Few words on g-mode asymptotic

T. Corbard, J. Provost, L. Bigot

1 Asymptotic model Provost & Berthomieu (1986)

Their Eq.(4a) writes:

$$P_{n,l} = \frac{P_0}{\sqrt{l(l+1)}}\left[n + \frac{l}{2} - \frac{1}{4} - \theta\right] + \frac{P_0^2}{P_{n,l}}\frac{l(l+1)V_1 + V_2}{l(l+1)}$$

- Use Cowling approximation
- ɵ is not a constant but is function of the frequency and l
- V_1 depends on the stratification near the center of the Sun
- V_2 and ɵ depends predominantly on the stratification and mixing processes at the base of the convection zone → Possible interplay between V_2 and ɵ

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$$LP \sim \left(|n| + \frac{1}{2}l + \alpha_g\right) P_0 - (A_g L^2 - B_g)\frac{P_0^2}{P} + \ldots,$$

Scherrer & Gough   L

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- V_1 depends on the stratification near the center of the Sun
- V_2 and ɵ depends predominantly on the stratification and mixing processes at the base of the convection zone

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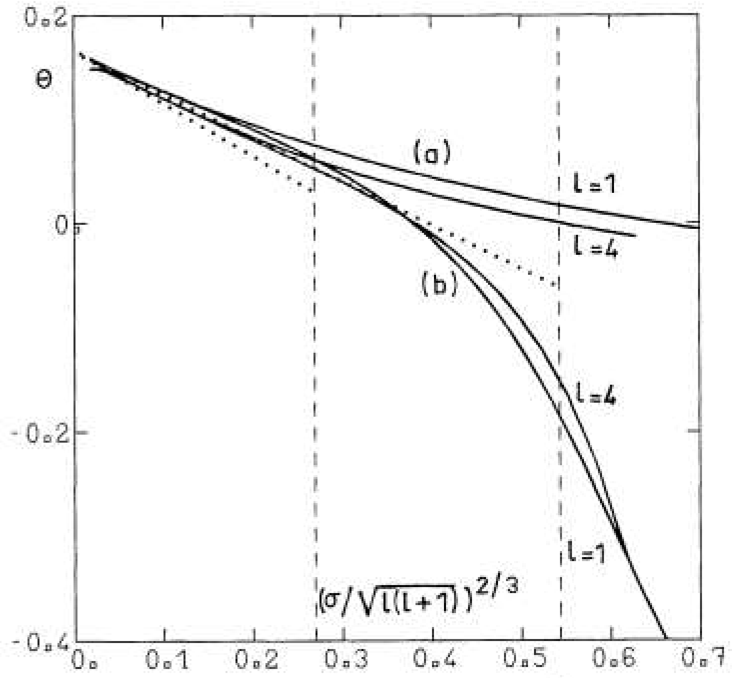


Fig. 2. Variation of θ relatively to $[\sigma^2/l(l+1)]^{1/3}$ for the standard solar model and for $l = 1, 4$ at first and second order relatively to σ, respectively curves (a) and (b). The dotted lines represent the asymptotic behavior (formula 8). The dashed vertical lines delimit the range of nondimensional frequencies corresponding approximately to modes of radial order $10 \leq k \leq 35$, $\varphi_0 = 138$, $\varphi_1 = -6.3$

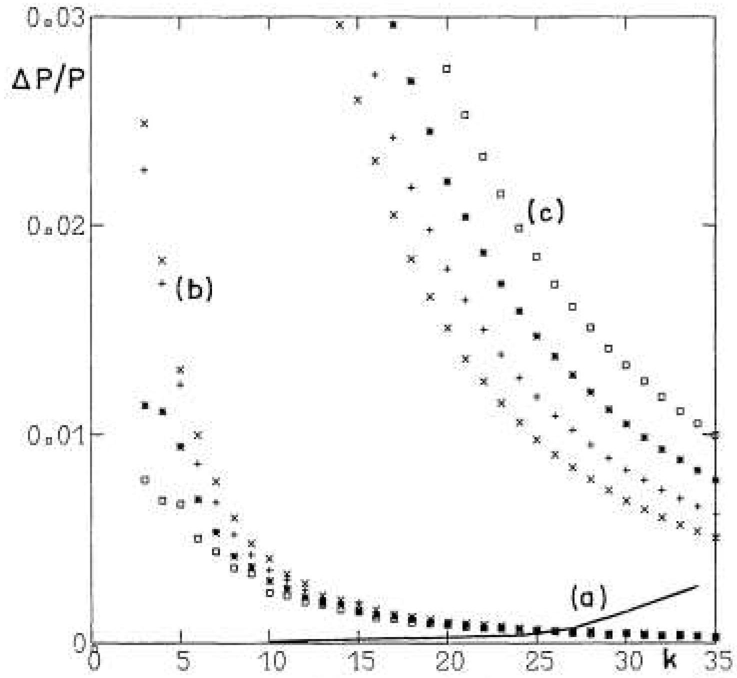


Fig. 4. Relative variation of the periods relatively to the radial order k, measuring: (a) the numerical errors; (b) the effect of the perturbation of gravitational potential; (c) the importance of second order terms in asymptotic formula(10) ($\times\, l = 1$; $+\, l = 2$; $*\, l = 3$; $\square\, l = 4$)

Provost & Berthomieu 1986

g-mode asymptotic : observations

2 Fossat et al. (2017)

Assuming that $n \gg 1$, $P_l = \frac{P_0}{\sqrt{l(l+1)}}$

$$P_{n,l} = P_{n_0,l} + (n - n_0)\left(P_l - \frac{\alpha}{n n_0}\right) + \mathrm{O}\left(\frac{1}{n^3}\right) \qquad \alpha = P_l(l(l+1)V_1 + V_2)$$

Fossat et al. 2017

$$\alpha_1 = 1350 \text{ s} \qquad P_1 \simeq 1443 \text{ s} \Rightarrow 2V_1 + V2 \simeq 0.94$$

Fossat & Schmider 2018

- $P_{n,1} = P_{-22,1} + (|n| - 22)(P_1 - 50/|n|)$ for $n = -95 \ldots -23$.
- $P_{n,2} = P_{-36,2} + (|n| - 36)(P_2 - 4/|n|)$ for $n = -142 \ldots -37$.

$$\alpha_1 = 1100 \text{ s} \qquad \qquad V_1 \simeq -0.147$$
$$\Longrightarrow$$
$$\alpha_2 = 144 \text{ s} \qquad \qquad V_2 \simeq 1.06$$

