Phase mixing of kink waves in solar flux tubes

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Transverse oscillations of coronal loops

- First observed with *TRACE* in 1999
  Nakariakov et al. (1999); Aschwanden et al. (1999)
- After an energetic disturbance (e.g., a flare), the whole loop displays a damped transverse oscillation

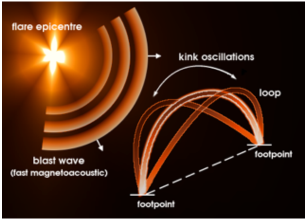
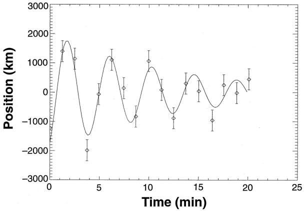


Image credit: E. Verwichte



Nakariakov et al. (1999)

Theoretical interpretation

1. Quasi-mode

- Transverse oscillation is the MHD Kink mode
  e.g., Edwin & Roberts (1983)
- Damping due to resonant absorption in the Alfvén continuum
  e.g., Ruderman & Roberts (2002); Goossens et al. (2002)

2. Spatial Fourier Expansion

- The superposition of Alfvén continuum modes builds up the global kink motion
- The damping of the global motion is caused by the phase mixing of the Alfvén modes
  e.g., Cally (1991); Soler & Terradas (2015)

3. Time-dependent numerical simulations

- Full temporal behavior
- Complicated effects: second order, nonlinear, etc...

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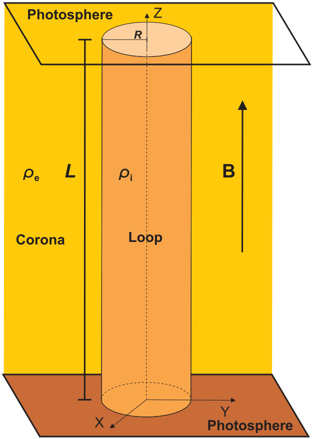
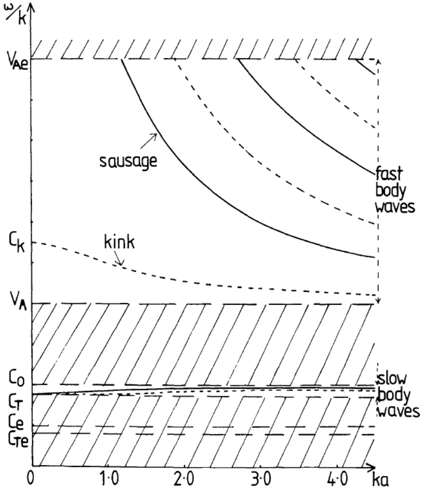
Quasi-mode

Homogeneous magnetic cylinder

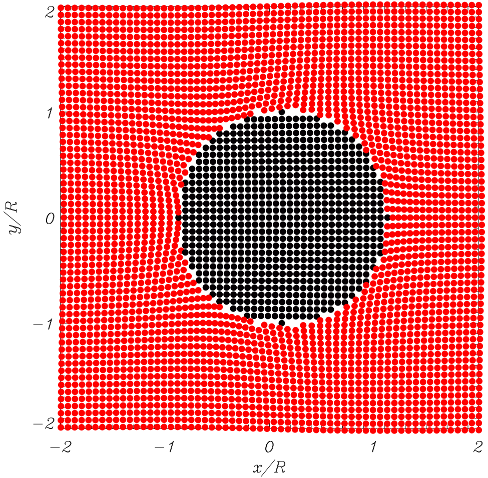




Edwin & Roberts (1983)

