Multi-fluid magnetohydrodynamics in the solar atmosphere

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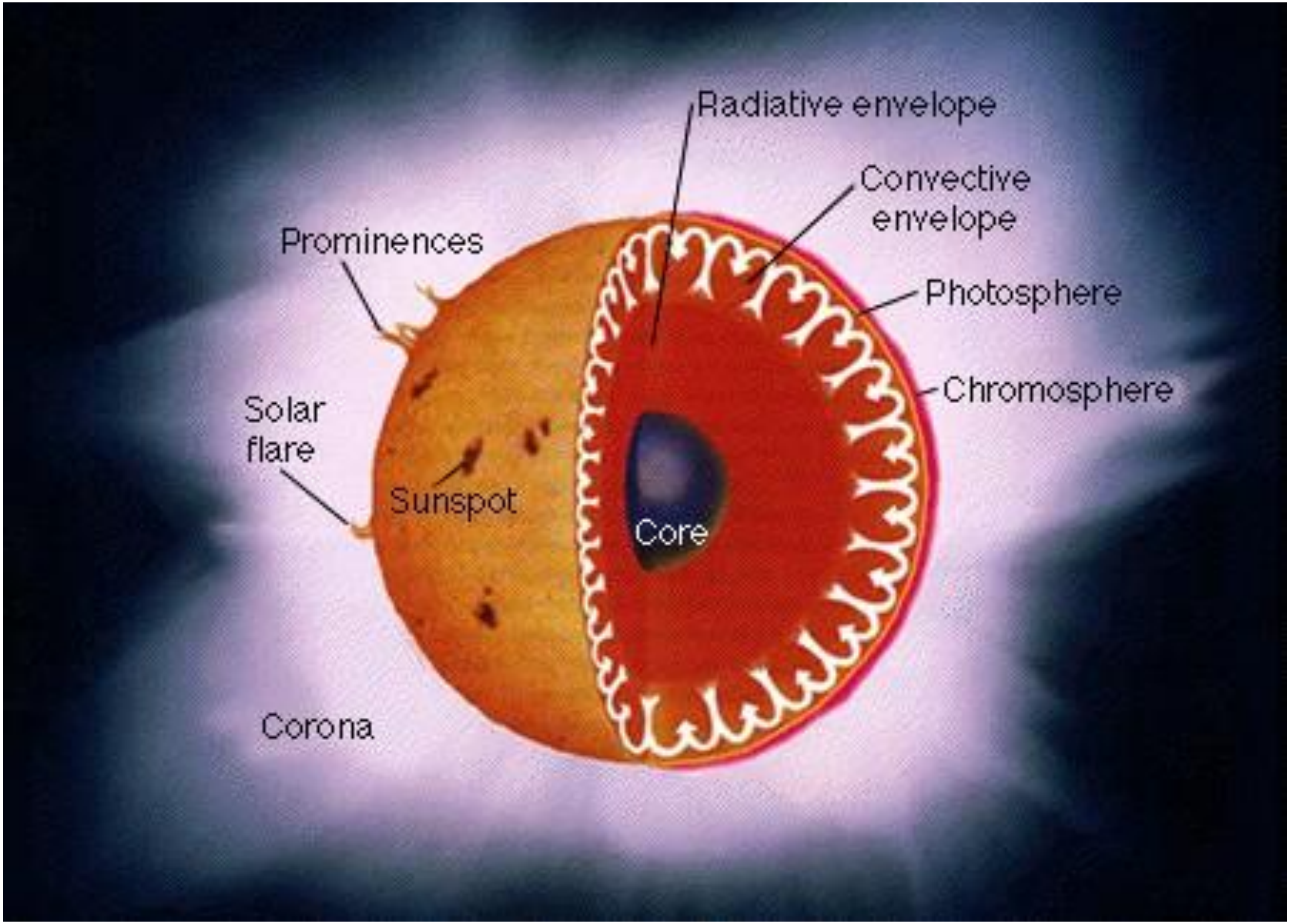
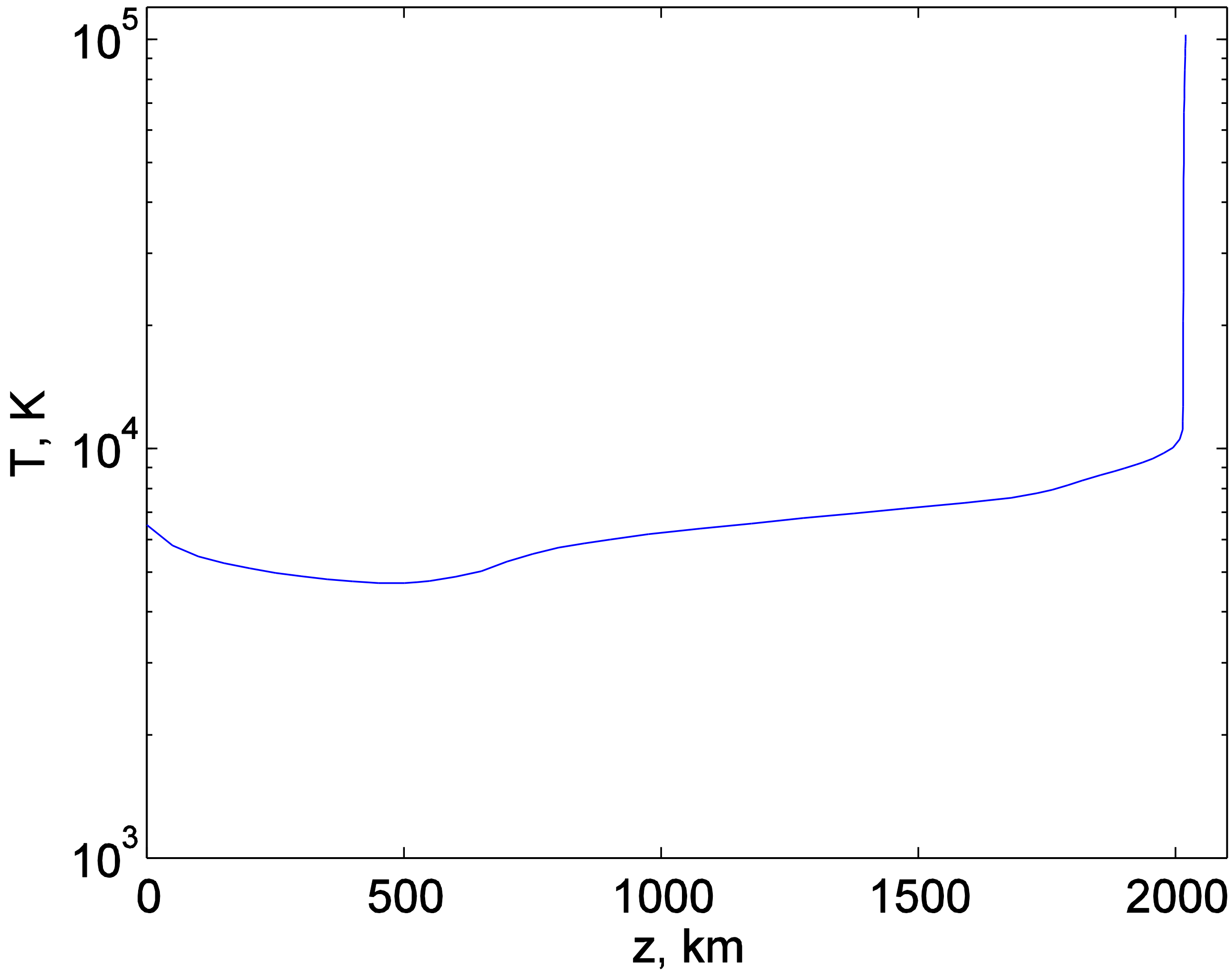

