Seismic inversions of solar structure and rotation

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Brief Outline

- Inversions of rotational profile.
- Inversions of structure.
- Constraints from p-modes inversions and neutrinos on g-mode period spacing.

Formalism of inversions of rotation

Classical formalism based on variational principle of adiabatic stellar oscillations (see e.g. Chandrasekhar 1964, Lynden-Bell & Ostriker 1967):

$$\delta \omega_{n,\ell,m} = \int_0^R \int_0^\pi K_{n,\ell,m}(r,\theta)\Omega(r,\theta) r dr d\theta, \quad (1)$$

with $K_{n,\ell,m}$ the rotational kernel associated with the splitting (See Thompson 2013 for a review).

Can also be expressed in terms of a_j coefficients following the decomposition of

$$\frac{\omega_{n,l,m}}{2\pi} = \nu_{n,l} + \sum_{j=1}^{j_{max}} a_j(n,l)\mathscr{P}_j^l(m) \quad (2)$$

where the $\mathscr{P}_j^l(m)$ the polynamial basis is related to the Clebsch-Gordan coefficients (see Pijpers 1997 for their expression).

Formalism of inversions of rotation

In this expression: rotation contributes to coefficients of odd j. Hence one may carry out the inversion for the a_{2j+1} coefficients:

$$2\pi a_{2j+1}(n,l) = \int_0^R \int_0^\pi K^a_{n,\ell,j}(r,\theta)\Omega(r,\theta) r dr d\theta \tag{3}$$

where the kernels are recomputed using the linear relation between the coefficients and the splittings.

$$K^a_{n,\ell,j}(r,\theta) = \sum_m \gamma_{2s+1}(l,m) K_{n,\ell,m}(r,\theta) \tag{4}$$

Also: rotational splitting is sensitive to the rotational part that is symmetric with respect to the equator.
Note: even coefficients contain information about asphericity (see e.g. Antia 2003 for an application to structure inversions).

Formalism of inversions of rotation

If one assumes spherical symmetry of the rotation profile:

$$\delta\omega_{n,\ell,m} = m\beta_{n,\ell}\int_0^R K_{n,\ell}(r)\Omega(r)rdr \quad (5)$$

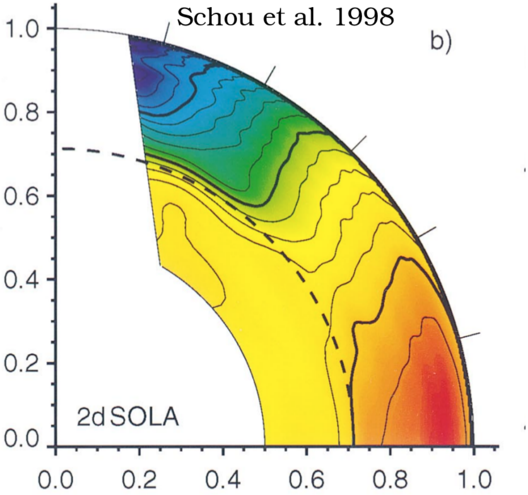
where $\beta_{n,\ell}$ is a constant and $K_{n,\ell}$ the rotation kernel.

Both quantities depend on the stratification of the model (density present in the expressions) and the eigenfunctions. This expression ensures the unimodularity of the kernel functions.

Dedicated numerical techniques (RLS or Tikhonov method, MOLA method from Backus & Gilbert 1967 or SOLA method Pijpers & Thompson 1994). Complementary nature of RLS and SOLA (Sekii 1997).

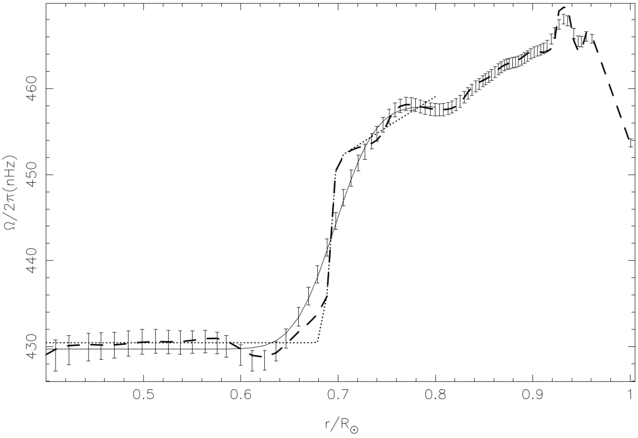
Extensively studied over the years (Brown & Morrow 1987, Kosovichev et al. 1988, Thompson 1990, Schou et al. 1994, ...).

The rotational profile of the Sun



- Still a debate today: IGW (Kumar, Talon, and Zahn 1999, Charbonnel and Talon 2005) or magnetic fields (Gough & McIntyre 1998, Spruit 2002, Eggenberger, Maeder, and Meynet 2005)

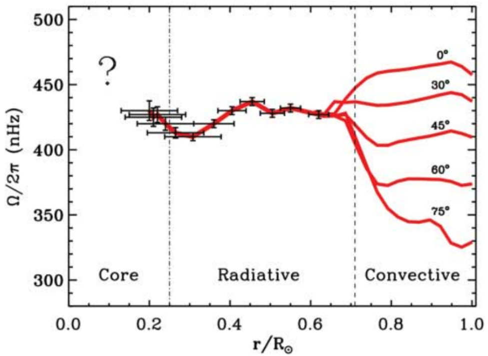
Rotation in the tachocline



Corbard et al. 1999

Non-linear RLS inversions to reproduce the rapid transition in the tachocline.

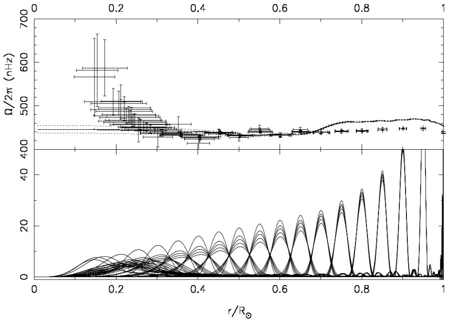
Pushing for the core I

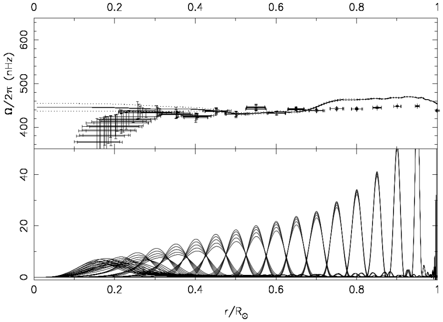


Garcia et al. 2007

- Unaccessible below 0.2 $r/R_\odot$ and at the poles.
- Elsworth et al. 1995 attempts and inferred solid-body rotating core.
- But: rapid increase in the core is not ruled out and possible (T-S dynamo, Spruit 2002)

Pushing for the core II



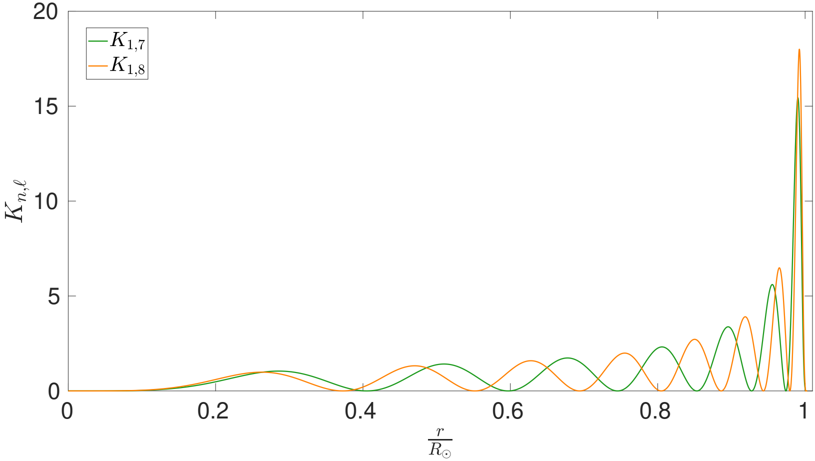


Corbard et al. 1998 (GOLF data)

Corbard et al. 1998 (GONG data)

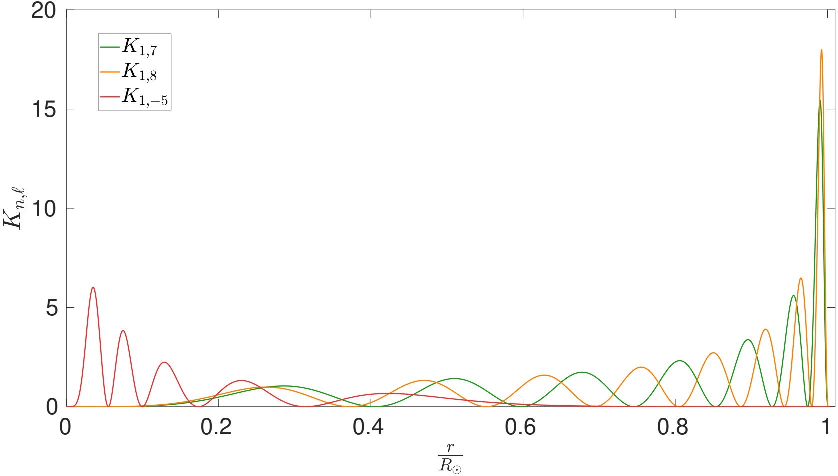
Results are not robust enough to draw conclusions.