The kink mode of a homogeneous tube

- Global transverse motion of the flux tube
- No damping

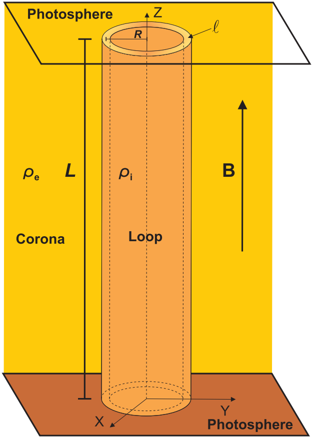


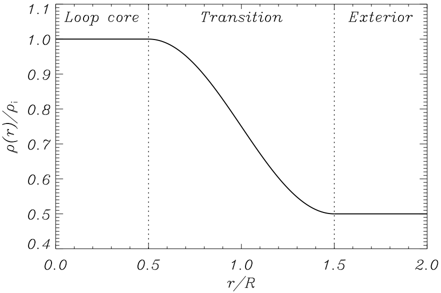
- Thin tube ($L/R \gg 1$) approximation:

$$P = \frac{2L}{v_{\mathrm{A,i}}}\sqrt{\frac{(\rho_{\mathrm{i}} + \rho_{\mathrm{e}})}{2\rho_{\mathrm{i}}}}$$

Transversely non-uniform cylinder

- A nonuniform layer is added



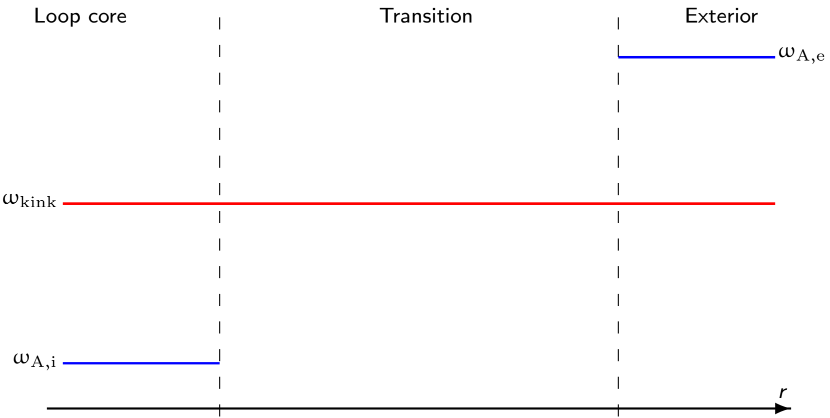


- $l = 0 \longrightarrow$ Abrupt density jump
- $l = 2R \longrightarrow$ Fully nonuniform tube

The quasi-mode

Assumptions

1. The kink mode still exists as a global mode (quasi-mode) $\sim \exp(-i\omega t)$
2. The kink mode is resonant in the Alfvén continuum



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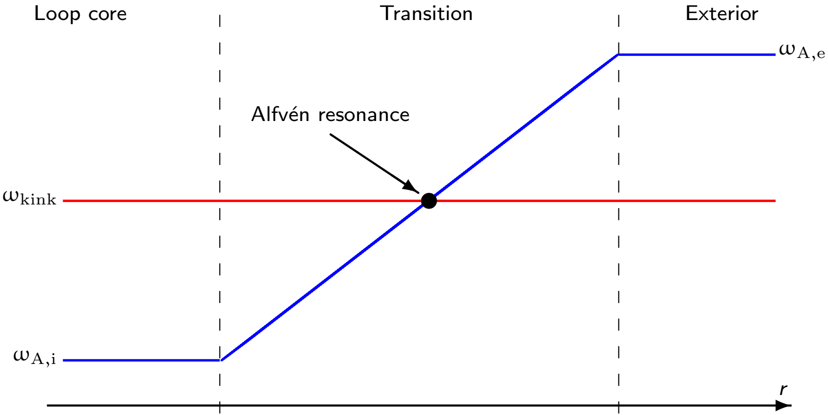

Computing the quasi-mode

1. Thin tube ($L/R \gg 1$) + thin boundary ($l/R \ll 1$) approximations
   (e.g., Ruderman & Roberts 2002; Goossens et al. 2002)
   - Correction to the kink mode of the homogeneous tube
   - Analytic expressions for P and τ_{D}/P
   - TT is OK: observations typically show that $L/R \sim 10^2$
   - There is no observational support for TB!

