\cong 31 \pm 7$km/sec.**

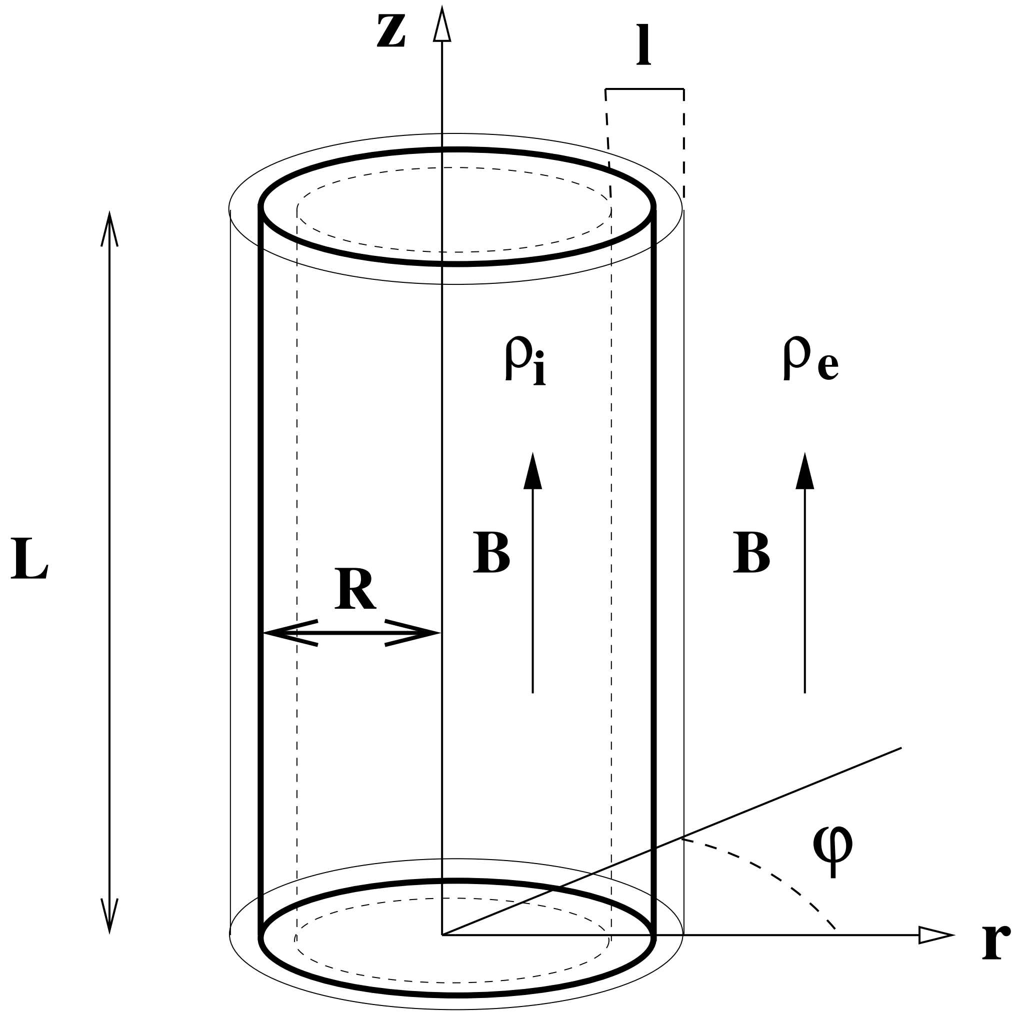
• Effects of flows on phase velocities and damping lengths of propagating MHD waves.

'Listen very carefully. I shall say this only once.'
'Well, in that case, could you please speak slowly.'
Michelle Dubois and Rene Artois.
Allo Allo.

2.Resonantly damped MHD waves on flux tubes in the presence of flow

2.1 Equilibrium and spatial and temporal dependence of waves

- **Cylindrical plasma column in stationary equilibrium.**
- **Loop of radius R:** $0 \leq r \leq R : i; r \geq R : e$
- **System of cylindrical coordinates** (r, z, φ).
- **Equilibrium magnetic field** $\vec{B}_0 = B_0 \, \vec{1}_z$, **equilibrium density** $\rho_0(r)$.
- **Field aligned equilibrium flow with velocity** $\vec{v}_0 = v_0(r) \, \vec{1}_z$
- $\exp(-i\omega t), \;\; \exp(i(m\varphi + k_z z)) \quad m, k_z =$ **azimuthal and axial wave numbers.**
- **Kink modes** $m = 1$
- $\vec{\xi} =$ **displacement,** $P' =$ **Eulerian perturbation of total pressure.**
- **Eigenmodes: dispersion relation DR**$(\omega, m, k_z) = 0$.



- **Inhomogeneity in interval**

$$b = R - l/2 \leq r \leq a = R + l/2$$

- $l = 2R$ **corresponds to a fully inhomogeneous loop**
- **Resonant damping due to non-uniformity.**
- **Standing waves: real wave number** k_z**: complex frequencies** ω**.**

$$\exp(-i\omega t), \quad \omega = \omega_R + i\gamma$$
$$\exp(-i\omega t) = \exp(-i\omega_R t)\exp(\gamma t) = \exp(-i\omega_R t)\exp(-t/\tau_d)$$

- γ = **the decrement,** $\tau_d = 1/\mid\gamma\mid =$ **the damping time.**
- **Propagating waves: real frequency** ω**: complex wavenumbers** k_z **.**

$$\exp(ik_z z), \quad k_z = k_{zR} + k_{zI}$$
$$\exp(ik_z z) = \exp(ik_{zR} z)\exp(-k_{zI} z) = \exp(ik_{zR} z)\exp(-z/L_D)$$

- **Phase velocity** $v_{ph} = \omega/k_{zR}$
- $L_D = 1/\mid k_I\mid =$ **the damping length.**

2.2 Resonant damping due to non-uniformity in presence of flow.

- $\Omega(r)$ = **Doppler shifted frequency,** $\omega_f(r)$ = **flow frequency**

$$\Omega = \omega - \omega_f, \quad \omega_f = \vec{k}\cdot\vec{v}_0 = k_z v_0(r)$$

- $\omega_A(r)$ = **local Alfvén frequency,** $\omega_C(r)$ = **local cusp frequency**

$$\omega_A^2(r) = \frac{(\vec{k}\cdot\vec{B})^2}{\mu\rho_0(r)} \; = \; k_z^2\, v_A^2(r), \quad \omega_C^2(r) = \frac{v_S^2}{v_S^2 + v_A^2}\omega_A^2(r)$$

