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Recovery of the ejecta velocity distribution by remote spacecraft measurements

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ABSTRACT

Impact-generated dust clouds were recognized by spacecraft observations around several planetary satellites, including the Moon. Here we propose a method of recovering the initial velocity distribution of ejecta particles on a satellite surface by spacecraft measurements of dust densities at different altitudes. It is shown that this problem can be reduced to the Abelian integral equation. Solution of this equation allows us to restore the ejecta velocity distribution through the use of experimental data.

1. Introduction

Gravitationally bound ejecta dust clouds generated due to impacts of fast meteoroids onto the surface of atmosphereless celestial bodies were detected by spacecraft dust instruments around the satellites of Jupiter (Krüger et al., 1999, 2000) and Saturn (Spahn et al., 2006). Recent measurements of dust particles at the spacecraft orbit were made around the Moon during the Lunar Atmosphere and Dust Environment Explorer (LADEE) mission (Horányi et al., 2015). It was found a good correlation between processes of the tenuous dust cloud formation and collisions of meteoroids with the lunar surface. The number density of dust particles in the cloud observed within the LADEE mission is in the range of $(0.4-4)\times 10^{-9}\ cm^{-3}$. The average number density as a function of altitude was found. A theoretical model of an impact-generated, steady-state, spherically symmetric dust cloud is developed in Krivov et al. (2003), where a power law $f_u(u)=\frac{\gamma}{u_0}\left(\frac{u}{u_0}\right)^{-\gamma-1}H(u-u_0)$ is adopted for the initial speed distribution of ejecta particles ($H(x)$ is the Heaviside step function equal to 1 for positive arguments and to zero otherwise, u is a velocity magnitude, γ is taken in the region 1.2–2.0). However, as it is shown in Horányi et al. (2015), the average number density deduced from experimental data as a function of height, and the results obtained with the initially assumed power-law speed distribution are not completely consistent. In present paper we propose a method of recovering the ejecta velocity distribution using the remote spacecraft measurements of dust cloud densities.

2. Method and results

We consider a spherically symmetric atmosphereless body with a radius R_m. We do not know how meteoritic showers are distributed in space and time. However, if we average the experimental data on the density of knocked-out dust particles over a long time and on different spacecraft trajectories, then their distribution can be regarded as stationary and spherically symmetric. A motion of ejecting dust particles in the central force field is considered. We use the conservation of angular momentum **M** and energy E

$$\mathbf{M}=[\mathbf{rp}]=Const,\quad |\mathbf{M}|\equiv M=mv_{\perp 0}R_M, \tag{1}$$

$$E=\frac{mv_r^2}{2}+\frac{M^2}{2mr^2}-\frac{\alpha}{r}=Const=\frac{mv_{r0}^2}{2}+\frac{M^2}{2mR_M^2}-\frac{\alpha}{R_M}, \tag{2}$$

where **r** is a radial vector with its origin in the center of the atmosphereless body under consideration; $\mathbf{p}=m\mathbf{v}$, m and **v** are particle mass and velocity, respectively; $v_{\perp 0}$ and v_{r0} are the initial ejecta velocity components perpendicular and parallel to **r**; $\alpha=gmR_M^2$ and g is a free fall acceleration on a body surface. Velocities of radial motion may be found from equation (2)

$$v_r(r)=\sqrt{\varepsilon_0-U(r)}, \tag{3}$$

where $\varepsilon_0=v_{r0}^2$ and $U(r)$ is proportional to the efficient potential energy that equals zero at the body surface

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$$U(r) = \frac{2}{m}\left(-\frac{\alpha}{r} + \frac{M^2}{2mr^2} + \frac{\alpha}{R_M} - \frac{M^2}{2mR_M^2}\right). \tag{4}$$

A conservation of particle flux provides a way to find a relation at an arbitrary altitude between the densities of particles with an initial ejecta velocity v_{r0}, an angular momentum M, mass m, and their energy spectrum $f(\varepsilon_0, M, m)$

$$n(r,M,m)\frac{r^2}{R_M^2} = \int_{U^*}^{U_{\max}} \frac{f(\varepsilon_0,M,m)d\varepsilon_0}{\sqrt{\varepsilon_0 - U(r)}} + \frac{1}{2}\int_{U_{\max}}^{\infty} \frac{f(\varepsilon_0,M,m)d\varepsilon_0}{\sqrt{\varepsilon_0 - U(r)}}. \tag{5}$$