Importance of g-modes

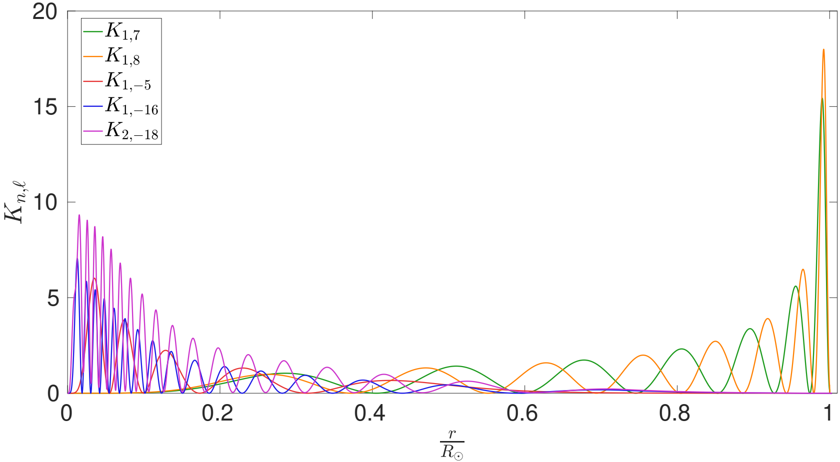


P-modes: Strong drop in amplitude below 0.2 $r/R_\odot$

Importance of g-modes

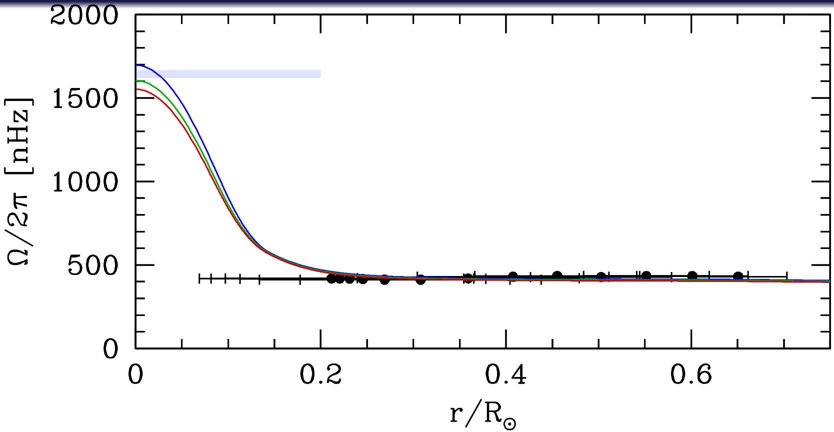


Importance of g-modes



Clearly, g-modes are required to infer in a robust way the properties of the solar core.

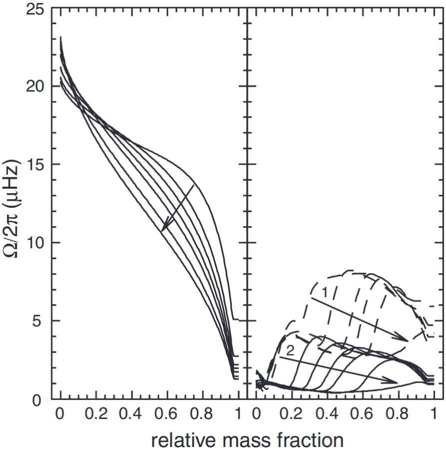
Why? Transport candidates.

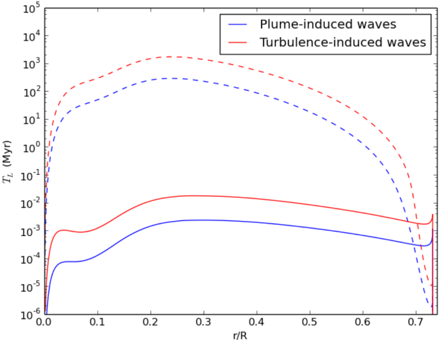


Eggenberger et al. (In prep)

With the T-S dynamo, rotation can increase sharply below 0.2 $r/R_\odot$ Spruit 2002, Eggenberger et al. 2005

Why? Transport candidates.

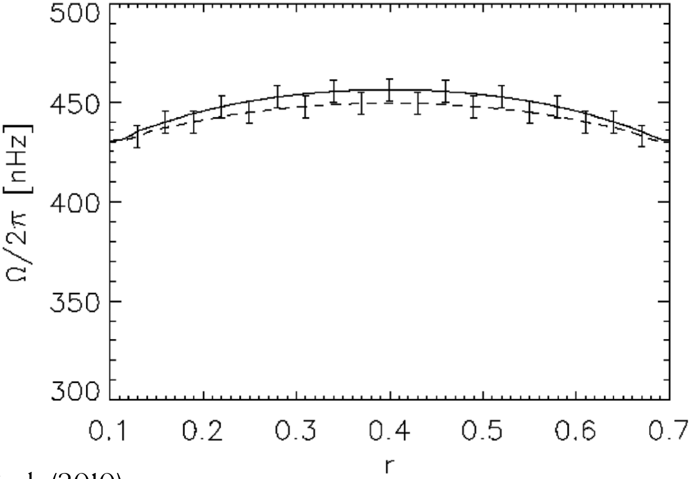




Gravity waves could produce a slowly rotating core. Charbonnel & Talon 2005, Pincon et al. 2016

Why? Transport candidates.

Fossil fields would produce a solid body rotation. (Charbonneau & MacGregor 1993, Gough & McIntyre 1998)



Spada et al. (2010)

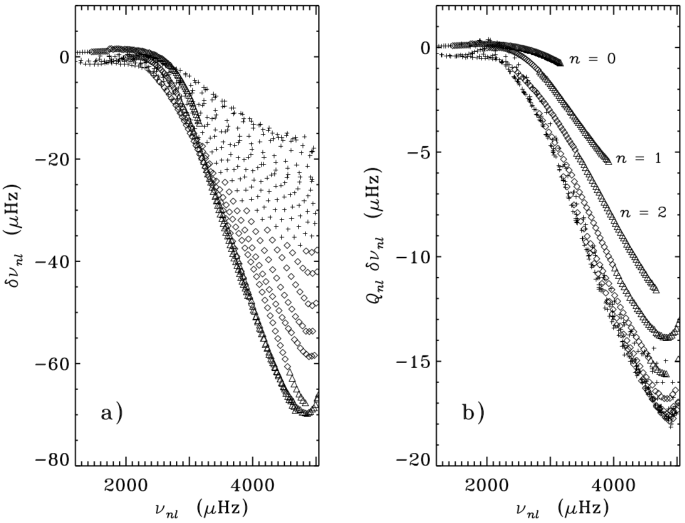
The variational equations

Application of the variational principle of adiabatic stellar pulsation (Chandrasekhar 1964, Lynden-Bell & Ostriker 1967) led to linear integral relations between frequency and structure (Dziembowski et al. 1990):

$$\frac{\delta \nu^{n,l}}{\nu^{n,l}} = \int_0^R K_{\rho,c^2}^{n,l} \frac{\delta \rho}{\rho} dr + \int_0^R K_{c^2,\rho}^{n,l} \frac{\delta c^2}{c^2} dr + \mathscr{F}(\nu) \qquad (6)$$

allowing for non-asymptotic structural inversions (e.g. Antia & Basu 1994, Marchenkov et al. 2000) with dedicated numerical techniques (RLS or Tikhonov method, MOLA method from Backus & Gilbert 1967 or SOLA method Pijpers & Thompson 1994).

Dealing with surface corrections



- Rescale the frequencies with $Q_{n,\ell}$.
- Model the surface effect as a combination of Legendre polynomials.