- $v_S =$ **local speed of sound,** $v_A =$ **the local Alfvén velocity,**

$$v_S^2(r) = \gamma p_0 \;/\; \rho_0, \quad v_A^2(r) = B^2/(\mu\;\rho_0(r))$$

- **Continuous parts in spectrum in absence of flow**

$$\omega^2 = \omega_A^2(r); \quad \omega^2 = \omega_C^2(r)$$

$$\omega = \pm\omega_A(r) \;\; \omega = \pm\omega_C(r)$$

- **Continuous parts in spectrum in presence of flow.**

$$\Omega^2(r) = \omega_A^2(r); \;\; \Omega^2(r) = \omega_C^2(r)$$

$$\omega = \omega_f(r) \pm \omega_A(r), \;\; \omega = \omega_f(r) \pm \omega_C(r)$$

- **Alfvén continuum and slow continuum modified by flow.**
- **Damping / amplification of MHD waves.**
- **Pressureless plasma** $v_S = 0, \;\; \Rightarrow \omega_C = 0.$
- **No slow waves, no slow continuum.**
- **Modified Alfvén continuum**

$$\omega = \omega_f(r) \pm \omega_A(r) = k_z \left(v_0(r) \pm v_A(r)\right)$$

- **Convention:** $\omega > 0.$
- **Minus sign in sub-Alfvenic flows:** $k_z < 0.$
- **Upstream propagation.**

3. Resonantly damped MHD waves on dense pressureless flux tubes

3.1 Mixed properties and singular solutions / resonant damping

- **Pressureless plasma**

$$p << \frac{B^2}{2\mu} \rightarrow v_S^2 = \gamma p_0/\rho_0 = 0$$

- $\omega_C = 0$ / **No slow waves / no slow continuum/** $\xi_z = 0$

$$\begin{aligned}
D\frac{d(r\,\xi_r)}{dr} &= -C_2\, r\, P', \\
\frac{dP'}{dr} &= \rho(\Omega^2 - \omega_A^2)\,\xi_r, \\
\rho(\Omega^2 - \omega_A^2)\xi_\varphi &= \frac{im}{r}\, P', \\
\xi_z &= 0, \\
\nabla \cdot \vec{\xi} &= \frac{-P'}{\rho v_A^2} \\
(\nabla \times \vec{\xi}) \cdot \vec{1}_z &= -i\frac{m}{r}\, P'\, \frac{1}{[\rho(\Omega^2 - \omega_A^2)]^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho(\Omega^2 - \omega_A^2)\right).
\end{aligned}$$

$$
\begin{aligned}
D &= \rho\, v_A^2\, \Omega^2\, (\Omega^2 - \omega_A^2), \\
C_2 &= \Omega^2\, (\Omega^2 - \omega_A^2 - \frac{m^2}{r^2}\, v_A^2)
\end{aligned} \qquad (1)
$$

- **Equations are coupled: Waves have mixed properties.**
- **Non-uniform plasma:** $\nabla \cdot \vec{\xi} \neq 0, \quad (\nabla \times \vec{\xi}) \cdot \vec{1}_z \neq 0$**.**
- **The 2nd order ODE for** P'**:**

$$
\rho(\Omega^2 - \omega_A^2) \frac{d}{dr} \left\{ \frac{r}{\rho(\Omega^2 - \omega_A^2)} \frac{dP'}{dr} \right\} = \left\{ \frac{m^2}{r^2} - \Gamma(\omega^2) \right\} r P'.
$$

- **Expresion for** $\Gamma(\omega^2)$

$$
\Gamma(\omega^2) = \frac{\Omega^2 - \omega_A^2}{v_A^2} = k_z^2 \;\; \Lambda(\omega^2) = \frac{\Omega^2}{v_A^2} - k_z^2
$$

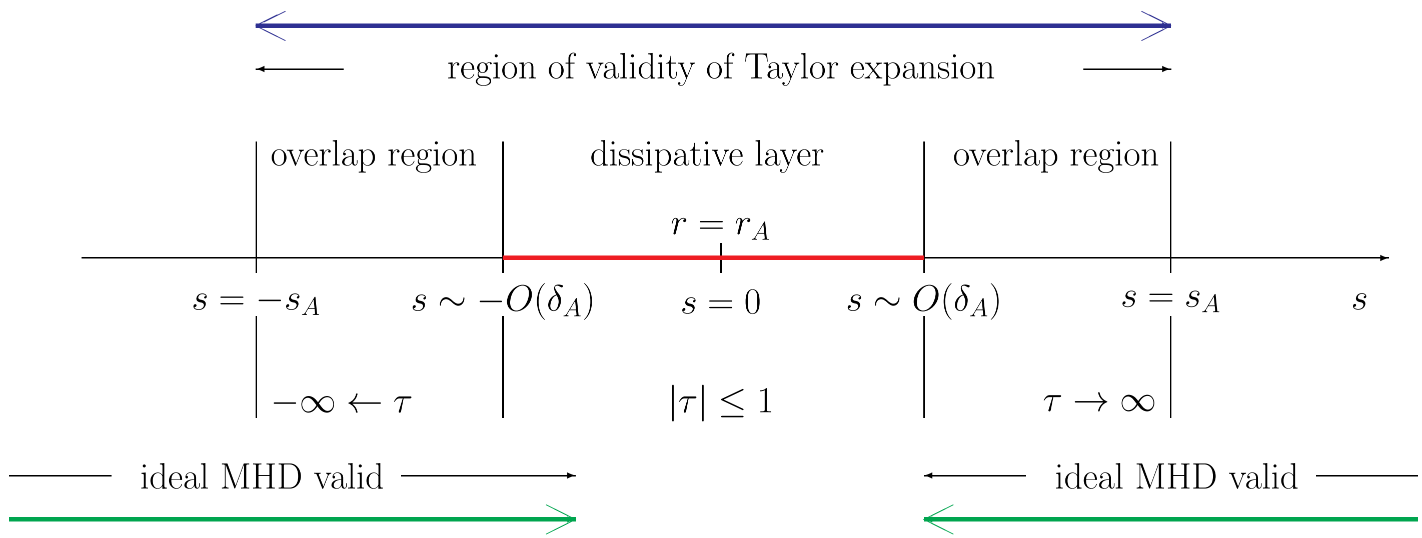
- **The sign of** Γ**, i.e. of** Λ **determines the local radial behavior of the MHD wave.**
- **Singular solutions for non-uniform equilibrium.**
- **Resonantly damped waves.**