Radiative envelope
Convective envelope
Prominences
Photosphere
Chromosphere
Solar flare
Sunspot
Core
Corona

Temperature in the solar atmosphere



FAL93-3 model (Fontenla et al. 1993)

Coronal heating problem

Coronal energy losses:

Coronal holes - $8 \cdot 10^5$ erg cm^{-2} s^{-1}

Quiet Sun - $3 \cdot 10^5$ erg cm^{-2} s^{-1}

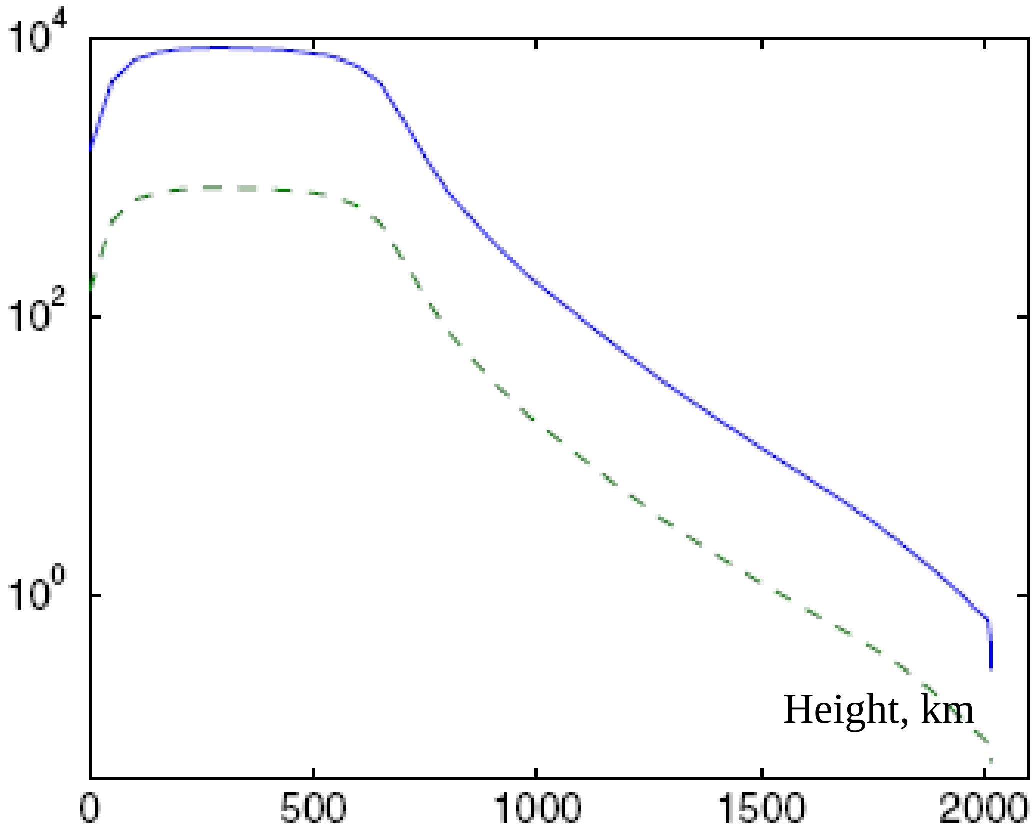
Active regions - 10^7 erg cm^{-2} s^{-1}

Chromospheric energy losses:

