

# Aspects of solar modelling

Jørgen Christensen-Dalsgaard  
Stellar Astrophysics Centre



# Selection of models

| Model                           | Age<br>(Gyr) | R<br>( $10^{10}$ cm) | L<br>( $10^{33}$ erg s $^{-1}$ ) | Opacity<br>tables | Surface<br>opacity | Surface<br>comp. | Other changes<br>(see caption) |
|---------------------------------|--------------|----------------------|----------------------------------|-------------------|--------------------|------------------|--------------------------------|
| [S; d.02c]                      | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [S'; d.30c]                     | 4.60         | 6.9599               | 3.846                            | OPAL96            | Alex94             | GN93             | Updated Model S                |
| [Age; d.06c]                    | 4.57         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [ $R_s$ ; d.05c]                | 4.60         | 6.95508              | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [ $L_s$ ; d.41c]                | 4.60         | 6.9599               | 3.828                            | OPAL92            | Kur91              | GN93             | -                              |
| [Liv05; d.40c]                  | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EOS Liv05                      |
| [Opc. 7.0; d.15c]               | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Local $\delta \log \kappa$     |
| [Opc. 6.5; d.16c]               | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Local $\delta \log \kappa$     |
| [OPAL96; d.07c]                 | 4.60         | 6.9599               | 3.846                            | OPAL96            | Kur91              | GN93             | -                              |
| [Surf. opac.; d.35c]            | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | GN93             | Surf. opac.                    |
| [GS98; d.37c]                   | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | GS98             | -                              |
| [OP05; d.36c]                   | 4.60         | 6.9599               | 3.846                            | OP05              | Fer05              | GS98             | -                              |
| [CM; d.24c]                     | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | CM conv.                       |
| [Adelb11; d.34c]                | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EnGen. Adelberger              |
| [NACRE; d.39c]                  | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EnGen. NACRE                   |
| [ $^3\text{He}$ eql.; d.02c_eq] | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | $^3\text{He}$ nucl. eql.       |
| [No el.scrn; d.20c]             | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | no electr. screen.             |
| [Dc; d.17c]                     | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Change diff.                   |
| [DVc; d.18c]                    | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Change diff., settl.           |
| [No diff.; 03c]                 | 4.60         | 6.9599               | 3.846                            | OPAL96            | Alex94             | GN93             | No diffusion                   |
| [AGS05; d.21c]                  | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGS05            | -                              |
| [AGSS09; d.22c]                 | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGSS09           | -                              |
| 17bi.d.os.83                    | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Smoothed undersh.              |
| ek16.d.598c                     | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGSS09           | Adjust. opac.                  |

# Selection of models

| Model                            | Age<br>(Gyr) | R<br>( $10^{10}$ cm) | L<br>( $10^{33}$ erg s $^{-1}$ ) | Opacity<br>tables | Surface<br>opacity | Surface<br>comp. | Other changes<br>(see caption) |
|----------------------------------|--------------|----------------------|----------------------------------|-------------------|--------------------|------------------|--------------------------------|
| [S; d.02c]                       | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [S'; d.30c]                      | 4.60         | 6.9599               | 3.846                            | OPAL96            | Alex94             | GN93             | Updated Model S                |
| [Age; d.06c]                     | 4.57         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [ $R_s$ ; d.05c]                 | 4.60         | 6.95508              | 3.846                            | OPAL92            | Kur91              | GN93             | -                              |
| [ $L_s$ ; d.41c]                 | 4.60         | 6.9599               | 3.828                            | OPAL92            | Kur91              | GN93             | -                              |
| [Liv05; d.40c]                   | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EOS Liv05                      |
| [Opc. 7.0; d.15c]                | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Local $\delta \log \kappa$     |
| [Opc. 6.5; d.16c]                | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Local $\delta \log \kappa$     |
| [OPAL96; d.07c]                  | 4.60         | 6.9599               | 3.846                            | OPAL96            | Kur91              | GN93             | -                              |
| [Surf. opac.; d.35c]             | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | GN93             | Surf. opac.                    |
| [GS98; d.37c]                    | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | GS98             | -                              |
| [OP05; d.36c]                    | 4.60         | 6.9599               | 3.846                            | OP05              | Fer05              | GS98             | -                              |
| [CM; d.24c]                      | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | CM conv.                       |
| [Adelb11; d.34c]                 | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EnGen. Adelberger              |
| [NACRE; d.39c]                   | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | EnGen. NACRE                   |
| [ $^3\text{He}$ eql.; d.02c_eql] | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | $^3\text{He}$ nucl. eql.       |
| [No el.scrn; d.20c]              | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | no electr. screen.             |
| [Dc; d.17c]                      | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Change diff.                   |
| [DVc; d.18c]                     | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Change diff., settl.           |
| [No diff.; 03c]                  | 4.60         | 6.9599               | 3.846                            | OPAL96            | Alex94             | GN93             | No diffusion                   |
| [AGS05; d.21c]                   | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGS05            | -                              |
| [AGSS09; d.22c]                  | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGSS09           | -                              |
| 17bi.d.os.83                     | 4.60         | 6.9599               | 3.846                            | OPAL92            | Kur91              | GN93             | Smoothed undersh.              |
| ek16.d.598c                      | 4.60         | 6.9599               | 3.846                            | OPAL96            | Fer05              | AGSS09           | Adjust. opac.                  |

[http://astro.phys.au.dk/~jcd/ASTEC\\_results/](http://astro.phys.au.dk/~jcd/ASTEC_results/)

# Small frequency separations

$$\nu_{nl} \sim \Delta\nu \left( n + \frac{l}{2} + \alpha \right) + \epsilon_{nl}$$

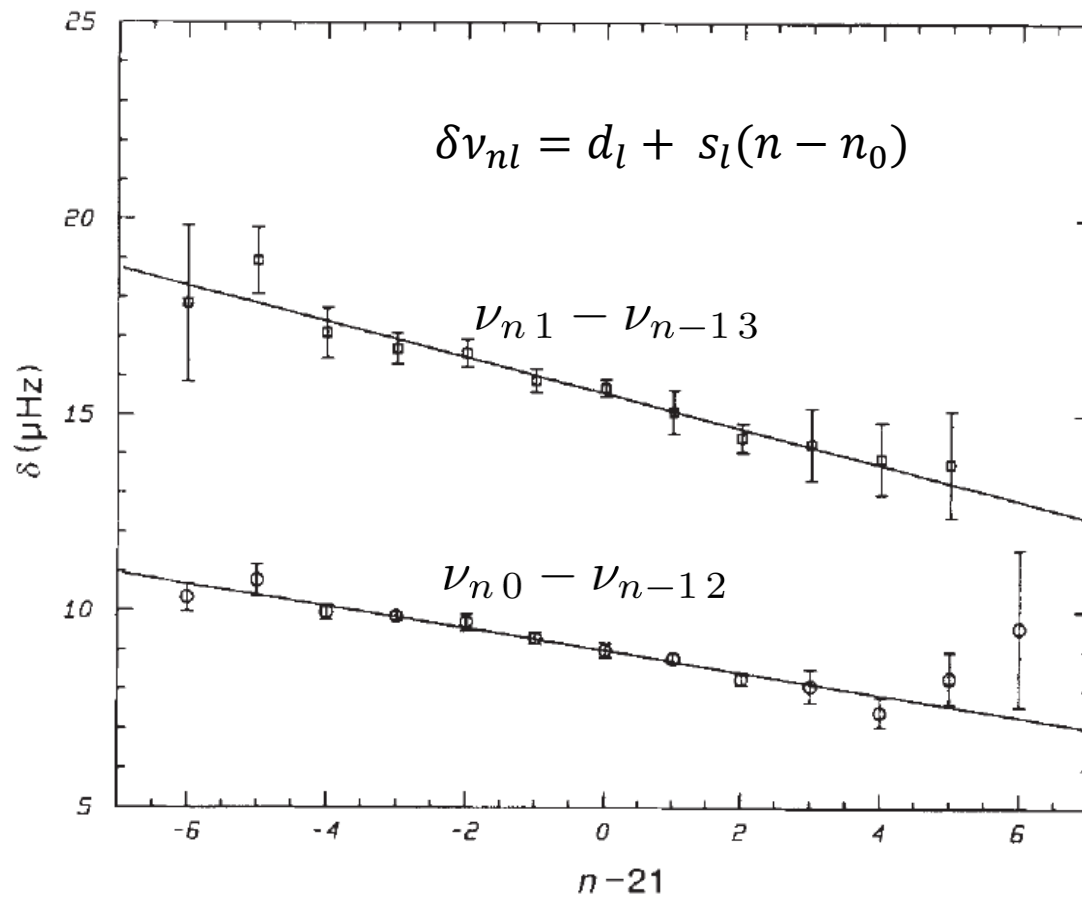
where

$$\epsilon_{nl} \simeq l(l+1) \frac{\Delta\nu}{4\pi^2 \nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}$$

Frequency separations:

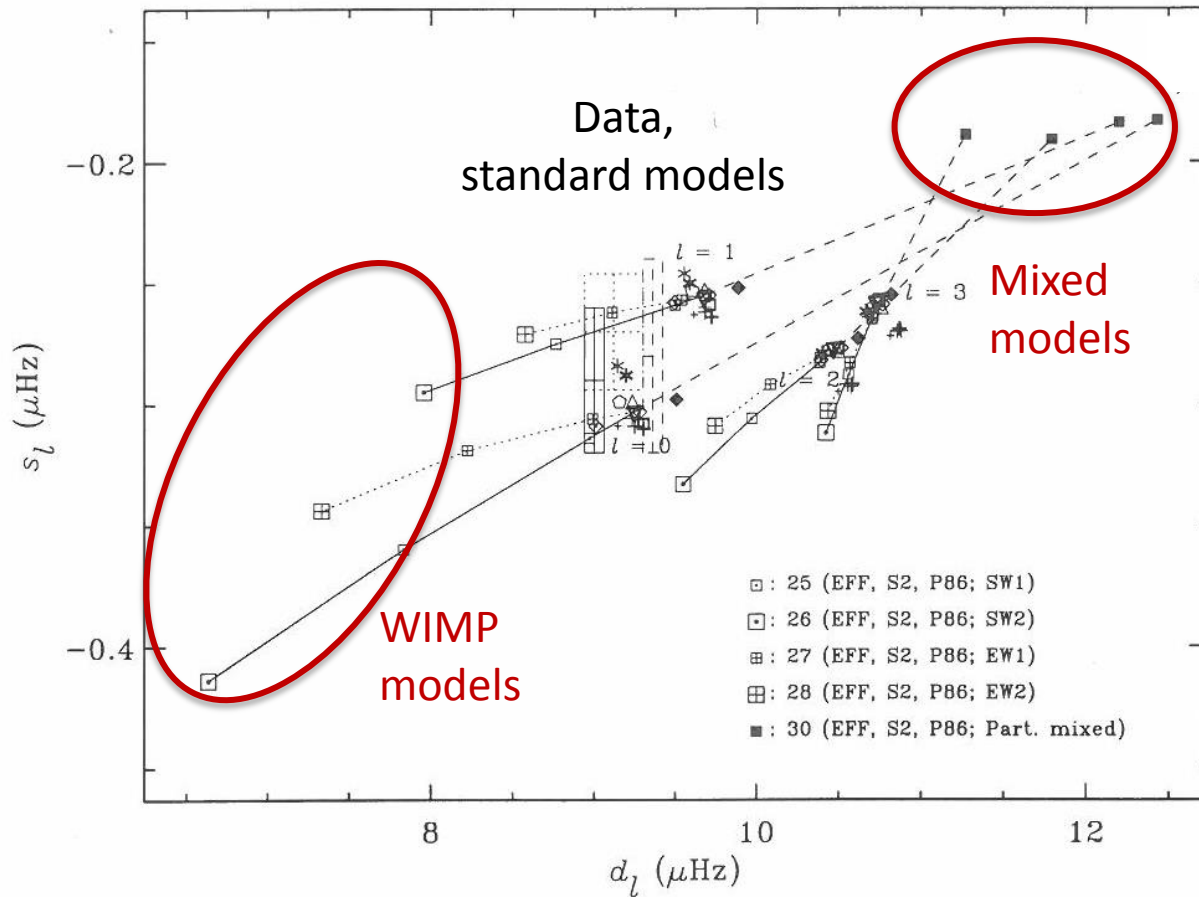
$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1, l+2} \simeq -(4l+6) \frac{\Delta\nu}{4\pi^2 \nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}$$

# Low-degree helioseismology



Elsworth et al. (1990; Nature 347, 536)

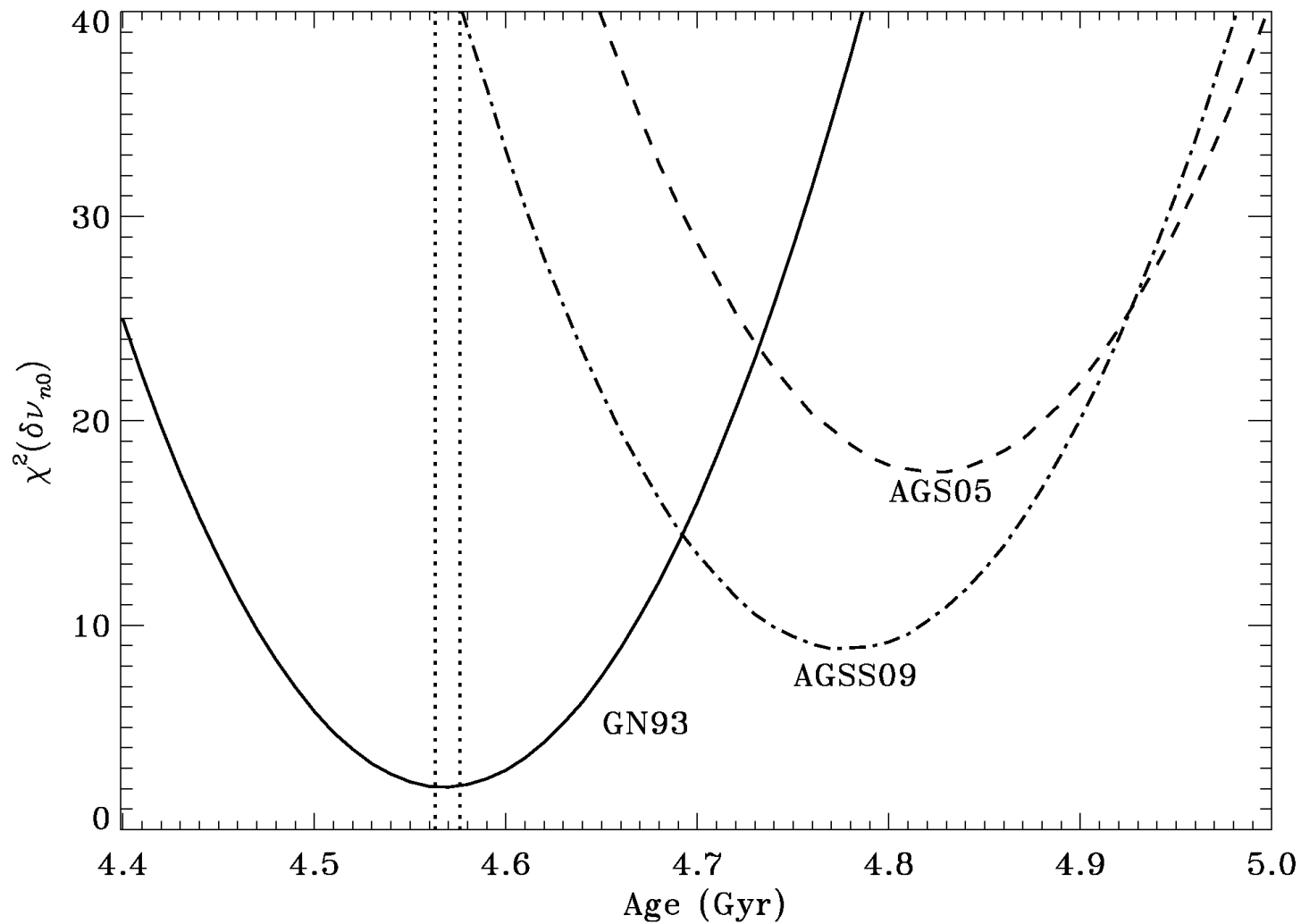
# Neutrinos and low-degree helioseismology



# Revising solar composition

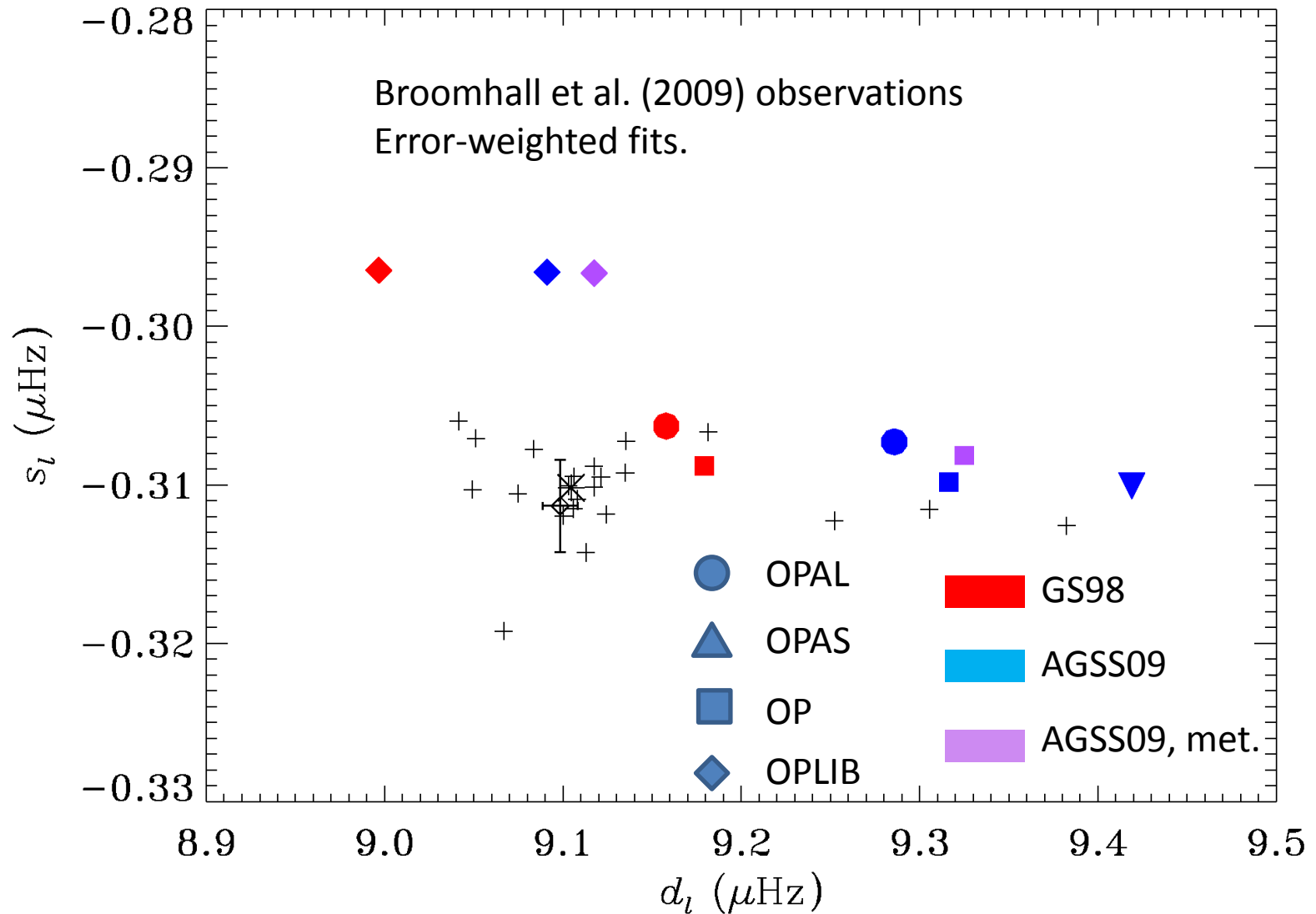
|           | AG89   | GN93   | GS98   | AGS05  | AGSS09 | C11         |
|-----------|--------|--------|--------|--------|--------|-------------|
| C         | 8.56   | 8.55   | 8.52   | 8.39   | 8.43   | 8.50        |
| N         | 8.05   | 7.97   | 7.92   | 7.78   | 7.83   | 7.86        |
| O         | 8.93   | 8.87   | 8.83   | 8.66   | 8.69   | 8.76        |
| Ne        | 8.09   | 8.07   | 8.08   | 7.84   | 7.93   | <i>8.05</i> |
| Na        | 6.33   | 6.33   | 6.33   | 6.17   | 6.24   | <i>6.29</i> |
| Mg        | 7.58   | 7.58   | 7.58   | 7.53   | 7.60   | <i>7.54</i> |
| Al        | 6.47   | 6.47   | 6.47   | 6.37   | 6.45   | <i>6.46</i> |
| Si        | 7.55   | 7.55   | 7.55   | 7.51   | 7.51   | <i>7.53</i> |
| Fe        | 7.67   | 7.51   | 7.50   | 7.45   | 7.50   | 7.52        |
| $Z_s/X_s$ | 0.0275 | 0.0245 | 0.0231 | 0.0165 | 0.0181 | 0.0209      |