2. Full solution of eigenvalue problem for arbitrary parameters
   - Resistive MHD eigenmode: Fully numerical
     (e.g., Van Doorsselaere et al. 2004; Arregui et al. 2005)
   - Ideal MHD quasi-mode: Semi-analytic
     (Soler et al. 2013, 2014)

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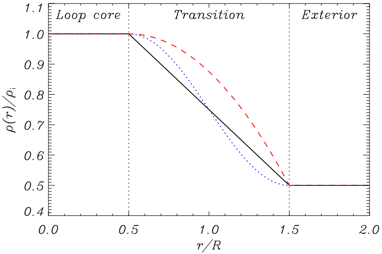
TT + TB approximations

- Thin tube ($L/R \gg 1$), thin boundary ($l/R \ll 1$) approximation

$$P = \frac{2L}{v_{\mathrm{A,i}}} \sqrt{\frac{(\rho_{\mathrm{i}} + \rho_{\mathrm{e}})}{2\rho_{\mathrm{i}}}}$$

$$\frac{\tau_{\mathrm{D}}}{P} = F \frac{R}{l} \frac{\rho_{\mathrm{i}} + \rho_{\mathrm{e}}}{\rho_{\mathrm{i}} - \rho_{\mathrm{e}}}$$



- Linear: $F = 4/\pi^2 \approx 0.405$
- Sinusoidal: $F = 2/\pi \approx 0.637$
- Parabolic: $F = 4\sqrt{2}/\pi^2 \approx 0.573$

Beyond TTTB

- Governing equation for P' in the $\beta = 0$ approximation

$$\frac{\partial^2 P'}{\partial r^2}+\left[\frac{1}{r}-\frac{\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)\right)}{\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)}\right]\frac{\partial P'}{\partial r}+\left(\frac{\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)}{B^2/\mu}-\frac{m^2}{r^2}\right)P'=0$$

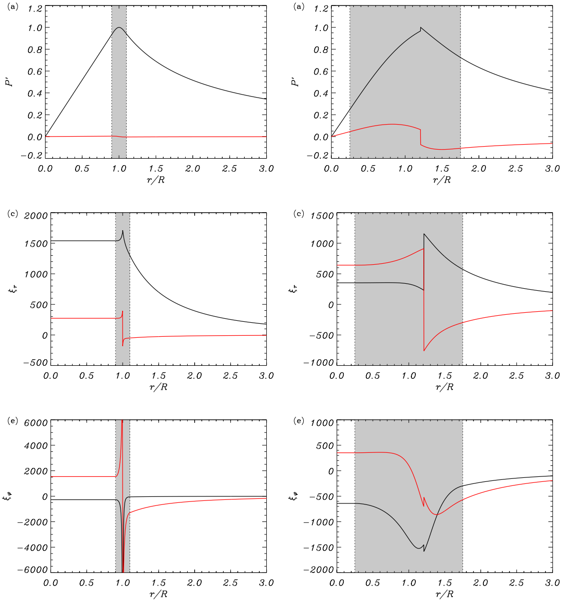
- General solution: $P'(r) = A_0 P_1'(r) + S_0 P_2'(r)$
- Frobenius series around the resonance position, $\omega = \omega_{\mathrm{A}}(r_{\mathrm{A}})$

$$P_1'(r) = (r - r_{\mathrm{A}})^2 \sum_{k=0}^{\infty} a_k (r - r_{\mathrm{A}})^k$$

$$P_2'(r) = \sum_{k=0}^{\infty} s_k (r - r_{\mathrm{A}})^k + \mathcal{C} P_1'(r) \ln (r - r_{\mathrm{A}})$$