Coronal holes - $4 \cdot 10^6$ erg cm^{-2} s^{-1}

Quiet Sun - $4 \cdot 10^6$ erg cm^{-2} s^{-1}

Active regions - $2 \cdot 10^7$ erg cm^{-2} s^{-1}



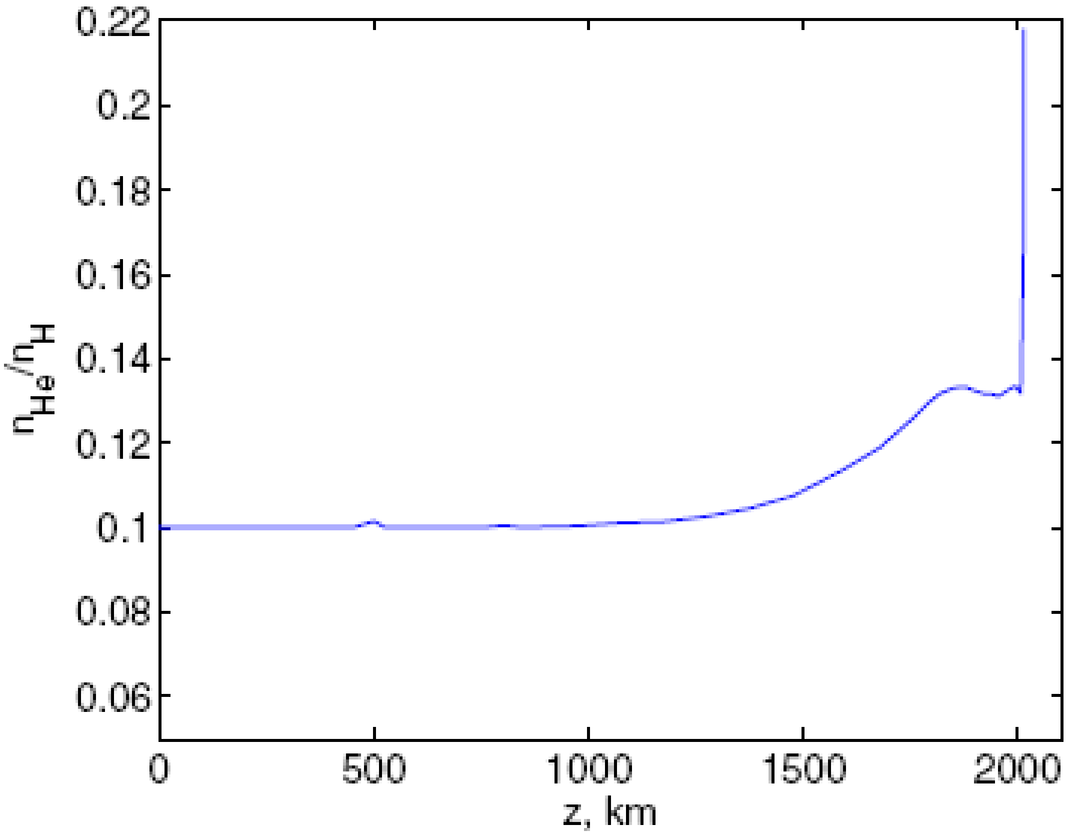
FAL93-3 model (Fontenla et al. 1993)

Blue solid line: ratio of neutral hydrogen and electron number densities.

Green dashed line: ratio of neutral helium and electron number densities.

Plasma is only weekly ionized in the photosphere, but becomes almost fully ionized in the transition region and corona.

The ratio of neutral helium and neutral hydrogen is around 0.1 in the lower heights. But it increases up to 0.22 near chromosphere/corona transition region i.e. at 2000 km.



FAL93-3 model (Fontenla et al. 1993)

Multi-Fluid equations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{V}_e) = 0,$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0,$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \vec{V}_n) = 0,$$

$$m_e n_e \left(\frac{\partial \vec{V}_e}{\partial t} + (\vec{V}_e \cdot \nabla) \vec{V}_e \right) = -\nabla p_e - \nabla \cdot \pi_e - e n_e \left(\vec{E} + \frac{1}{c} \vec{V}_e \times \vec{B} \right) + \vec{R}_e,$$

$$m_i n_i \left(\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \nabla) \vec{V}_i \right) = -\nabla p_i - \nabla \cdot \pi_i - e n_i \left(\vec{E} + \frac{1}{c} \vec{V}_i \times \vec{B} \right) + \vec{R}_i,$$

$$m_n n_n \left(\frac{\partial \vec{V}_n}{\partial t} + (\vec{V}_n \cdot \nabla) \vec{V}_n \right) = -\nabla p_n - \nabla \cdot \pi_n + \vec{R}_n,$$

$$\frac{3}{2} n_e k \left(\frac{\partial T_e}{\partial t} + (\vec{V}_e \cdot \nabla) T_e \right) + p_e \nabla \cdot \vec{V}_e + \pi_e : \nabla \vec{V}_e = -\nabla \cdot \vec{q}_e + Q_e,$$

$$\frac{3}{2} n_i k \left(\frac{\partial T_i}{\partial t} + (\vec{V}_i \cdot \nabla) T_i \right) + p_i \nabla \cdot \vec{V}_i + \pi_i : \nabla \vec{V}_i = -\nabla \cdot \vec{q}_i + Q_i,$$

$$\frac{3}{2} n_n k \left(\frac{\partial T_n}{\partial t} + (\vec{V}_n \cdot \nabla) T_n \right) + p_n \nabla \cdot \vec{V}_n + \pi_n : \nabla \vec{V}_n = -\nabla \cdot \vec{q}_n + Q_n,$$

$$p_e = n_e k T_e, \; p_i = n_i k T_i, \; p_n = n_n k T_n,$$

We consider partially ionized plasma, which consists of electrons (e), protons (i) and neutral atoms (n).