Asymptotic parameters from models

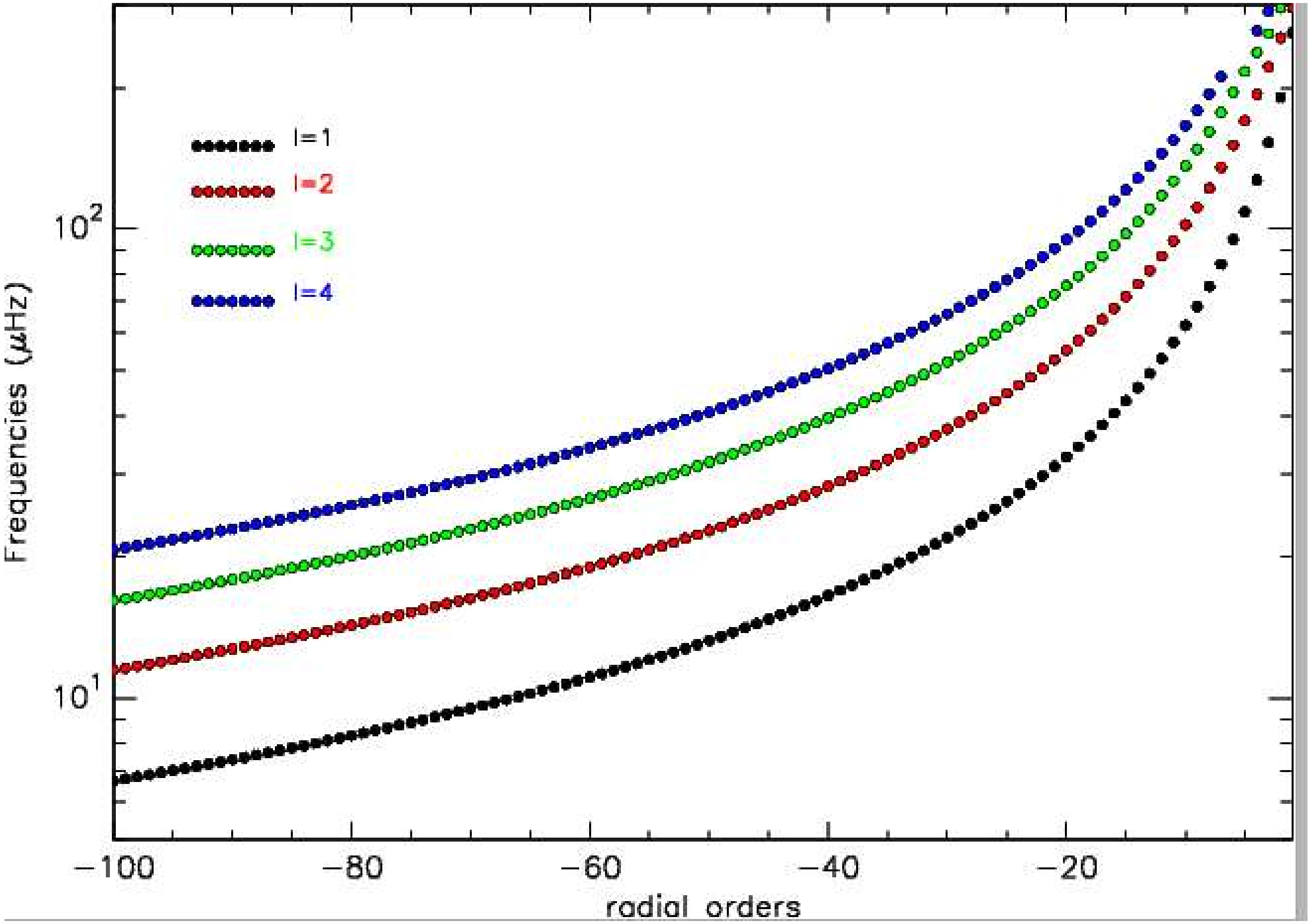
Two approaches :

(1) Fitting theoretical periods

=> model+oscillation code

(2) Directly from a model

Numerical frequencies : CESAM+GYRE



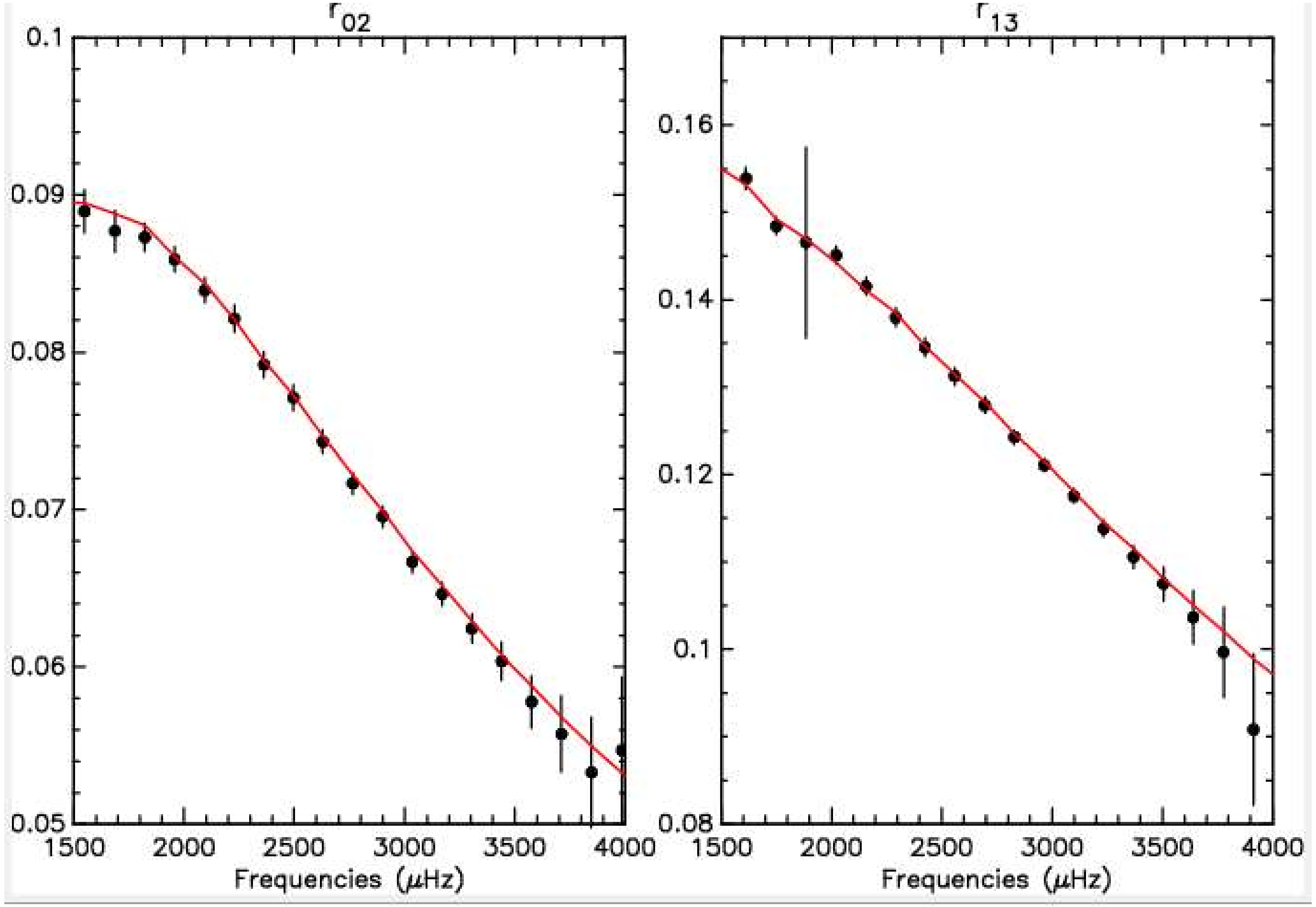
CESAM

M= 1.988475000000E+33 g
R= 6.958842022607E+10 cm
L= 3.822689401918E+33 erg/s
Teff = 5774.2 K
Y = 0.2423
Z/X=0.0244
Age = 4.57Gy
Base of cz = 0.712
P0 = 35.34 min

OPAL,
MLT Böhm-Vitesse (alpha=2.16),
diffusion Michaud-Proffitt,
NACRE for nuclear reactions,
MARCS atmosphere,
no rotation,
no overshoot.