# Fitting $\delta\nu_{nl}$

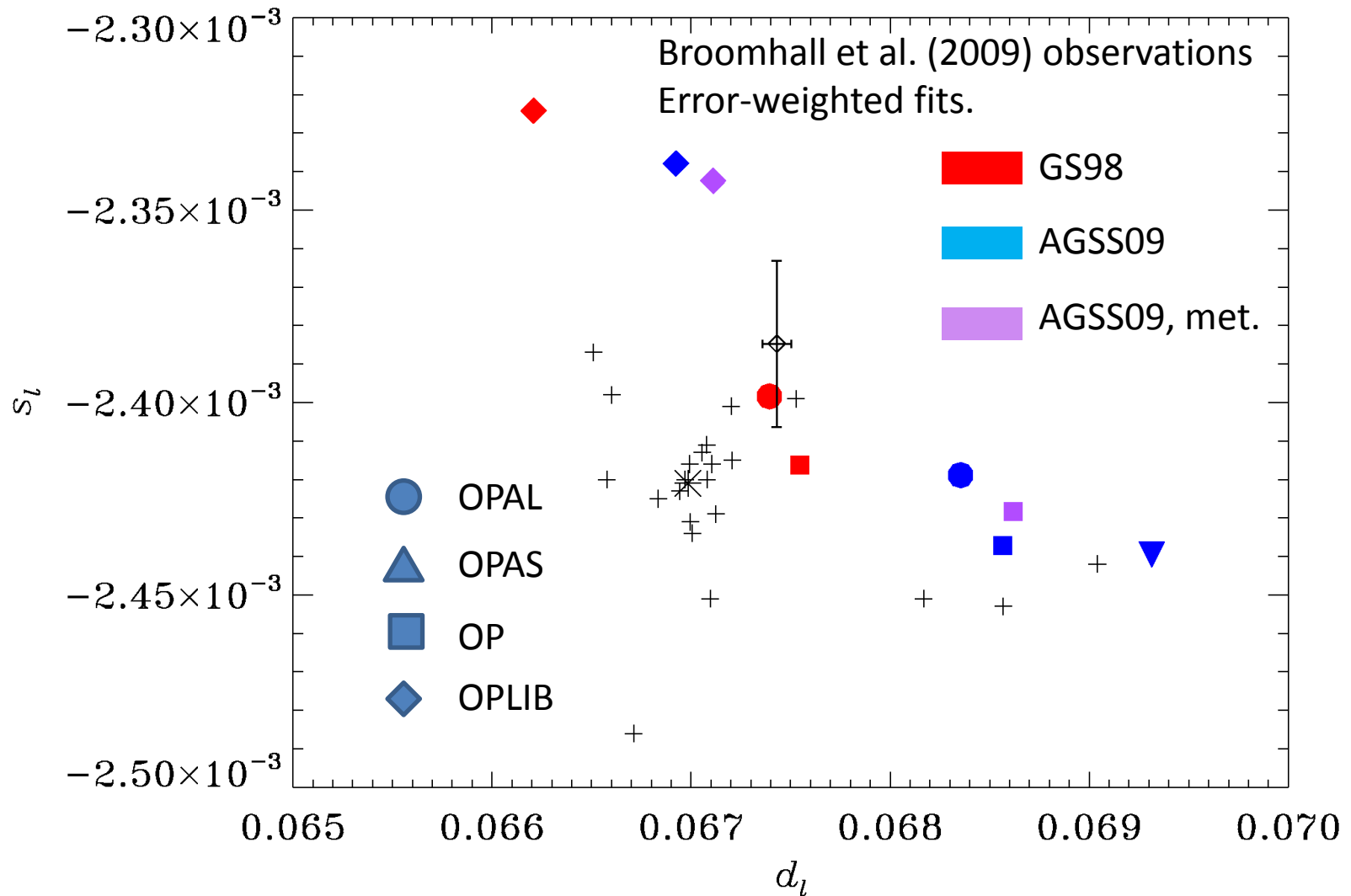




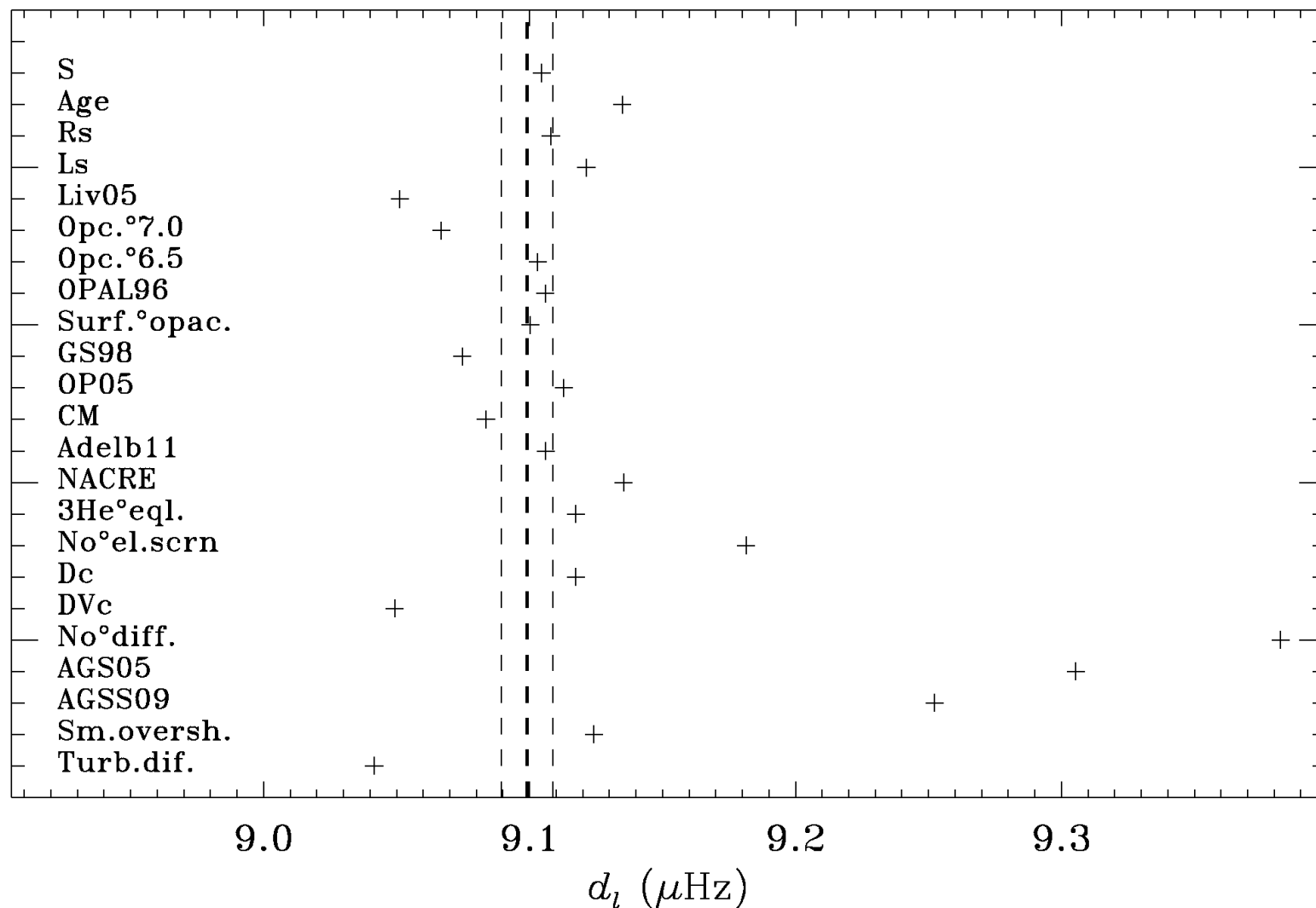
# Dependence of $d_{02}$ fit on physics



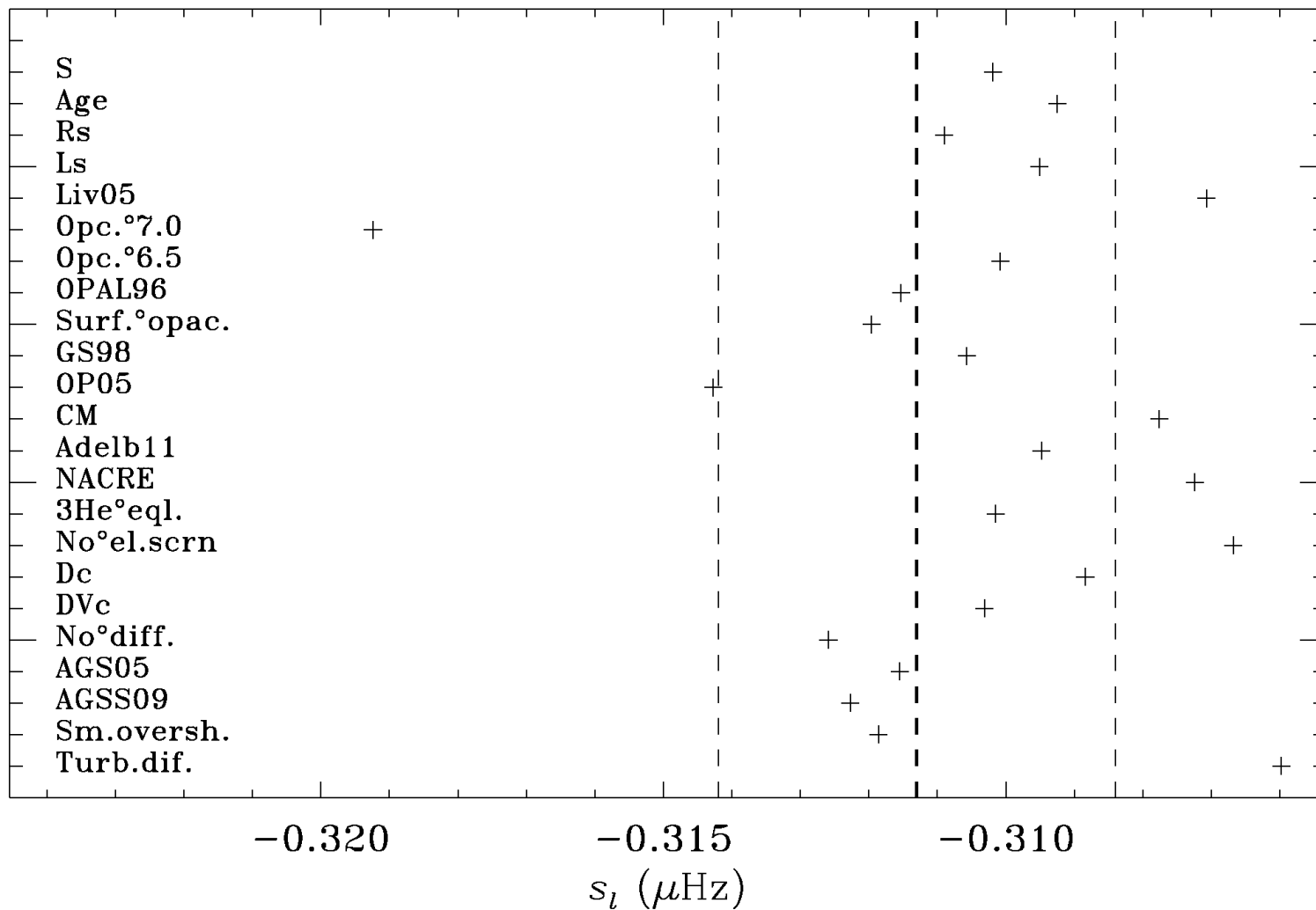
# Dependence of $r_{02}$ fit on