(Braginskii 1965)

$\vec{R}_a$ is the change of impulse, $\vec{q}_a$ is the heat flux density, Q_a is the heat production.

Impulse change and heat production

Impulse change and heat production can be expressed as (Braginskii 1965)

$$\vec{R}_e = -\alpha_{ei}\left(\vec{V}_e - \vec{V}_i\right) - \alpha_{en}\left(\vec{V}_e - \vec{V}_n\right),$$
$$\vec{R}_i = -\alpha_{ie}\left(\vec{V}_i - \vec{V}_e\right) - \alpha_{in}\left(\vec{V}_i - \vec{V}_n\right),$$
$$\vec{R}_n = -\alpha_{ne}\left(\vec{V}_n - \vec{V}_e\right) - \alpha_{ni}\left(\vec{V}_n - \vec{V}_i\right),$$
$$Q_e = \alpha_{ei}\left(\vec{V}_e - \vec{V}_i\right)\vec{V}_e + \alpha_{en}\left(\vec{V}_e - \vec{V}_n\right)\vec{V}_e,$$
$$Q_i = \alpha_{ie}\left(\vec{V}_i - \vec{V}_e\right)\vec{V}_i + \alpha_{in}\left(\vec{V}_i - \vec{V}_n\right)\vec{V}_i,$$
$$Q_n = \alpha_{ne}\left(\vec{V}_n - \vec{V}_e\right)\vec{V}_n + \alpha_{ni}\left(\vec{V}_n - \vec{V}_i\right)\vec{V}_n,$$

$\alpha_{ab} = \alpha_{ba}$ are coefficients of friction between different sort of particles.

Important: the fluid description is valid for the time scales which are longer than electron-electron, ion-ion and neutral-neutral collision times!

$$\tau_H >> \tau_i, \tau_e, \tau_n$$

Collision frequencies

At 1000 km:

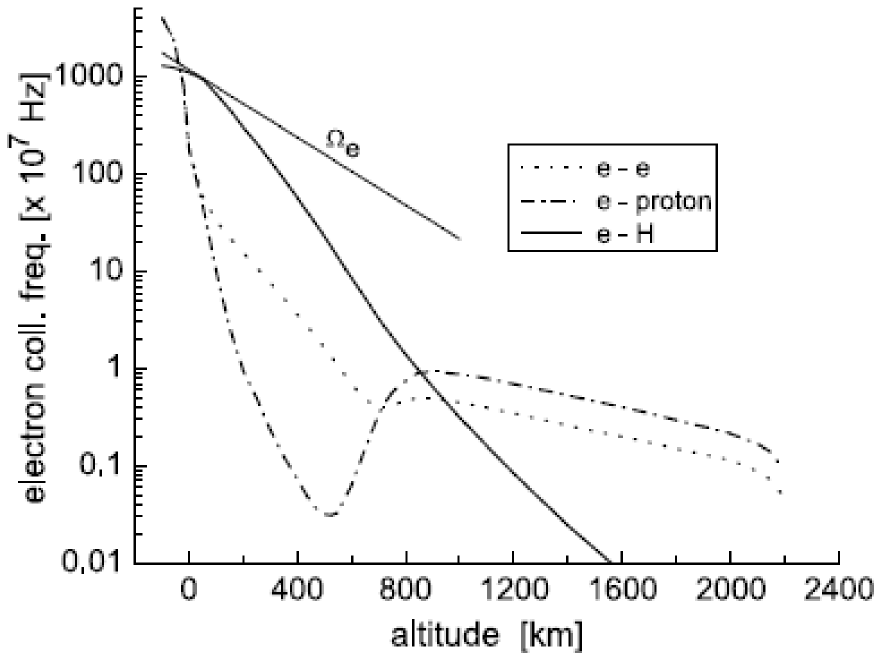
e-e: ~10^7 Hz
e-p: ~10^7 Hz
p-p: ~10^5 Hz

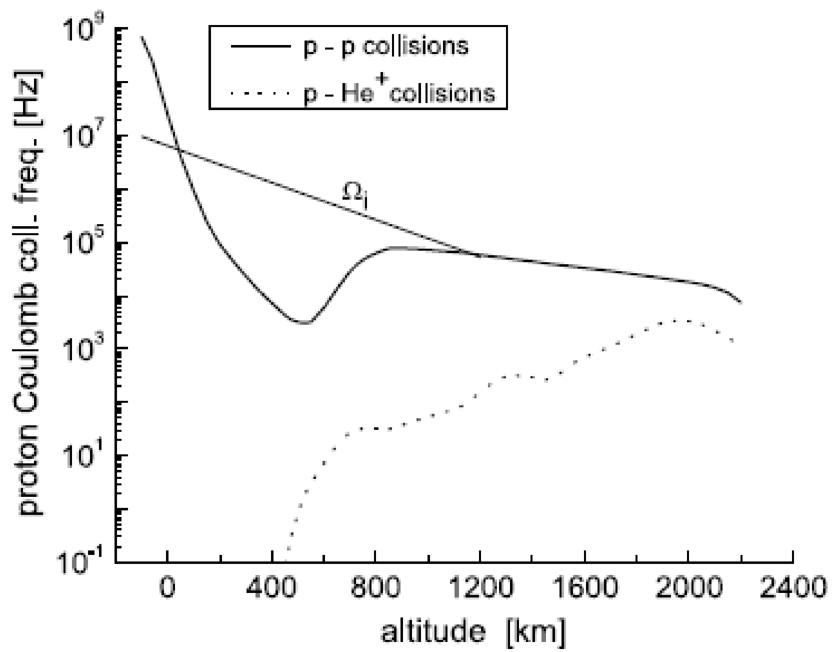
At 2000 km:

e-e: ~10^6 Hz
e-p: ~10^6 Hz
p-p: ~10^4 Hz

Multi-fluid description is valid for

$$\rightarrow \quad \tau_H >> 10^{-4} s$$





Vranjes and Krstic 2013

Maxwell equations

The system is completed by Maxwell equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$

$$\nabla \times \vec{B} = -\frac{4\pi}{c}\vec{j},$$

where

$$\vec{j} = -en_e\left(\vec{V}_e - \vec{V}_i\right)$$

is the current density and

$$\nabla \cdot \vec{B} = 0.$$

Plasma is supposed to be quasi neutral

$$n_e = n_i.$$

Electron–ion collision frequency

The coefficient of friction between ions and electrons can be expressed as (Braginskii 1965)

$$\alpha_{ei} = \frac{4\sqrt{2\pi}\,\lambda e^4 n_i n_e m_{ie}}{3\sqrt{m_{ie}}\left(kT_e\right)^{3/2}},$$

where λ is the Coulomb logarithm .

Electron-ion collision frequency is expressed by

$$\nu_{ei} = \frac{\alpha_{ei}}{m_e n_e} = \frac{4\sqrt{2\pi}\,\lambda e^4 n_i}{3\sqrt{m_e}\left(kT_e\right)^{3/2}}.$$

Fluid-fluid collision frequency

The coefficient of friction between ions and neutral hydrogen atoms is (Braginskii 1965)

$$\alpha_{in} = n_i n_n m_{in} \sigma_{in} \sqrt{\frac{8kT}{\pi m_{in}}},$$

where σ_{in} is ion-hydrogen collision cross-section and m_{in} is the reduced mass.

Ion-neutral collision frequency is different than neutral-ion collision frequency

$$\nu_{in} = \frac{\alpha_{in}}{m_i n_i} \neq \nu_{ni} = \frac{\alpha_{in}}{m_n n_n}.$$

The equation for relative velocity between ions and neutrals can be obtained as

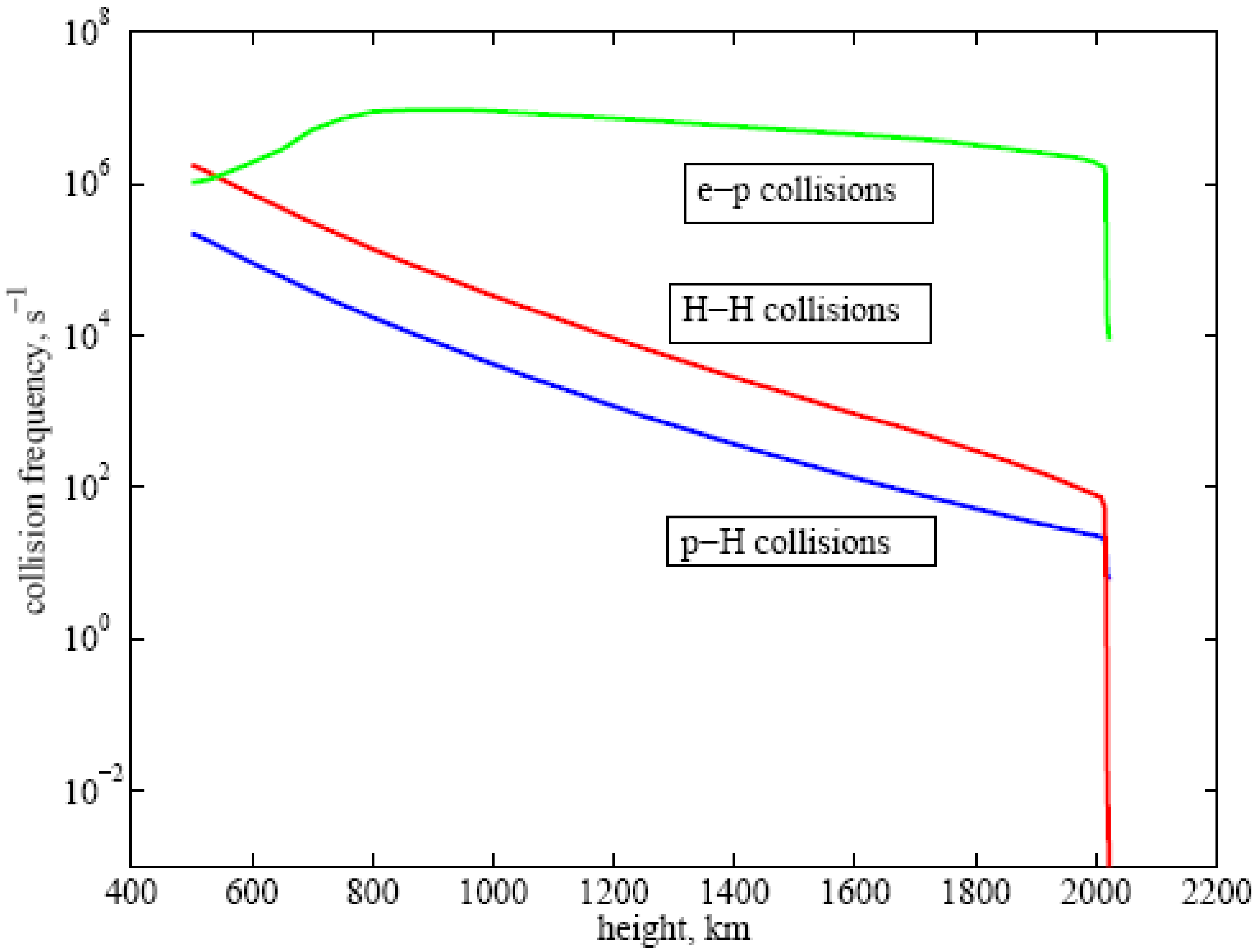
$$\frac{\partial\left(\vec{V}_i - \vec{V}_n\right)}{\partial t} = -\alpha_{in}\left(\frac{1}{m_i n_i} + \frac{1}{m_n n_n}\right)\left(\vec{V}_i - \vec{V}_n\right).$$