Changing the variables in the integral relations

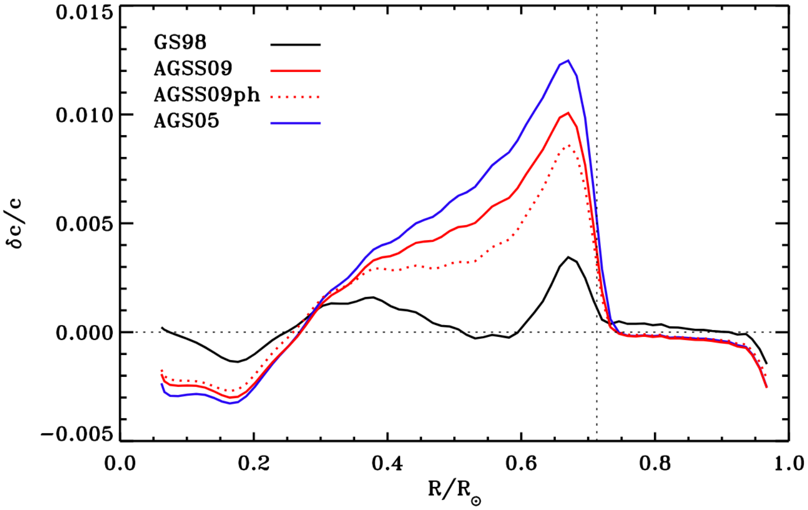
Inversions are not limited to ρ, c^2, Γ_1. One can generalize:

$$\int_0^R K^{n,l}_{s_1,s_2}\frac{\delta s_1}{s_1}dr+\int_0^R K^{n,l}_{s_2,s_1}\frac{\delta s_2}{s_2}dr=\int_0^R K^{n,l}_{s_3,s_4}\frac{\delta s_3}{s_3}dr+\int_0^R K^{n,l}_{s_4,s_3}\frac{\delta s_4}{s_4}dr$$

In practice, very general variables can be derived following two approaches: conjugated functions (see e.g. Elliott 1996 or Kosovichev 1999 for a full description) or "direct method" (Buldgen et al. 2017a, following Masters et al. 1979).

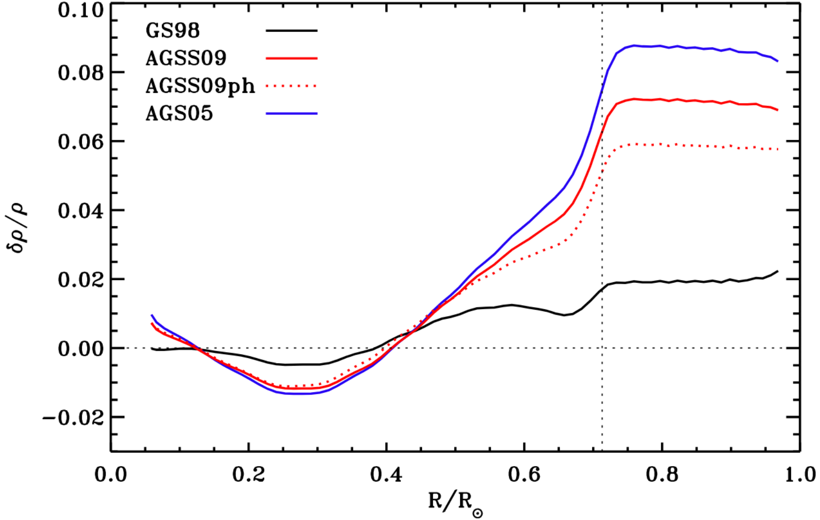
If E.O.S is assumed "secondary" variables (T,X,...) can be inverted (e.g. Gough & Kosovichev 1988).

Sound speed and density inversions



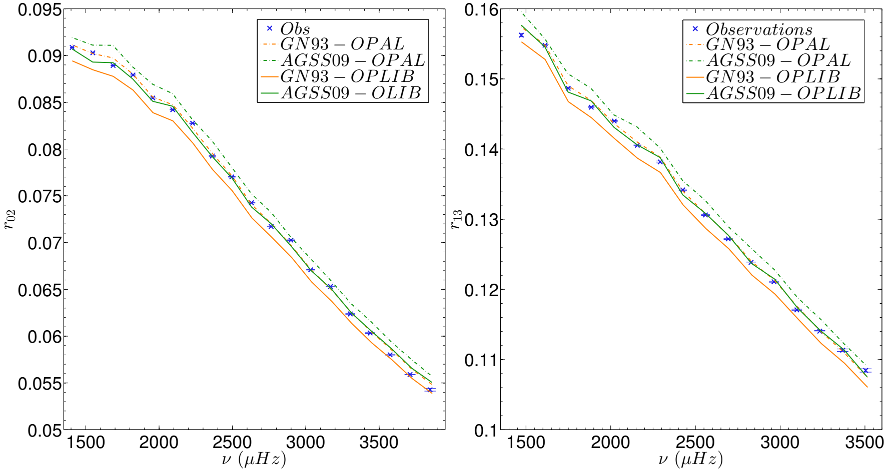
Serenelli et al. (2006)

Sound speed and density inversions



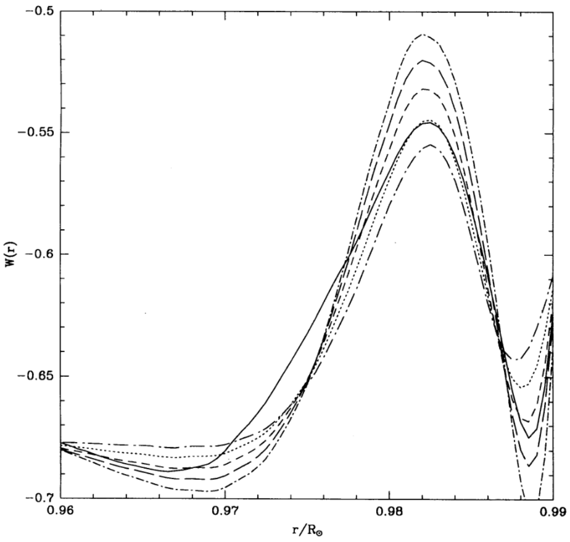
Serenelli et al. (2006)

Frequency ratios



Constraints on solar core but no direct constraints on abundances. (Ratios defined by Roxburgh & Vorontsov 2003)

Helium abundance determinations



Antia & Basu (1994)

Most difficult determination, very EOS dependent (Voronstov et al. 1991):

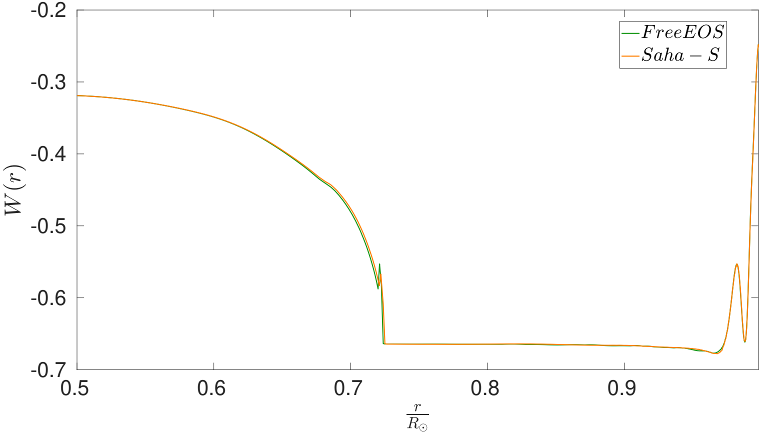
- Γ_1 variations.
- $W(r) = \frac{1}{g}\frac{dc^2}{dr}$.
- H_1 and H_2 functions.

Canonical value:
$Y_{\odot,\mathrm{Conv}} = 0.2485 \pm 0.0035$
(Basu & Antia 1995, 2004)

Also : $[0.24, 0.255]$ (Vorontsov et al. 2013) + other studies (Dziembowski et al. 1991, Richard et al. 1998, Vorontsov et al. 2013, ...)

Helium abundance determinations

W(r) is slightly more efficient to determine helium (noticed by D. Gough):



Other structural inversions I: metallicity inversions

Constraints from seismic inversions:

Inversions can only constrain variables from the *acoustic structure* : ρ, c^2, A or $\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S$ for example.