3.2 How to compute resonantly damped waves.

- **Numerical code that integrates the resistive/viscous MHD equations.**
- **Generalized Frobenius method, Hollweg 1990, Soler et al. 2013, 2014.**
- **Use connection formulae: SGH-method.**
- **Dissipation is important only in a narrow layer around the resonant point.**
- **Thickness of dissipative layer is measured by**

$$\delta_A = \left(\frac{\omega \eta}{| \Delta |} \right)^{1/3}, \quad \Delta = \frac{d}{dr}(\omega^2 - \omega_A^2), \quad \delta_A \sim (R_m)^{-1/3}.$$

- **Magnetic Reynolds number** R_m **is very large** $\Rightarrow$ $s_A/\delta_A \gg 1.$
- $[r_A - 5\,\delta_A, \;\; r_A + 5\,\delta_A \;\;]$
- **Circumvent the numerical integration of the non-ideal MHD equations.**
- **Numerical integration of the linear ideal MHD equations.**
- $[0, \;\; r_A - 5\,\delta_A]$ **and** $[r_A + 5\,\delta_A, +\infty \;\; [$ **.**
- **Use jump conditions to connect the ideal solutions.**


region of validity of Taylor expansion
overlap region
dissipative layer
overlap region
$r = r_A$
$s = -s_A$
$s \sim -O(\delta_A)$
$s = 0$
$s \sim O(\delta_A)$
$s = s_A$
s
$-\infty \leftarrow \tau$
$|\tau| \leq 1$
$\tau \rightarrow \infty$
ideal MHD valid
ideal MHD valid

- **Further simplification: the thin boundary (TB) approximation.**
- **The non-uniform layer is thin** $l/R << 1$
- **Plasma is uniform in** $[0, \;\; R-l/2]$ **and** $[R+l/2, \;\; +\infty[$
- **Use analytic linear ideal MHD solutions for a uniform plasmas in** $[0, \;\; R-l/2]$ **and** $[R+l/2, \;\; +\infty[$ **: Bessel functions**
- **Use jump conditions to connect solutions from** $r = R - l/2$ **to** $r = R + l/2$**.**
- **Straight field:** P' **is continuous, see e.g GHS1992.**
- **Add an additional term to the dispersion relation.**
- **Takes into account the jump in** ξ_r **across the resonant layer where the real part of the kink eigenmode is equal to the local Alfvén frequency** $\omega = \omega_f(r_A) + \omega_A(r_A)$**.**
- $r_A =$ **the resonant position. In the thin boundary approximation** $r_A \approx R$**.**
- **The jump in** ξ_r **is**

$$[\xi_r] = -i\pi \frac{m^2/r_A^2}{\rho \mid \Delta \mid} P', \;\; [P'] = 0$$

$$\Delta = \frac{d}{dr}(\Omega^2(r) - \omega_A^2(r)) \mid_{r_A}$$

3.3 Dispersion relation

- **Continuity of P' and ξ_r**

$$\begin{aligned} P_i'(R) &= P_e'(R) \\ \xi_i(R) + [\xi_r] &= \xi_e(R) \end{aligned}$$

- **The dispersion relation.**

$$F\,\frac{J_m'(x_0)K_m(y_0)}{J_m(x_0)K_m'(y_0)} - i\,G\,\frac{K_m(y_0)}{K_m'(y_0)} = 1$$

$$F = \frac{k_i}{k_e}\frac{\rho_e(\Omega_e^2 - \omega_{Ae}^2)}{\rho_i(\Omega_i^2 - \omega_{Ai}^2)}, \quad G = \pi\frac{m^2/r_A^2}{\rho \mid \Delta \mid}\frac{\rho_e(\Omega_e^2 - \omega_{Ae}^2)}{k_e}$$

- **Differs from the dispersion relation for the piece-wise constant density by factor containing G.**
- **G contains the effect of the resonance.**
- **Complex dispersion relation because of effect of resonance.**
- **Dispersion relation involves Bessel functions.**

- **The thin tube (TT) approximation.**
  - **Standing waves (long wavelength approximation):** $k_z R << 1$
  - **Propagating waves (low frequency approximation):** $(\omega\ R)/v_{Ai} << 1$
- **Replace $J_m(x)$ and $K_m(y)$ by their asymptotic approximations.**
- **Thin tube and thin boundary approximation.**
- **Dispersion Relation / Eq 74 of GHS1992**

$$\rho_e(\Omega_e^2 - \omega_{Ae}^2) + \rho_i(\Omega_i^2 - \omega_{Ai}^2) = i\pi \frac{\mid m \mid /R}{\rho(r_A) \mid \Delta \mid}\ \ \rho_i(\Omega_i^2 - \omega_{Ai}^2)\rho_e(\Omega_e^2 - \omega_{Ae}^2)$$

- **Effect of resonance is contained in RHS**
- **Group terms in ω^2 and k_z^2 in LHS and divide by $\rho_i + \rho_e$**

$$k_z^2(v_k^2 - v_{KE}^2) + 2k_z\ \omega\ v_{cm} - \omega^2 = -i\pi \frac{\mid m \mid /r_A}{\rho(r_A) \mid \Delta \mid}\ \frac{\rho_i(\Omega_i^2 - \omega_{Ai}^2)\ \rho_e\ (\Omega_e^2 - \omega_{Ae}^2)}{(\rho_i + \rho_e)}$$

- $$v_k^2 = \left(\frac{\rho_i v_{Ai}^2 + \rho_e v_{Ae}^2}{\rho_i + \rho_e}\right) = \frac{2}{\rho_i + \rho_e}\frac{B_z^2}{\mu}, \quad v_{cm} = \frac{\rho_i v_i + \rho_e v_e}{\rho_i + \rho_e}, \quad v_{KE}^2 = \frac{\rho_i v_i^2 + \rho_e v_e^2}{\rho_i + \rho_e}$$
- v_k **= kink velocity,** v_{cm} **= velocity of centre of mass .**
- v_{KE} **= measures energy in flow.**
- **Propagating waves: given frequency** ω**, determine wave number** k_z
- **Wave number** k_z **is complex because of damping due to resonant absorption**

$$k_z = k_{z,R} + k_{z,I}$$

- $k_{z,R}$ **determines the phase velocity** $v_{ph} = \omega / k_{z,R}$.
- $k_{z,I}$ **determines the damping length** $L_D = 1/ \mid k_{z,I} \mid$ **.**
- **Focus on** $k_{z,R}$ **and forget about resonant damping.**
- **Put RHS = 0.**
- **Piece wise constant density and velocity: no resonant damping.**

