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Hilbert–Huang Transform analysis of solar quasi-periodicities

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Hilbert–Huang Transform (HHT)

Empirical Mode Decomposition (EMD)+Hilbert transform

Skeleton of the method:

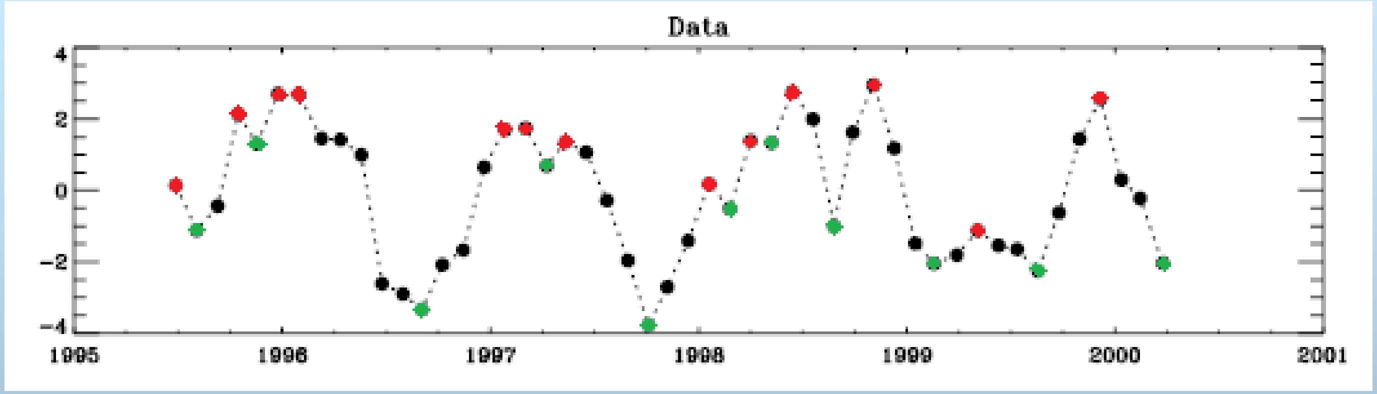
- Decomposition of the initial data set into a number of intrinsic modes by **EMD**

- Subsequent **Hilbert transformation** of the identified modes

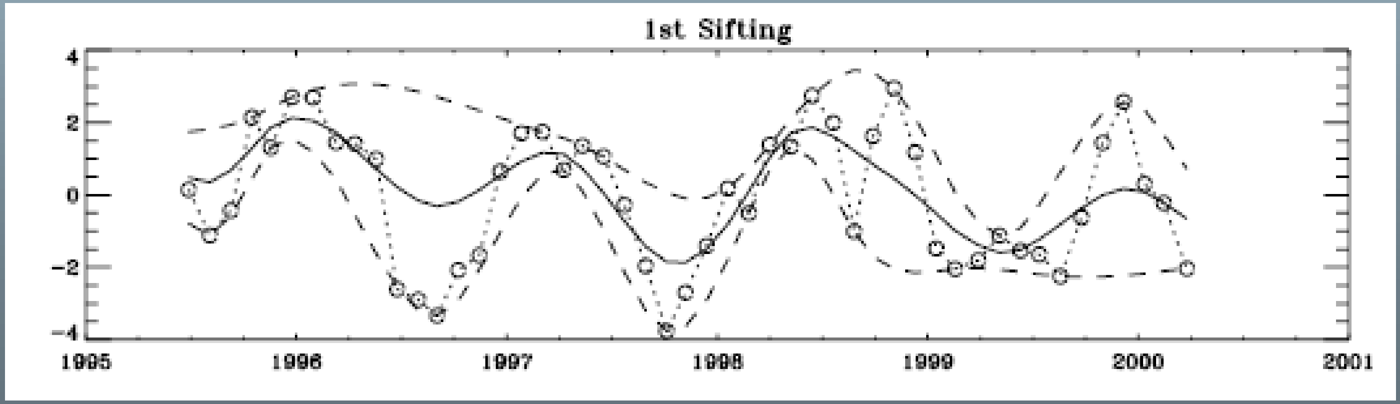
Huang, N. E. et al. 1998, Royal Soc. of London Proc. Series A, 454, 903

Empirical Mode Decomposition (EMD)

1. Identifying of all local extrema of the original data *X(t)*

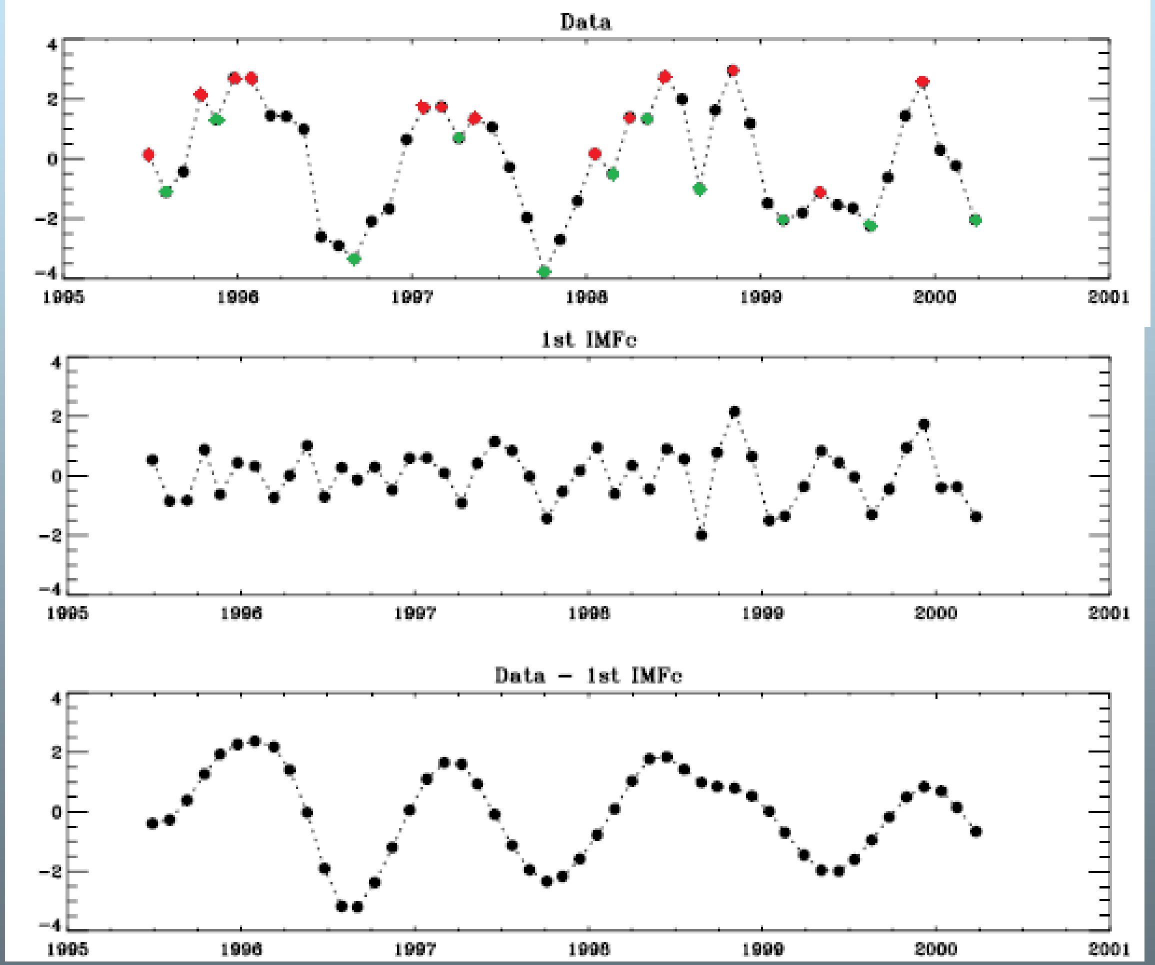


2. Define upper/lower envelopes connecting all maxima/minima and their mean m_1. Then $h_1(t) = X(t) - m_1(t).$



Empirical Mode Decomposition (EMD)

Sufficient number of repetitions allows the residue to become the first intrinsic mode function (IMF):