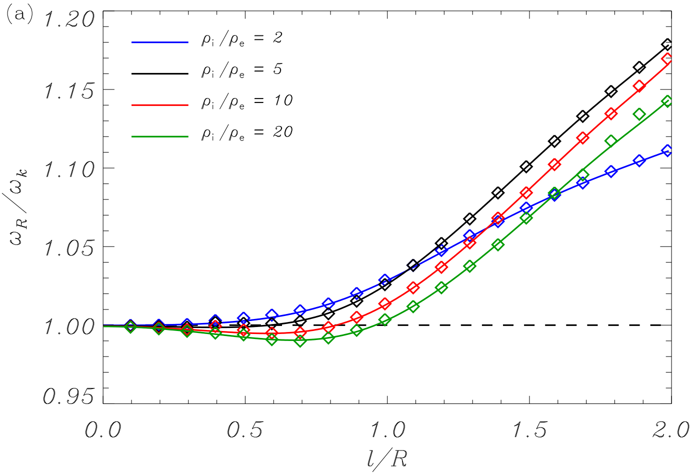
$$\mathcal{C} = \frac{m^2}{2 r_{\mathrm{A}}^2}$$

Quasi-mode perturbations



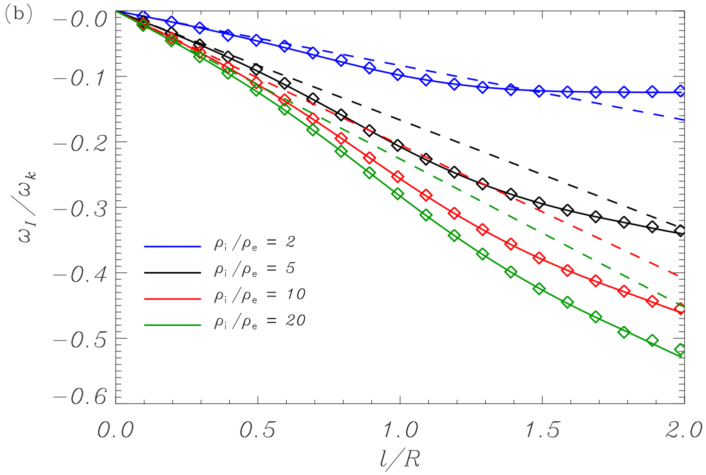
Quasi-mode frequency

- Sinusoidal transition

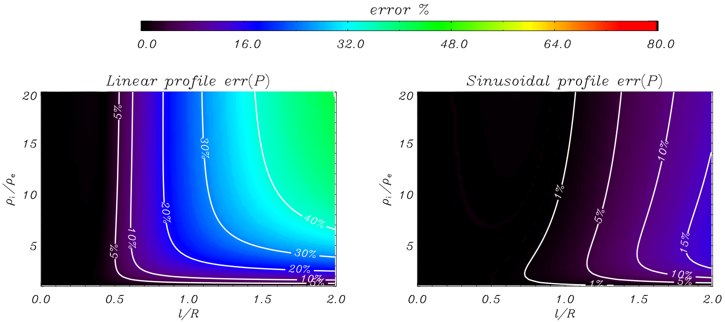


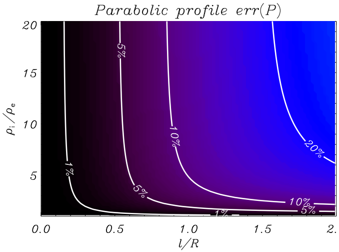
Quasi-mode damping rate

- Sinusoidal transition



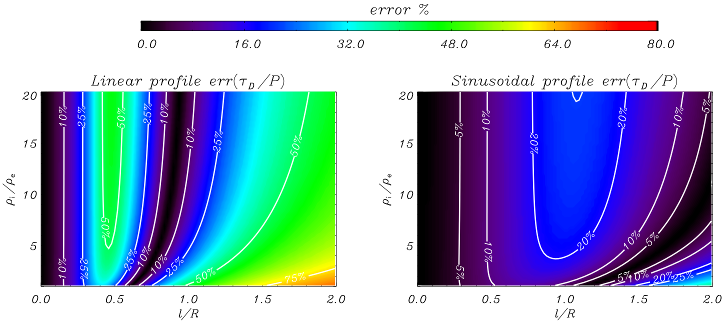
Error due to the TB approximation: P

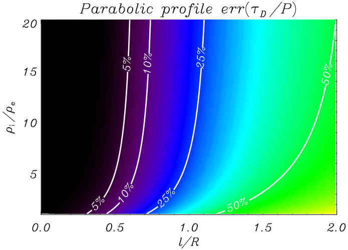




- Weak dependence on ρ_i/ρ_e
- The larger l/R, the larger the error
- Some differences between density profiles

Error due to the TB approximation: τ_{D}/P



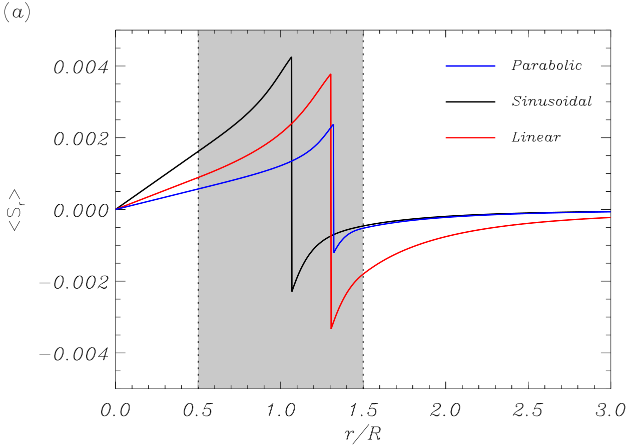


- Weak dependence on $\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$
- Complicated dependence on l/R
- **Strong influence of the density profile!**

Flux of energy

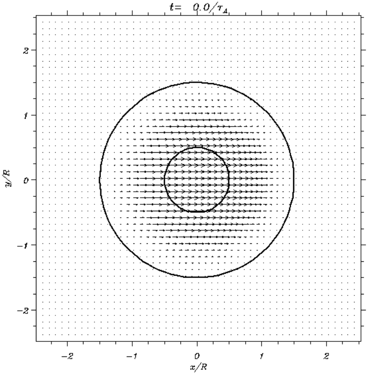
- Resonant absorption is an ideal process $\rightarrow$ Wave energy is conserved
- Damping $\neq$ Dissipation
- Radial flux of energy

$$\langle S_r \rangle = -\frac{1}{2}\mathrm{Re}\left(i\omega\xi_r P'^*\right)$$



Linear numerical simulation

- Energy transfer from transverse (kink) motions to azimuthal (rotational) motions $\rightarrow$ **"Mode conversion"**
- Phase mixing of rotational motions



Goossens et al. (2014)

Spatial Fourier Expansion

An alternative view

- Based on the paper *Phase mixing and surface waves: a new interpretation* by P. S. Cally, J. Plasma Physics 45, 1991
- Adapted to a cylindrical flux tube

- No global mode, no assumed coordinated motion of the flux tube
- Plasma motions described by the superposition of Alfvén continuum modes
- Full temporal evolution is retained

- Damping of the kink oscillation due to **phase mixing**
- Conversion from transverse to rotational motions consistently described

Method

- Linear incompressible MHD equations
- Compressibility of the kink mode (Goossens et al. 2009)

$$\nabla \cdot \mathbf{v} \sim (k_z R)^2 \sim \left(\frac{R}{L}\right)^2$$

- Governing equation for ξ_r

$$\mathcal{L}_\mathrm{A} \mathcal{L}_\mathrm{S} \xi_r + \left(k_z^2 + \frac{m^2}{r^2}\right) \frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \frac{\partial^2}{\partial t^2} \frac{1}{r} \frac{\partial (r \xi_r)}{\partial r} = 0$$