3.4 Phase velocities, $k_{z,R}$, $v^{\pm}_{ph}$

- **STG2011**
- **Dispersion Relation**
- **Version 1**

$$\rho_e(\Omega_e^2 - \omega_{Ae}^2) + \rho_i(\Omega_i^2 - \omega_{Ai}^2) = 0$$

- **Version 2**

$$k_z^2(v_k^2 - v_{KE}^2) + 2k_z\,\omega\,v_{cm} - \omega^2 = 0$$

- **Quadratic equation in k_z and ω.**
- **In absence of flow $k^{\pm}_{zR} = \pm\,\omega/v_k$.**
- **Convention: frequency $\omega > 0$.**
- **$\pm$ sign: a wave propagating in the positive/negative $z-$ direction.**
- **Phase velocity v_{ph} of wave with frequency ω and wave number k_{zR}**

$$v_{ph} = \omega/k_{zR}$$

- **Rewrite version 2 as an equation for $v_{ph} = \omega/k_z$. Divide by $k_z^2, k_z \neq 0$.**
- **Version 3**

$$v_{ph}^2 - 2v_{cm}\, v_{ph} - (v_k^2 - v_{KE}^2) = 0$$

- **Phase velocity of the propagating wave in a static plasma in the thin tube approximation, $v_{cm} = 0, v_{KE} = 0$**

$$v_{ph,0}^{\pm} = \pm\, v_k = \pm\, \textbf{kink velocity}$$

- **Include flow in the full DR**

$$D = v_{cm}^2 + \ (v_k^2 - v_{KE}^2) = v_k^2(1 - \beta^2),$$

$$\beta^2 = \frac{v_{KE}^2 - v_{cm}^2}{v_k^2} = \frac{\rho_i \rho_e}{(\rho_i + \rho_e)^2} \frac{(\Delta v)^2}{v_k^2},$$

$$\Delta v = v_i - v_e.$$

$$\alpha = \frac{v_{KE}}{v_k}$$

- β **: stability,** α**: direction of propagation.**

- β **stability**
- $\beta^2 > 1, \quad D < 0 \Longrightarrow$ **two complex conjugate solutions for** k_z
- **Kelvin-Helmholtz instability**
- $\beta^2 = 1, \quad D = 0, \Longrightarrow$ **two equal real solutions**, $v^-_{ph,f} = v^+_{ph,f} = v_{cm}$
- $\beta^2 < 1 \Longrightarrow$ **two unequal real solutions**, $v^{\pm}_{ph,f}$

$$v^-_{ph,f} + v^+_{ph,f} = 2v_{cm}, \quad v^-_{ph,f} \times v^+_{ph,f} = v^2_{KE} - v^2_k$$

- α **direction of propagation.**
- $\alpha < 1, \quad v^2_{KE} - v^2_k < 0:$
- $v^-_{ph,f} < 0$ **backward propagating wave, wave propagates upstream in the negative z-direction.**
- $v^+_{ph,f} > 0$ **forward propagating wave, wave propagates downstream in the positive z-direction.**
- $\alpha = 1, \quad v^2_{KE} - v^2_k = 0: \quad v^-_{ph,f} = 0, \; v^+_{ph,f} = 2v_{cm}$, **no backward propagating wave, only a forward propagating wave.**
- $\alpha > 1, \quad v^2_{KE} - v^2_k > 0:$ **two forward propagating waves**

- **General solution:**

$$\boxed{v_{ph,f}^{\pm} = v_{cm} \pm v_k \, (1-\beta^2)^{1/2}}$$

- **Result is independent of** m **and holds for any non-axisymmetric wave (?).**
- **Simple expression for** $v_{ph,f}^{\pm}$**. Can we replace** $(1-\beta^2)^{1/2}$ **with 1?**

$$\boxed{v_{ph,f}^{\pm} = v_{cm} \pm v_k}$$

- **Introduce the density contrast** $\zeta = \frac{\rho_i}{\rho_e}$
- **Rewrite** β^2 **as**

$$\beta^2 = f(\zeta)\frac{(\Delta v)^2}{v_k^2}, \quad f(\zeta) = \frac{\zeta}{(1+\zeta)^2}$$

- $f(\zeta)$ **is a strictly decreasing and attains its maximum for** $\zeta = 1$ **and** $f(\zeta) \leq 1/4$**.**
- $$\beta^2 \leq \frac{1}{4}\left(\frac{\Delta v}{v_k}\right)^2$$

- **For** $\frac{\Delta v}{v_k} = 1/2 : \; \beta^2 < 1/16.$
- **The absolute value of the difference between the exact and the approximate value of** $v_{ph,f}^{\pm}$ **for sufficiently small values of** β^2

$$< v_k \times \frac{1}{8}\frac{(\Delta v)^2}{v_k^2}$$

- **For** $\frac{\Delta v}{v_k} = 1/2$ **the relative error is less than** 3%**.**
- **Low shear approximation.**
- **Remember TT and TB approximations.**
- **Special case** $v_e = 0$**.**

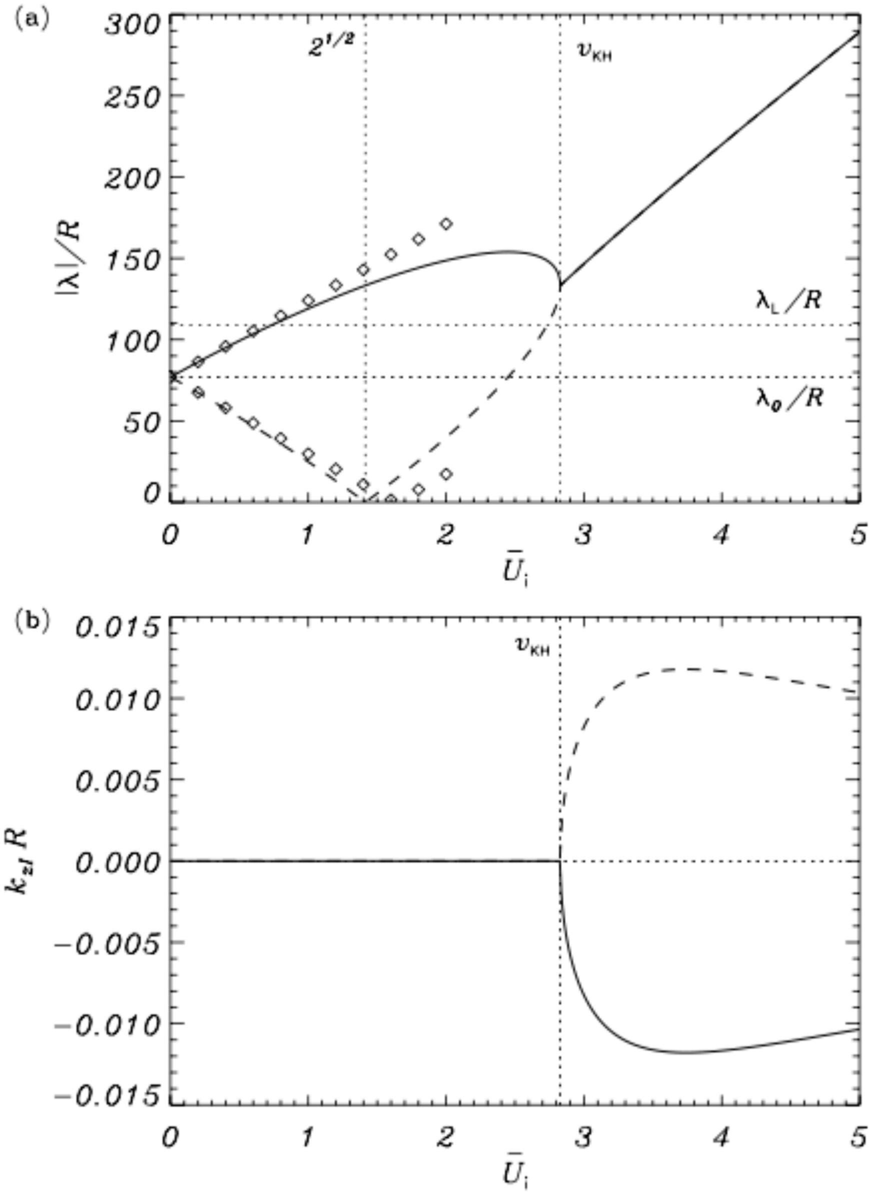
$$v_{cm} = \frac{\rho_i}{\rho_i + \rho_e} v_i = \frac{\zeta}{1+\zeta} v_i, \quad v_i = \frac{\rho_i + \rho_e}{\rho_i} v_{cm} = \frac{1+\zeta}{\zeta} v_{cm},$$