L. Bigot

Numerical frequencies : CESAM+GYRE



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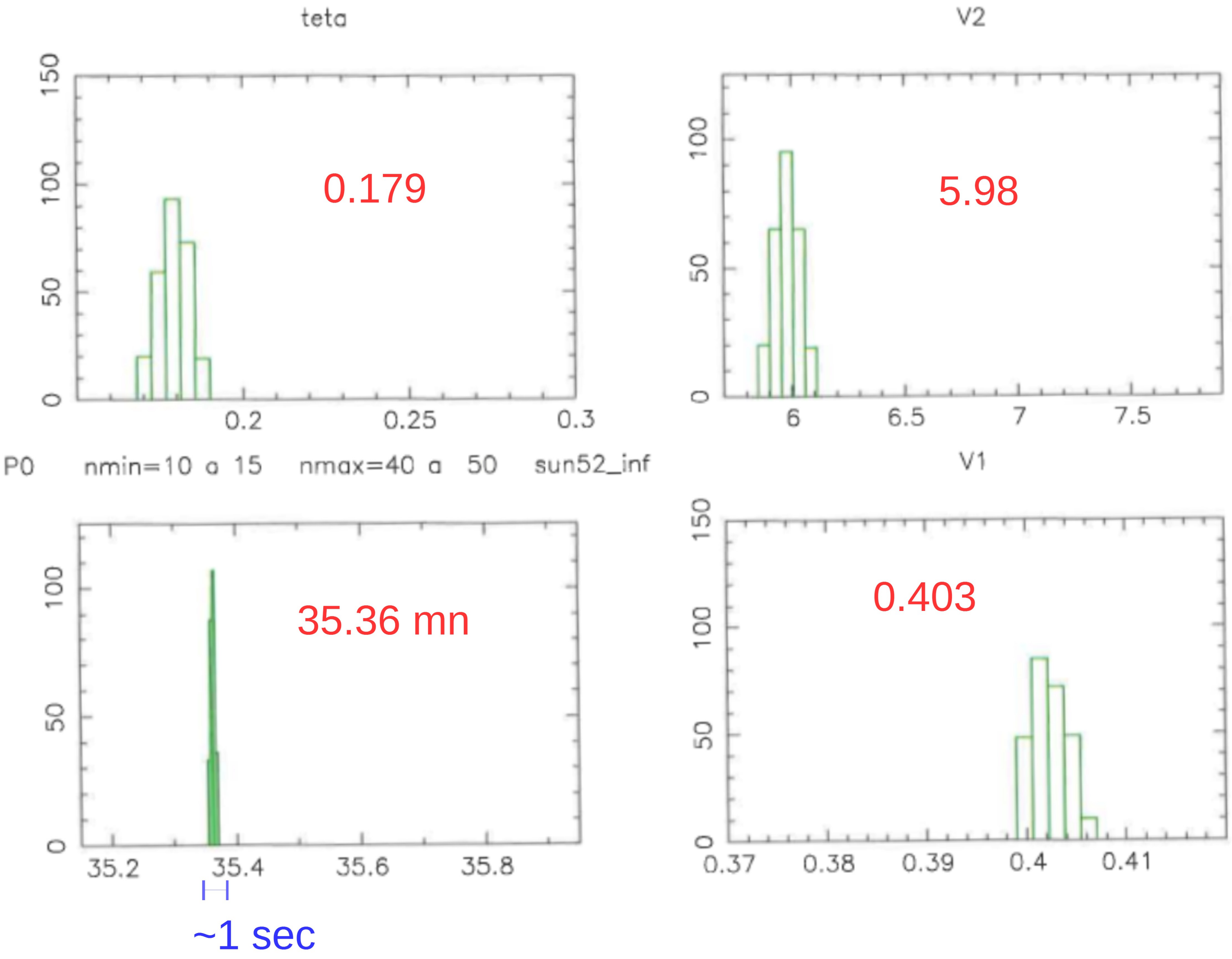
Fitting the 2d order asymptotic to numerical frequencies



g-mode asymptotic : observations

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Assuming that $n \gg 1$, $P_l = \frac{P_0}{\sqrt{l(l+1)}}$

$$P_{n,l} = P_{n_0,l} + (n - n_0)\left(P_l - \frac{\alpha}{n n_0}\right) + \mathrm{O}\left(\frac{1}{n^3}\right) \qquad \alpha = P_l(l(l+1)V_1 + V_2)$$

<u>Fossat et al. 2017</u>

$$\alpha_1 = 1350 \text{ s} \qquad P_1 \simeq 1443 \text{ s} \Rightarrow 2V_1 + V2 \simeq 0.94$$ (~6.8)