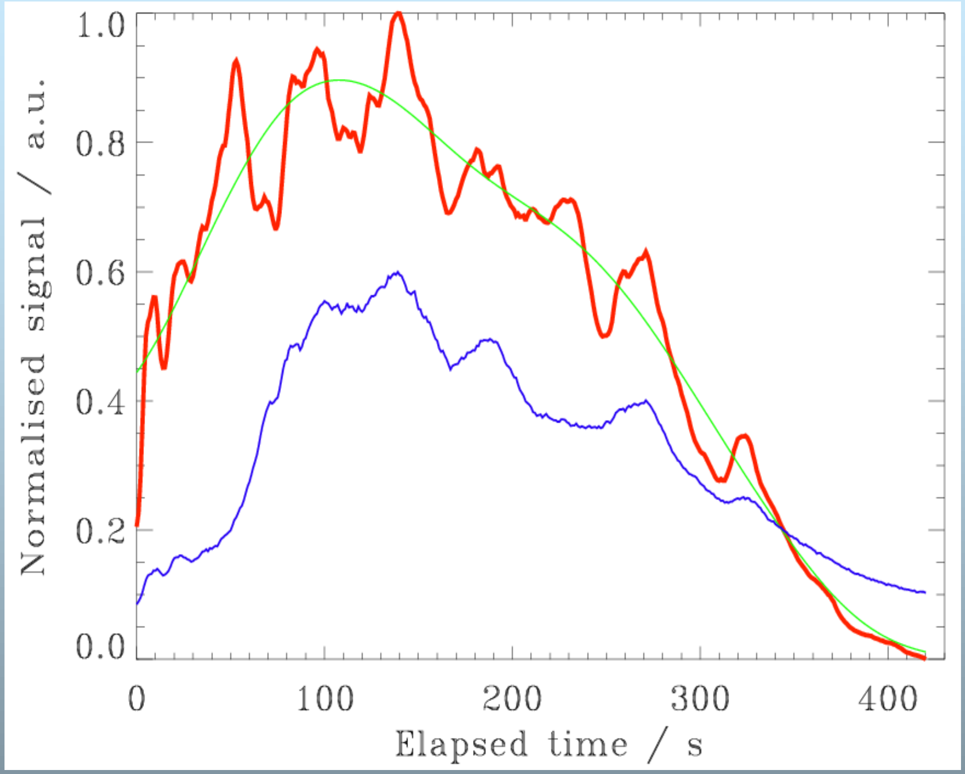
Original data

The first IMF

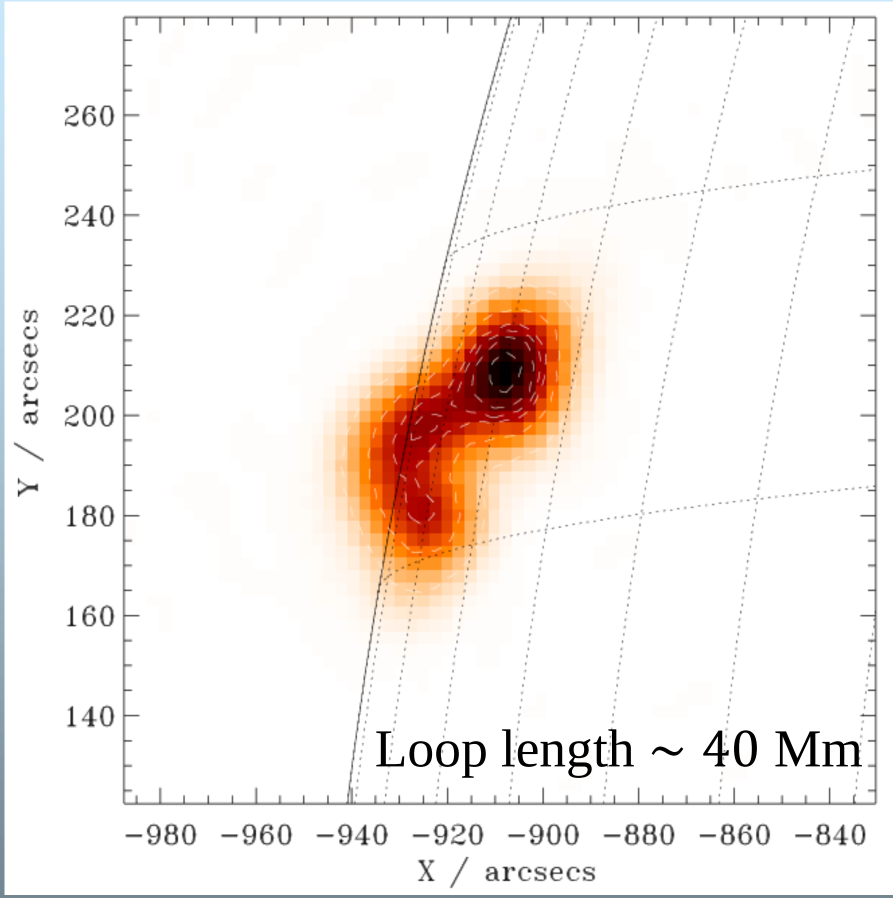
Data with the first IMF subtracted

Example of the EMD expansion

Solar flare on 14 May 2013, *Kolotkov et al. (2015)*



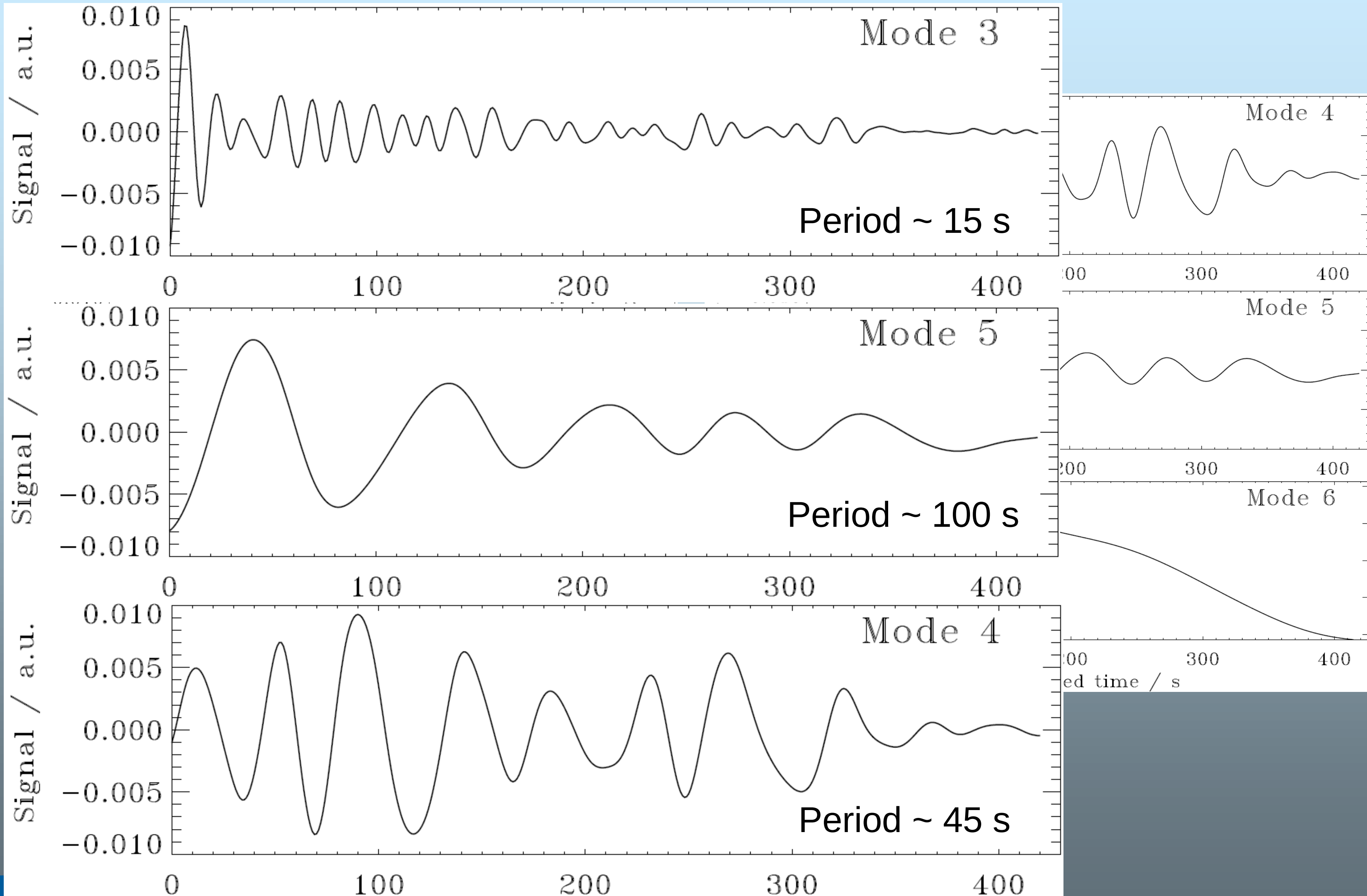
Correlation curve (red) and R+L flux (blue) at 17 GHz detected in solar flare 14 May 2013, with NoRH and NoRP, respectively