$$v_{KE}^2 = \frac{\rho_i}{\rho_i + \rho_e} v_i^2 = \frac{\zeta}{1+\zeta} v_i^2, \quad v_k^2 = \frac{2\rho_i}{\rho_i + \rho_e} v_{Ai}^2 = \frac{2\zeta}{1+\zeta} v_{Ai}^2$$

$$\alpha = \frac{v_{KE}}{v_k} = \frac{1}{\sqrt{2}} \frac{v_i}{v_{Ai}}$$

$$\beta^2 = \frac{\rho_i \rho_e}{(\rho_i + \rho_e)^2} \frac{v_i^2}{v_k^2} = \frac{\zeta}{(1+\zeta)^2} \frac{v_i^2}{v_k^2} = \frac{1}{2} \frac{\rho_e}{\rho_i + \rho_e} \frac{v_i^2}{v_{Ai}^2} = \frac{1}{2} \frac{1}{1+\zeta} \frac{v_i^2}{v_{Ai}^2}$$

- $\alpha = 1: \quad v_i = \sqrt{2}\, v_{Ai}$**: Super-Alfvénic.**
- $\beta = 1: \quad v_i = \sqrt{2}\, (1+\zeta)^{1/2}\, v_{Ai}$**: Super-Alfvénic. Ex.** $\zeta = 3, \;\; v_i = 2\sqrt{2}\, v_{Ai}$



(a)
$2^{1/2}$
v_{KH}
$|\lambda|/R$
λ_L/R
λ_0/R
$\bar{U}_i$
(b)
v_{KH}
$k_{zI}\,R$
$\bar{U}_i$

- **Low shear approximation.**
- $$\alpha = \frac{v_{KE}}{v_k}, \quad \beta^2 = \frac{\rho_i \rho_e}{(\rho_i + \rho_e)^2} \frac{(\Delta v)^2}{v_k^2}, \quad \Delta v = v_i - v_e$$
- **Sub-Alfvénic flows:** $\alpha < 1, \quad \beta < 1$ **are definitely low shear.**
- **Super-Alfvénic flows can be low shear flows: Ex** $v_i = 3/2 v_k, \quad v_e = v_k$
- **Dependence of** $v_{ph,f}^{\pm}$ **on** $\rho_i/\rho_e, \ v_i, \ v_e$ **is hidden in** β.
- **Two special cases**
- **Case 1: Slow flows**

$$v_{ph,f}^{\pm} = \pm v_k + v_{cm}, \quad k_{zR}^{\pm} = \frac{\omega}{\pm v_k + v_{cm}}$$

- $\pm$ **sign corresponds to the forward/backward propagating wave**
- **Slow flows: the waves propagate in opposite directions with velocity** $v_k + v_{cm} > 0$ **downstream in the positive** z**-direction and velocity** $-v_k + v_{cm} < 0$ **upstream in the negative** z**-direction.**
- **Case 2: Morton et al. 2015** $v_i = v_e = v_0$
- $$v_{KE} = v_{cm} = v_0, \quad \alpha = v_0/v_k, \quad \beta = 0$$

$$v_{ph,f}^{\pm} = \pm v_k + v_0, \quad k_{zR}^{\pm} = \frac{\omega}{\pm v_k + v_0}$$

- **No restriction on the speed of the flow.**
- **Sub-Alfvénic** $(\alpha < 1)$**, super-Alfvénic** $(\alpha > 1)$
- **Sub-Alfvénic flow** $(\alpha < 1)$**: the waves propagate in opposite directions, with velocity** $v_k + v_0 > 0$ **downstream in the positive** z**-direction and velocity** $-v_k + v_0 < 0$ **upstream in the negative** z**-direction.**
- **Super-Alfvénic flow** $(\alpha > 1)$**: both waves propagate downstream with velocities** $v_k + v_0 > 0$ **and** $-v_k + v_0 > 0$**.**
- **Observers often quote positive velocities.**
- **In case of the backward propagating wave in a sub-Alfvénic flow**

$$v_{ph,f,obs}^{-} = -v_{ph,f}^{-}.$$

- **Continue to use** $v_{ph,f}^{-}$**.**
- **Phase velocities of the downstream propagating and the upstream propagating wave for a sub-Alfvénic flow**

$$v_{ph,f}^{+} = v_k + v_{cm}, \quad v_{ph,f}^{-} = -v_k + v_{cm}.$$

3.5 Seismology with phase velocities ; $v_{ph}^{\pm}$

- **Substraction and addition**

$$v_k = \frac{v_{ph,f}^{+} - v_{ph,f}^{-}}{2}, \quad v_{cm} = \frac{v_{ph,f}^{+} + v_{ph,f}^{-}}{2}$$

- **Morton et al. (2015) :**

$$v_{ph,f}^{+} = 444 \pm 2 \ \mathbf{km/sec} \ , \ v_{ph,f,obs}^{-} = 365 \pm 76 \ \mathbf{km/sec}$$

$$v_k = 405 \pm 38 \ \mathbf{km/sec} \ , \ v_{cm} = 40 \pm 38 \ \mathbf{km/sec}$$