Here $U_{\max} = \frac{2\alpha}{mR_M} - \frac{M^2}{m^2R_M^2}$. This value is determined from (4), when r tends to infinity. The particle density, described by the expression (5), is contributed both by ejecta particles executing the finite motion (first term at the right side) and the infinite motion (second term). The finite motion is possible only under condition $U_{\max} > 0$ that gives $v_{\perp 0} < v_2$, where $v_2 = \sqrt{2gR_M}$ is the escape velocity from the celestial body. The lower limit U^* is equal to $U(r)$ when $v_{\perp 0} < v_1$, $v_1 = \sqrt{gR_M}$ is the orbital velocity. When $v_{\perp 0} > v_1$, then $U^* = 0$ if $U(r) < 0$, and $U^* = U(r)$ if $U(r) > 0$.

It is possible to assume that a dust cloud around an atmosphereless body mainly consists of particles that perform the finite motion. This assumption is strongly supported by experimental data obtained during the LADEE mission (Horányi et al., 2015) which show that the characteristic velocities of dust particles in the cloud are of the order of hundreds of meters per second. This value is small compared to the lunar orbital velocity which is 1.68 km s^{-1}. The equation (5) without the second term at the right side and in the region where $U(r) > 0$ is a well-known Abelian integral equation that may be solved in a way similar to (Landau and Lifshitz, 1969). Both sides of this equation should be divided by $\sqrt{U(r) - \varepsilon_1}$, where ε_1 is a parameter, and integrated over U from ε_1 to $U_{\max}$

$$\int_{r(\varepsilon_1)}^{\infty} \frac{n(r,M,m)\frac{r^2}{R_M^2}\frac{dU}{dr}dr}{\sqrt{U(r)-\varepsilon_1}} = \int_{\varepsilon_1}^{U_{\max}} dU \int_{U(r)}^{U_{\max}} \frac{f(\varepsilon_0,M,m)d\varepsilon_0}{\sqrt{\varepsilon_0 - U(r)}\sqrt{U(r)-\varepsilon_1}}. \tag{6}$$

The lower limit of integration at the left side of (6) $r(\varepsilon_1)$ is determined from the condition $U(r) = \varepsilon_1$ that gives

$$r(\varepsilon_1) = \frac{\alpha}{mZ}\left(1 + \sqrt{1 - \frac{M^2 Z}{\alpha^2}}\right); \qquad Z = v_2^2 - v_{\perp 0}^2 - \varepsilon_1. \tag{7}$$

For the case when $v_{\perp 0} << v_2$ we can simplify (7) and get $r(\varepsilon_1) = R_M\left(1 + \frac{\varepsilon_1}{v_2^2}\right)$.

Changing the order of integration on the right side of (6) we have

$$\int_{r(\varepsilon_1)}^{\infty} \frac{n(r,M,m)\frac{r^2}{R_M^2}\frac{dU}{dr}dr}{\sqrt{U(r)-\varepsilon_1}} = \int_{\varepsilon_1}^{U_{\max}} d\varepsilon_0 \int_{\varepsilon_1}^{\varepsilon_0} \frac{f(\varepsilon_0,M,m)dU}{\sqrt{\varepsilon_0 - U(r)}\sqrt{U(r)-\varepsilon_1}}.$$

The integral over U can be easily calculated and turns out to be equal to π. The integral on the left side of (6) is integrated by parts and finally we have

$$-2\int_{r(\varepsilon_1)}^{\infty} \sqrt{U - \varepsilon_1}\frac{d}{dr}\left(n(r,M,m)\frac{r^2}{R_M^2}\right)dr = \int_{\varepsilon_1}^{U_{\max}} \pi f(\varepsilon_0,M,m)d\varepsilon_0. \tag{8}$$

Differentiating both parts of (8) with respect to ε_1, we find the desired relation

$$\pi f(\varepsilon_1,M,m) = -\int_{r(\varepsilon_1)}^{\infty} \frac{d}{dr}\left(n(r,M,m)\frac{r^2}{R_M^2}\right) \frac{dr}{\sqrt{U(r)-\varepsilon_1}}. \tag{9}$$