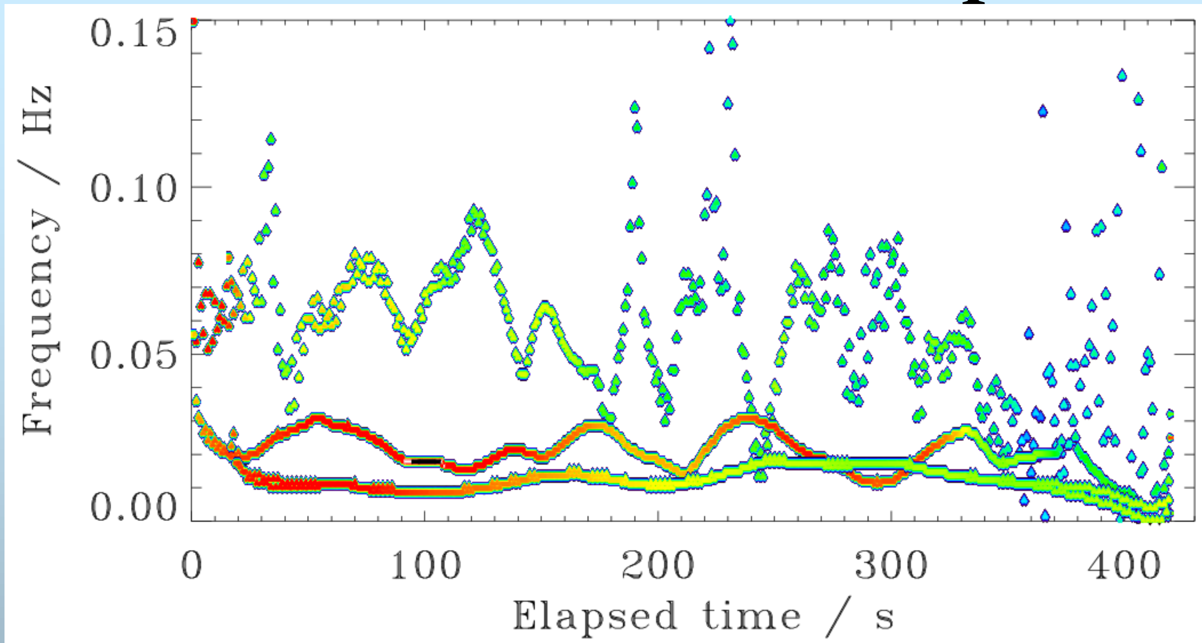
Radio map at 17 GHz, 14 May 2013, NoRH

Example of the EMD expansion

Set of intrinsic empirical modes in the correlation curve



Hilbert spectrum



Mode 3 – upper line

Mode 4 – middle line

Mode 5 – bottom line

$$Z(t) = X(t) + \mathrm{i}Y(t) = a(t)\mathrm{e}^{\mathrm{i}\theta(t)}$$

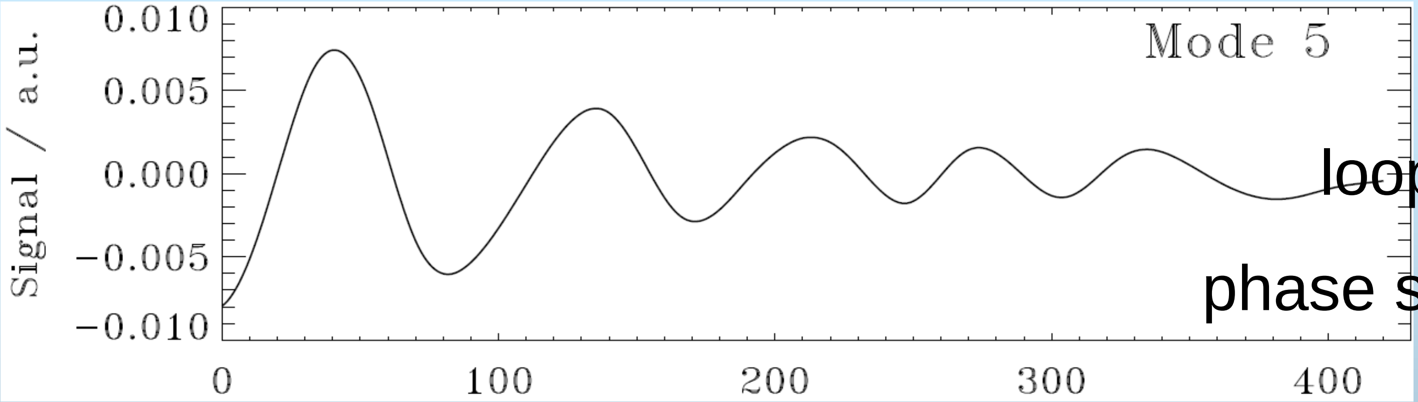
$$a(t) = [X^2(t) + Y^2(t)]^{1/2},$$

$$Y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{X(t')}{t - t'}\, \mathrm{d}t'$$

$$\omega = \frac{\mathrm{d}\theta(t)}{\mathrm{d}t}.$$

Hilbert transformation is used to obtain the frequency-power-time distribution for each IMF, designated as the **Hilbert spectrum**

Origination of mode 5

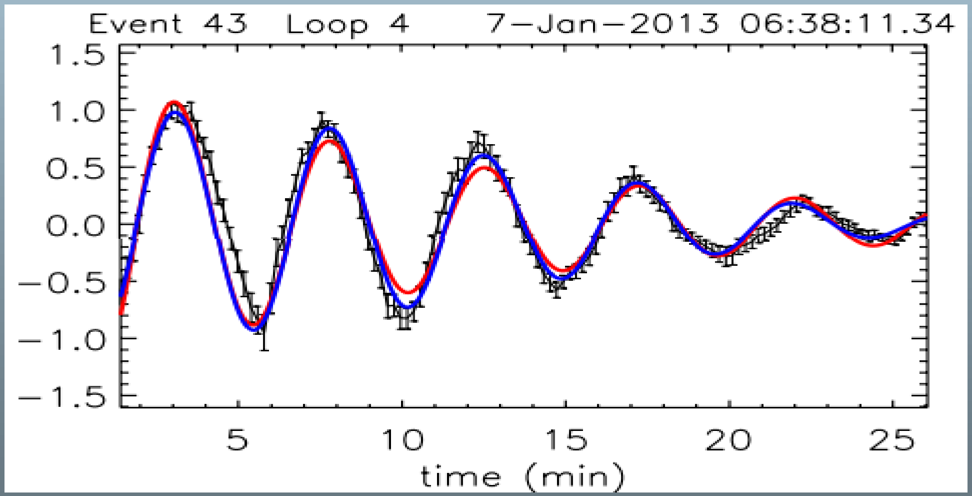


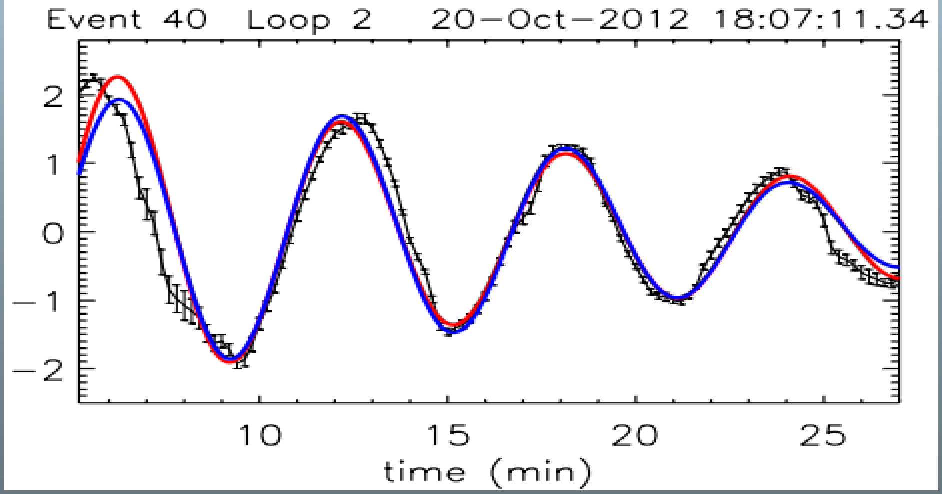
period ~ 100 s

loop length ~ 40 Mm

phase speed ~ 800 km/s

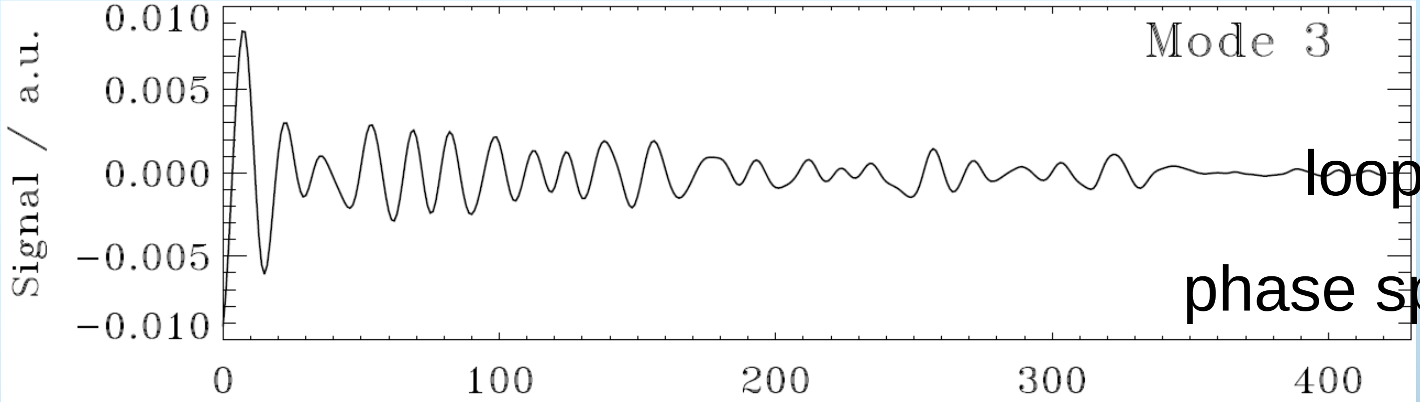
Kink modes: typical speed ~ 1000 km/s, periods ~ a few minutes





Pascoe et al. (2016)

