ISSI team “Quasi-periodic Pulsations in Stellar Flares:
a Tool for Studying the Solar-Stellar Connection”,
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WARWICK

Nonlinear oscillations of coalescing magnetic ropes

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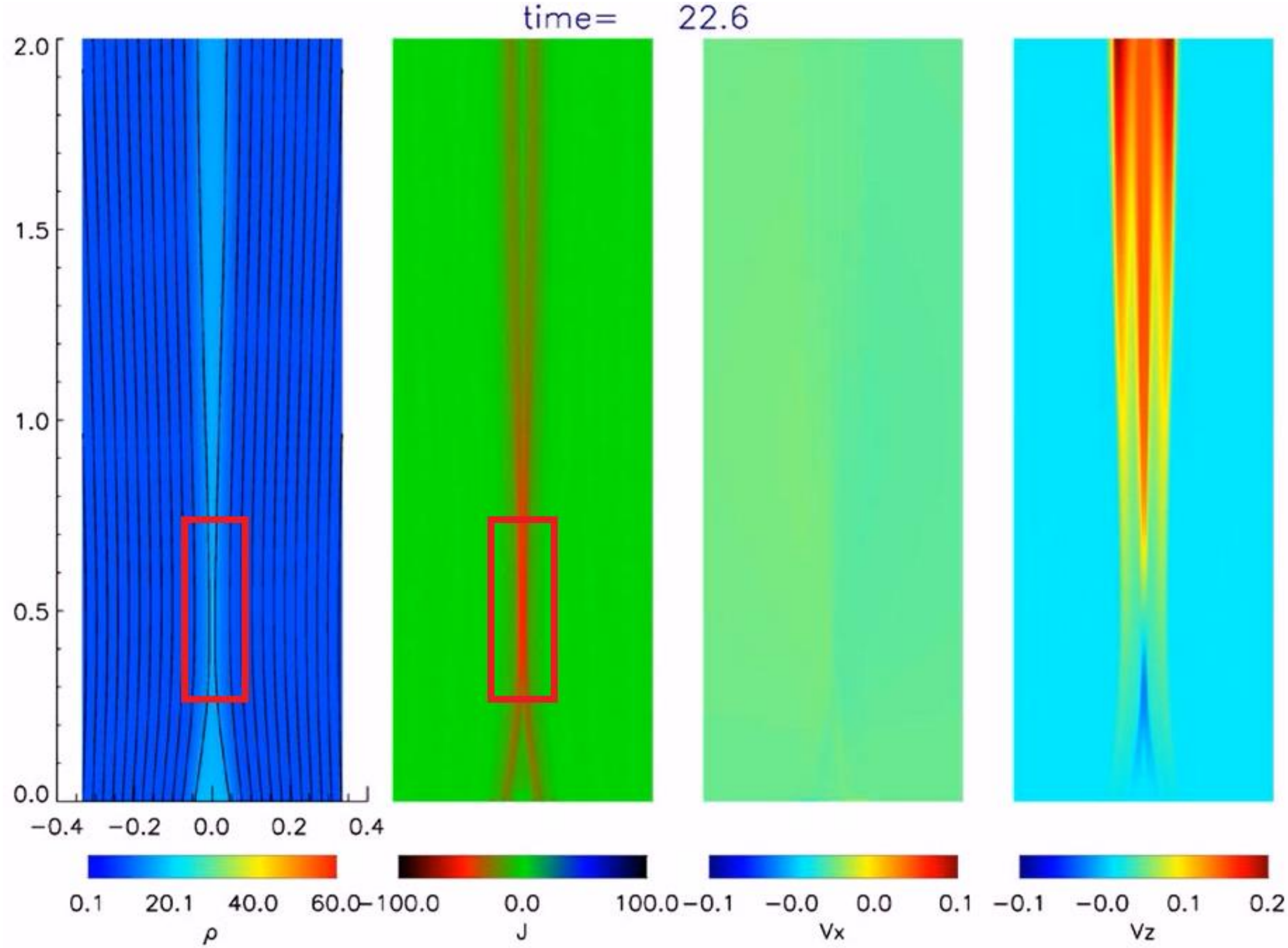
Possible mechanisms for quasi-periodicity