This equation gives a single value for the ion-neutral collision frequency (Zaqarashvili et al. 2011)

$$\nu_{in} = \nu_{ni} = \alpha_{in}\left(\frac{1}{m_i n_i} + \frac{1}{m_n n_n}\right) = 2(n_i + n_n)\sigma_{in}\sqrt{\frac{kT}{\pi m_i}}.$$

Fluid–fluid collision frequency

For elastic hard sphere collision, the ion-hydrogen collision cross-section is (Braginskii 1965) $\sigma_{in} = 2\pi(r_i + r_n)^2$, which equals atomic cross section.



At 1000 km:

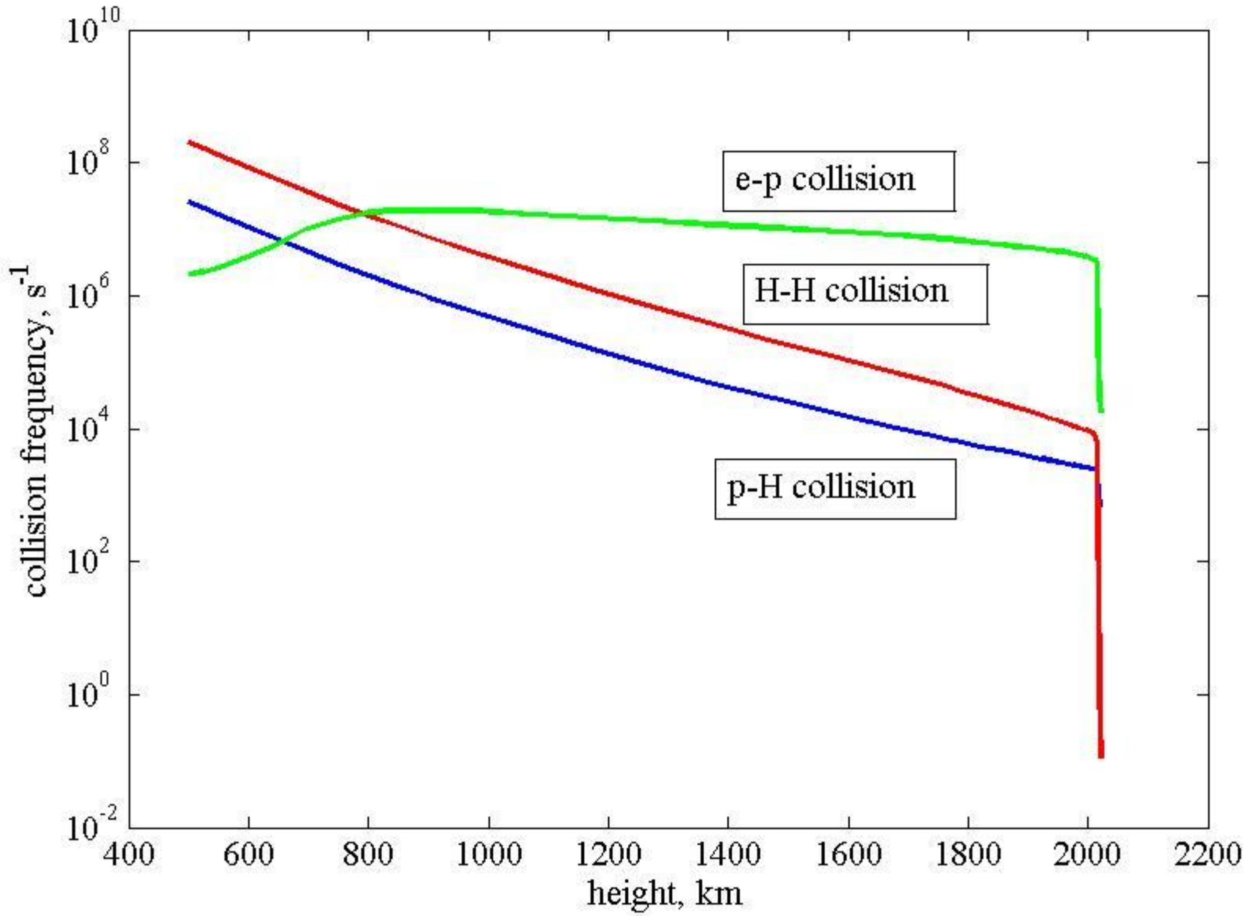
e-p: ~10^7 Hz
H-H: ~10^5 Hz
p-H: ~5 10^3 Hz

At 2000 km:

e-p: ~10^6 Hz
H-H: ~5 10^2 Hz
p-H: ~50 Hz

Fluid–fluid collision frequency

Quantum-mechanical approach leads to the different values for the ion-hydrogen collision cross-section, which gives different collision frequency.



At 1000 km:

e-p: ~5 10^7 Hz
H-H: ~5 10^6 Hz
p-H: ~5 10^5 Hz

At 2000 km:

e-p: ~10^6 Hz
H-H: ~5 10^4 Hz
p-H: ~ 5 10^3 Hz

Two-fluid equations

For time scales longer than ion-electron collision time, the ion-electron gas can be considered as a single fluid.