- **Flow is sub-Alfvénic.**
- **Compare v_{cm} with average speed of outward moving flow $= 30$ km/sec**
- $v_{Ai} =$ **internal Alfvén velocity,** $v_{Ae} =$ **the external Alfvén velocity .**
-

$$v_{Ai} = \frac{v_k}{\sqrt{2}} \left\{ \frac{\zeta + 1}{\zeta} \right\}^{1/2}, \quad v_{Ae} = \sqrt{\zeta}\, v_{Ai} = \frac{v_k}{\sqrt{2}} \ \{\zeta + 1\}^{1/2},$$

- $\zeta = \rho_i/\rho_e =$ **the ratio of the internal to the external density.**

- v_k **known: a parametrization of** v_{Ai} **and** v_{Ae} **in terms of** ζ.
-

$$v_k/\sqrt{2} < v_{Ai} < v_k, \quad v_k < v_{Ae} < \infty$$

- **Morton et al. (2015):**

$$286 \ \mathbf{km/sec} < v_{Ai} < 405 \ \mathbf{km/sec}, \quad 405 \ \mathbf{km/sec} < v_{Ae} < \infty$$

- **Value of** ζ **is usually not accurately known.**
- **Take** $\zeta = 3$**:** $v_{Ai} = \sqrt{2/3}\, v_k, \quad v_{Ae} = \sqrt{2}\, v_k$
- **Morton et al. (2015):** $v_{Ai} = 331$ **km/sec**, $v_{Ae} = 573$ **km/sec**.
- **Continuous variation of** v_A **in a layer of thickness** l**.**
- **Wave undergoes damping by resonant absorption due to variation of** v_A **.**
- $v_e = 0: \quad \Delta v = v_i.$
- **Contimuous variation of** v_0 **in a layer of thickness** $l^\star$**.**
- **Damping by resonant absorption due to velocity shear.**
- **Slow flows or uniform flows: Effect of flow is linear in ratio** v_{cm}/v_k
- **The effect is not necessarily small even for slow flows.**

3.6 Damping lengths $Ł_D$, $k_{z,I}$

- **STG2011, GSV2016**
- **Recall full dispersion relation**

$$k_z^2(v_k^2 - v_{KE}^2) + 2k_z\,\omega\,v_{cm} - \omega^2 = -i\pi \frac{\mid m \mid / r_A}{\rho(r_A) \mid \Delta \mid}\ \frac{\rho_i(\Omega_i^2 - \omega_{Ai}^2)\ \rho_e\ (\Omega_e^2 - \omega_{Ae}^2)}{(\rho_i + \rho_e)}$$

- **Solve this equation for $k_{z,I}$ by a standard perturbation method.**
- **Drop the quadratic terms in $k_{z,I}$ in the LHS.**
- $$\mathbf{LHS} = 2i\ \frac{k_{z,I}^{\pm}}{k_{z,R}^{\pm}}\ \omega(\omega - k_{z,R}^{\pm}\ v_{cm})$$

- **Replace k_z with $k_{z,R}$ in the RHS and recall that for $k_z = k_{z,R}^{\pm}$ (i.e. in absence of damping)**

$$\rho_i\ (\Omega_i^2 - \omega_{Ai}^2) + \rho_e\ (\Omega_e^2 - \omega_{Ae}^2) = 0$$

- $$\mathbf{RHS} = i\pi \frac{\mid m \mid / r_A}{\rho(r_A) \mid \Delta \mid}\ \frac{\left\{\rho_i(\Omega_i^2 - \omega_{Ai}^2)\right\}^2}{(\rho_i + \rho_e)}$$

•

$$\frac{k^{\pm}_{z,I}}{k^{\pm}_{z,R}} = \frac{\pi}{2} \, \frac{\mid m \mid / r_A}{\rho(r_A) \mid \Delta \mid} \, \frac{\rho_i^2}{\rho_i + \rho_e} \, \frac{(\Omega_i^2 - \omega_{Ai}^2)^2}{\omega(\omega - k^{\pm}_{z,R} \, v_{cm})}$$

• **RHS** > 0

$$\frac{k^{\pm}_{z,I}}{k^{\pm}_{z,R}} > 0$$

• **Downstream propagating waves with** $k^{+}_{z,R} > 0, \quad k^{+}_{z,I} > 0$ **are damped since** $\exp(-k^{+}_{z,I} z)$ **decays exponentially for** $z \geq 0$**.**

• **Upstream propagating waves with** $k^{-}_{z,R} < 0, \quad k^{-}_{z,I} < 0$ **are damped since** $\exp(-k^{-}_{z,I} z)$ **decays exponentially for** $z \leq 0$**.**

• **Intermediate results:**

$$\rho(r_A) \mid \Delta \mid \; = \; \mid \Omega^2(r_A) \, (\frac{d\rho}{dr})_{r_A} - 2\rho(r_A) \, \Omega(r_A) \, k_{zR} \, (\frac{dv_0}{dr})_{r_A} \mid$$

$$\mid \frac{d\rho}{dr} \mid_{r_A} \; = \; F \, \frac{\pi^2}{4} \, \frac{\rho_i - \rho_e}{l}, \quad \mid \frac{dv_0}{dr} \mid_{r_A} = F \, \frac{\pi^2}{4} \, \frac{v_i - v_e}{l_\star}.$$

• $F =$ **a constant** $= 4/\pi^2$ **for a linear variation,** $= 2/\pi$ **for a sinusoidal variation.**

• l ($l_\star$) **is length scale for variation of density (velocity).**

- **Two cases**
- **Case 1:** **Uniform flow** $v_i = v_e = v_{cm} = v_0$

$$\boxed{\frac{L_D^{\pm}}{R} = 2\pi\ \xi_E\ \ \frac{1}{f_*}\ (1 \pm v_0/v_k)^2 \approx 2\pi\ \xi_E\ \ \frac{1}{f_*}\ \ (1 \pm 2v_0/v_k)}$$

- f_* = **dimensionless frequency,** ξ_E = **dimensionless quantity:**

$$f_* = \frac{\omega R}{v_k}, \quad \xi_E = \frac{F}{\mid m \mid}\ \frac{R}{l}\ \frac{\xi + 1}{\xi - 1}, \quad f = \frac{\omega R}{v_{Ai}}$$