However, assuming an E.O.S, one has:

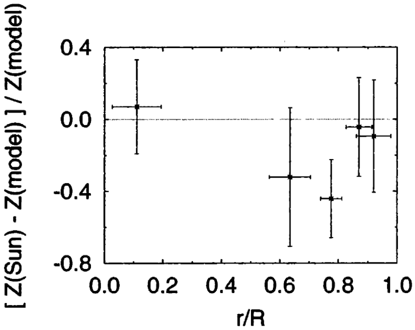
$$\frac{\delta \Gamma_1}{\Gamma_1} = \left(\frac{\partial \ln \Gamma_1}{\partial \ln P}\right)_{\rho,Y,Z} \frac{\delta P}{P} + \left(\frac{\partial \ln \Gamma_1}{\partial \ln \rho}\right)_{P,Y,Z} \frac{\delta \rho}{\rho} + \left(\frac{\partial \ln \Gamma_1}{\partial Y}\right)_{P,\rho,Z} \delta Y + \left(\frac{\partial \ln \Gamma_1}{\partial Z}\right)_{P,\rho,Y} \delta Z,$$

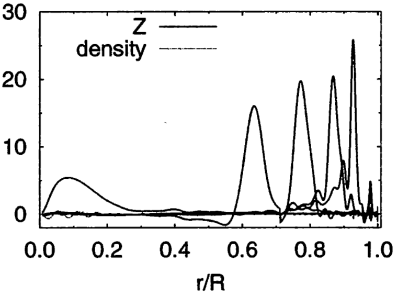
thus allowing for inversions of Y, the helium abundance, or Z, the metallicity.

Previous studies by Takata & Shibahashi (2001), Antia & Basu (2006) and Vorontsov et al. (2013).

Other structural inversions I: metallicity inversions I

Initial attempts in Takata and Shibahashi 2001 using density and Γ_1 kernels.

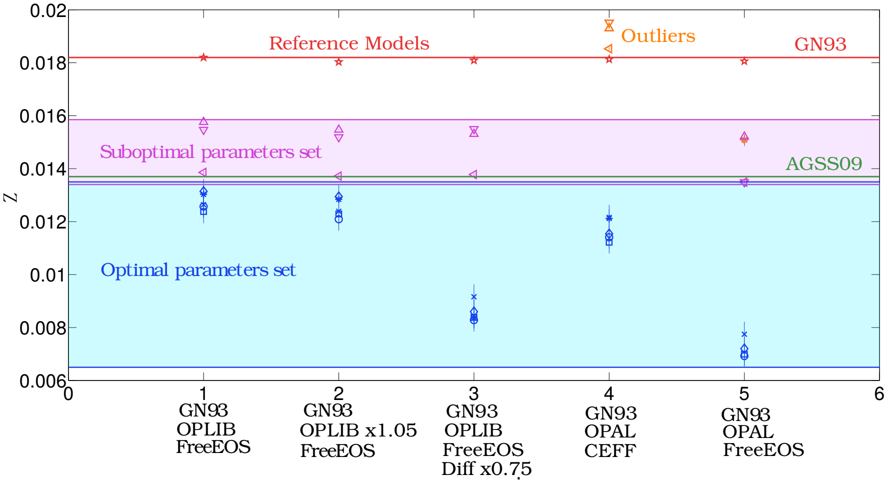




Impossibility to conclude because of the errors bars of the results. However, the conclusion states that the three last point are consistent with a 30% reduction of the solar metallicity.

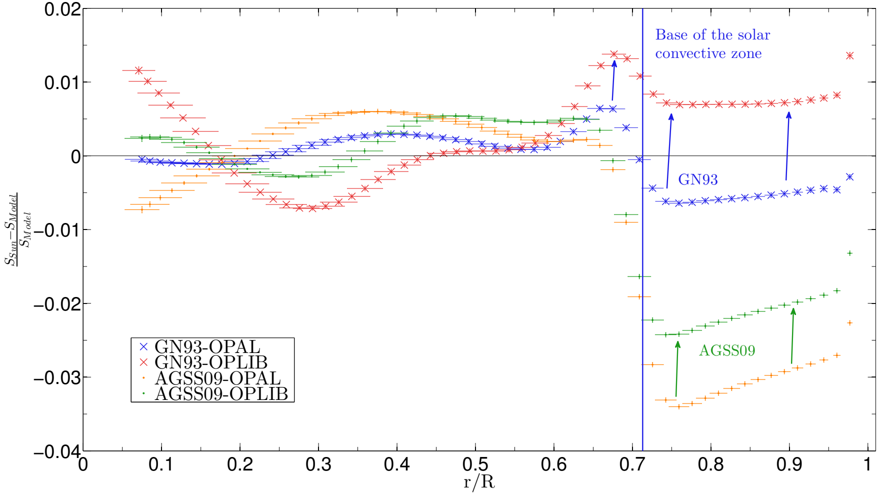
Other structural inversions I: metallicity inversions II

This inversion (Buldgen et al. 2017c) favours a low metallicity (as in Vorontsov et al. 2013).



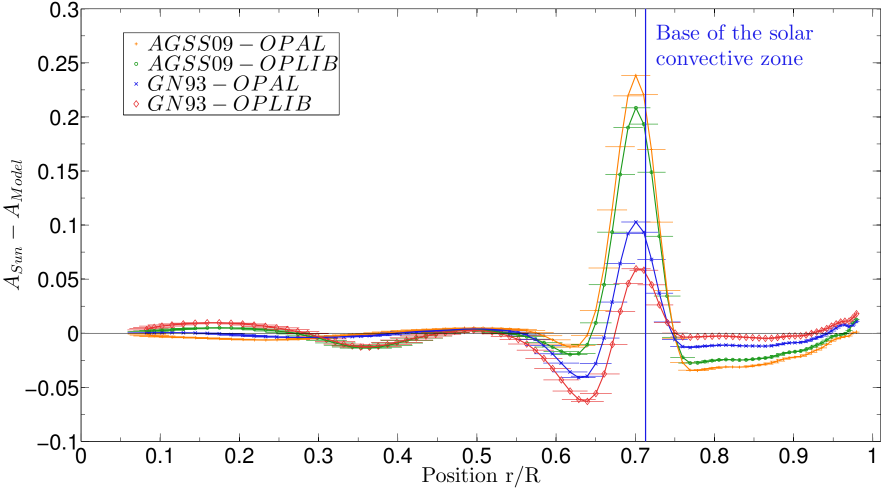
Other structural inversions II: Entropy proxy inversions

Inversions for $S_{5/3} = \frac{P}{\rho^{5/3}}$ (P/ρ^{Γ_1} also possible). (Buldgen et al. 2017b)

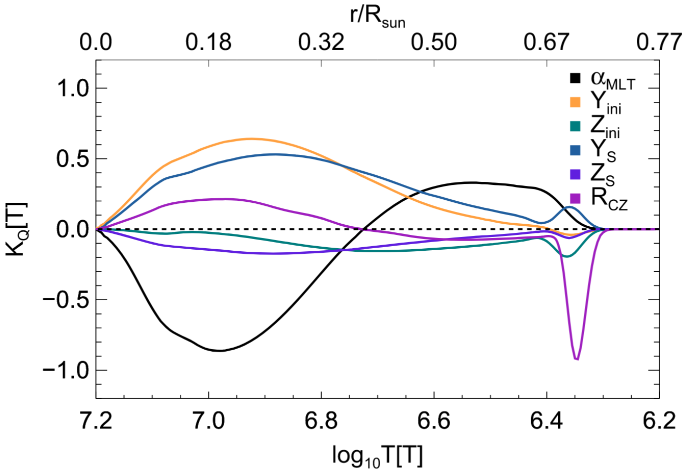


Other structural inversions III: Ledoux discriminant inversions

$A = \frac{1}{\Gamma_1}\frac{d\ln P}{d\ln r} - \frac{d\ln \rho}{d\ln r}$, related to the Brunt-Väisälä frequency. (Buldgen et al. 2017d)



Opacity kernels

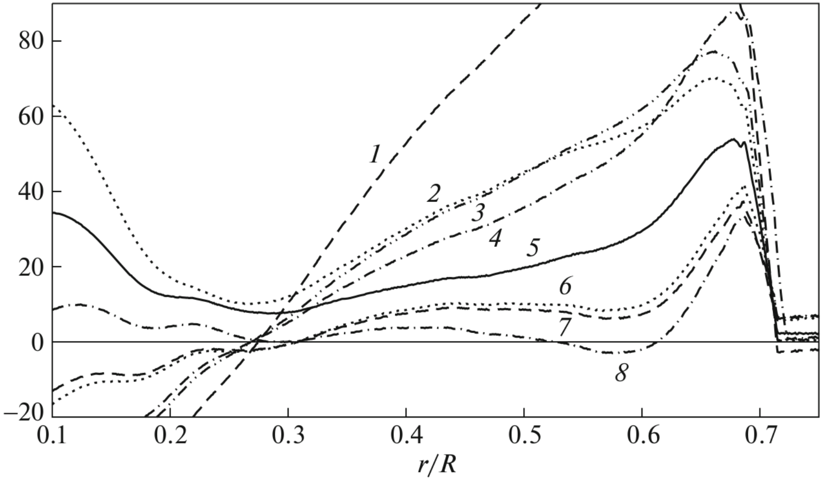


- Based on Tripathy & Christensen-Dalsgaard (1998).
- Assumes linear behaviour with respect to small κ perturbation.