Any additional sort of neutral atoms can be treated as a separate fluid.

The approximation is valid if

$$\tau_{in} \geq \tau_H > \tau_{ie}, \tau_{nn}.$$

It means that the considered time scales should be more than ion-electron and neutral-neutral collision time and comparable or less than ion-neutral collision times.

For the time scales of

$$\tau_H >> \tau_{in}, \tau_{ii}$$

the ion-electron and neutral fluids are collisionally coupled and the single-fluid approximation can be used.

Two-fluid equations

This approximation is valid in the solar chromosphere for the time scales of

$$2\cdot 10^{-4} \geq \tau_H > 10^{-5} s$$ at 1000 km height

$$0.02 \geq \tau_H > 2\cdot 10^{-3} s$$ at 2000 km height

for hard sphere collisions and

$$2\cdot 10^{-5} \geq \tau_H > 2\cdot 10^{-7} s$$ at 1000 km height

$$2\cdot 10^{-3} \geq \tau_H > 2\cdot 10^{-4} s$$ at 2000 km height

for the quantum-mechanical cross-sections

In the later case, the two-fluid approximation is not valid for the heights lower than 700 km because

$$\tau_{in} < \tau_{nn}.$$

Two-Fluid MHD equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0,$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \vec{V}_n) = 0,$$

$$m_i n_i \left(\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \nabla)\vec{V}_i \right) = -\nabla p_{ie} - \frac{1}{c}\vec{j} \times \vec{B} + \frac{\alpha_{en}}{e n_e}\vec{j} - (\alpha_{in} + \alpha_{en})(\vec{V}_i - \vec{V}_n),$$

$$m_n n_n \left(\frac{\partial \vec{V}_n}{\partial t} + (\vec{V}_n \cdot \nabla)\vec{V}_n \right) = -\nabla p_n - \frac{\alpha_{en}}{e n_e}\vec{j} + (\alpha_{in} + \alpha_{en})(\vec{V}_i - \vec{V}_n),$$

$$\frac{\partial p_{ie}}{\partial t} + (\vec{V}_i \cdot \nabla) p_{ie} + \gamma p_{ie} \nabla \cdot \vec{V}_i = (\gamma - 1)\frac{\alpha_{ei}}{e^2 n_e^2} j^2 + (\gamma - 1)\alpha_{in}\left(\vec{V}_i - \vec{V}_n\right)\cdot \vec{V}_i +$$

$$+ (\gamma - 1)\alpha_{en}\left(\vec{V}_e - \vec{V}_n\right)\cdot \vec{V}_e + \frac{\left(\vec{j} \cdot \nabla\right) p_e}{e n_e} + \gamma p_e \nabla \cdot \frac{\vec{j}}{e n_e} - (\gamma - 1)\nabla \cdot (\vec{q}_i + \vec{q}_e),$$

$$\frac{\partial p_n}{\partial t} + (\vec{V}_n \cdot \nabla) p_n + \gamma p_n \nabla \cdot \vec{V}_n = -(\gamma - 1)\alpha_{in}\left(\vec{V}_i - \vec{V}_n\right)\cdot \vec{V}_n - (\gamma - 1)\alpha_{en}\left(\vec{V}_e - \vec{V}_n\right)\cdot \vec{V}_n - (\gamma - 1)\nabla \cdot \vec{q}_n,$$

$p_{ie} = p_i + p_e$ is the pressure of electron-ion gas.

Induction equation in two-fluid approach

Ohm's law is obtained from the electron equation after neglecting the electron inertia

$$\vec{E}+\frac{1}{c}\vec{V}_i\times\vec{B}+\frac{1}{en_e}\nabla p_e=\frac{\alpha_{ei}+\alpha_{en}}{e^2n_e^2}\left(\vec{V}_i-\vec{V}_n\right)+\frac{1}{cen_e}\vec{j}\times\vec{B}.$$

Faraday's law and Ohm's law lead to the induction equation

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\left(\vec{V}_i\times\vec{B}\right)+\nabla\times\left(\frac{c\nabla p_e}{en_e}\right)-\nabla\times\left(\eta\nabla\times\vec{B}\right)-\nabla\times\left(\frac{\vec{j}\times\vec{B}}{en_e}\right)+\nabla\times\left(\frac{c\alpha_{en}\left(\vec{V}_i-\vec{V}_n\right)}{en_e}\right),$$

where

$$\eta=\frac{c^2}{4\pi\sigma}=\frac{c^2\left(\alpha_{ei}+\alpha_{en}\right)}{4\pi e^2n_e^2}$$

is the coefficient of magnetic diffusion.

Single-Fluid equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0,$$

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right) = -\nabla p + \frac{1}{c}\vec{j}\times\vec{B} - \nabla\cdot(\xi_i \xi_n \rho \vec{w}\vec{w}),$$

$$\cancel{\frac{\partial \vec{w}}{\partial t}} + \cancel{(\vec{w}\cdot\nabla)\vec{V}} + \cancel{(\vec{V}\cdot\nabla)\vec{w}} + \cancel{\xi_n(\vec{w}\cdot\nabla)\vec{w}} - \cancel{(\vec{w}\cdot\nabla)\xi_i\vec{w}} = -\left(\frac{\nabla p_{ie}}{\rho\xi_i} - \frac{\nabla p_n}{\rho\xi_n}\right) + \frac{1}{c\rho\xi_i}\vec{j}\times\vec{B} + \frac{\alpha_{en}}{en_e\rho\xi_i\xi_n}\vec{j} - \frac{\alpha_{in}+\alpha_{en}}{\rho\xi_i\xi_n}\vec{w},$$