physics



# Dependence $d_{02}$ fit on physics

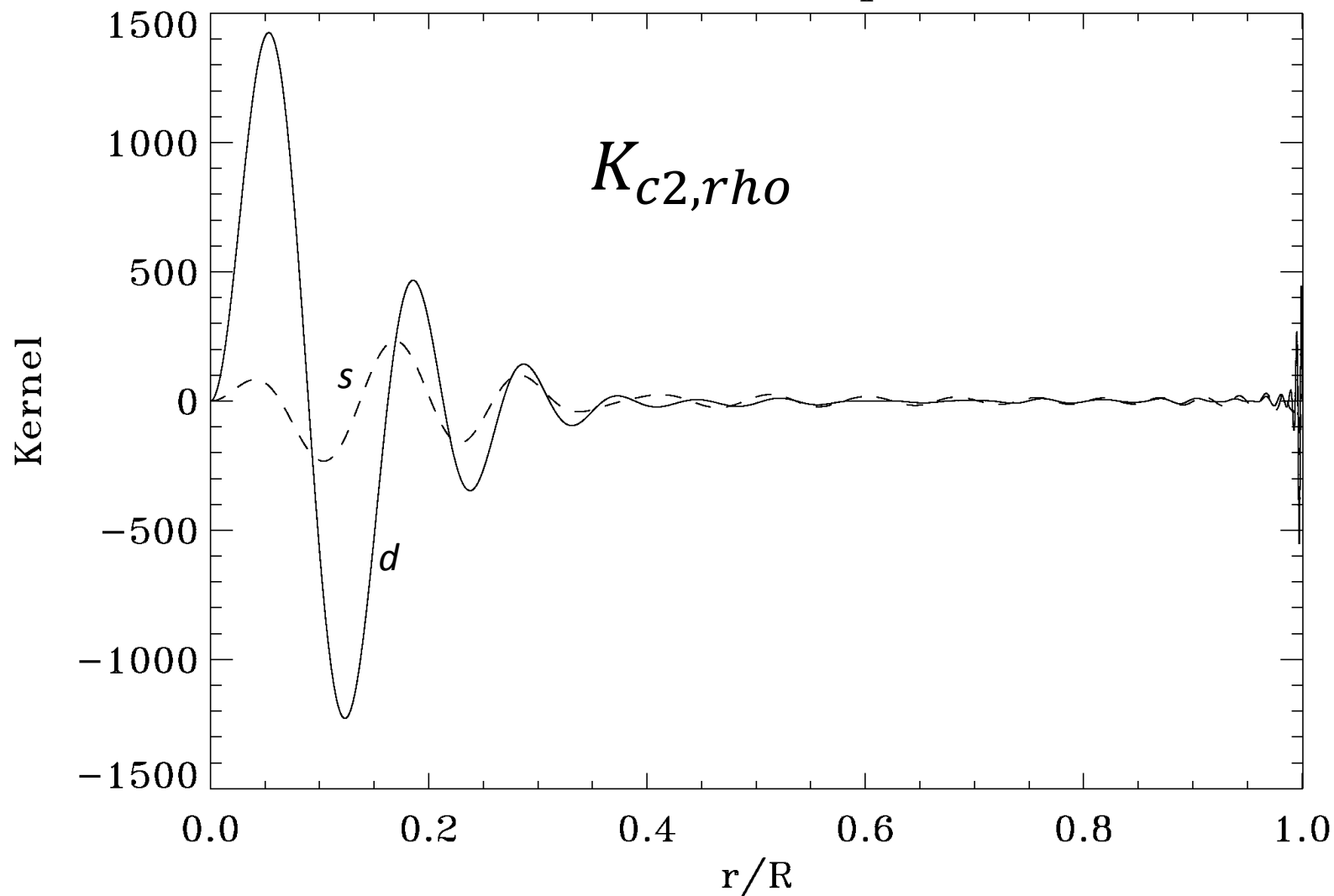


# Dependence $d_{02}$ fit on physics



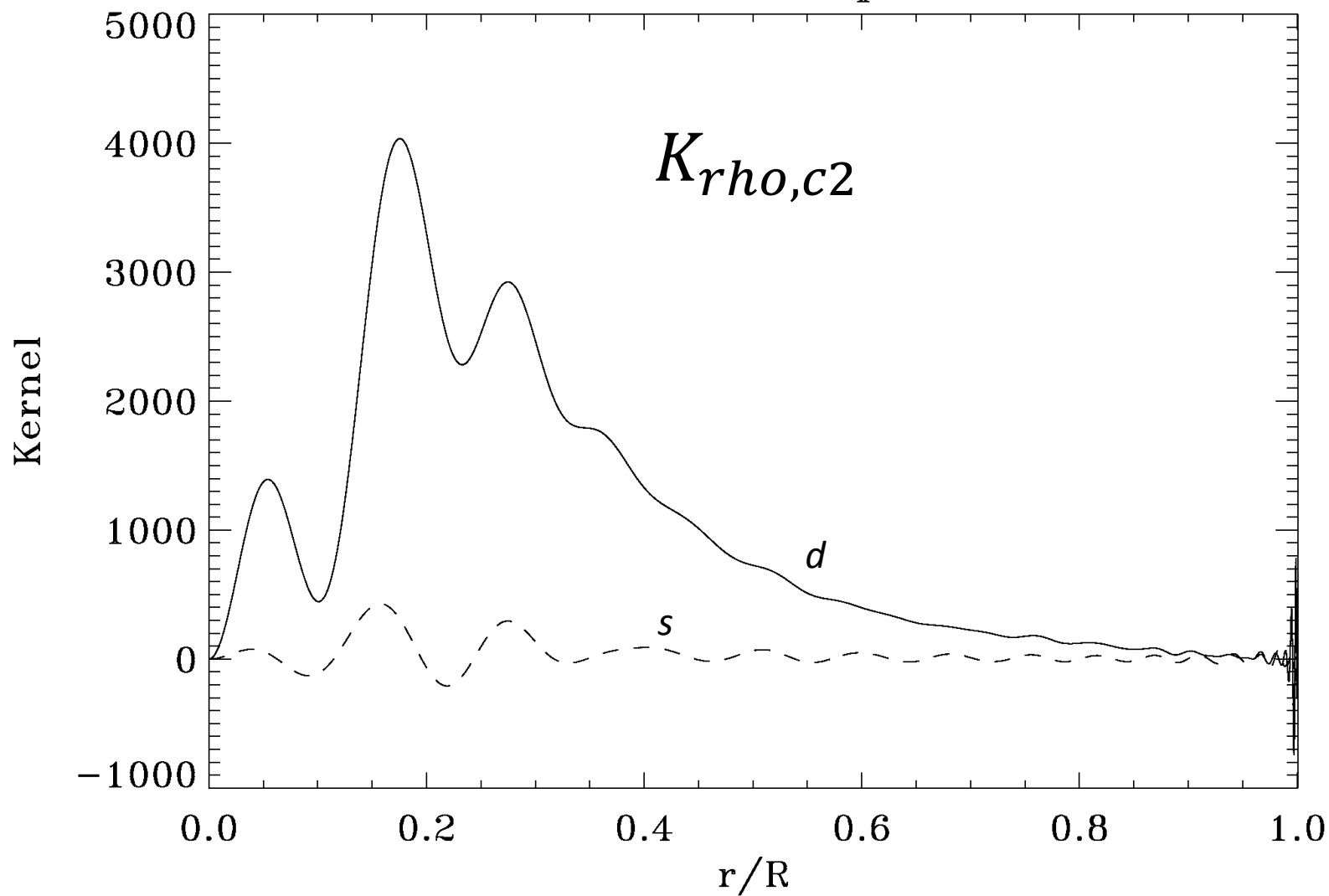
# Kernels for $d_{02}$ fit

19bi.d.02c.p2

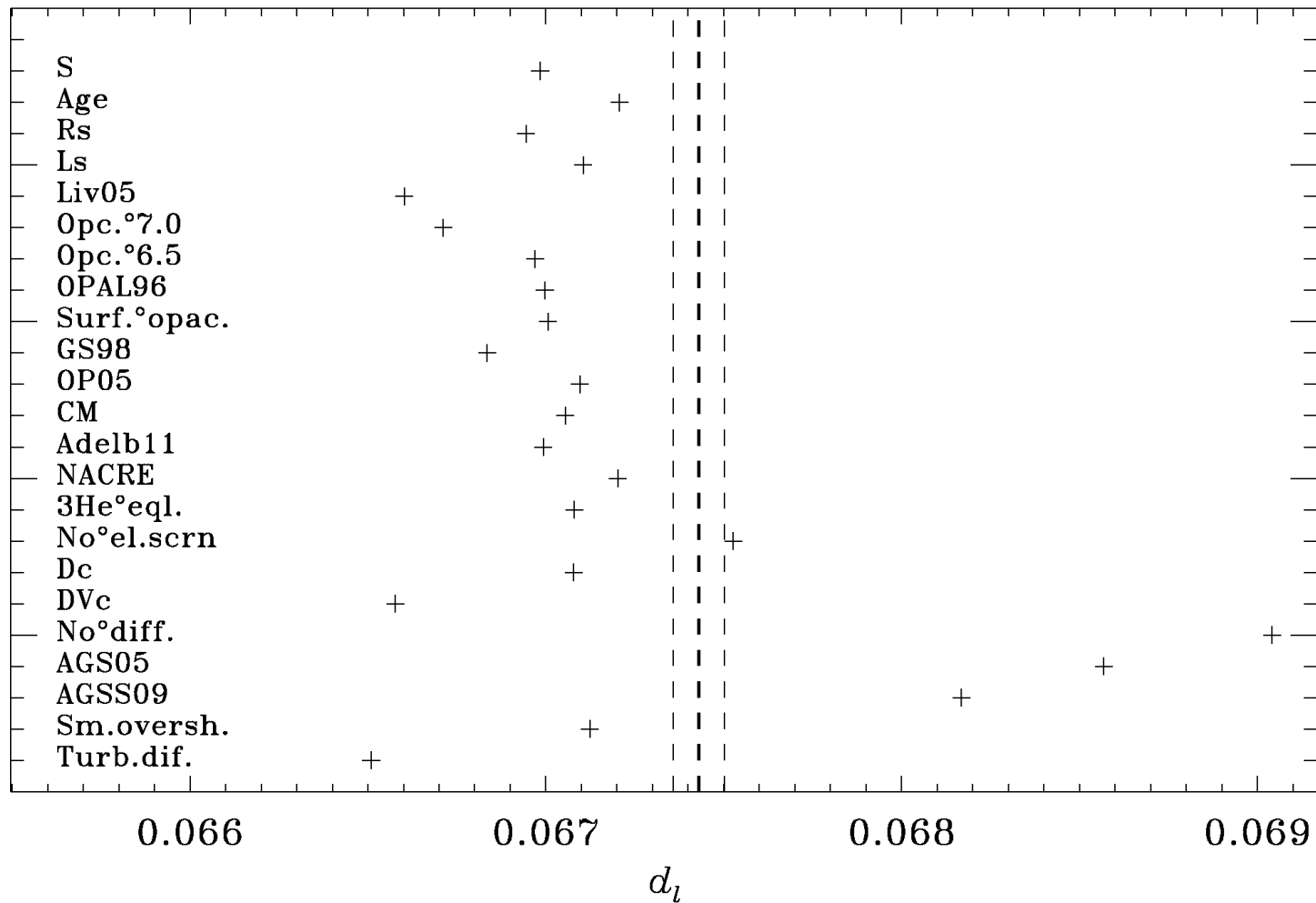