Origination of mode 3

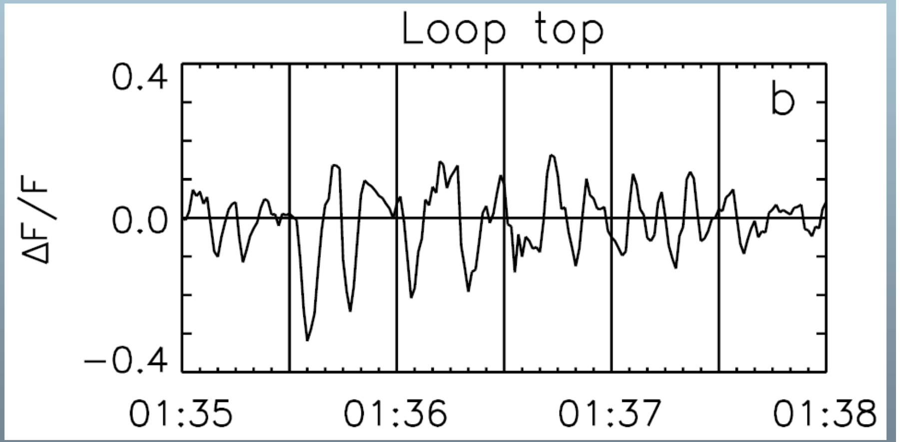


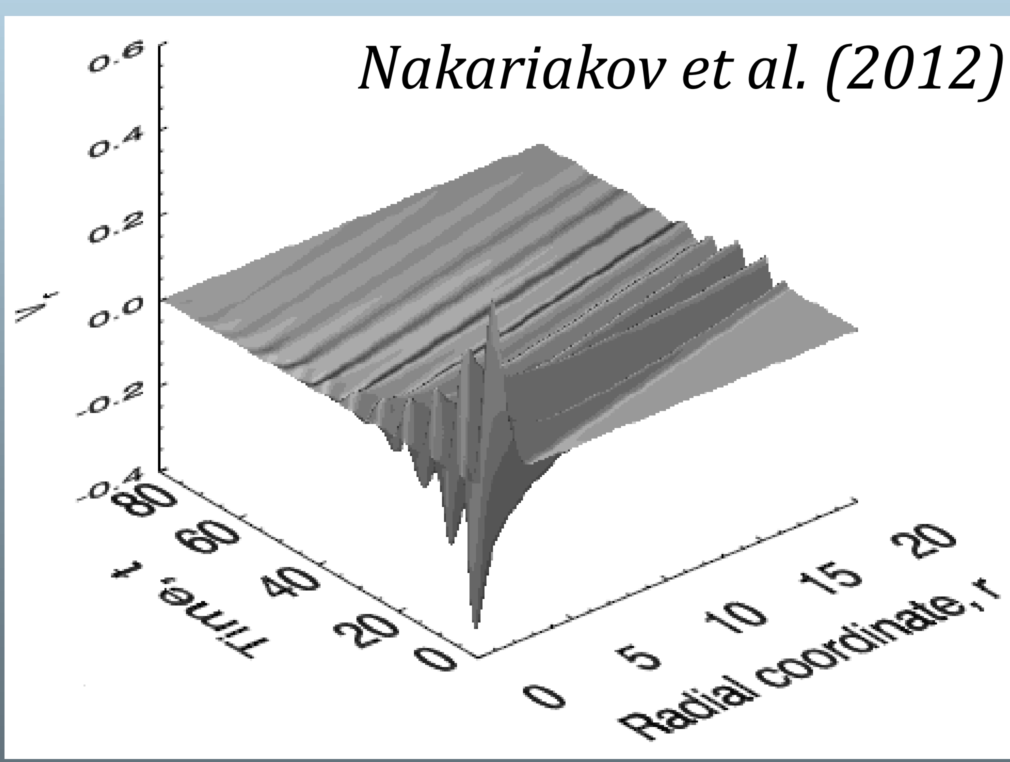
period ~ 15 s

loop length ~ 40 Mm

phase speed ~ 5.3 Mm/s

Sausage mode:





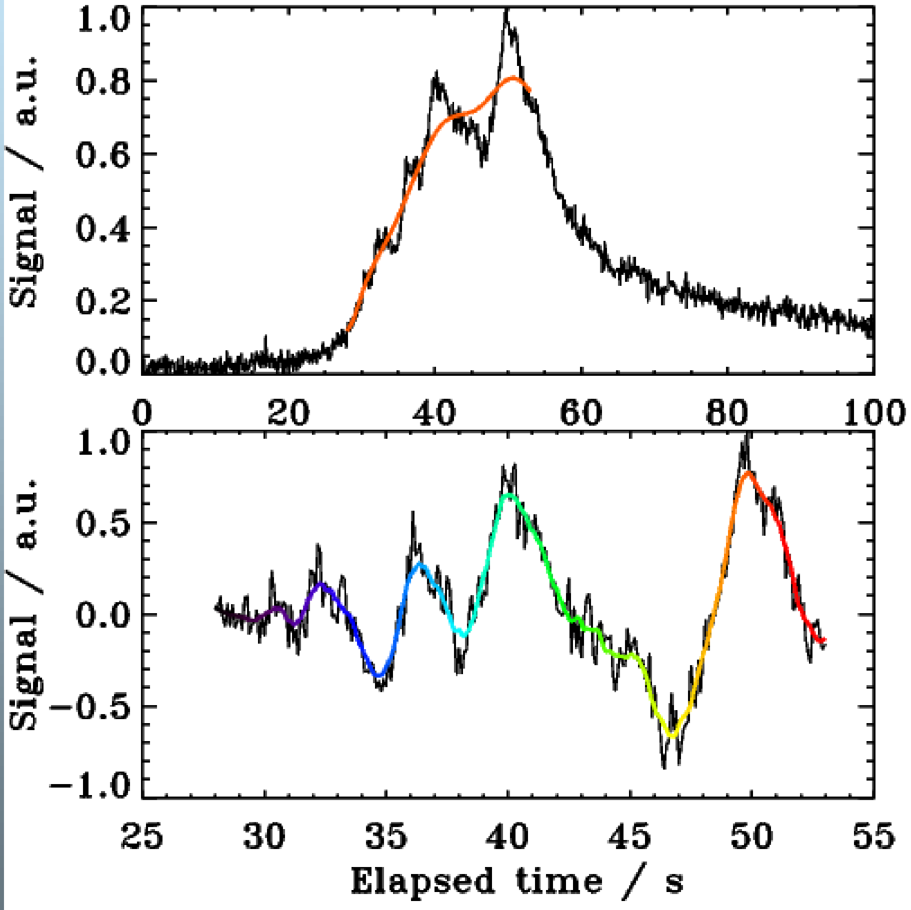
Nakariakov et al. (2012)

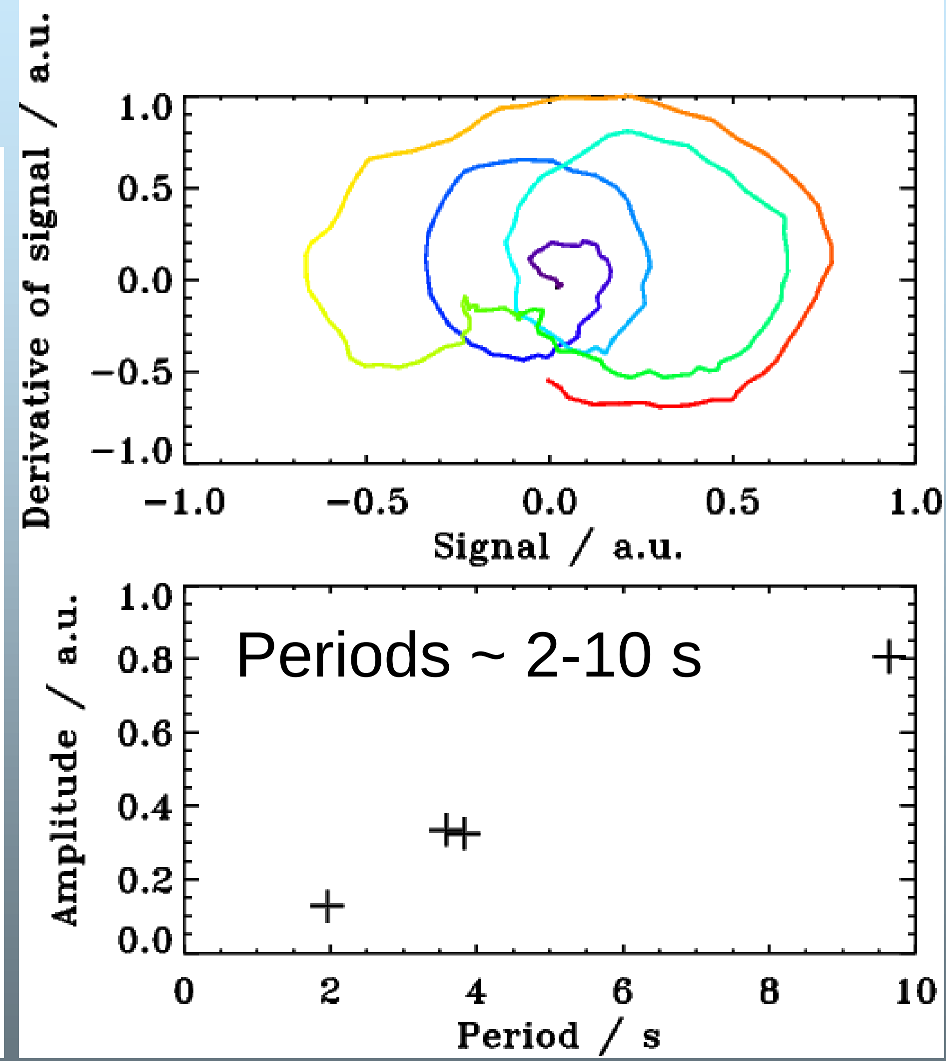
Period ~ 16 s,

Melnikov et al. (2005)