Vinyoles et al. (2017)

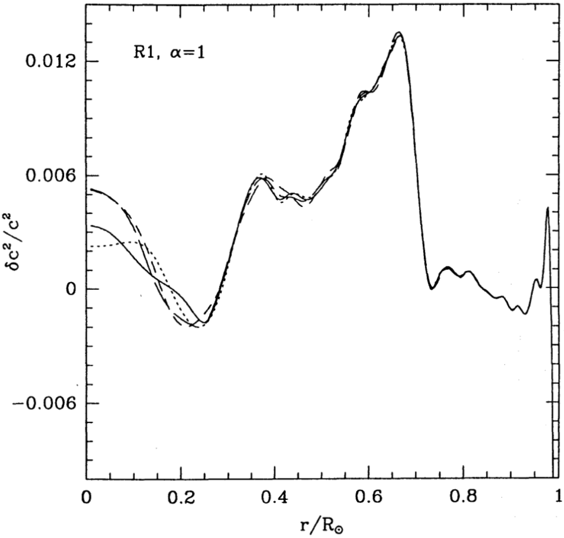
Allow for a static analysis of the required changes in opacity to match helioseismic constraints.

Extended solar calibrations



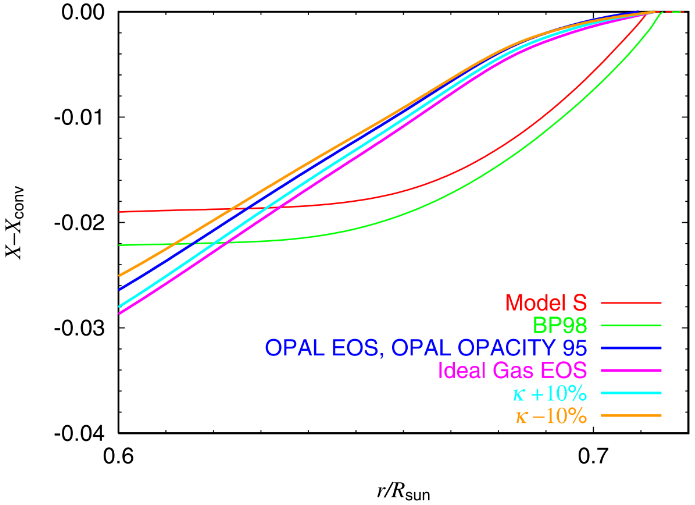
Add additional free parameters and constraints to the solar models to expand the calibration procedure. (Ayukov & Baturin 2013, 2017)

Seismic models and detailed inferences



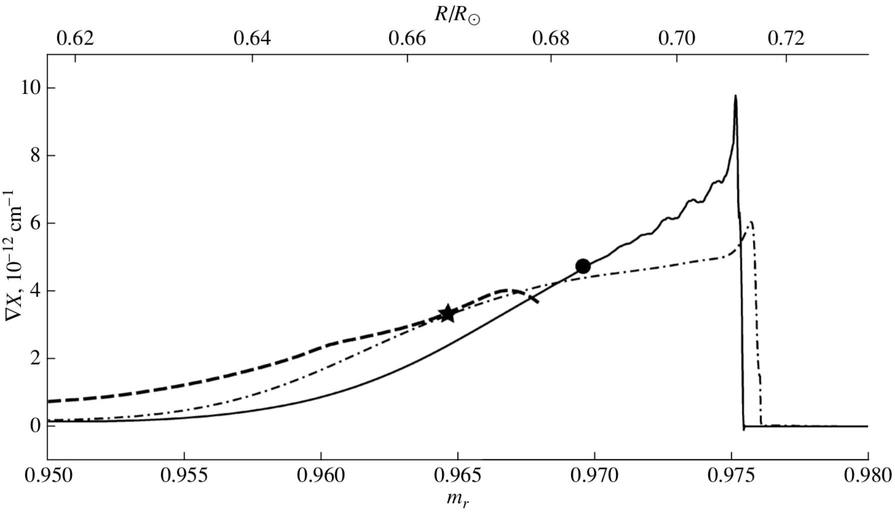
Use a static approach to compute a structure from the helioseismic data. (Basu & Thompson 1996, Takata & Shibahashi 1998, Gough 2006, Mathur et al. 2007)

Seismic models and detailed inferences



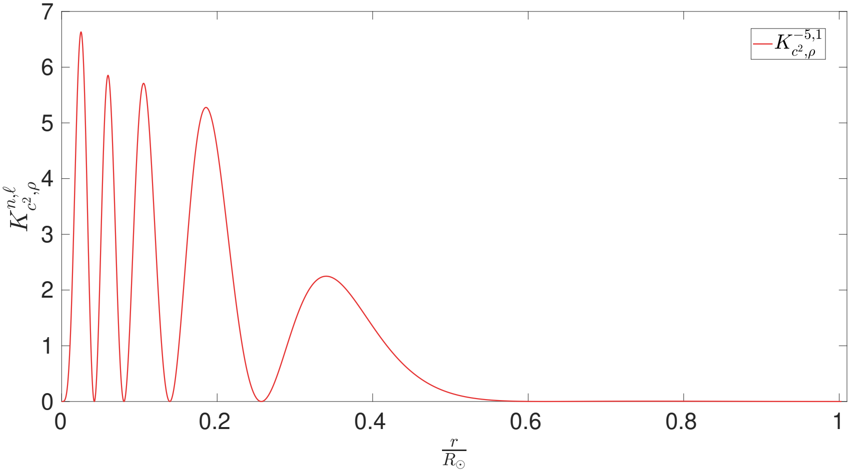
Analysis of the chemical properties of the tachocline (Takata & Shibahashi 2003)

Seismic models and detailed inferences



Attempts to determine the chemical history of the Sun from helioseismic analyses of the BCZ. (Baturin et al. 2015)

As for g modes... (Preliminary)



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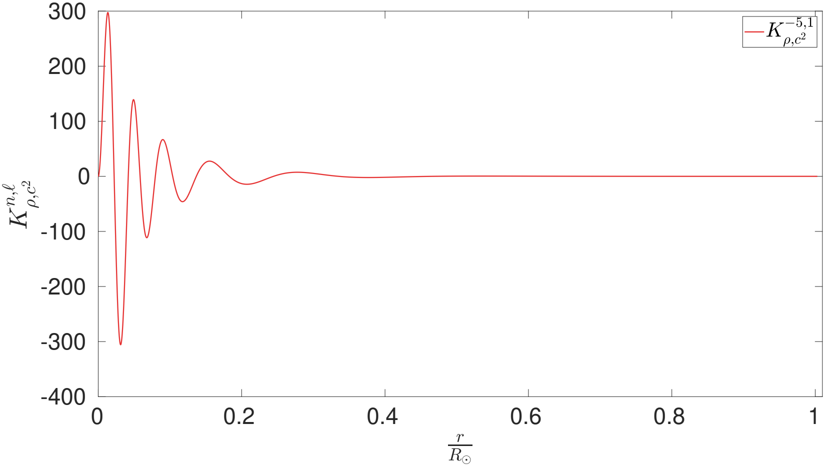

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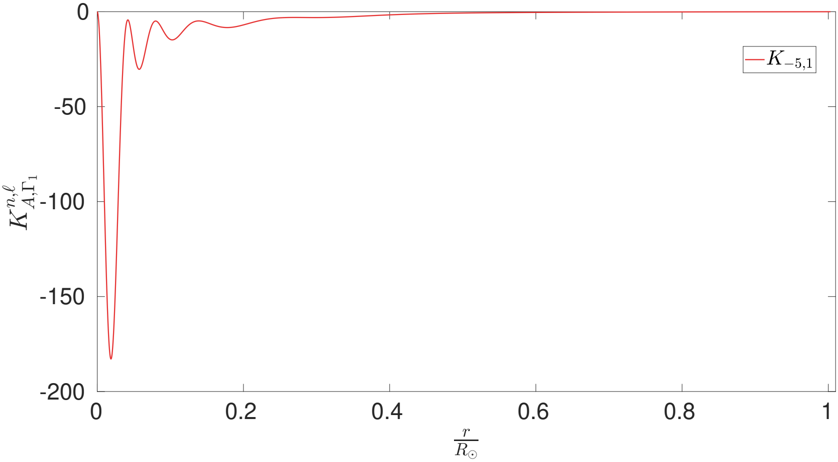


Potential issues

Preliminary conclusions from these tests

- Requires sufficient numerical quality for the models + eigenfunctions.
- Changing the variables: using the conjugated functions approach to avoid high-order derivatives.
- Use variational equations to determine limits of linear regime.
- The ability to localize kernels will strongly depend on the radial order of the detected modes (low $|n|$ modes are better).