# Kernels for $d_{02}$ fit

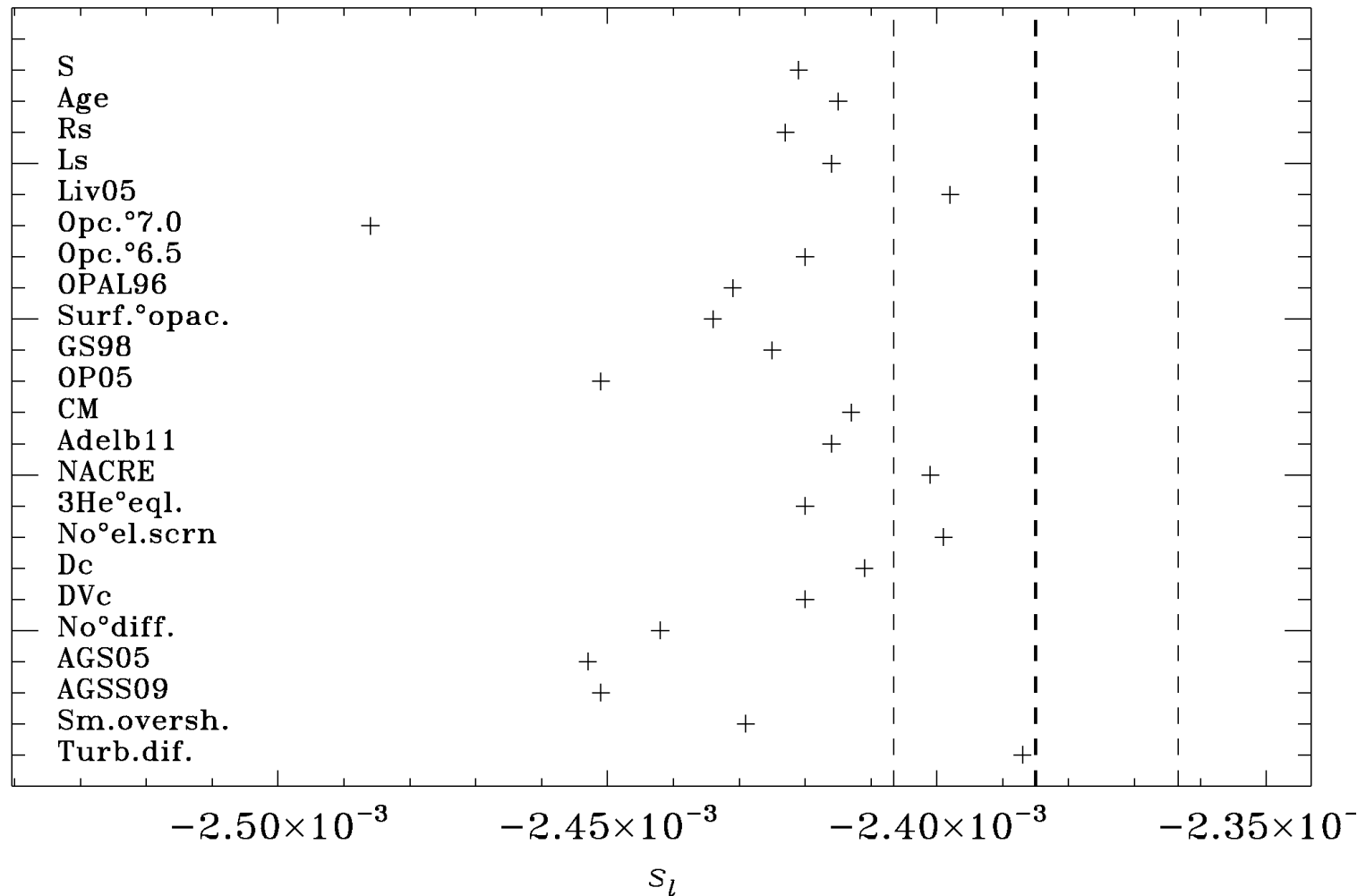
19bi.d.02c.p2



# Dependence $r_{02}$ fit on physics



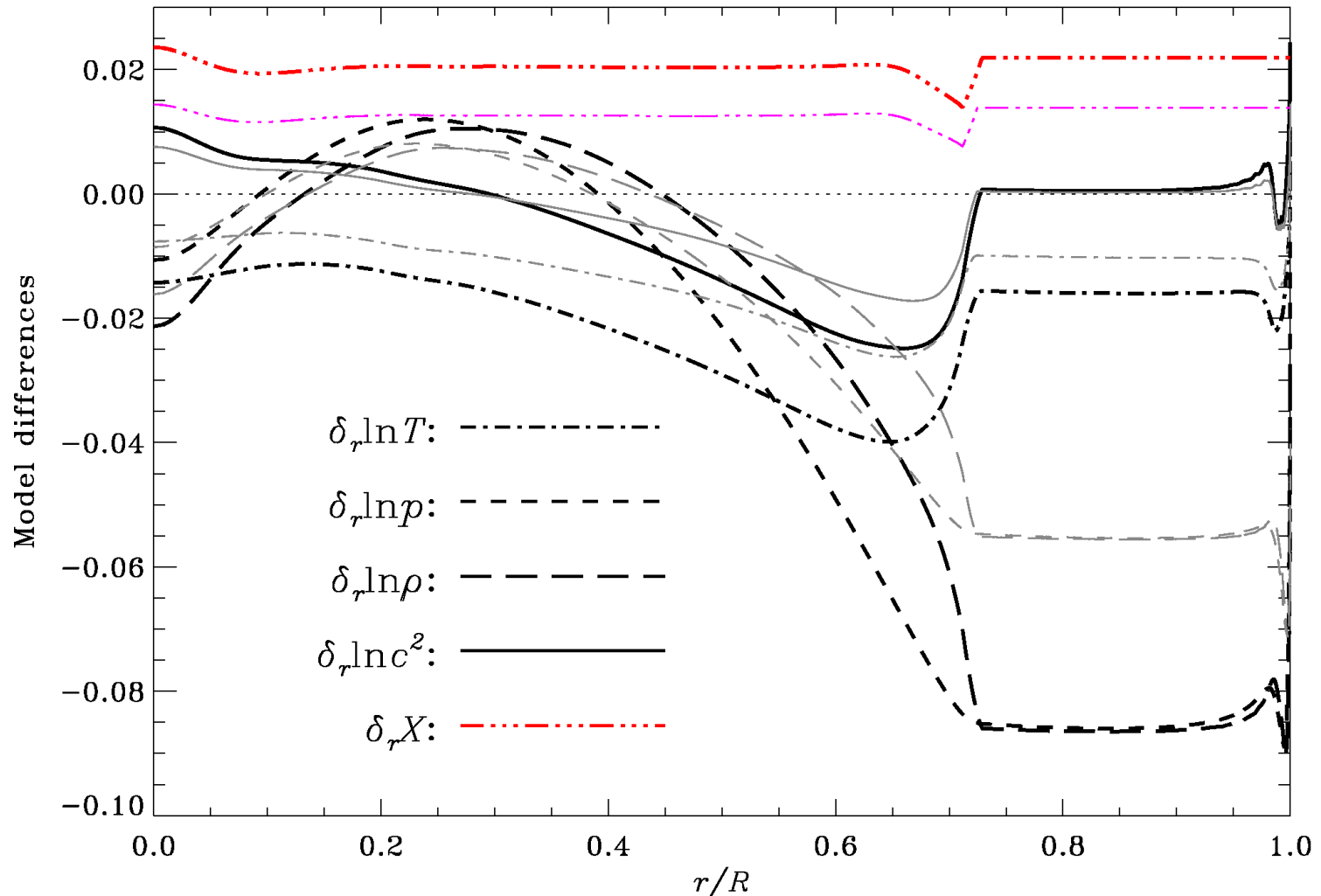
# Dependence $r_{02}$ fit on physics



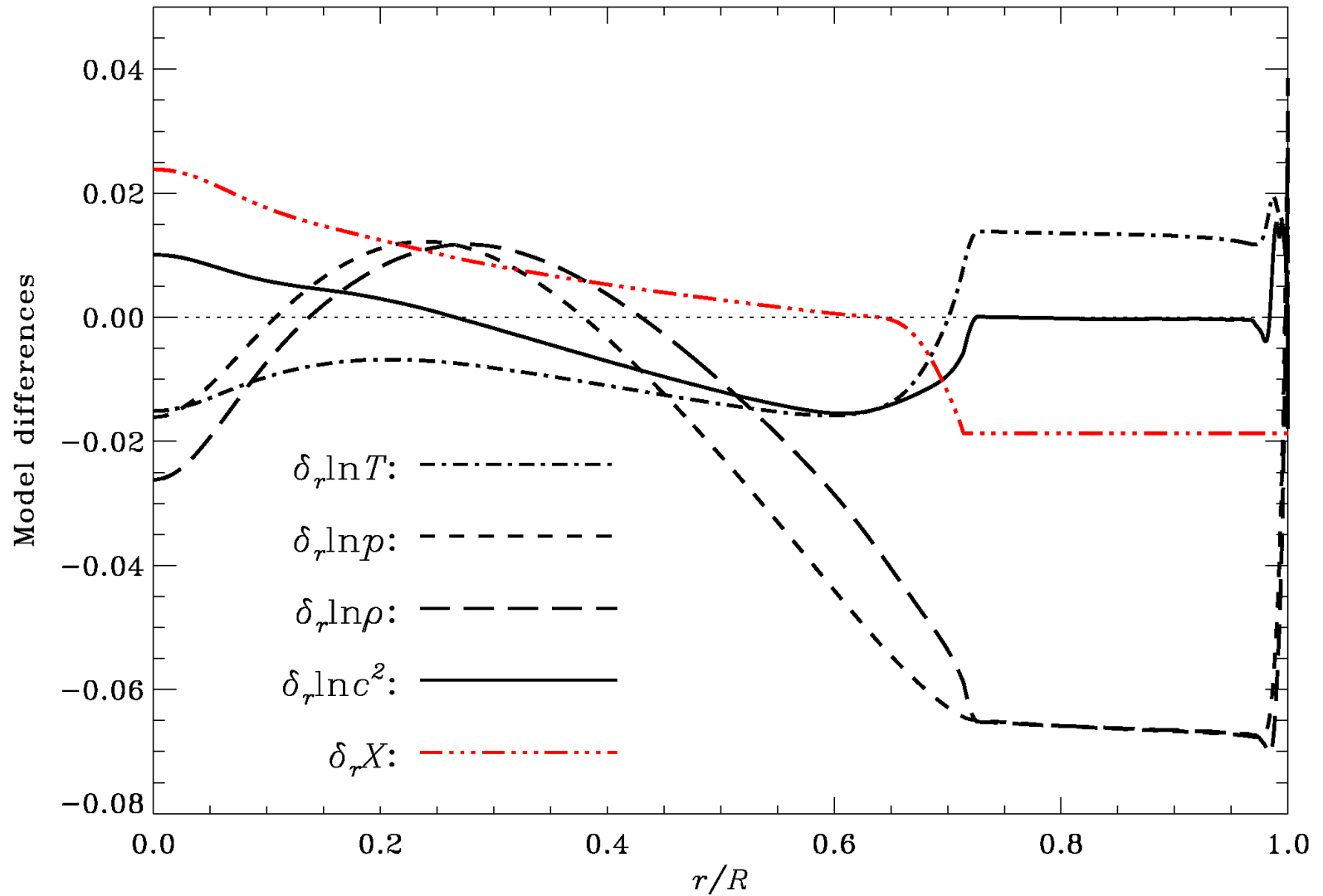