A few more examples on the detection of underlying trends with EMD…

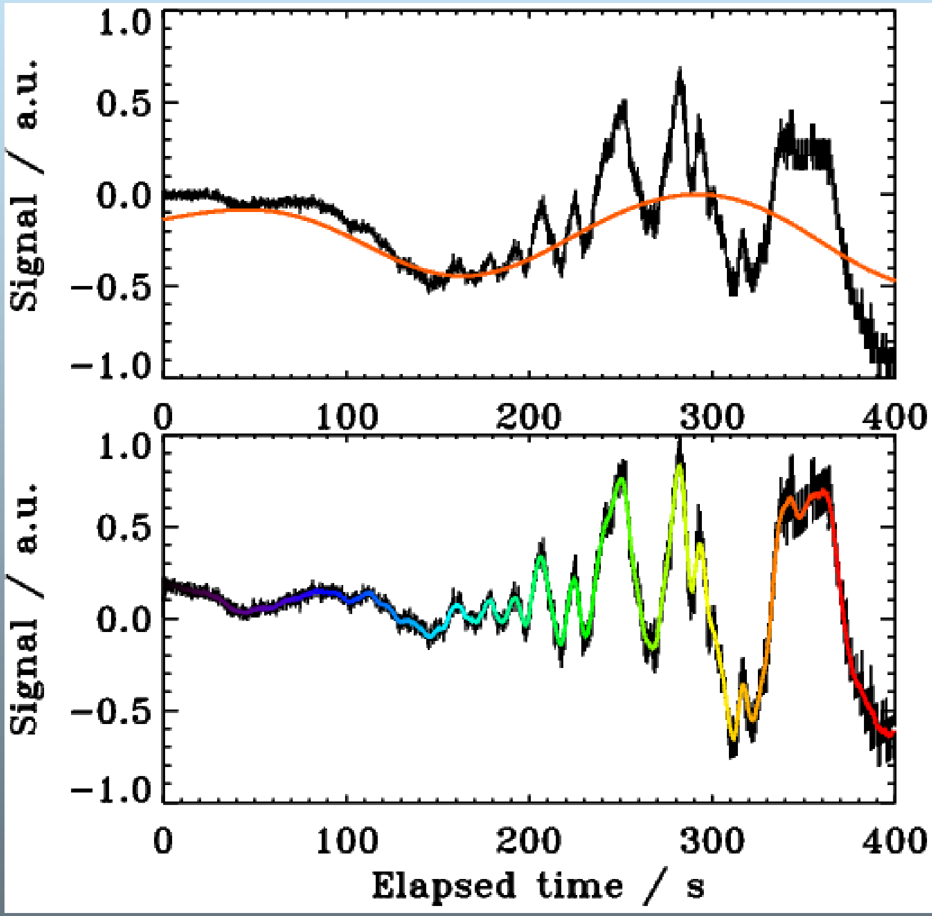
Solar flare on 04 Feb 2014,
R+L flux at 17 GHz, NoRP

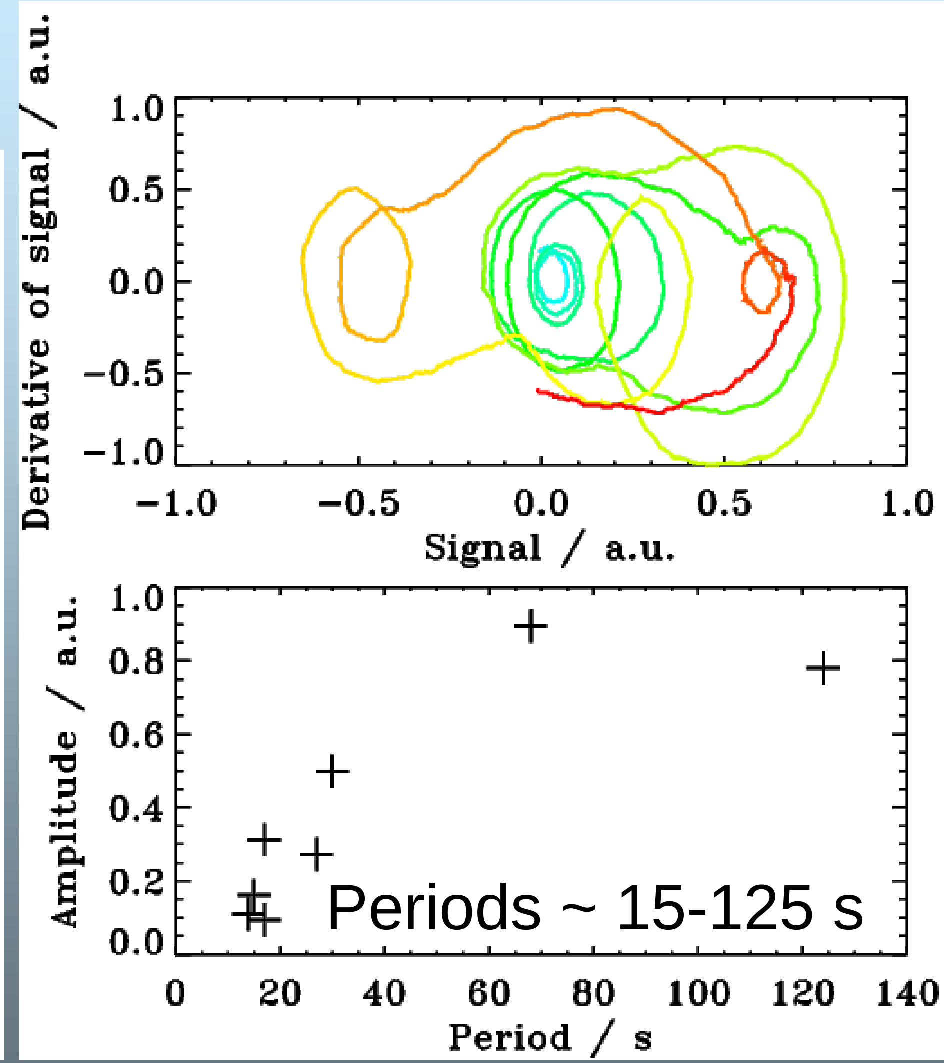




A few more examples on the detection of underlying trends with EMD...

Solar flare on 25 Feb 2014,
R-L flux at 17 GHz, NoRP





Significance of EMD modes in comparison with coloured noises of $S \propto f^{-\alpha}$ (*Kolotkov et al. 2016*)

- Check the noise colour, i.e. the power law index α, in the original data set:

$$E_m P_m^{1-\alpha} = const,$$

where $E_m = \sum X_i^2$ is the total energy of the mth mode, and P_m is its main period.

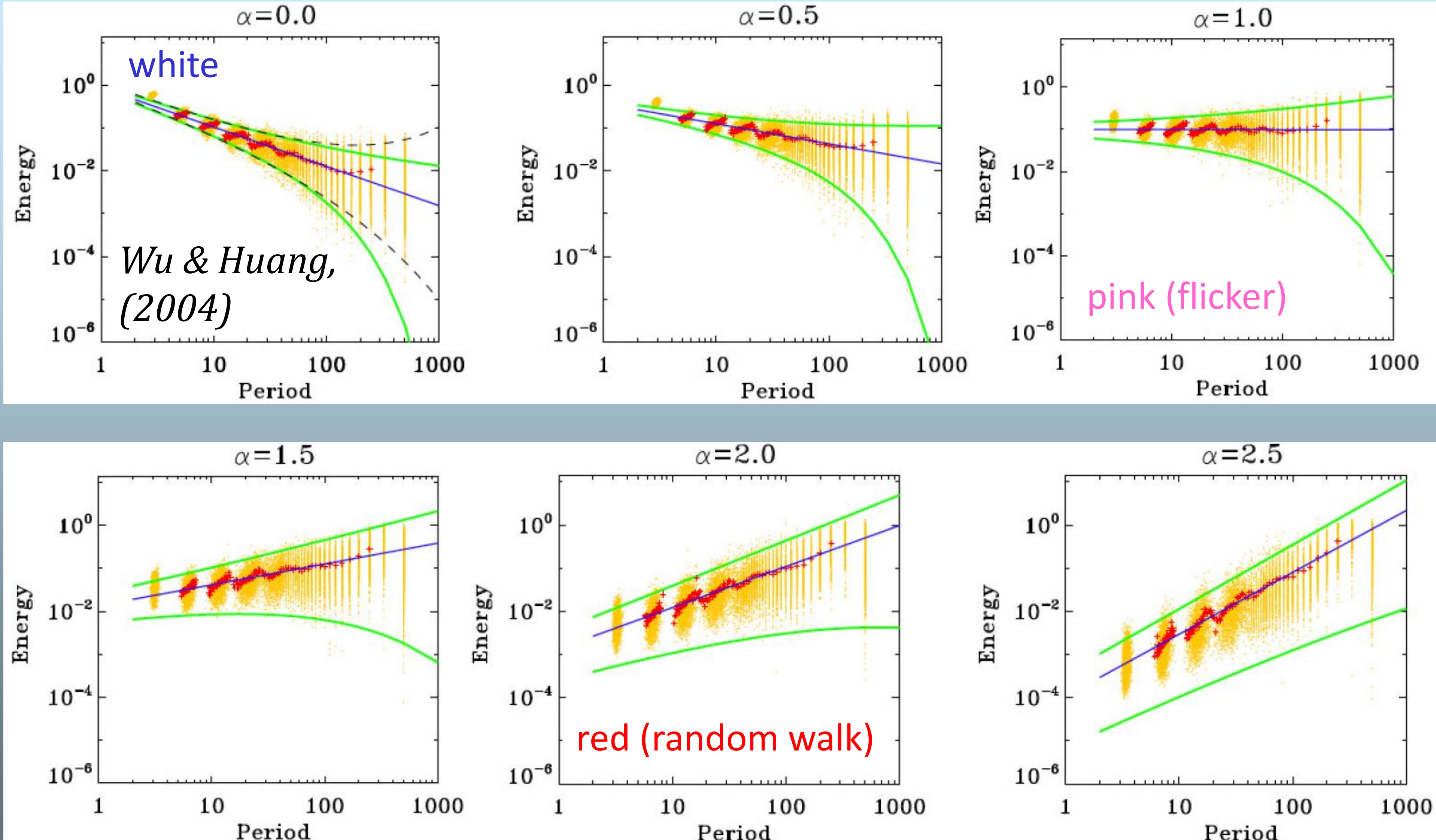
- For the chosen sort of noise, calculation of the confidence intervals using the chi-squared distribution:

$$f(E_m) = \frac{k_m}{\bar{E}_m} \chi^2 \left(\frac{k_m E_m}{\bar{E}_m}, k_m \right),$$

where $\bar{E}_m(\alpha)$ is the average total energy of the mth mode, and $k_m(\alpha)$ is the number of degrees of freedom.