Using p-modes to constrain g-modes (Preliminary)

Fossat's et al. 2017 **results:**

- Rapidly rotating solar core;
- Period-spacing value;
- g-mode spectrum that could be tested for inversions.

In the context of the solar modelling problem:

Fossat's $P_0 \approx 100s$ different than a SSM's P_0 (Provost et al. 2000). Is this value reconciliable with p-modes constraints?

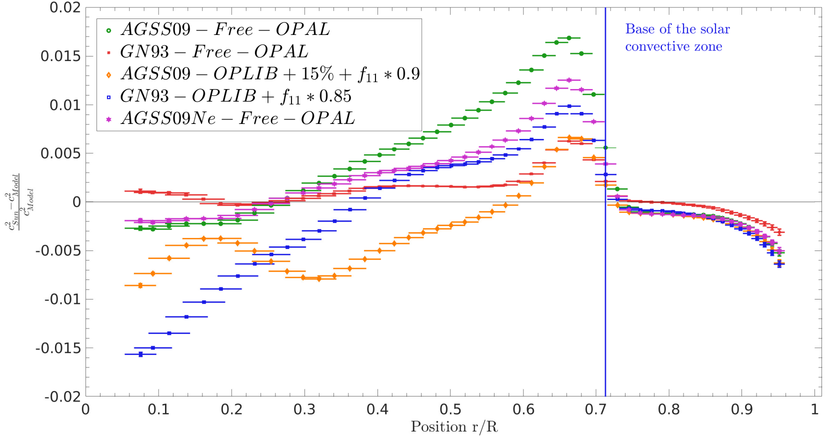
Using p-modes to constrain g-modes (Preliminary)

$(r/R)_{BCZ}$	Y_{CZ}	P_0 **(s)**	**Opacity**	**Abundances**	**f11**
0.7227	0.2374	2170	OPAL	AGSS09	f11
0.7209	0.2360	2172	OPAL	AGSS09	f11
0.7243	0.2322	2195	OPAS	AGSS09	f11
0.7134	0.2463	2150	OPAL	GN93	f11
0.7106	0.2374	2046	OPLIB∗1.15	AGSS09	f11∗0.9
0.7165	0.2341	2041	OPLIB	GN93	f11∗0.85

One way to alter the period spacing to cope with Fossat's results is to alter the nuclear reactions (perhaps motivated by Däppen & Mussack 2012?), but the models don't agree with usual constraints.

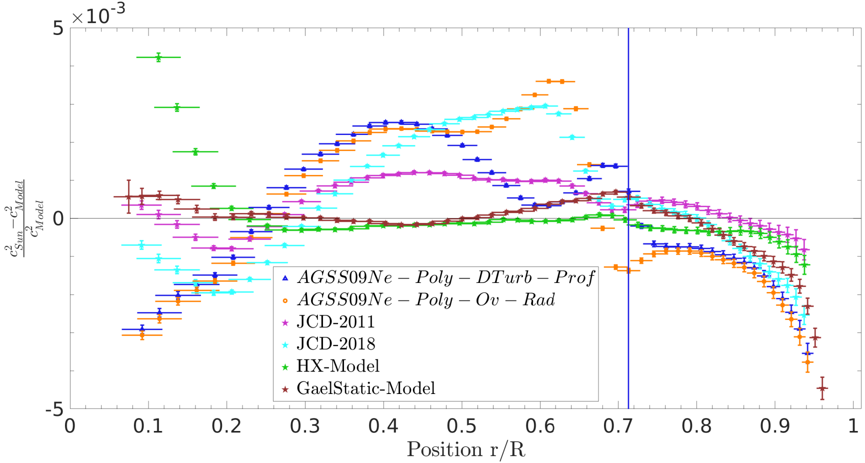
Using p-modes to constrain g-modes (Preliminary)

The models also fail when looking at inversions...



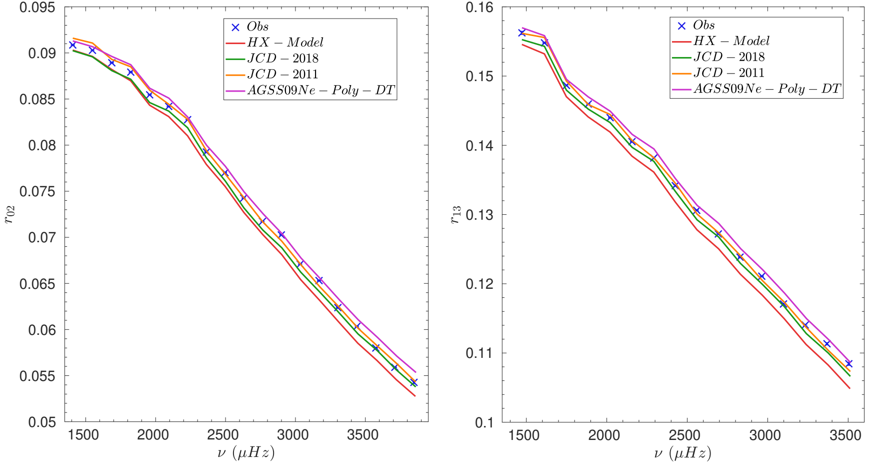
Using p-modes to constrain g-modes (Preliminary)

Reversing the issue, could a seismic model fit Fossat's P_0 value?



Only the H-X model is closer to Fossat's value (2069s), but it fails for c^2 in the core, as confirmed from the ratio values.

Using p-modes to constrain g-modes (Preliminary)



Only the H-X model is close to Fossat's value, but it fails for c^2 in the core, as confirmed from the ratio values.

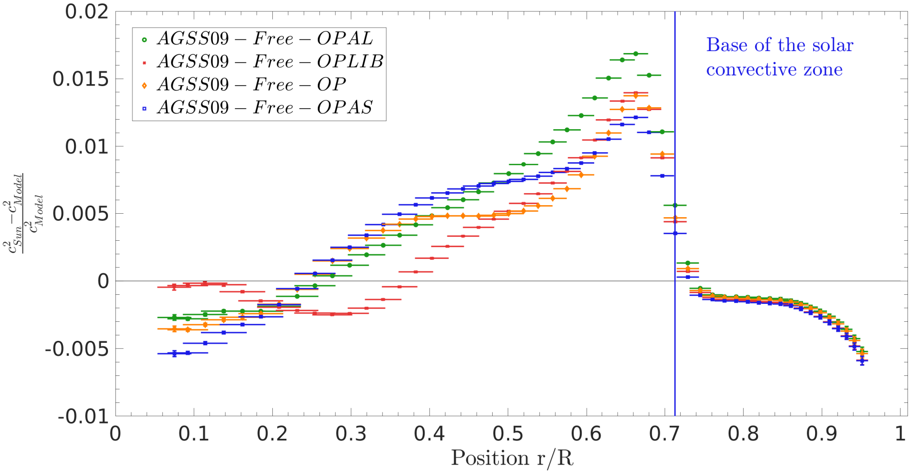
Using p-modes to constrain g-modes (Preliminary)

Open question

- Can we use the knowledge of the solar structure from p-modes and neutrinos to constrain as much as possible the expected behaviour of the g-modes?
- Could this be used as a prior information to detect g-modes in the asymptotic regime?

Thank you for your attention!