# Effect of changed composition

AGS05 and AGSS09 – GN93



# Effect of no diffusion



# g-mode asymptotics

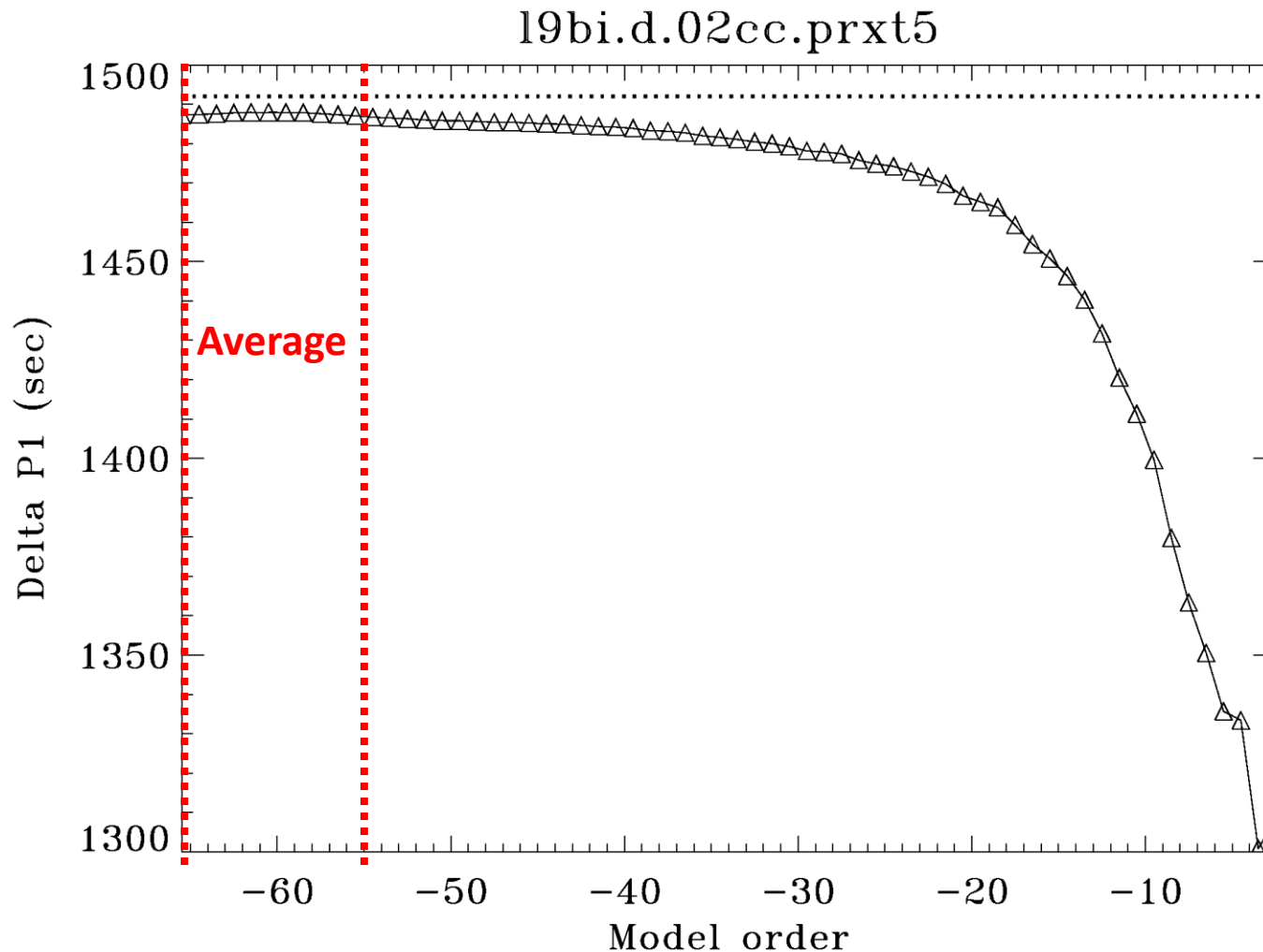
$$\Pi_{nl} = \frac{\Pi_0}{L} (|n| + \epsilon_g)$$