EMD analysis of synthetic coloured noises

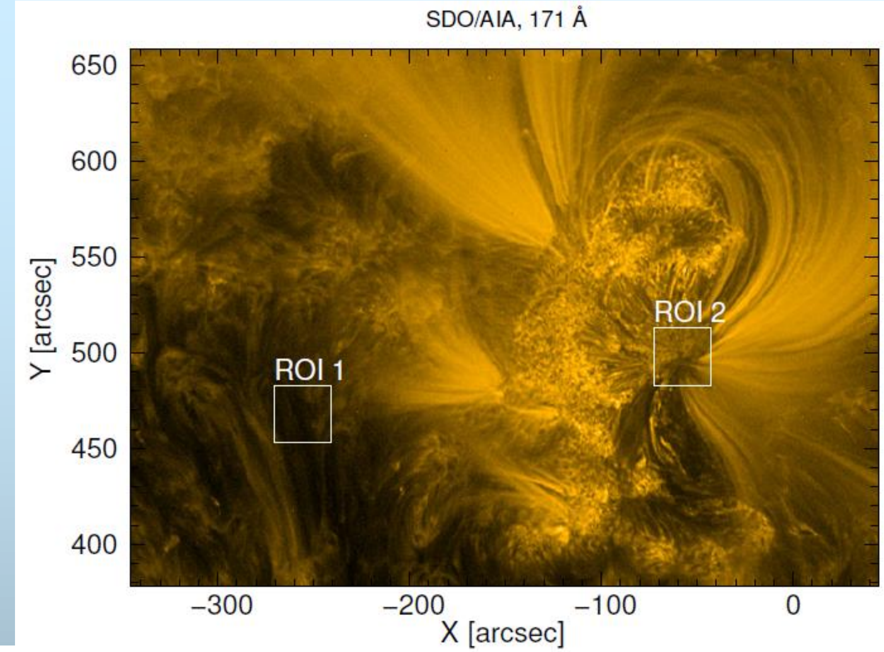
(*Kolotkov et al. 2016*)

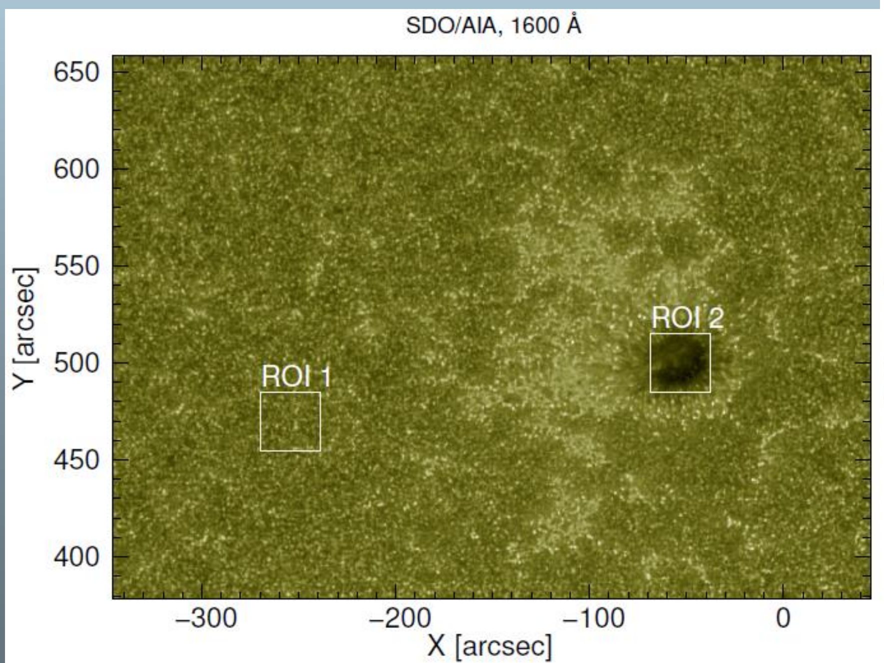


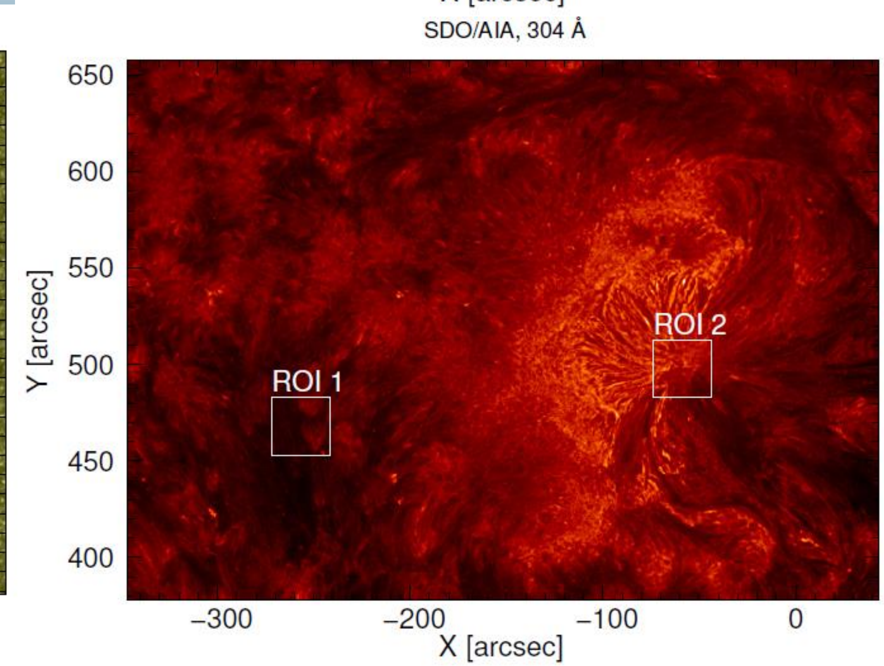
8 Dec 2010 (NOAA 11131)