$$\frac{\partial p}{\partial t} + (\vec{V}\cdot\nabla)p + \gamma p \nabla\cdot\vec{V} - \xi_i(\vec{w}\cdot\nabla)p - \gamma p\nabla\cdot(\xi_i\vec{w}) + (\vec{w}\cdot\nabla)p_{ie} + \gamma p_{ie}\nabla\cdot\vec{w} = (\gamma-1)\frac{\alpha_{ei}+\alpha_{en}}{e^2 n_e^2}j^2 + (\gamma-1)(\alpha_{in}+\alpha_{en})w^2 -$$

$$-(\gamma-1)\frac{2\alpha_{en}}{en_e}\vec{j}\vec{w} + \frac{(\vec{j}\cdot\nabla)p_e}{en_e} + \gamma p_e\nabla\cdot\frac{\vec{j}}{en_e} - (\gamma-1)\nabla\cdot(\vec{q}_i + \vec{q}_e + \vec{q}_n),$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times(\vec{V}\times\vec{B}) + \nabla\times\left(\frac{c\nabla p_e}{en_e}\right) - \nabla\times(\eta\nabla\times\vec{B}) - \nabla\times\left(\frac{\vec{j}\times\vec{B}}{en_e}\right) + \nabla\times\left(\frac{c\alpha_{en}\vec{w}}{en_e}\right) + \nabla\times(\xi_n\vec{w}\times\vec{B}),$$

where $\vec{V} = \dfrac{\rho_i\vec{V}_i + \rho_n\vec{V}_n}{\rho_i + \rho_n}$ is the total velocity, $\vec{w} = \vec{V}_i - \vec{V}_n$ is the relative velocity.

For time scales longer than ion-neutral collision time inertial terms can be neglected.

$$\vec{w} = -\frac{\vec{G}}{\alpha_{in}+\alpha_{en}} - \left(\frac{\nabla p_{ie}}{\rho\xi_i} - \frac{\nabla p_n}{\rho\xi_n}\right) + \frac{\xi_n}{c(\alpha_{in}+\alpha_{en})}\vec{j}\times\vec{B} + \frac{\alpha_{en}}{en_e(\alpha_{in}+\alpha_{en})}\vec{j}.$$

Induction equation in single-fluid approach

Then we obtain

$$\vec{w} = -\frac{\vec{G}}{\alpha_{in}+\alpha_{en}} - \left(\frac{\nabla p_{ie}}{\rho\xi_i} - \frac{\nabla p_n}{\rho\xi_n}\right) + \frac{\xi_n}{c(\alpha_{in}+\alpha_{en})}\vec{j}\times\vec{B} + \frac{\alpha_{en}}{en_e(\alpha_{in}+\alpha_{en})}\vec{j},$$

and the induction equation becomes

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{V}\times\vec{B}\right) + \frac{c}{e}\nabla\times\left(\frac{c\nabla p_e - \varepsilon\vec{G}}{n_e}\right) - \nabla\times\left(\eta_T\nabla\times\vec{B}\right) - \frac{c}{4\pi e}\nabla\times\left(\frac{1-2\varepsilon\xi_n}{n_e}\left(\nabla\times\vec{B}\right)\times\vec{B}\right) -$$

$$-\nabla\times\left(\frac{\xi_n}{\alpha_{in}+\alpha_{en}}\vec{G}\times\vec{B}\right) + \nabla\times\left(\frac{\xi_n^2}{4\pi(\alpha_{in}+\alpha_{en})}\left(\left(\nabla\times\vec{B}\right)\times\vec{B}\right)\times\vec{B}\right),$$

where

$$\eta_T = \frac{c^2}{4\pi e^2 n_e^2}\left(\alpha_{ei}+\alpha_{en}-\frac{\alpha_{en}^2}{\alpha_{in}+\alpha_{en}}\right), \qquad \vec{G} = \xi_n\nabla p_{ie} - \xi_i\nabla p_n, \qquad \varepsilon = \frac{\alpha_{en}}{\alpha_{in}+\alpha_{en}}.$$

Conclusion: 1

- Multi-fluid MHD approximation is valid for the time scales which are longer than ion-ion, electron-electron and neutral-neutral collision times:

- In the lower chromosphere (at 1000 km):

$$\tau_H >> 10^{-5} s.$$

- In the upper chromosphere (at 2000 km):

$$\tau_H >> 10^{-4} s.$$

Conclusions: 2

- Two-fluid equations are valid when the time scale is more than electron-ion and neutral-neutral collision times:
- In the lower chromosphere (at 1000 km, for hard sphere collision):

$$\tau_H >> 2 \cdot 10^{-5} s.$$

- In the upper chromosphere (at 2000 km for hard sphere collision):

$$\tau_H >> 2 \cdot 10^{-3} s.$$

- In the lower chromosphere (at 1000 km, for quantum collision):

$$\tau_H >> 2 \cdot 10^{-7} s.$$

- In the upper chromosphere (at 2000 km for quantum collision):

$$\tau_H >> 2 \cdot 10^{-5} s.$$

- For quantum-mechanical cross-sections, two-fluid approximation is not valid in lower heights < 700 km.

Conclusions: 3

- Single-fluid approach is valid:

- At 1000 km height:
- hard sphere $\tau_H >> 2 \cdot 10^{-4} s,$
- quantum - $\tau_H >> 10^{-5} s.$

- At 2000 km height:
- hard sphere - $\tau_H >> 2 \cdot 10^{-2} s,$
- Quantum - $\tau_H >> 10^{-4} s.$

Collision cross-sections are important!