This relation makes it possible to find the ejecta energy spectrum for each group of particles with a specified mass m and angular moment M by spacecraft remote measurements of particle densities as functions of their distance to the center of the celestial body. Taking into account that satellite observations are commonly made at altitudes much lower than a celestial body radius (i. e. the LDEX experiment was carried out at altitudes 20–200 km above the lunar surface), we may simplify the expression for the efficient potential energy (3). Taking $r = R_M + h$, and $h << R_M$, we get that, firstly, $U(r) = 2gh$, and, secondly, the velocity component $v_\perp$ does not change with altitude as follows from the conservation of angular momentum under the assumption that $h << R_M$. Thus equation (9) can be rewritten as

$$\pi f(\varepsilon_1, v_{\perp 0}, m) = -\int_{\varepsilon_1/2g}^{\infty} \frac{dh}{\sqrt{2gh - \varepsilon_1}} \frac{dn(h, v_{\perp 0}, m)}{dh}. \tag{10}$$

Integrating (10) over the transverse velocities $v_{\perp 0}$ and masses m, we find a relation between the averaged radial velocity distribution function $\tilde{f}(\varepsilon_1) = 2\pi\int f(\varepsilon_1, v_{\perp 0}, m)v_{\perp 0}dv_{\perp 0}dm$ and the total number density of particles $n_\Sigma(h) = 2\pi\int n(h, v_{\perp 0}, m)v_{\perp o}dv_{\perp 0}dm$

$$\pi\tilde{f}(\varepsilon_1) = \int_{\varepsilon_1/2g}^{\infty} \frac{dh}{\sqrt{2gh-\varepsilon_1}} \frac{dn_\Sigma(h)}{dh}. \tag{11}$$

Let us consider first the height-limited density distribution: $n_{\sum}(h) = n_0\left(1 - \frac{h}{L}\right)$. Substituting this expression in (11), we obtain $\tilde{f}(\varepsilon_1) = \frac{n_0}{\pi Lg}\sqrt{2gL - \varepsilon_1}$. Since the altitude limit is specified in this case, then there is an energy limit. For a case of a power density decrease with altitude: $n_{\sum}(h) = n_0/\left(1 + \frac{h}{L}\right)^s$, we have $\tilde{f}(\varepsilon_1) = \frac{sn_0}{\sqrt{2\pi gL}} \frac{1}{(1+\varepsilon_1/2gL)^{s+1/2}} \frac{\Gamma(s+1/2)}{\Gamma(s+1)}$ where $\Gamma(s+1/2)$ and $\Gamma(s+1)$ are gamma-functions, and when $s = 2$ we have $\tilde{f}(\varepsilon_1) = \frac{2n_0}{\sqrt{2gL}} \frac{1}{(1+\varepsilon_1/2gL)^{5/2}}$. The empirical relation for the altitude distribution of particle densities in the dust cloud around the Moon is presented in paper (Popel et al., 2017): $n_{\sum}(h) = 4.43\exp(-h/L) \times 10^{-9}\ cm^{-3}$, where h is measured in kilometers and $L = 200\ km$. Substituting this empirical relation in (11), we find the averaged radial velocity distribution function

$$\tilde{f}(\varepsilon_1) = \frac{4.43 \times 10^{-9}}{\sqrt{2\pi gL}}\exp\left(-\frac{\varepsilon_1}{2gL}\right) \tag{12}$$

which corresponds to the Maxwell distribution with a thermal velocity of the order of 570 m/s. The characteristic velocities of particles in the dust cloud around the Moon are of the order of hundreds of meters per second (Horányi et al., 2015), the latter agree well with the values obtained with a help of (12). It is also possible to estimate the total flux of particles knocked out from a square centimeter of a lunar surface per second $nv = \frac{1}{2}\int_0^\infty f(\varepsilon_1)d\varepsilon_1 \approx 10^{-4}\ cm^{-2}s^{-1}$. It should be noted that for the case, when the characteristic lift height of dust particles L is much smaller than the radius of the celestial body, particles reaching this height come from a limited area of the body surface with a radius $2L$. So, it is sufficient to average the experimental data along a spacecraft trajectory over a band of width $\sim 4L$ to satisfy our assumption that the distribution of impact-generated particles can be regarded as spherically symmetric.