<u>Fossat & Schmider 2018</u>

$$\alpha_1 = 1100 \text{ s}$$

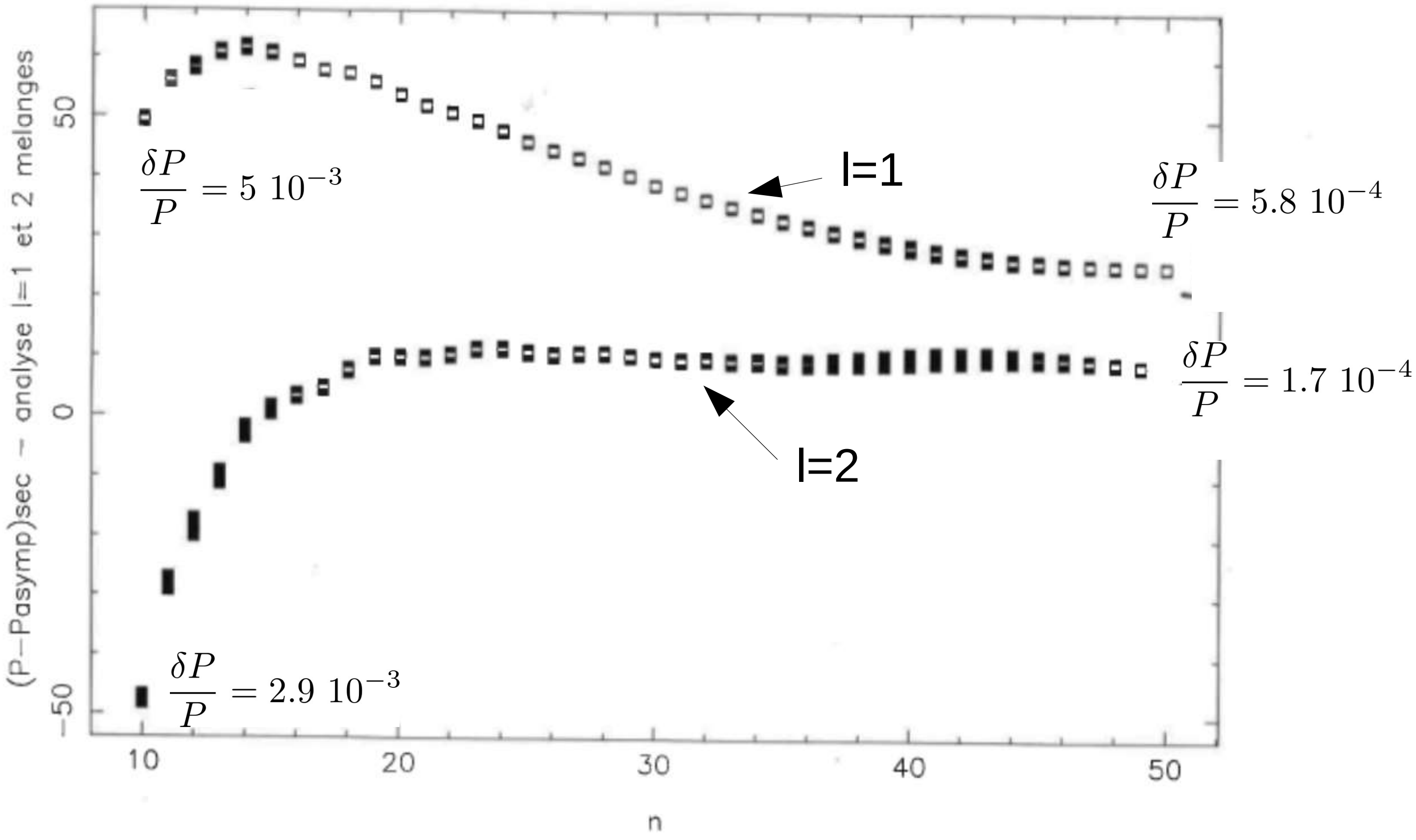
$$\alpha_2 = 144 \text{ s}$$

$$\Longrightarrow$$

$$V_1 \simeq -0.147$$ (~0.4)

$$V_2 \simeq 1.06$$ (~6)

Comparision Numeric / Asymptotic



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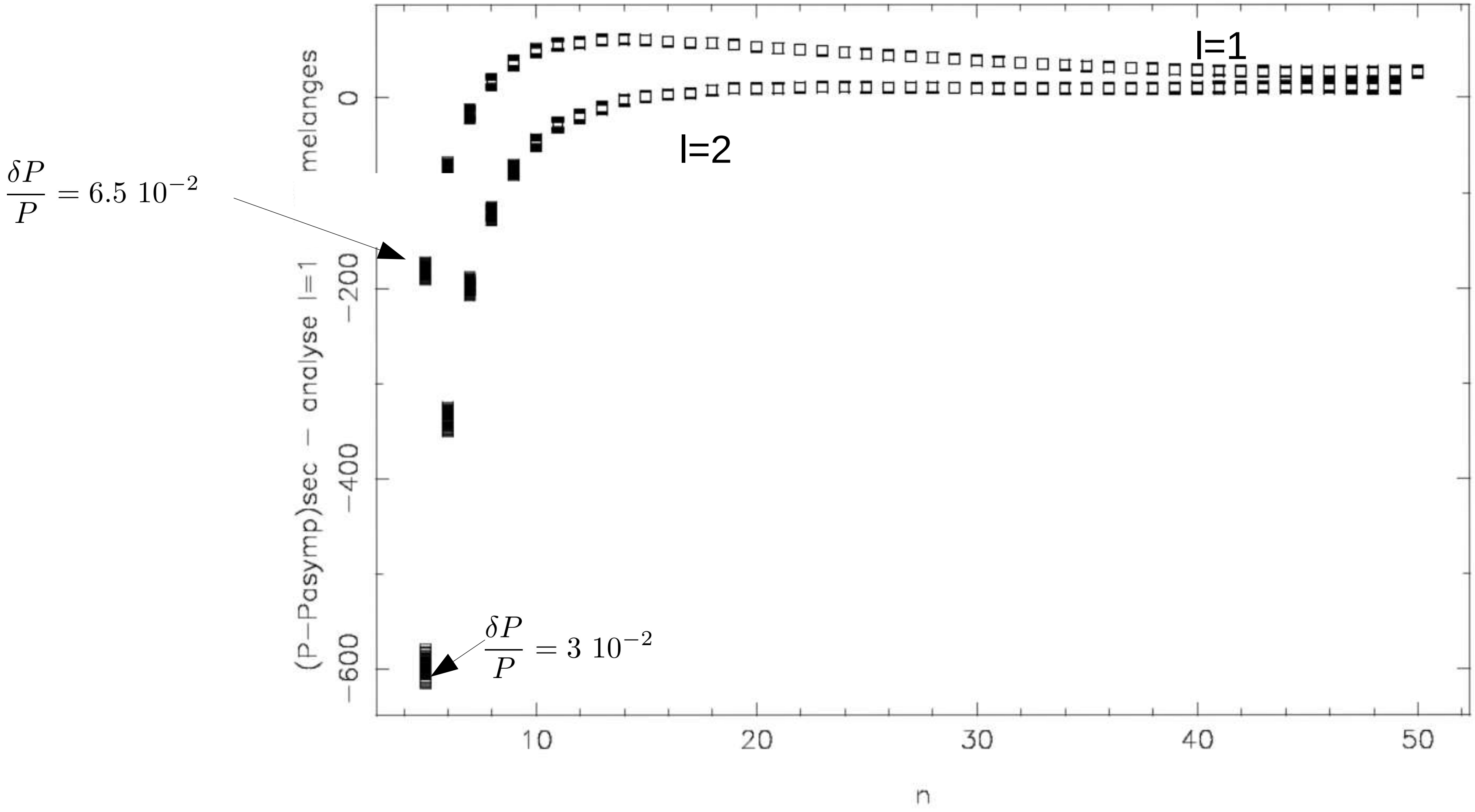


Asymptotic parameters from models

Two approaches :