ROI 1 – quiet sun region

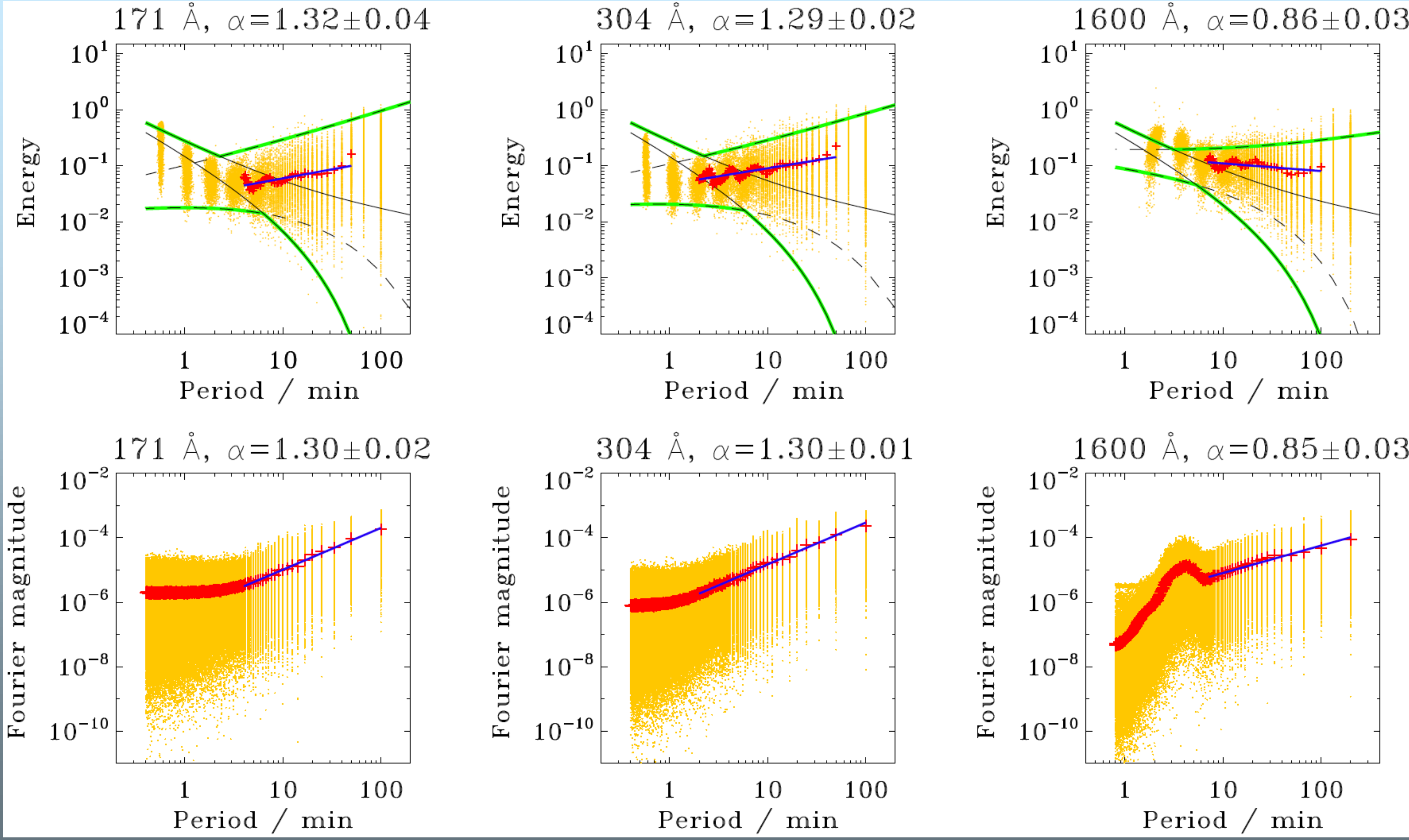
ROI 2 – sunspot umbra



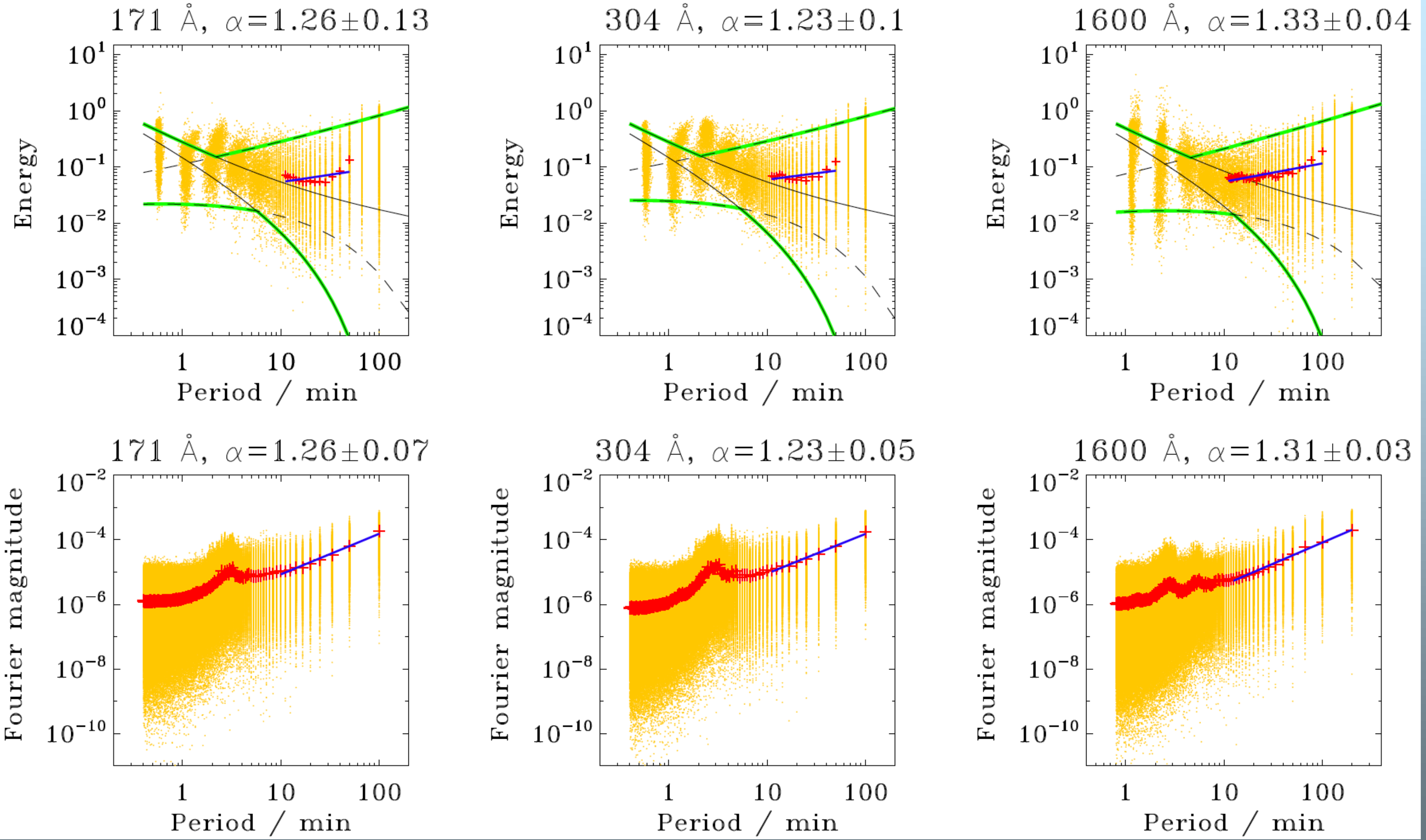




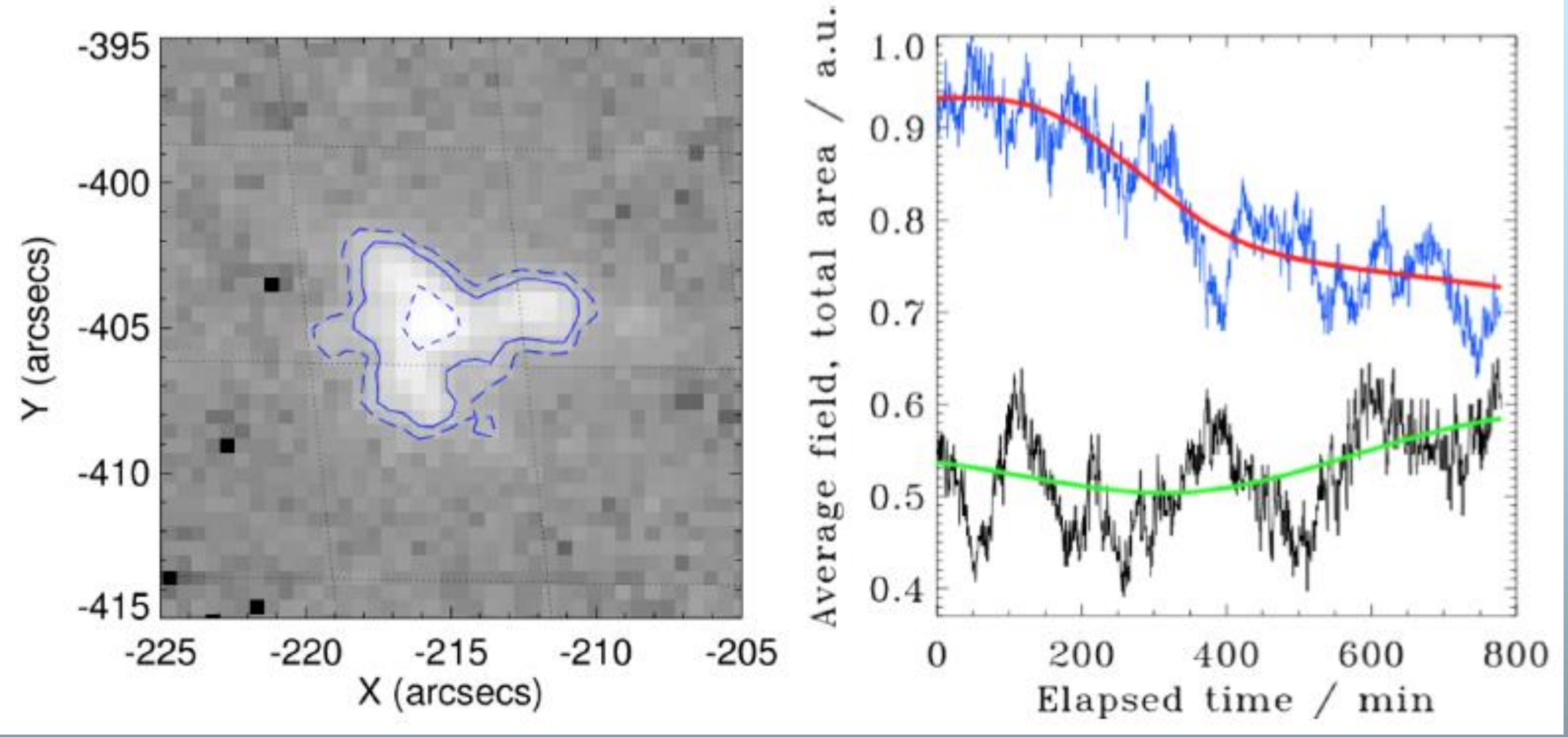
Quiet sun region



Sunspot umbra



Quasi-periodic oscillations of a small-scale magnetic structure (*Kolotkov et al. 2017*)