$$L^2 = l(l + 1)$$

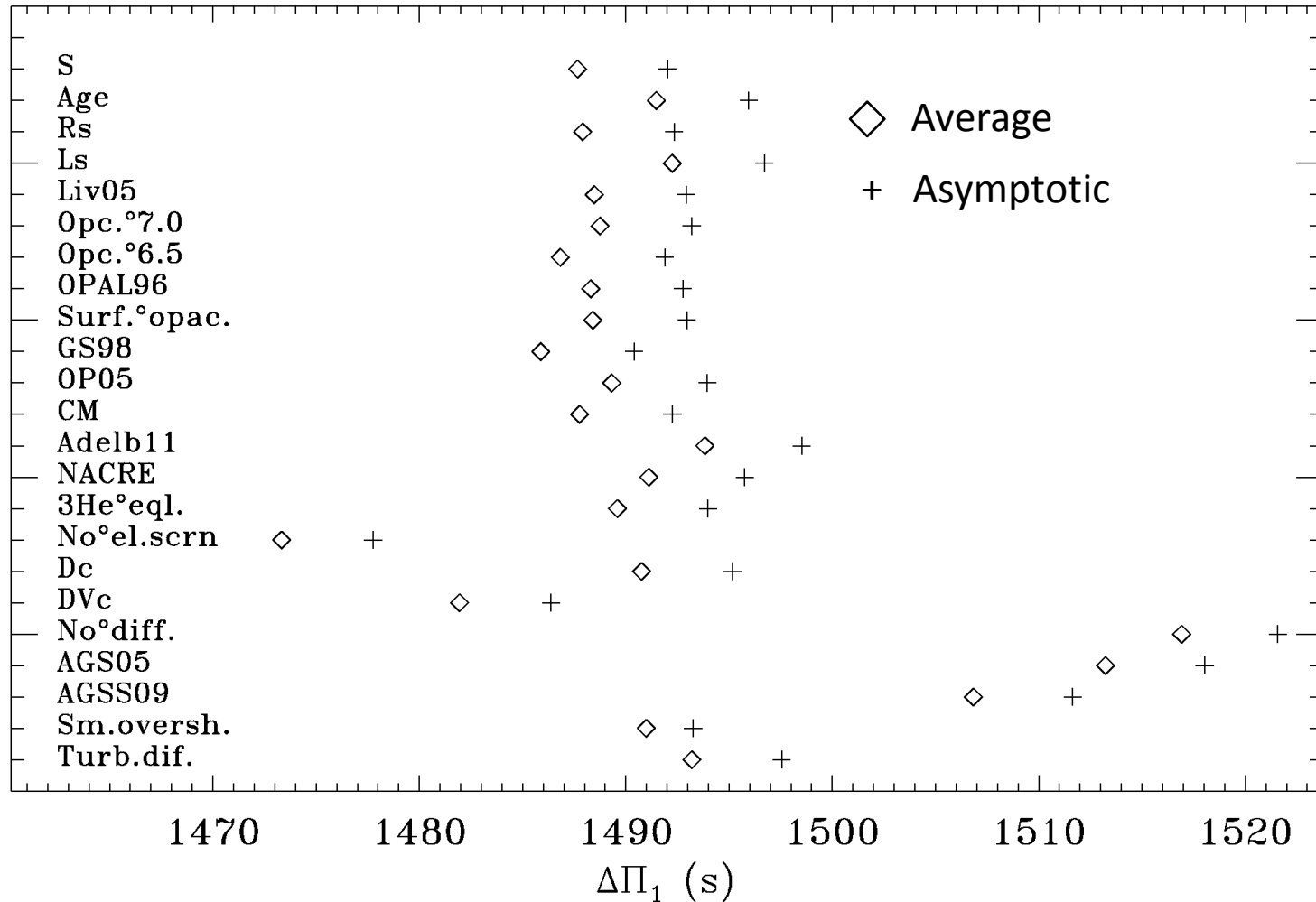
$$\Pi_0 = 2\pi^2 \left( \int N \frac{dr}{r} \right)^{-1}$$

$$N^2 = g \left( \frac{1}{\Gamma_1} \frac{d \ln p}{dr} - \frac{d \ln \rho}{dr} \right)$$

# Period spacings, $l = 1$ , Model S



# Period spacings, $l = 1$



# Origin of differences in period spacings

$$\Pi_{nl} = \frac{\Pi_0}{L} (|n| + \epsilon_g) \qquad \Pi_0 = 2\pi^2 \left( \int N \frac{dr}{r} \right)^{-1}$$

$$\Pi_0 = 2\pi^2 \mathcal{I}(r_{\text{bcz}})^{-1} \qquad \mathcal{I}(r) = \int_{r_1}^r N \frac{dr}{r}$$

$$\delta \ln \Pi_0 \simeq - \frac{\delta_r \mathcal{I}(r_{\text{bcz}})}{\mathcal{I}(r_{\text{bcz}})}$$

$$N^2 = \frac{g^2 \rho}{p} \left( \frac{1}{\Gamma} - \frac{1}{\Gamma_1} \right)$$

$$\frac{1}{\Gamma} = \frac{d \ln \rho}{d \ln p} .$$

# Origin of differences in period spacings

$$N^2 = \mathcal{B}\mathcal{G}$$

$$\mathcal{B} = \frac{g^2 \rho}{p} \ , \quad \mathcal{G} = \left( \frac{1}{\Gamma} - \frac{1}{\Gamma_1} \right)$$

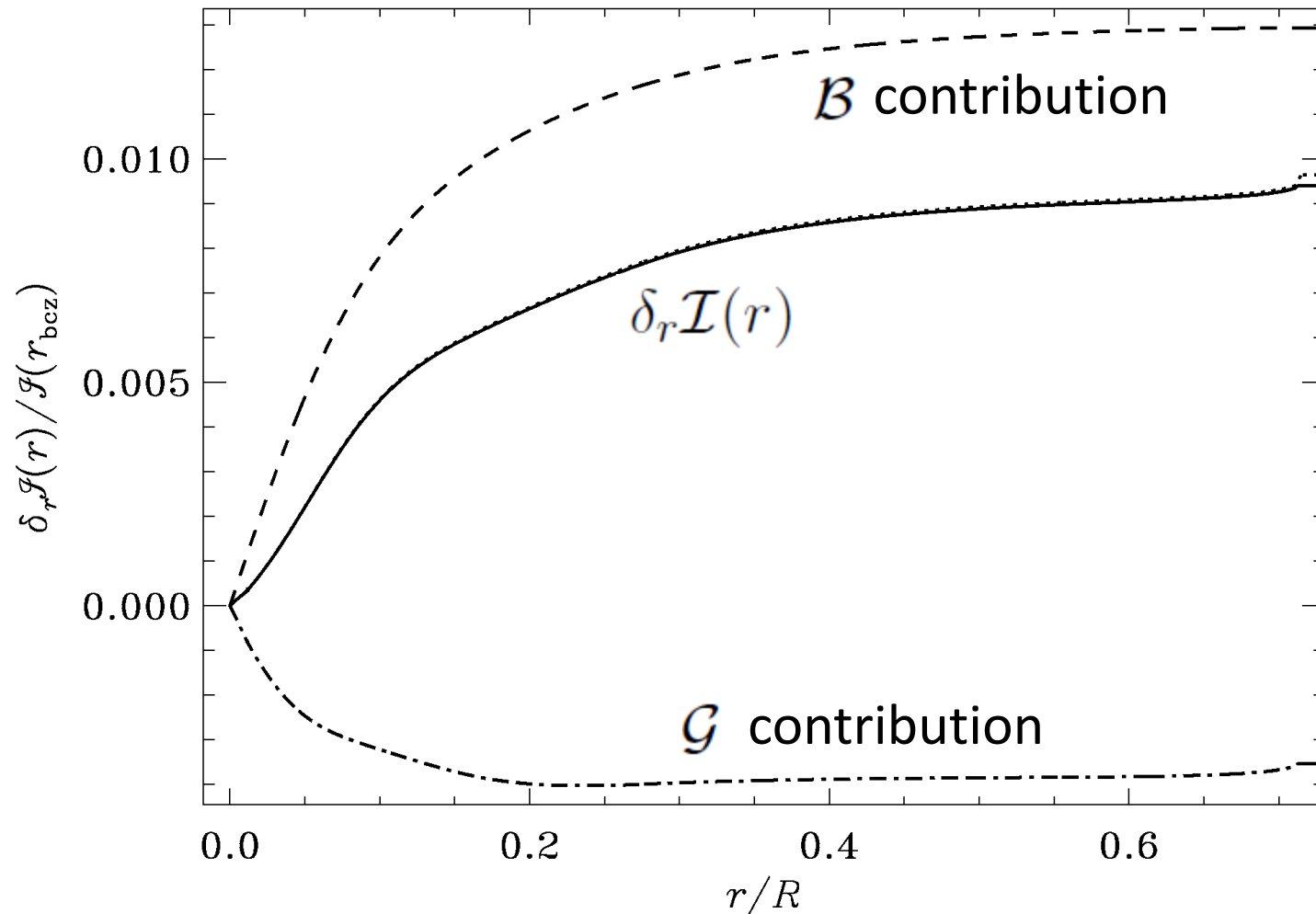
$$\mathcal{G} = \delta \mathcal{D} + \phi \nabla_\mu \ , \quad \mathcal{D} = \nabla_{\text{ad}} - \nabla$$