(1) Fitting theoretical periods

=> model+oscillation code

(2) Directly from a model

=> compute V_1 , V_2 directly as a function of the model quantities

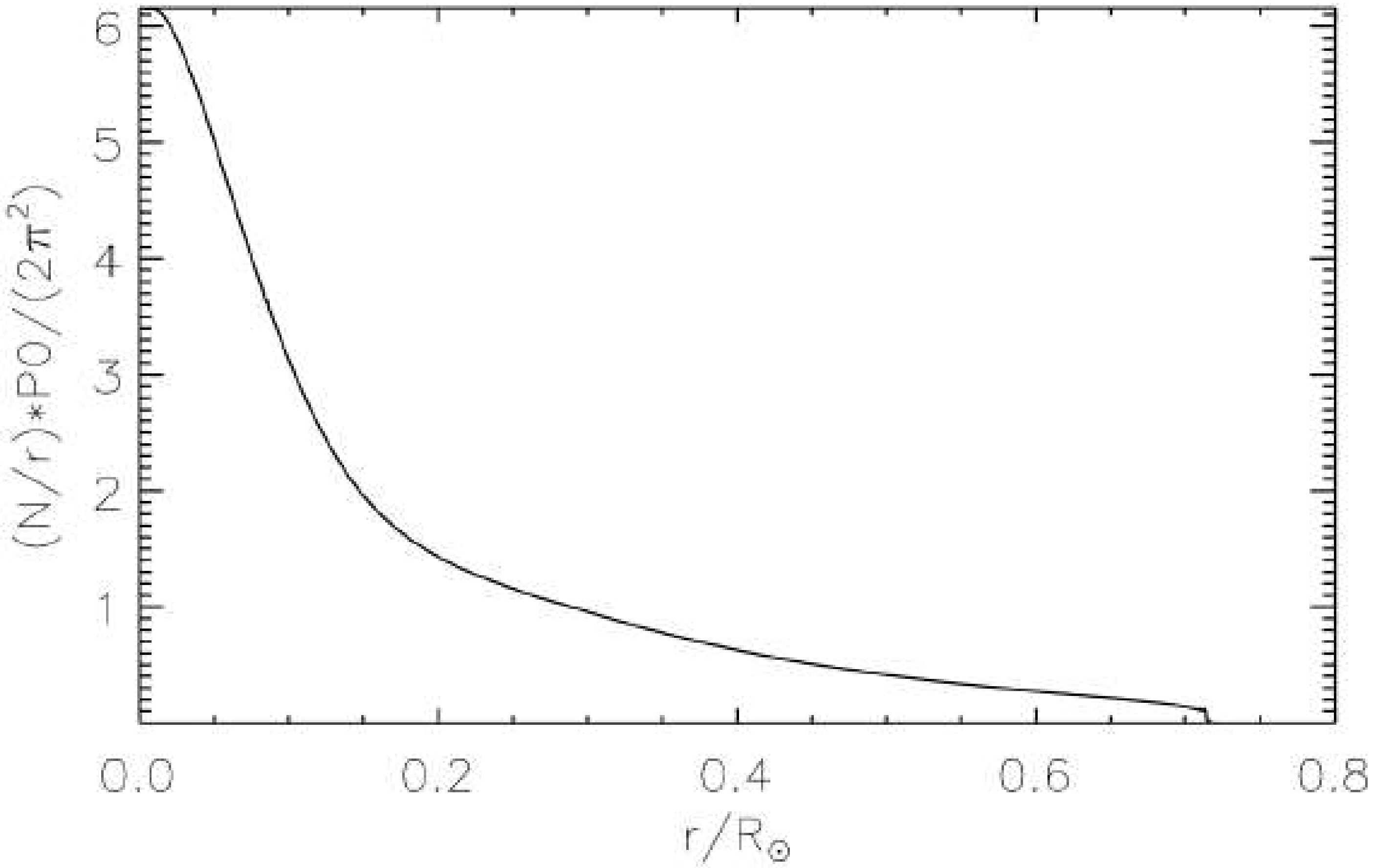
$$P_{n,l} = \frac{P_0}{\sqrt{l(l+1)}}\left[n + \frac{l}{2} - \frac{1}{4} - \theta\right] + \frac{P_0^2}{P_{n,l}}\frac{l(l+1)V_1 + V_2}{l(l+1)}$$

P_0

$$P_0 = 2\pi^2 \left(\int_0^R \frac{N}{r} \mathrm{d}r \right)^{-1}$$

~35.355 mn

Lower bound of
previous histogram



V_1

$$V_1 = \frac{1}{P_0} \lim_{\epsilon \to 0} \left(\int_{\epsilon}^{r_c} \frac{\mathrm{d}r}{Nr} - \frac{1}{N(\epsilon)} \right)$$

$$N(r) \underset{r \to 0}{\simeq} \alpha r$$

$$\int_{\epsilon}^{r_1} \frac{\mathrm{d}r}{Nr} = \int_{\epsilon}^{r_1} \frac{\mathrm{d}r}{\alpha r^2} = -\frac{1}{\alpha r_1} + \frac{1}{\alpha \epsilon} = -\frac{1}{N(r_1)} + \frac{1}{N(\epsilon)}$$

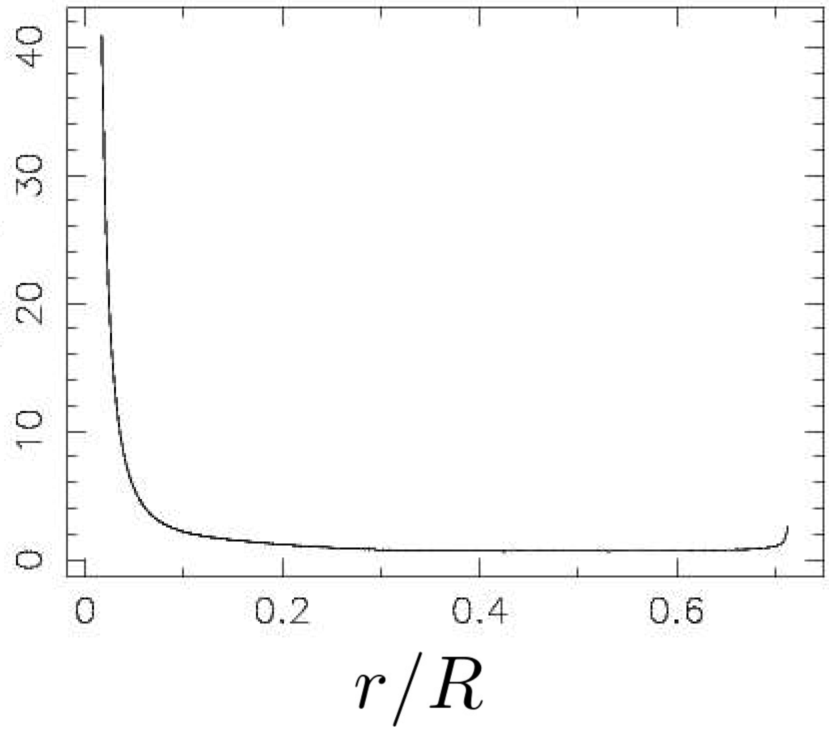
$$V_1 = \frac{1}{P_0} \left(\int_{r_1}^{r_c} \frac{\mathrm{d}r}{Nr} - \frac{1}{N(r_1)} \right)$$