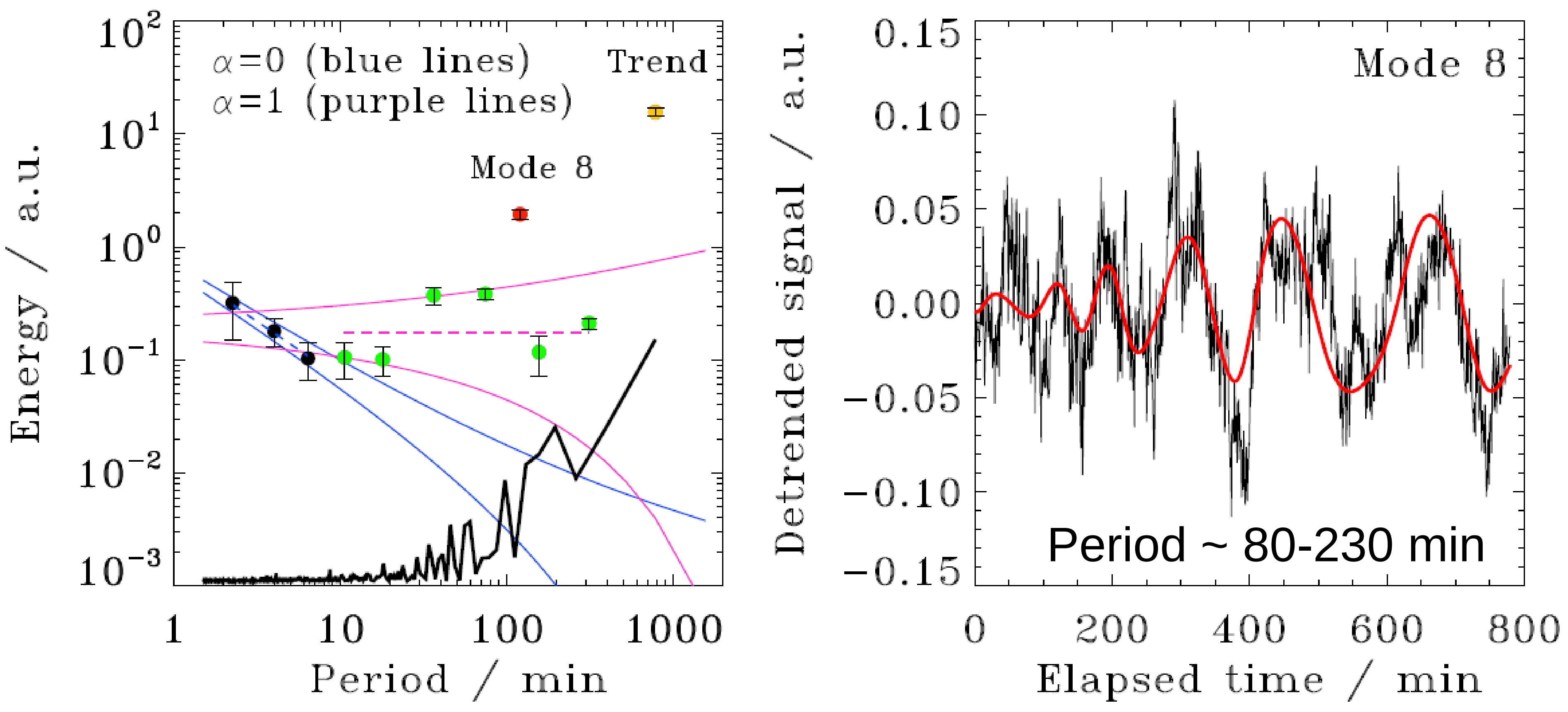


SDO/HMI line-of-sight magnetogram at 23:00:45 UT on 6 July 2013

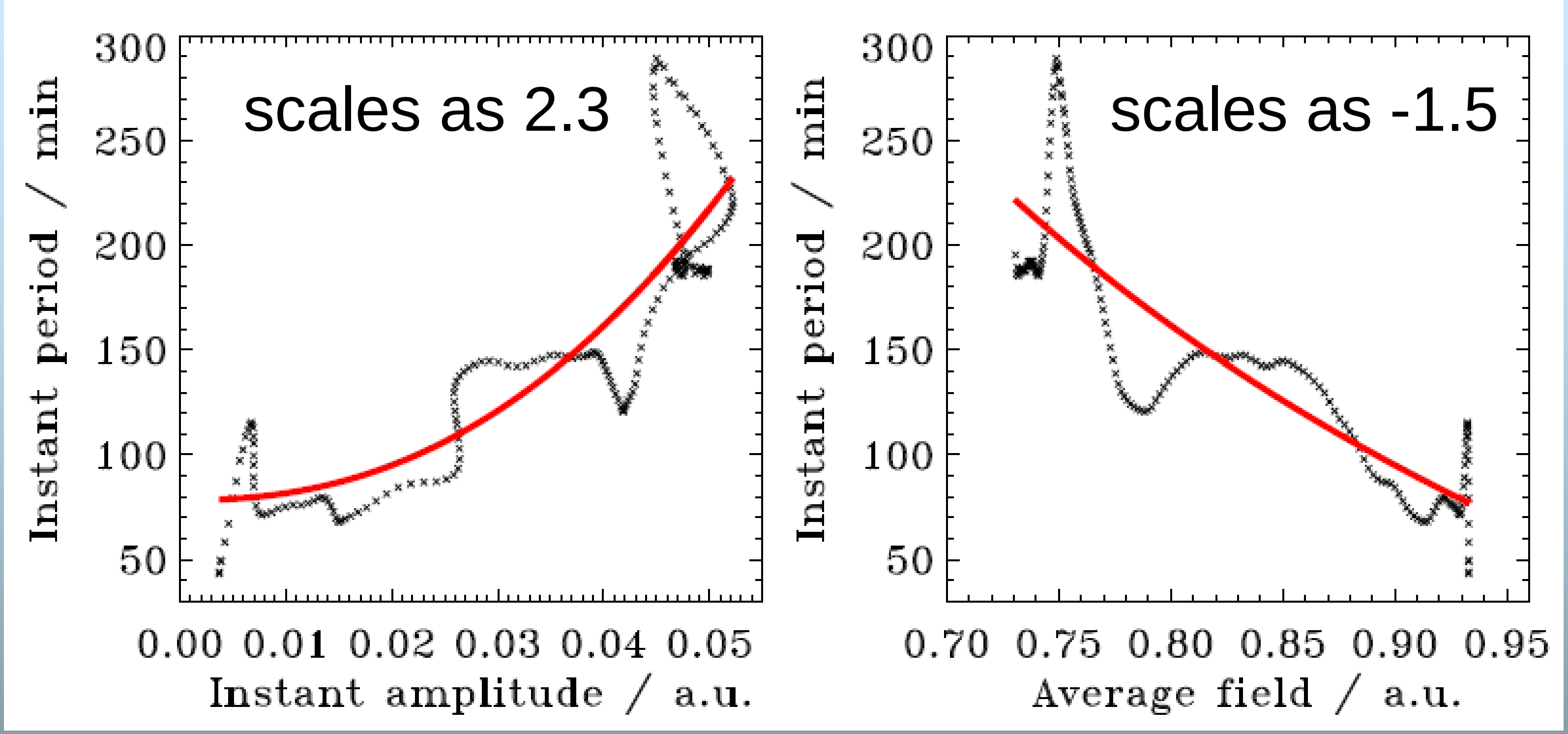
Average field (blue) in the structure, and its total area (black) in 100 G contour

In the context of Sun-like stars: e.g. *Karoff et al. (2013).*

EMD modes of the average field signal

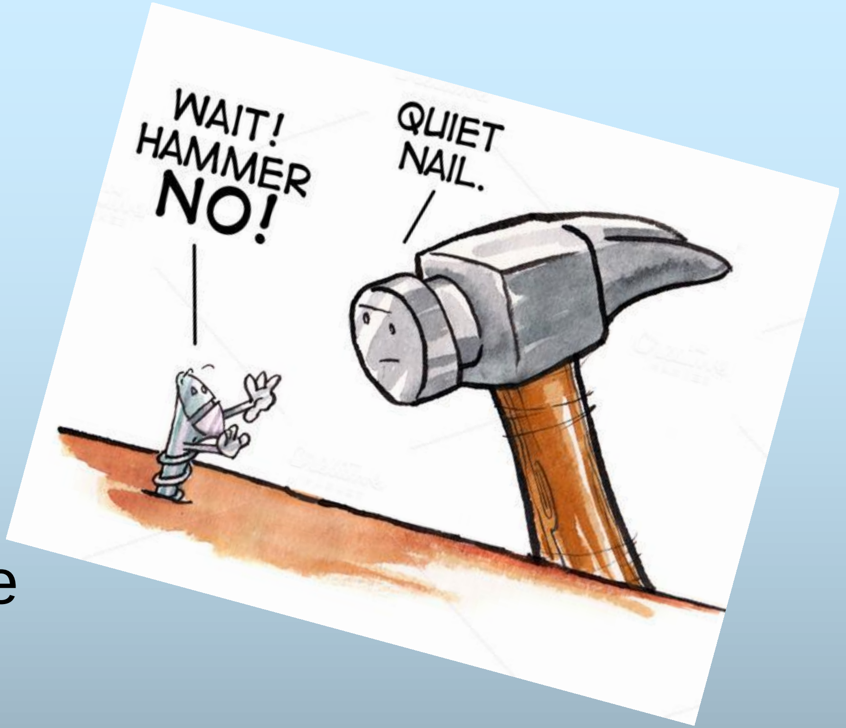


Scaling of the instant period of Mode 8



Such a behaviour could be explained e.g. by the dynamical interaction with the boundaries of supergranula cells or in terms of the vortex shedding appearing during the magnetic flux emergence. However, a specific mechanism is still to be revealed.

Conclusions



- HHT (EMD) is a good tool for studying non-stationary and non-regular time-series
- It operates self-adaptively, and hence is readily able to determine natural intrinsic modes, including underlying trends
- It works well on different time scales: from a few seconds to tens of years (e.g. 11 yr solar cycle), and is worth applying for stellar quasi-periodicities
- This "hammer" should be used correctly, e.g. accounting for the appearance of frequency-dependent components.