- Alfvén wave operator:

$$\mathcal{L}_\mathrm{A} \equiv \rho(r) \frac{\partial^2}{\partial t^2} + \frac{B^2}{\mu} k_z^2$$

- Surface wave operator:

$$\mathcal{L}_\mathrm{S} \equiv \left(k_z^2 + \frac{m^2}{r^2}\right) \frac{\partial^2}{\partial r^2} + \left(k_z^2 + \frac{3m^2}{r^2}\right) \frac{1}{r} \frac{\partial}{\partial r} - \left[\left(k_z^2 + \frac{m^2}{r^2}\right)^2 + \left(k_z^2 - \frac{m^2}{r^2}\right) \frac{1}{r^2}\right]$$

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Solution in the uniform regions

- Decoupled Alfvén waves and surface waves

$$\mathcal{L}_{\mathrm{A}} \xi_r = 0$$

$$\mathcal{L}_{\mathrm{S}} \xi_r = 0$$

- $m = 1$ surface wave (kink mode)
- Internal plasma ($r \leq R - \frac{l}{2}$):

$$\xi_r(r,t) = A_{\mathrm{i}}(t) I_1'(k_z r) \sim A_{\mathrm{i}}(t) \qquad \text{if} \qquad k_z R \ll 1$$

- External plasma ($r \geq R + \frac{l}{2}$):

$$\xi_r(r,t) = A_{\mathrm{e}}(t) K_1'(k_z r) \sim A_{\mathrm{e}}(t) \frac{1}{r^2} \qquad \text{if} \qquad k_z R \ll 1$$

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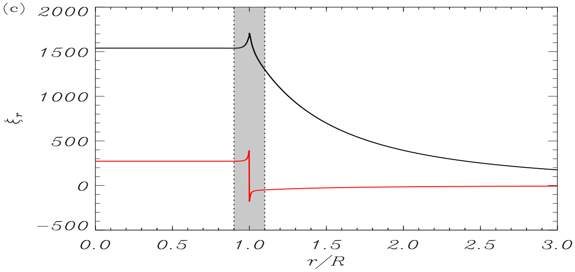


Solution in the nonuniform boundary

- Alfvén waves and surface waves are unavoidably coupled
- Generalized Fourier series

$$\xi_r(r,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(r)$$

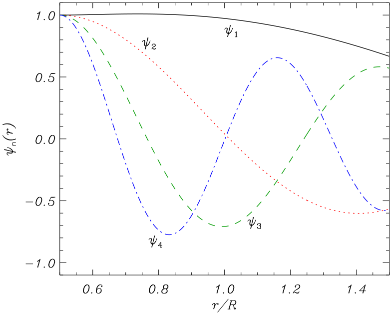
$$a_n(t=0) = \int_{R-l/2}^{R+l/2} \xi_r(r,t=0)\psi_n(r) r \mathrm{d}r$$

- Base functions $\psi_n(r)$

$$\frac{\mathrm{d}^2\psi_n}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\psi_n}{\mathrm{d}r} + \left(\lambda^2 - \frac{1}{r^2}\right)\psi_n = 0$$

$$\frac{\mathrm{d}\psi_n}{\mathrm{d}r} = 0 \qquad \text{at} \qquad r = R - \frac{l}{2}$$

$$\frac{\mathrm{d}\psi_n}{\mathrm{d}r} + \frac{2}{r}\psi_n = 0 \qquad \text{at} \qquad r = R + \frac{l}{2}$$