r_1 : first point of the model

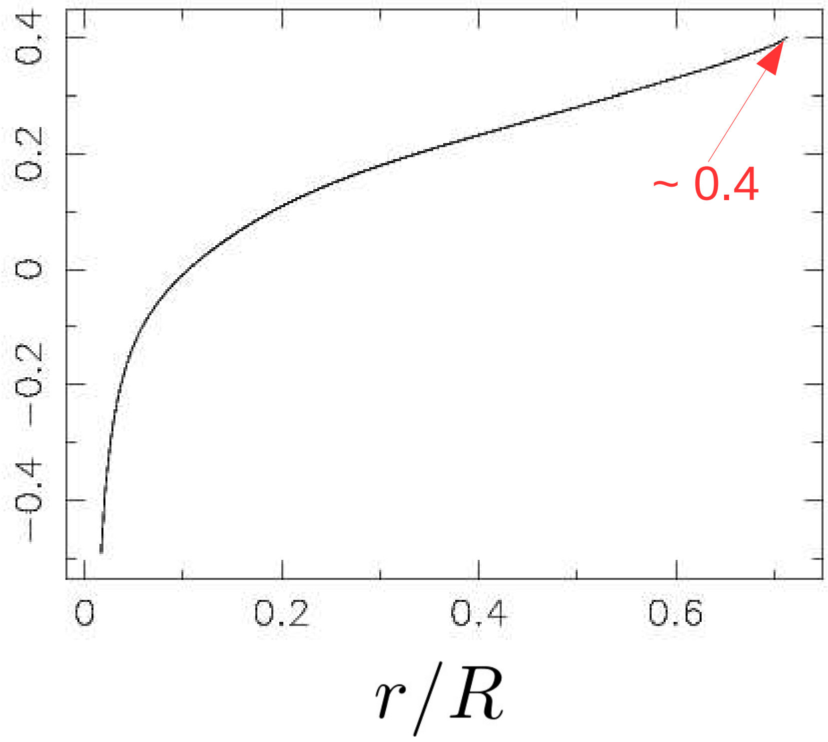
V_1

$$V_1 = \frac{1}{P_0}\left(\int_{r_1}^{r_c}\frac{\mathrm{d}r}{Nr} - \frac{1}{N(r_1)}\right)$$

$1/(Nr)$



construction V1 = [int$_{r_{min}}^{r_c}$(dx/(x N))−1/N(r_{min})]/ P_0



V_2

$$V_2 = \frac{1}{P_0} \lim_{\epsilon \to 0} \left(\int_0^{r_c - \epsilon} \left[\frac{1}{4} \left(\frac{\rho'}{\rho} \right)^2 - \frac{1}{2} \left(\frac{\rho'}{\rho} \right)' + A' - \frac{1}{16} \left(\frac{\varphi'}{\varphi} \right)^2 + \frac{1}{4} \left(\frac{\varphi'}{\varphi} \right)' \right] \frac{\mathrm{d}r}{\varphi^{\frac{1}{2}}} + \frac{5}{24} \frac{1}{u(r_c - \epsilon)} \right) \tag{1}$$

$$\varphi = \frac{N^2}{r^2}; \qquad A = \frac{\rho'}{\rho} - \frac{p'}{\Gamma p} = -\frac{N^2}{g}; \qquad u(r_c - \epsilon) = \frac{3}{2} \int_{r_c - \epsilon}^{r_c} \varphi^{\frac{1}{2}} \mathrm{d}r \tag{2}$$

$$V_2 = V_2^1 + V_2^2 \tag{3}$$

$$V_2^1 = \frac{1}{P_0} \left(\int_0^{r_c} \frac{r \mathrm{d}r}{N} \left[\frac{1}{4} \left(\frac{\rho'}{\rho} \right)^2 - \frac{1}{2} \left(\frac{\rho'}{\rho} \right)' + A' \right] \right) \tag{4}$$

$$V_2^2 = \frac{1}{P_0} \lim_{\epsilon \to 0} \left(\int_0^{r_c - \epsilon} \left[-\frac{1}{16} \left(\frac{\varphi'}{\varphi} \right)^2 + \frac{1}{4} \left(\frac{\varphi'}{\varphi} \right)' \right] \frac{\mathrm{d}r}{\varphi^{\frac{1}{2}}} + \frac{5}{24} \frac{1}{u(r_c - \epsilon)} \right) \tag{5}$$

V_2^2

$$V_2^2 = \frac{1}{P_0} \lim_{\epsilon \to 0} \left(\int_0^{r_c - \epsilon} \left[-\frac{1}{16} \left(\frac{\varphi'}{\varphi} \right)^2 + \frac{1}{4} \left(\frac{\varphi'}{\varphi} \right)' \right] \frac{\mathrm{d}r}{\varphi^{\frac{1}{2}}} + \frac{5}{24} \frac{1}{u(r_c - \epsilon)} \right) \quad (5)$$

$$\frac{1}{4} \int_0^{r_c - \epsilon} \left(\frac{\varphi'}{\varphi} \right)' \varphi^{-\frac{1}{2}} \mathrm{d}r = \frac{1}{4} \left[\frac{\varphi'}{\varphi^{\frac{3}{2}}} \right]_0^{r_c - \epsilon} + \frac{1}{8} \int_0^{r_c - \epsilon} \left(\frac{\varphi'}{\varphi} \right)^2 \frac{\mathrm{d}r}{\varphi^{\frac{1}{2}}} \quad (6)$$

$$\left[\frac{\varphi'}{\varphi^{\frac{3}{2}}} \right]_{r=0} = 0 \quad (7)$$

$$V_2^2 = \frac{1}{P_0} \lim_{\epsilon \to 0} \left(\frac{1}{16} \int_0^{r_c - \epsilon} \left(\frac{\varphi'}{\varphi} \right)^2 \frac{\mathrm{d}r}{\varphi^{\frac{1}{2}}} + \frac{1}{4} \left(\frac{\varphi'}{\varphi^{\frac{3}{2}}} \right)_{r_c - \epsilon} + \frac{5}{24} \frac{1}{u(r_c - \epsilon)} \right) \quad (8)$$

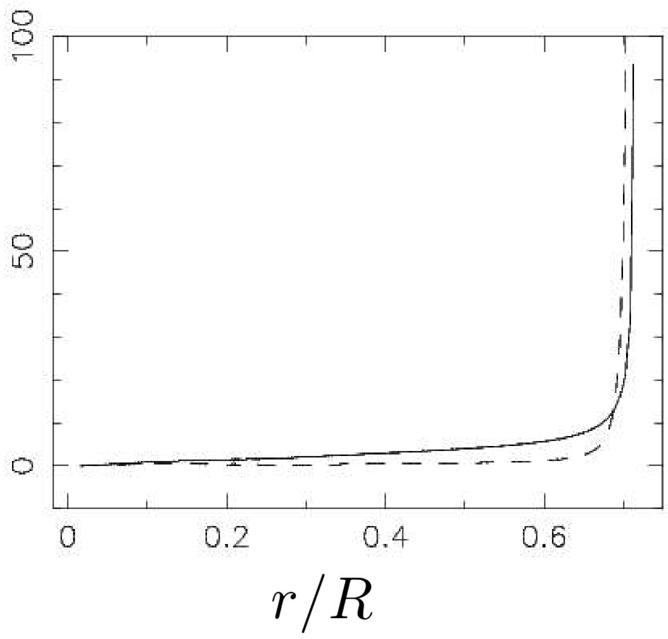
$$\varphi(r) \underset{r \to r_c}{\simeq} \varphi_1 (r_c - r); \qquad \varphi' = -\varphi_1; \qquad \frac{\varphi'}{\varphi} = -\frac{1}{r_c - r} \quad (9)$$