- **Case 2:** $v_e = 0, \quad l = l_\star$

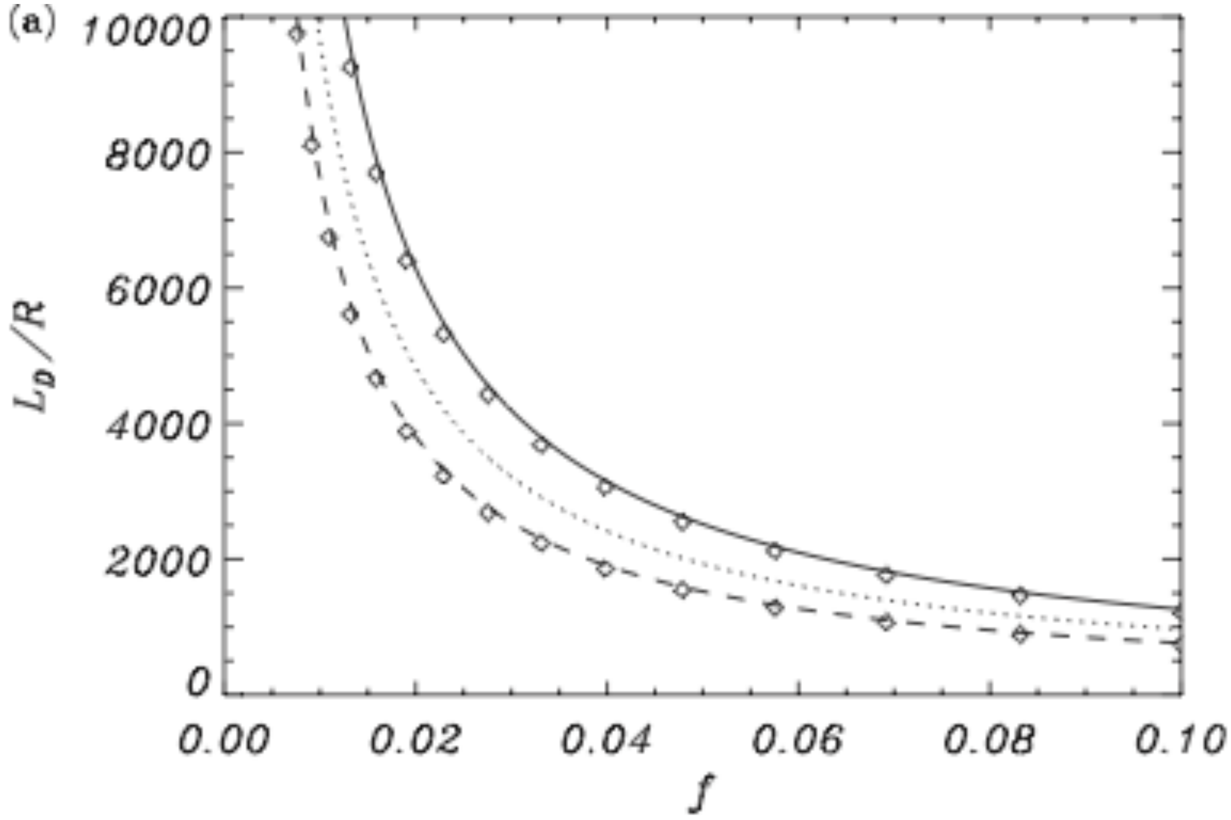
$$\boxed{\frac{L_D^{\pm}}{R} = 2\pi\ \xi_E\ \ \frac{1}{f_*}\ \left(1 \pm \frac{v_i}{v_k}\ \frac{2\zeta}{\zeta - 1}\right)}$$

- **Recall static case TGV2010**

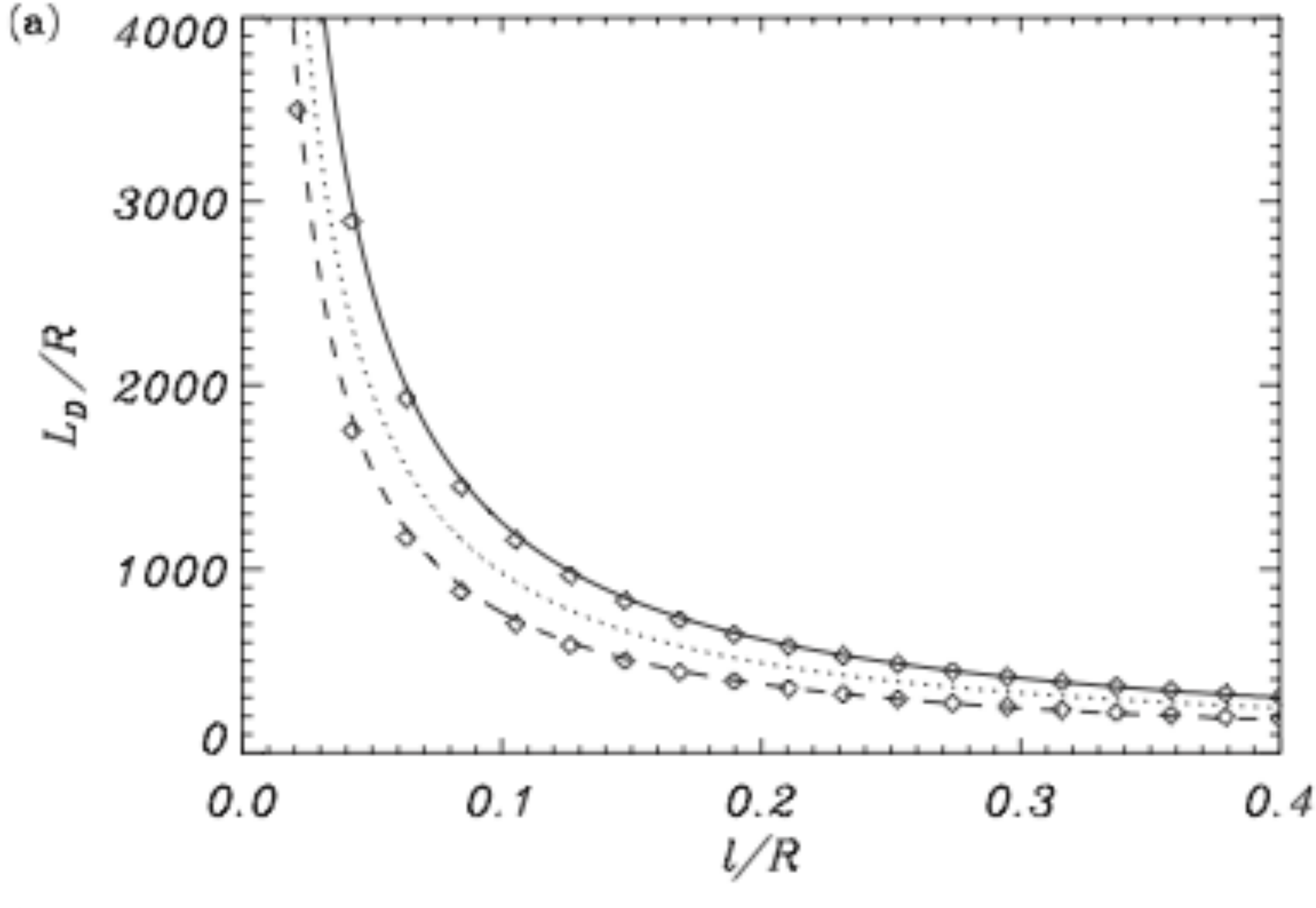
$$\boxed{\frac{L_D^{\pm}}{R} = 2\pi\ \xi_E\ \ \frac{1}{f_*}}$$