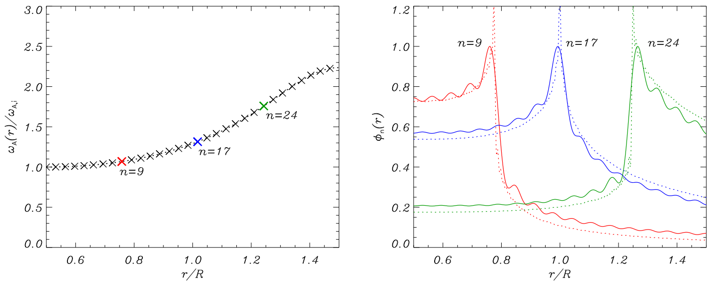


Alfvén continuum modes

- Temporal evolution of coefficients $a_n(t) \sim \exp(-i\omega t)$
- Generalized eigenvalue problem: $\mathcal{H}\mathbf{a} = \omega^2 \mathcal{M}\mathbf{a}$

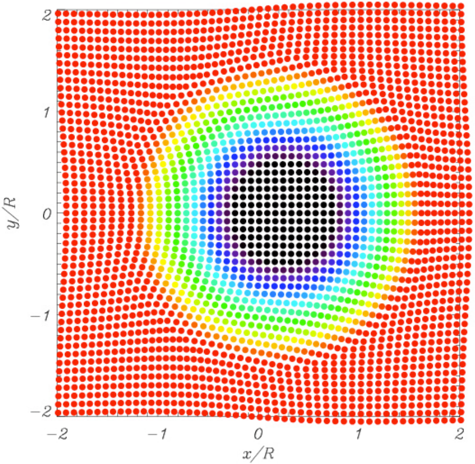
$$\mathcal{H}_{nn'} = k_z^2 \frac{B^2}{\mu} \frac{1}{l} \int_{R-l/2}^{R+l/2} \psi_n(r) \mathcal{L}_\mathrm{S} \psi_{n'}(r) r \mathrm{d}r$$

$$\mathcal{M}_{nn'} = \frac{1}{l} \int_{R-l/2}^{R+l/2} \left[\rho(r) \mathcal{L}_\mathrm{S} \psi_{n'}(r) + \frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \left(k_z^2 + \frac{m^2}{r^2} \right) \left(\frac{\mathrm{d}\psi_{n'}(r)}{\mathrm{d}r} + \frac{1}{r}\psi_{n'}(r) \right) \right] \psi_n(r) r \mathrm{d}r$$

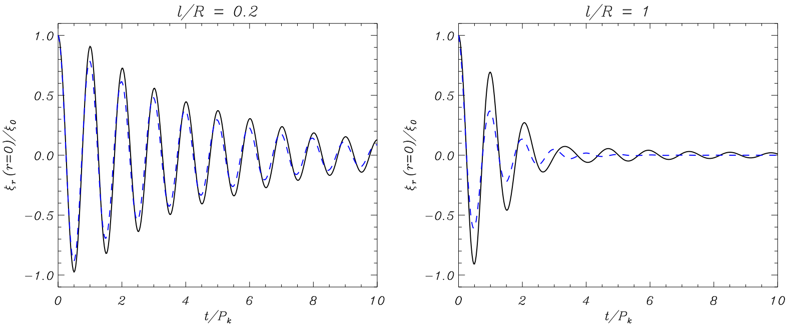


Time-dependent solution

- Phase 1: Damping of the global transverse motion
- Phase 2: Motions become rotational in the nonuniform layer

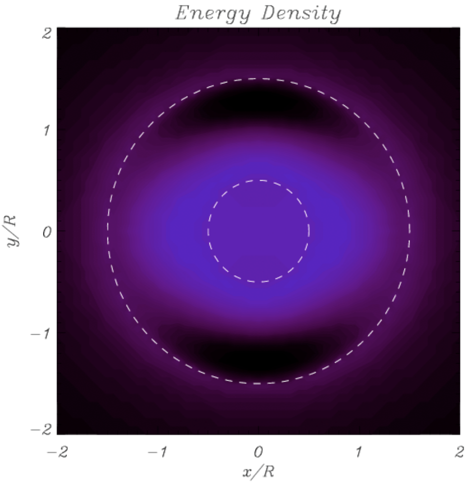


Damping of the global motion (quasi-mode)