The current state of the issue



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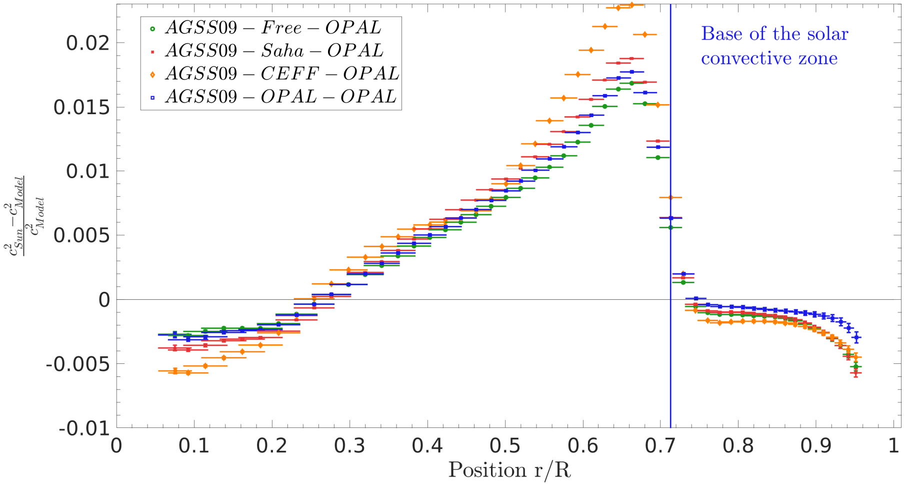

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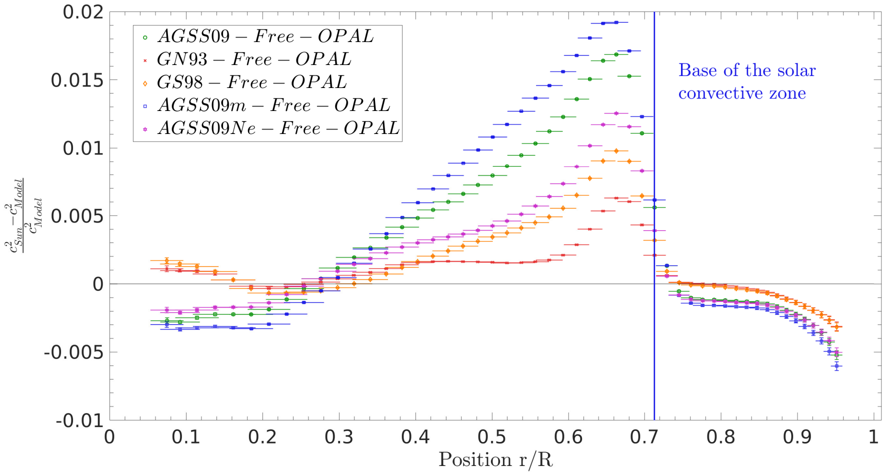
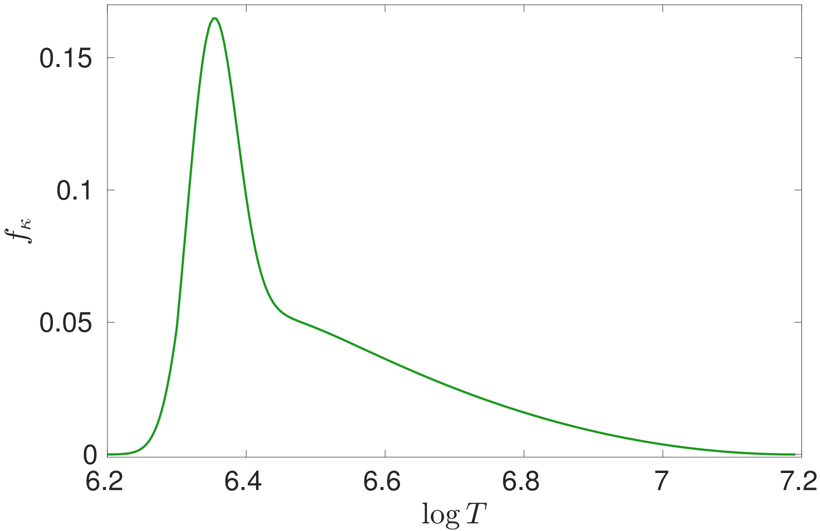


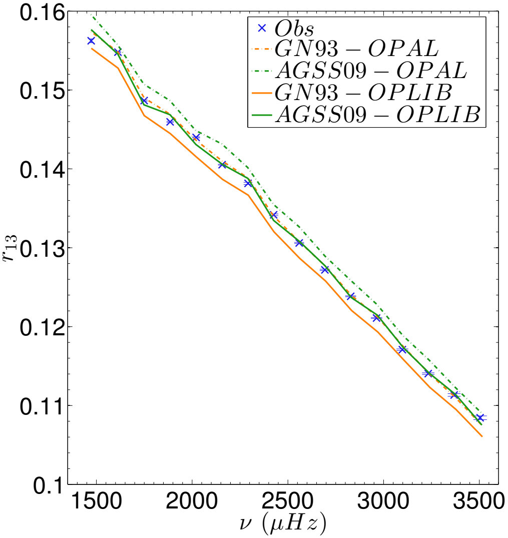
Considered opacity modification



Other classical diagnostics

	$r_{Conv}/R_{\odot}$	Y_{Conv}
Helioseismic measurements	0.713 ± 0.001	0.2485 ± 0.0035
SSM (AGSS09, Free, OPAL)	0.720	0.236
SSM (AGSS09, Free, OPLIB)	0.718	0.230
SSM (AGSS09, Free, OPAS)	0.717	0.232
SSM (GN93, Free, OPAL)	0.711	0.245
SSM (GN93, Free, OPLIB)	0.708	0.240

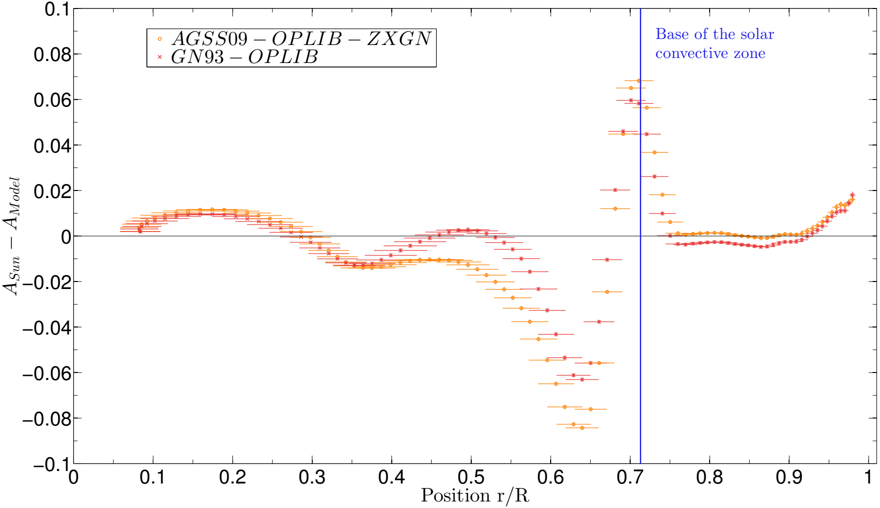
Standard Models with new opacities - Frequency ratios



- r_{02}, r_{13} $\Rightarrow$ AGSS09 favoured!
- c^2 inversions still favour GN93.
- BCZ wrong for both AGSS09 and GN93.
- Y_S very low for AGSS09.

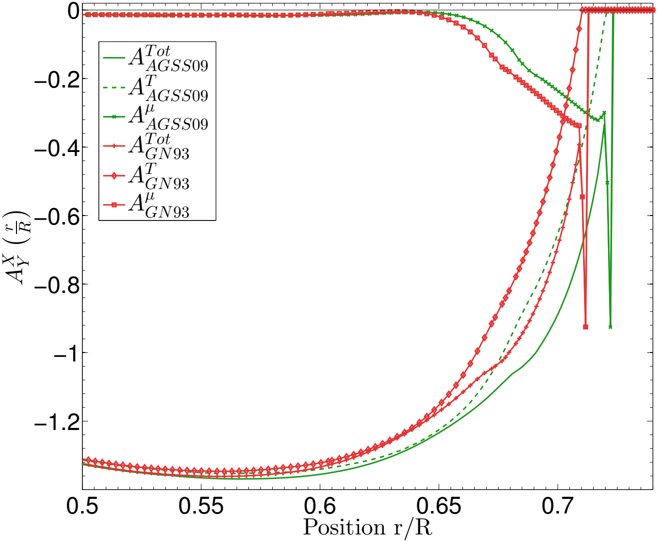
$\Rightarrow$ **Need new diagnostics.**

Inversions of the convective parameter for Standard Solar Models

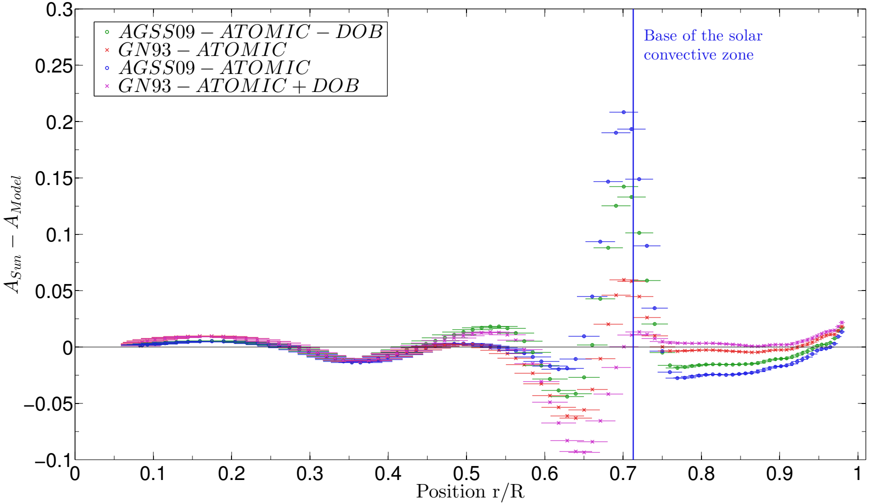


The compensation is related to the heavy-element mixture.