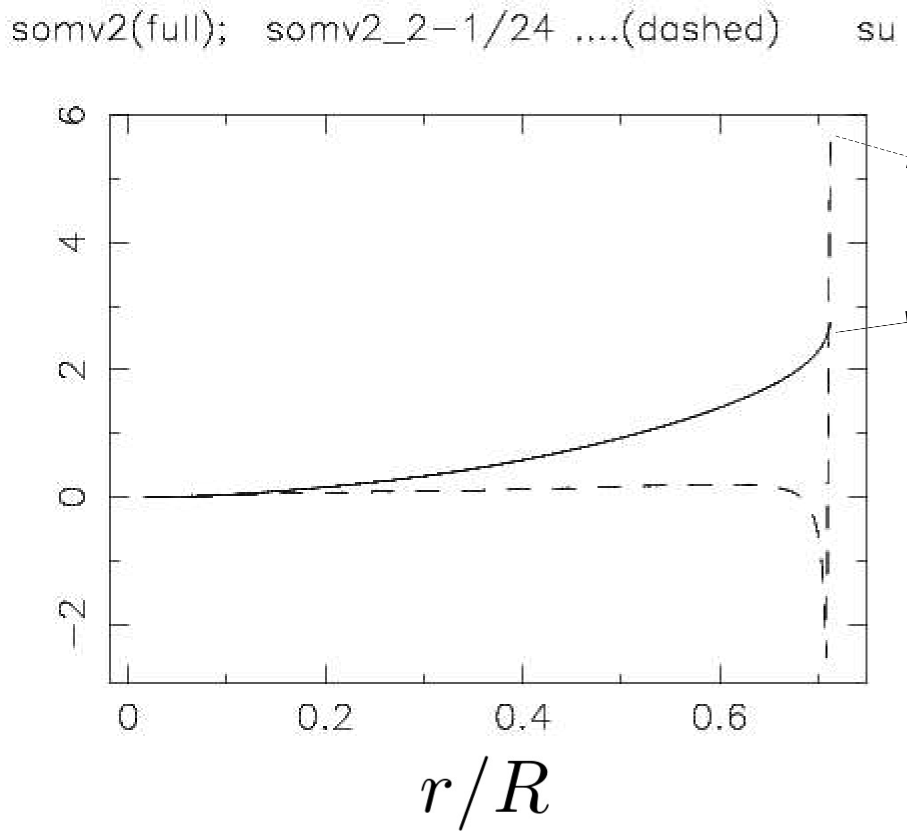
$$V_2^2 = \frac{1}{P_0} \left(\frac{1}{16} \int_0^{r_{\max}} \frac{\varphi'^2}{\varphi^{\frac{5}{2}}} \mathrm{d}r - \frac{1}{24 \varphi(r_{\max})(r_c - r_{\max})} \right)$$

r_{max} : first point below the convective zone

V_2

$$V_2^1 = \frac{1}{P_0}\left(\int_0^{r_c} \frac{r\mathrm{d}r}{N}\left[\frac{1}{4}\left(\frac{\rho'}{\rho}\right)^2 - \frac{1}{2}\left(\frac{\rho'}{\rho}\right)' + A'\right]\right)$$