$$\delta = - \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_{p, \mu} \ , \quad \phi = \left( \frac{\partial \ln \rho}{\partial \ln \mu} \right)_{p, T}$$

$$\delta \ln \Pi_0 \simeq - \frac{\delta_r \mathcal{I}(r_{\text{bcz}})}{\mathcal{I}(r_{\text{bcz}})}$$

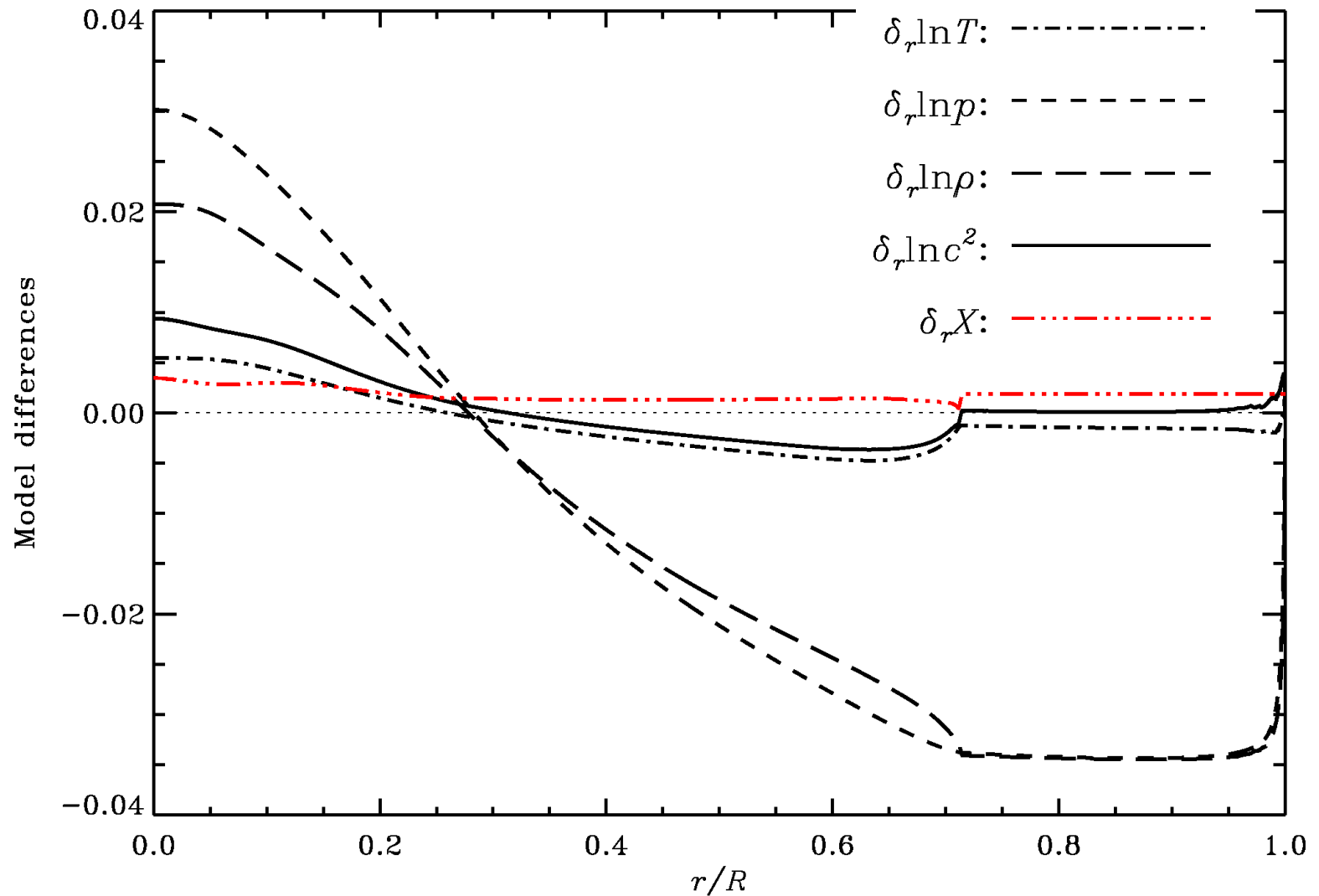
$$\delta_r \mathcal{I}(r) \simeq \frac{1}{2} \int_0^r N(\delta_r \ln \mathcal{B} + \delta_r \ln \mathcal{G}) \frac{dr}{r} \ ,$$

# Effect of no electron screening



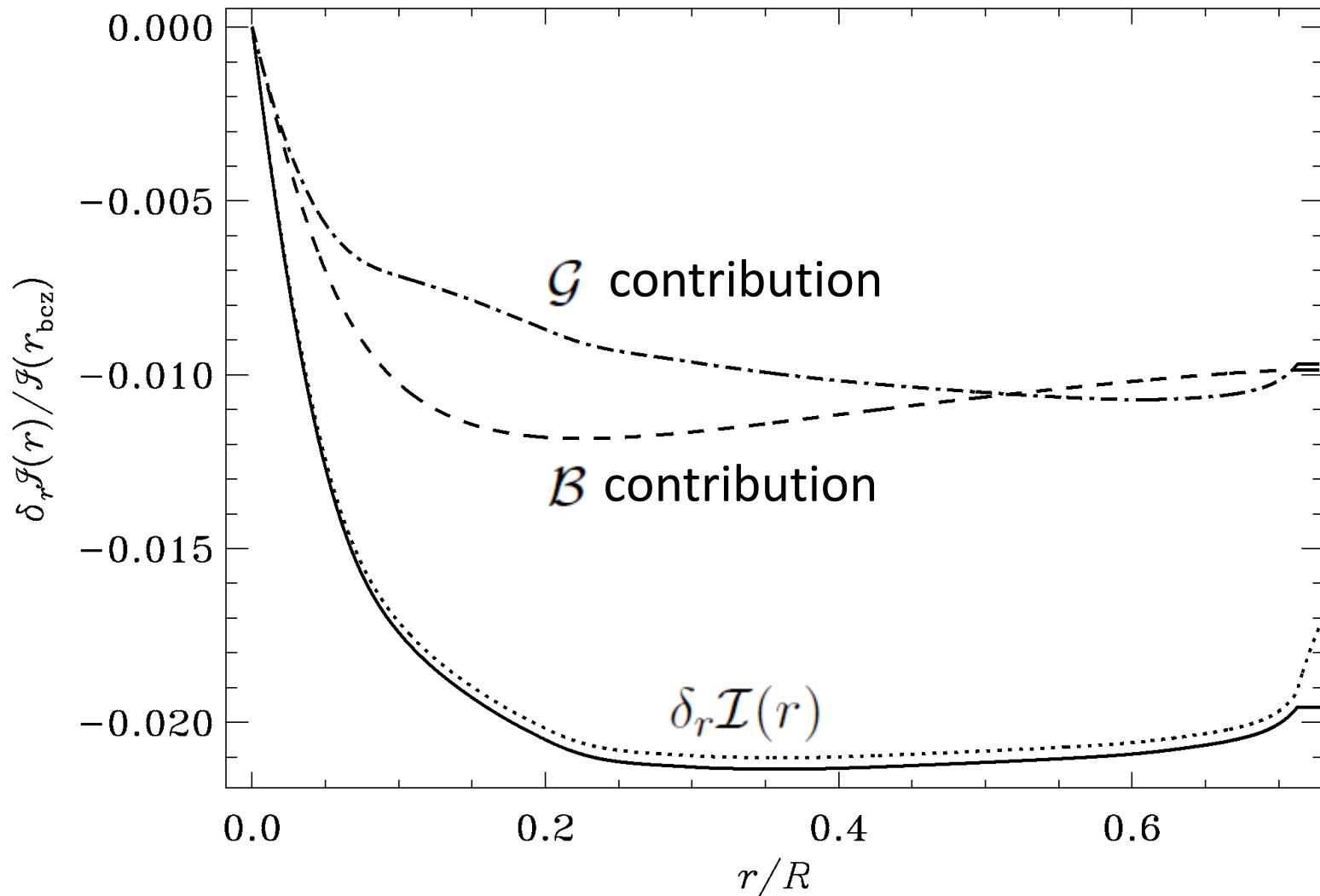


# Effect of no electron screening



# Effect of changed composition

AGS05– GN93



# Effect of changed composition

AGS05 and AGSS09 – GN93