- **Selective (frequency dependent) spatial damping of propagating MHD waves:** $1/f_\star-$ **dependency**
- **VTG2010**

- **What does flow do?**
- **Selective (frequency dependent) spatial damping of propagating MHD waves.**



- $\zeta = 3, \;\; l/R = 0.1, \;\; v_i/v_{A,i} = 0.1$



- $\zeta = 3, \quad f = 0.1, \quad v_i/v_{A,i} = 0.05$

- **Selective spatial damping of propagating MHD waves**
- **Spatial damping depends on direction of propagation**
- L_D^+ **increases** / L_D^- **decreases with velocity.**
- **Forward / backward propagating waves are less /more damped than in the static case.**
- **Focus on the decrease of** L_D^- **with velocity.**
- **Extrapolation of the linear variation.**
- **Case 1 :** $L_D^- = 0$ **for** $v_0/v_k = 1/2.$ **(**$\zeta = 3,\ v_i/v_{Ai} = 0.612$**)**
- **Case 2 :** $L_D^- = 0$ **for** $v_i/v_k = (\zeta - 1)/(2\zeta)$
- **Take** $\zeta = 3\ L_D^- = 0$ **for** $v_i/v_k = 1/3, (v_i/v_{Ai} = 0.41)$
- **Ratio** L_D^-/L_D^+
- **Case 1: Take** $v_0/v_k = 1/10,\ \ 1/5,\ \ 2/5$ **and find**

$$\frac{L_D^-}{L_D^+} = 2/3,\ \ 3/7,\ \ 1/9$$

- **Case 2: Take** $\zeta = 3$**,** $v_i/v_k = 1/10,\ \ 1/5, (v_i/v_{Ai} = 0.25)$ **and find**

$$\frac{L_D^-}{L_D^+} = 7/13,\ \ 1/4$$

- **Linear approximation ?**

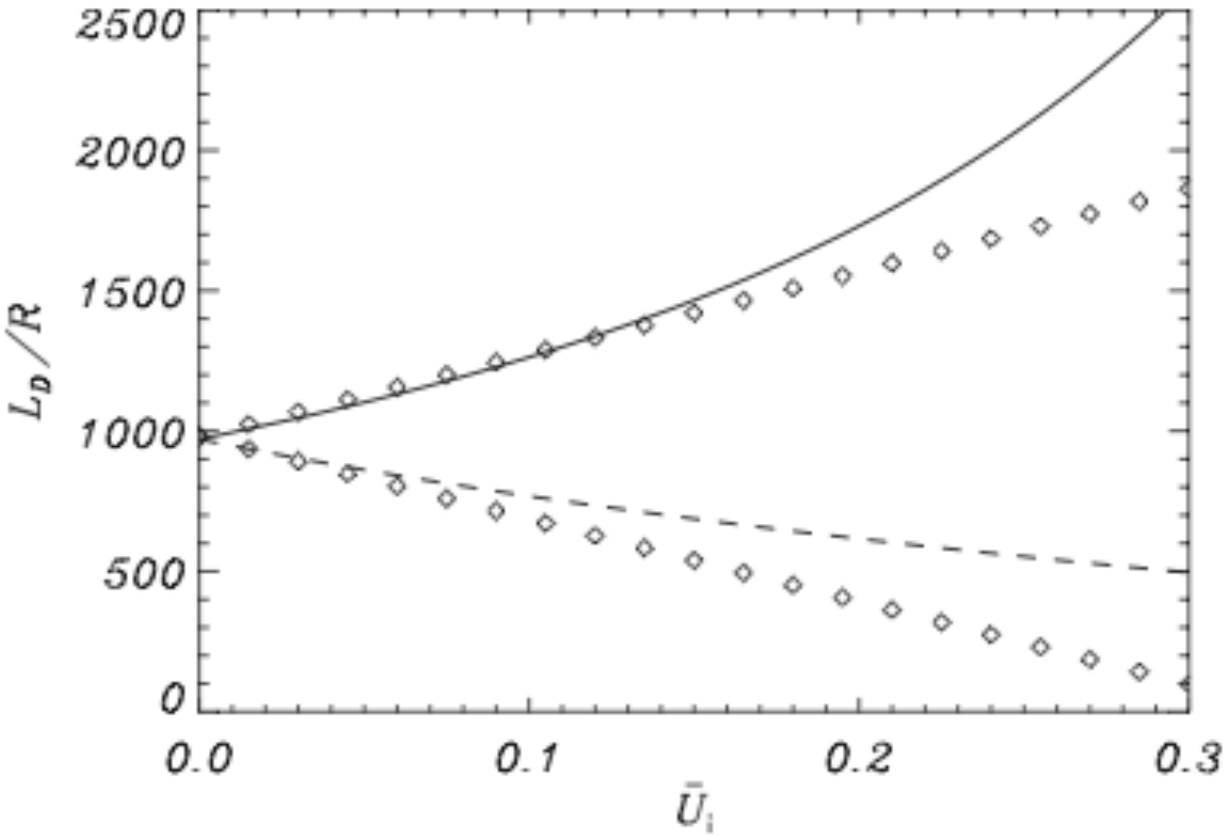


Figure 4. Ratio of the damping length to the radius, L_D/R, vs. the flow velocity normalized to the internal Alfvén velocity, $\bar{U}_i$, for the forward (solid line) and backward (dashed line) kink waves. The symbols are the linear approximation given in Equation (33). We have used $l/R = l^\star/R = 0.1$, $U_e = 0$, $f = 0.1$, and $\zeta = 3$.

• **Increased damping of the backward propagating waves.**

• **Natural explanation for the fact that counterstreaming waves are hard to observe. Even for slow sub-Alfvénic flows they might be very rapidly damped by resonant absorption.**

"Has anything escaped me?" I asked with some self-importance.
"I trust that there is nothing of consequence which I have overlooked?"
"I am afraid my dear Watson, that most of your conclusions were erroneous."
The Hound of the Baskervilles.
A. Conan Doyle.