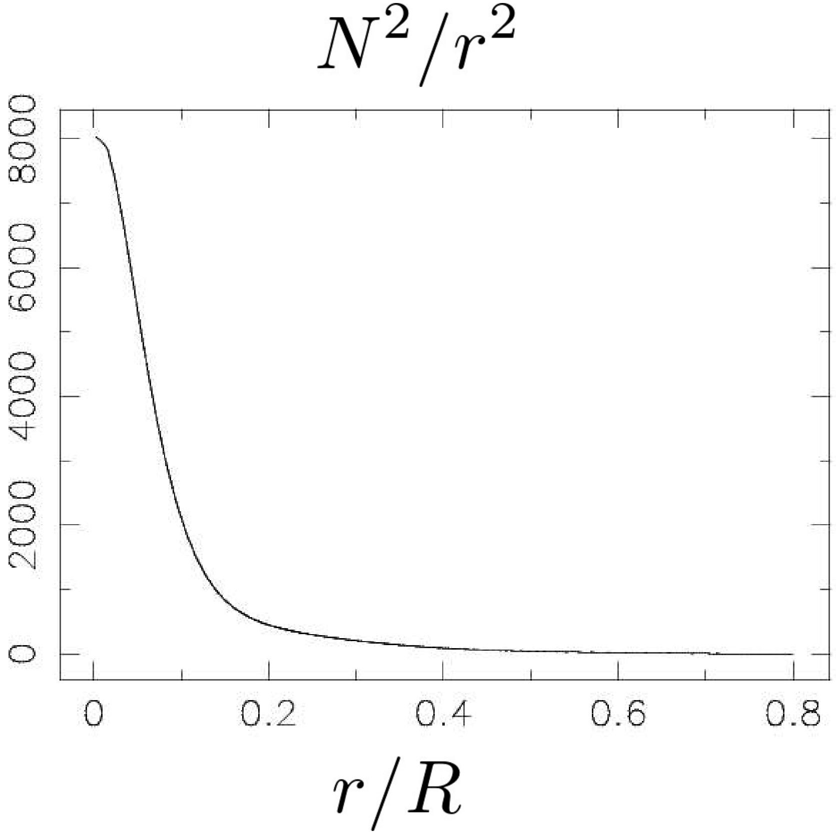
$$V_2^2 = \frac{5.57}{P_0} = 4.16$$

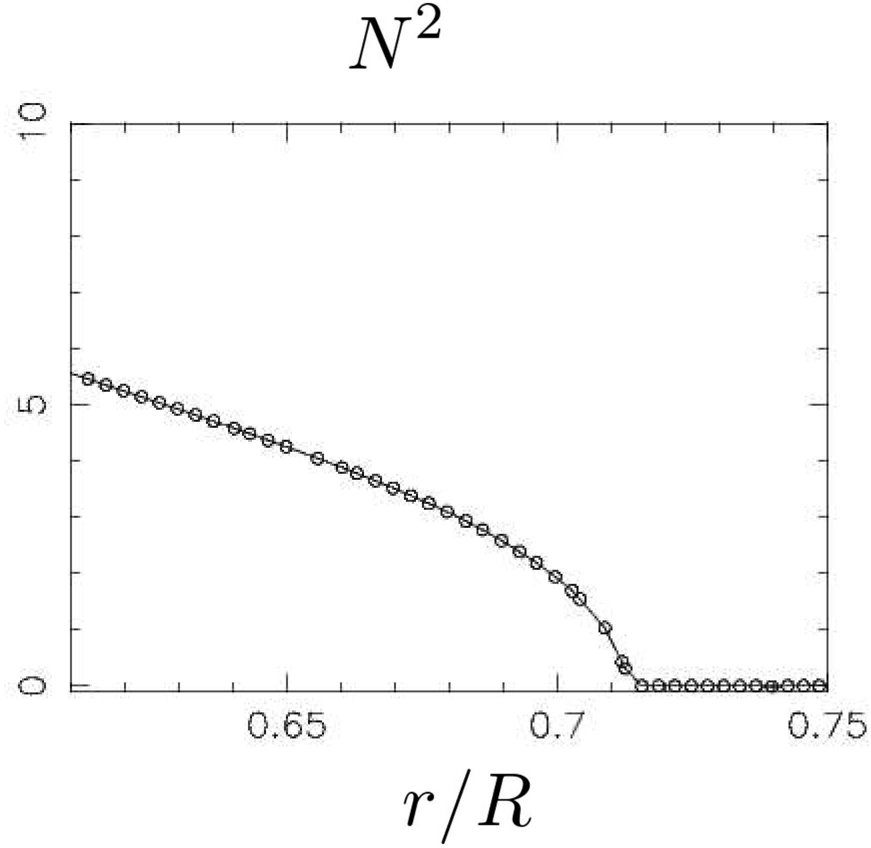
$$V_2^1 = \frac{2.73}{P_0} = 2.04$$

$$V_2 \simeq 6.2$$

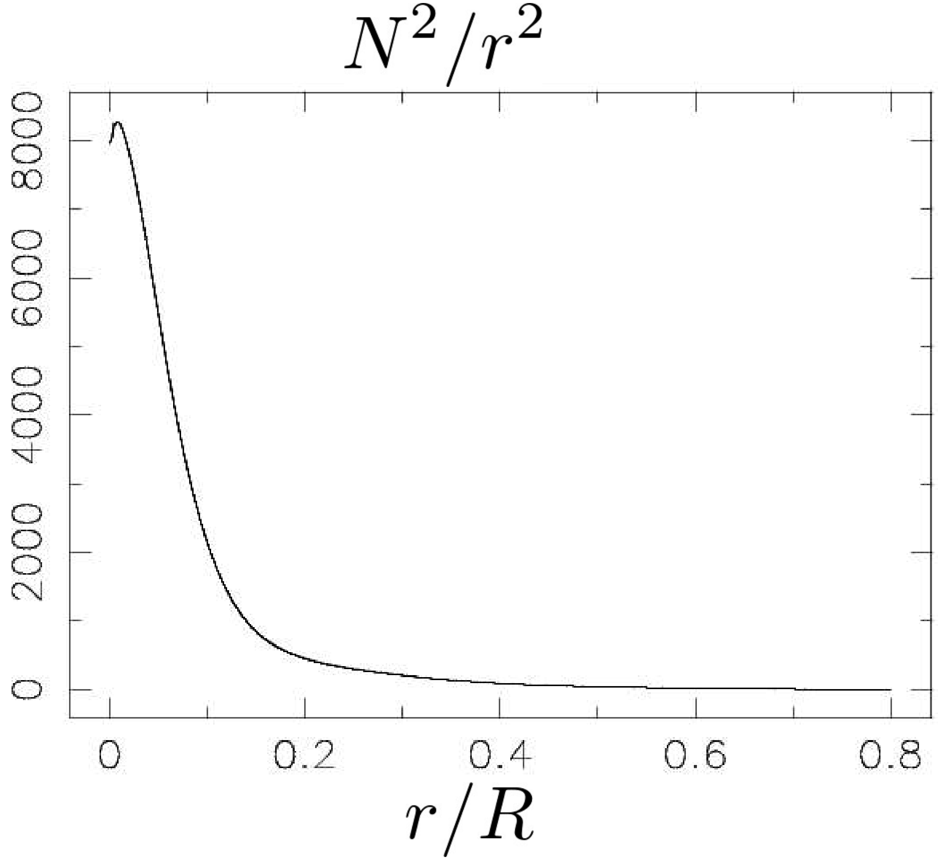
$$V_2^2 = \frac{1}{P_0}\left(\frac{1}{16}\int_0^{r_{\max}} \frac{\varphi'^2}{\varphi^{\frac{5}{2}}}\mathrm{d}r - \frac{1}{24\varphi(r_{\max})(r_c - r_{\max})}\right)$$

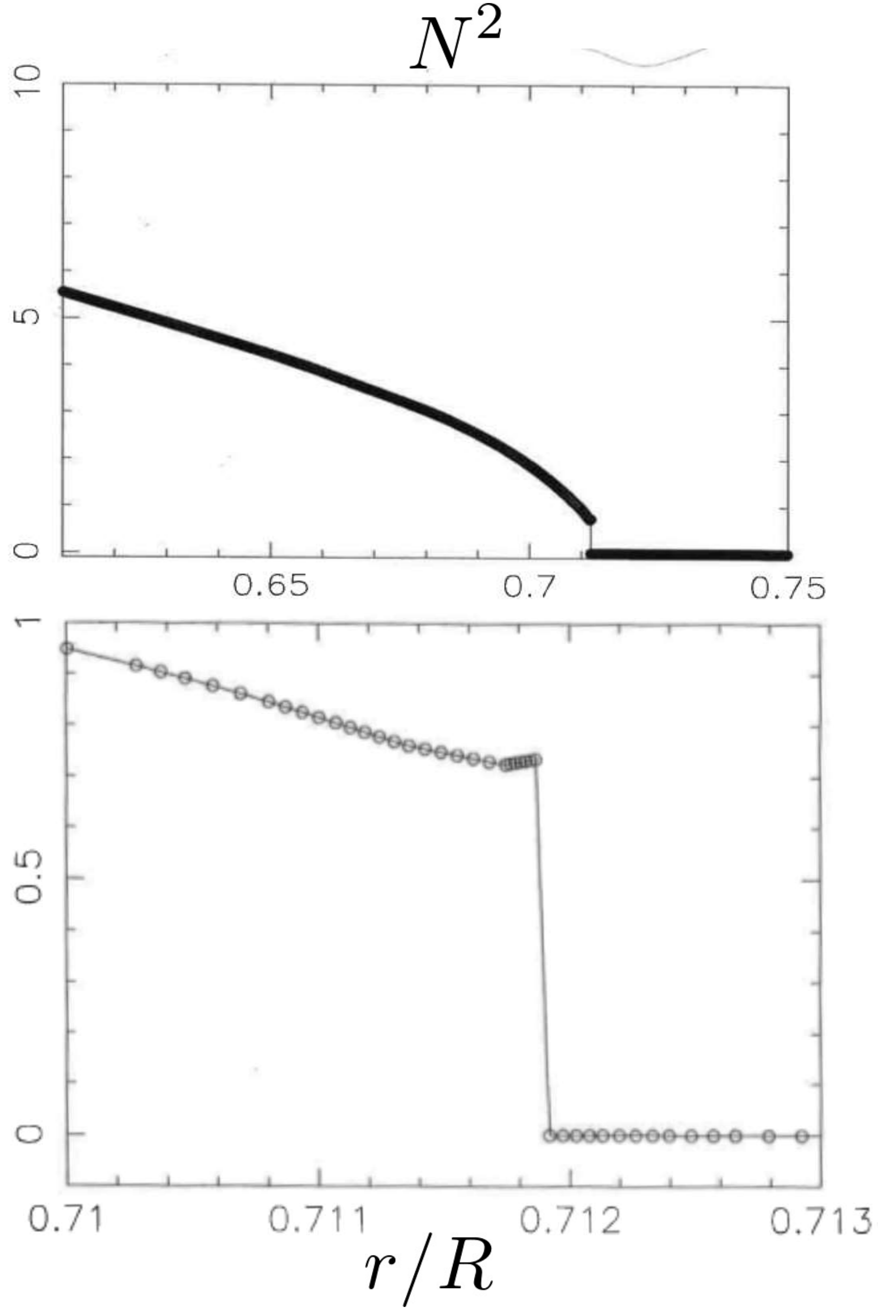
1000 points CESAM model





10 000 points CESAM model





Numerical errors (for a given model)