Inversions of the convective parameter for Standard Solar Models

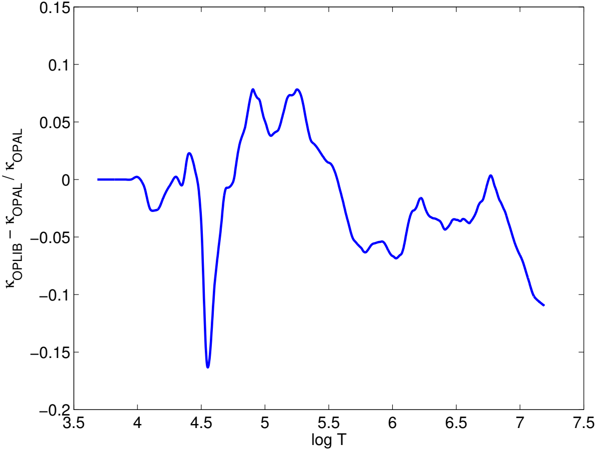


Inversions of the convective parameter for Standard Solar Models



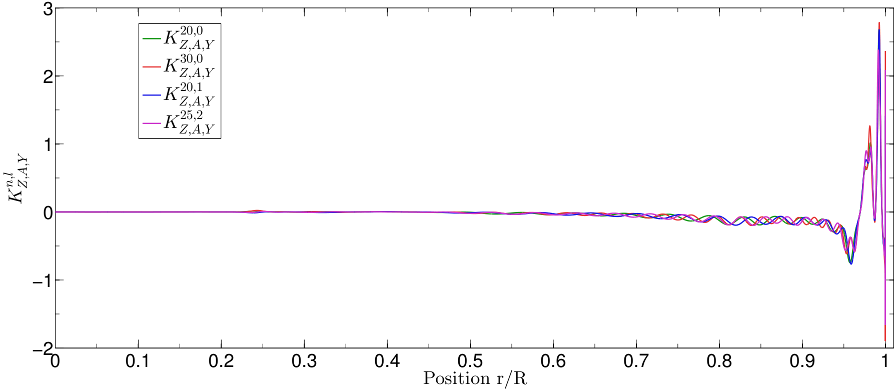
The compensation is also related to the temperature gradient.

Relative differences OPLIB-OPAL

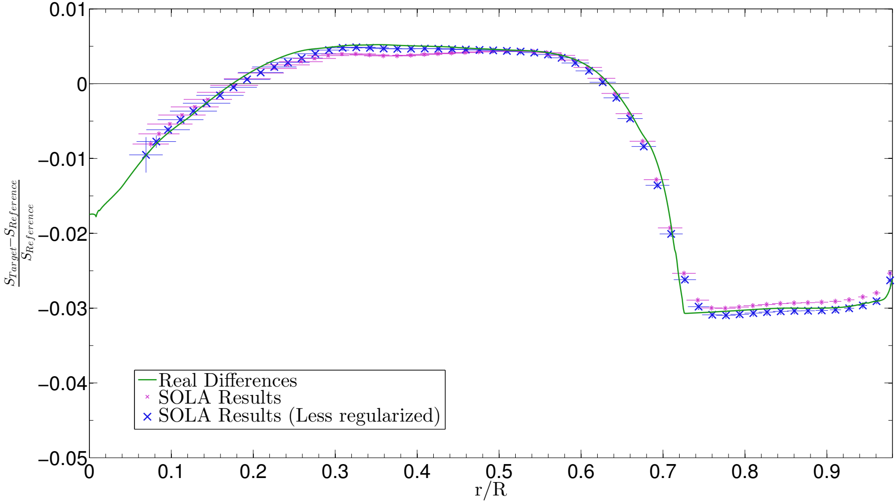


Metallicity Inversions for the Solar Envelope

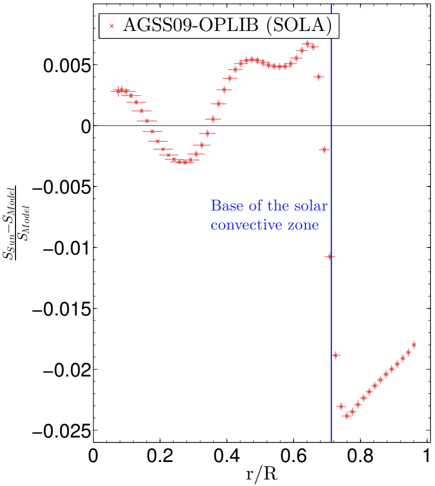
Metallicity kernels can thus be derived to estimate Z in the envelope.

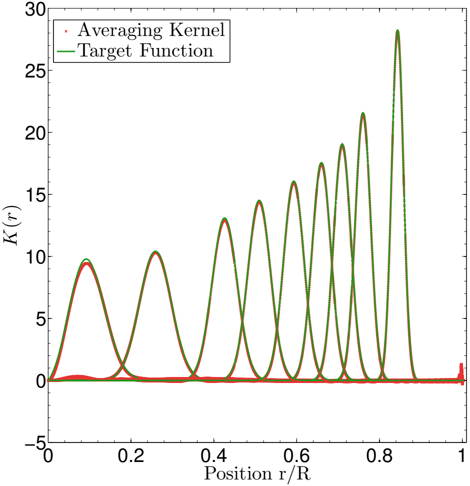


Appendices Helioseismology - Hare-and-Hounds exercises

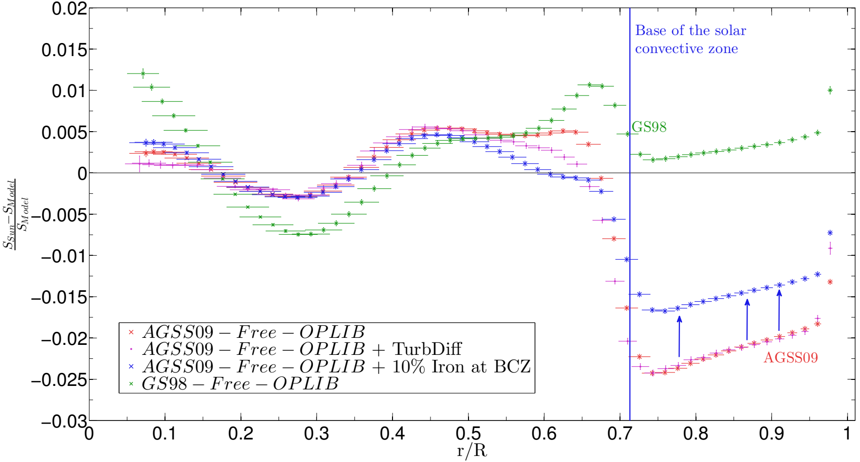


Appendices Helioseismology - Kernel fits





Links with opacity and chemical composition



Entropy inversions hint directly at inaccuracies in the radiative zone.

Parameters of the solar models with modified opacities and additional mixing used in this study

$(r/R)_{BCZ}$	$(m/M)_{CZ}$	Y_{CZ}	Z_{CZ}	Y_0	Z_0	**Opacity**	**Abundances**	**Diffusion**
0.7122	0.9757	0.2416	0.01385	0.2692	0.01494	OPAL+Poly	AGSS09Ne	Thoul
0.7129	0.9761	0.2427	0.01383	0.2678	0.01483	OPAL+Poly	AGSS09Ne	Paquette
0.7106	0.9762	0.2425	0.01383	0.2685	0.01466	OPAL+Poly	AGSS09Ne	Thoul+D_{Turb}
0.7106	0.9762	0.2374	0.01359	0.2645	0.01490	OPAS+Poly	AGSS09	Thoul+D_{Turb}
0.7121	0.9756	0.2460	0.01376	0.2696	0.01500	OPAL+Poly	AGSS09Ne	Thoul+D_{Turb} − Prof
0.7118	0.9757	0.2437	0.01381	0.2692	0.01495	OPAL+Poly	AGSS09Ne	Thoul+Ov − Rad
0.71056	0.9751	0.2438	0.01381	0.2700	0.01506	OPAL+Poly	AGSS09Ne	Thoul+Ov − Ad