3. Summary and conclusions

In the present paper a method of recovering the ejecta velocity distribution of impact-generated particles by remote spacecraft measurements is proposed. The motion of ejecting dust particles is considered in the central gravitational field. It is necessary to note that the influence of electric fields, existing in the vicinity of the lunar surface, at the charged dust particle motion is not taken into account. The reason is that electric fields (arising under action of the solar electromagnetic radiation, the solar wind plasma, the plasma of the terrestrial magnetosphere tail and so on) are not sufficiently strong, and they are located in space region of the order of several Debye radius (several tens of meters) (Burinskaya, 2015). It is shown in Popel et al. (2018) that the electrostatically ejected dust population can exist only in the near surface layer while the dust appearing in the lunar exosphere owing to impacts of meteoroids with the lunar surface exist at all altitudes. The electric force acting in the near surface layer can play some role in the motion of fairly light charged particles with a radius less than 100 nm. For these particles the ejecta velocity distribution derived in the present investigation corresponds to the upper boundary of the spatial region, where there is an electric field. It worth noting that LADEE mission have found no evidence for high-altitude electrostatically lofted nanoparticles, the dust populations measured by LADEE have been ejecta particles from meteoroid bombardment (Horányi et al., 2015). In the present investigation we show that, if the bulk of dust particles executes a finite motion, it is possible to reconstruct the ejecta energy spectrum for each group of particles with a specified mass m and an angular moment M by remote measurements of particle densities as functions of their distance to the center of the celestial body using the expression (9), when densities of each group are known from experimental data. When the data set is limited, this expression can be used to obtain the averaged velocity distribution function, integrating (9) over the angular momentums or masses, or over both of them. Using the empirical relation for the altitude distribution of particle densities in the dust cloud around the Moon (Popel et al., 2017) we find the averaged radial velocity distribution function that gives characteristic physical parameters corresponding to experimental observations.

At present, there are no direct spacecraft measurements of particle densities in planetary dust clouds. The density is mainly estimated by means of impact dust detectors under the assumption about the initial velocity distribution. Theoretical method of reconstruction of the ejecta velocity distribution proposed in the present paper can be used to derive dust density and velocity distributions in a self-consistent way from available experimental data. The obtained velocity distribution functions can be used to develop models for the dust particle generation due to impacts of meteoroids on a body surface. It should be noted that the research conducted and the results obtained are applicable for any atmosphereless body such as planetary satellites or asteroids.

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References

Burinskaya, T.M., 2015. Non-monotonic potentials above the day-side lunar surface exposed to the solar radiation. Planet. Space Sci. 115, 64–68.

Horányi, M., Szalay, J.R., Kempf, S., Schmidt, J., Grün, E., Srama, R., Sternovsky, Z., 2015. A permanent, asymmetric dust cloud around the Moon. Nature 522, 324–326.

Krivov, A.V., Sremčević, M., Spahn, F., Dikarev, V.V., Kholshevnikov, K.V., 2003. Impact-generated dust clouds around planetary satellites: spherically symmetric case. Planet. Space Sci. 51, 251–269.

Krüger, H., Krivov, A., Hamilton, D., Grün, E., 1999. Detection of an impact –generated dust cloud around Ganymede. Nature 399, 558–560.

Krüger, H., Krivov, A., Grün, E., 2000. A dust cloud of Ganymede maintained by hypervelocity impacts of interplanetary micrometeoroids. Planet. Space Sci. 48, 1457–1471.

Landau, L.D., Lifshitz, E.M., 1969. Course of Theoretical Physics, vol. 1. Pergamon Press.

Popel, S.I., Golub, A.P., Zelenyi, L.M., Horányi, M., 2017. Impacts of fast meteoroids and a plasma-dust cloud over the lunar surface. JETP Lett. (Engl. Transl.) 105, 635–640.

Popel, S.I., Golub, A.P., Zelenyi, L.M., Dubinskii, A.Yu, 2018. Lunar dust and dusty plasmas: recent developments, advances, and unsolved problems. Planet. Space Sci. 156, 71–84.

Spahn, F., Thiessenhusen, K.-U., Colwell, J.E., Srama, R., Grün, E., 2006. Cassini dust measurements at Enceladus and implications for the origin of the E ring. Science 311, 1416–1418.