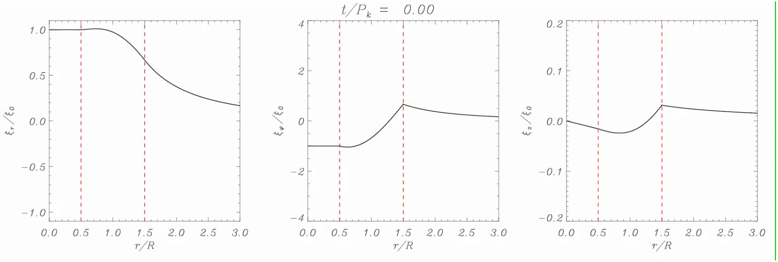
- Excellent agreement, specially for thin nonuniform layers
- For thick layers, undamped oscillations after the global motion is damped

Flux of wave energy to the nonuniform boundary



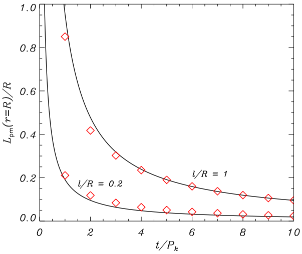
Phase mixing

- Smaller and smaller spatial scales are generated as time increases



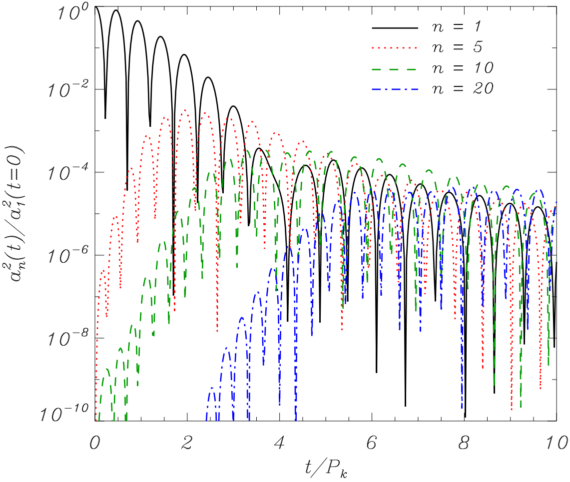
- Phase mixing length scale (Mann et al. 1995):

$$L_{\mathrm{ph}} = \frac{2\pi}{|\partial\omega_A/\partial r|\, t}$$



Energy cascade

$$\xi_r(r,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(r) \qquad \rightarrow \qquad E \sim a_n^2(t)$$



Conclusion

Comparison

1. Quasi-mode

- The kink oscillation is understood as a global mode that is damped due to **resonant absorption** in the Alfvén continuum
- Only phase 1 (damping of global motion) is described
- Simple expressions for P and τ_{D} are obtained in the TTTB approximation

2. Spatial Fourier Expansion

- Temporal evolution of kink oscillations built up as the superposition and **phase mixing** of Alfvén continuum modes
- Both phase 1 (damping of global motion) and phase 2 (generation of small-scale rotational motions) are described
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Final Remarks

What have we learnt?

- Resonant absorption and phase mixing are two aspects of the same underlying physical process
- **Energy cascade from large scales to small scales**

Missing Ingredients

- Dissipation of the small scales
- Associated heating
- Nonlinearity: KHI can generate shorter scales and enhance dissipation and heating

- References:
Soler, Goossens, Terradas, & Oliver 2013, ApJ 777, 158
Soler, Goossens, Terradas, & Oliver 2014, ApJ 781, 111
Soler & Terradas 2015, ApJ 803, 43

Final Remarks

What have we learnt?

- Resonant absorption and phase mixing are two aspects of the same underlying physical process
- **Energy cascade from large scales to small scales**

Missing Ingredients

- Dissipation of the small scales
- Associated heating
- Nonlinearity: KHI can generate shorter scales and enhance dissipation and heating

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