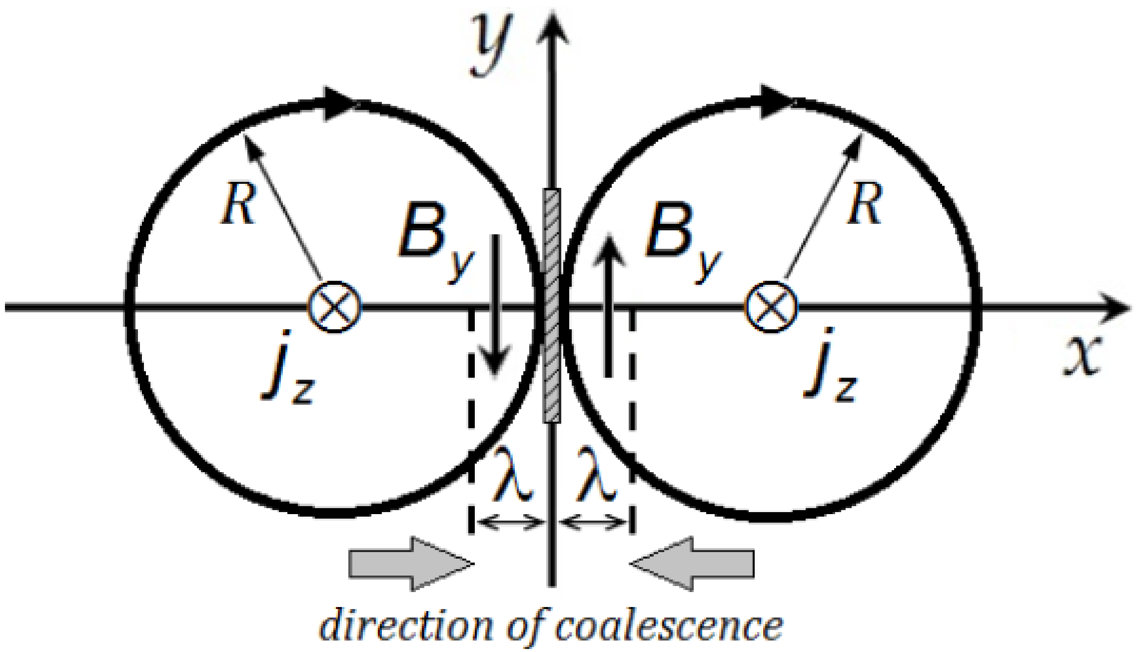
- Direct modulation of the non-thermal electron dynamics by MHD oscillations (*Nakariakov, V. M. & Melnikov, V. F. 2009; Zaitsev, V. V. & Stepanov, A. V. 1982*)
- Periodic triggering of energy releases by external MHD waves (*e.g. Nakariakov et al., 2006*)
- Repetitive regimes of a self-induced magnetic reconnection or so-called "magnetic dripping models" (*e.g. Tajima et al., 1987; Kliem et al., 2000, Murray et al., 2009*)

Stochastic reconnection



C. Shen, J. Lin, and N. A. Murphy, 2011 ApJ 737, 14

1.5D magnetohydrodynamical model



$$\nabla = \{\frac{\partial}{\partial x}, 0, 0\}$$

$$\mathbf{B} = \{0, B_{\rm y}, 0\}$$

$$\mathbf{V} = \{V_{\rm x}, 0, V_{\rm z}\}$$

$$\mathbf{E} = \{E_{\rm x}, 0, E_{\rm z}\}$$

$E_{\rm x}$ is the electrostatic field

$E_{\rm z}$ is the induced field

T. Tajima et al., 1987 ApJ; D. Kolotkov et al., 2016 Phys. Rev. E

Ideal MHD limit, $n_\mathrm{e} = n_\mathrm{i}$ *(Tajima et al., 1987)*

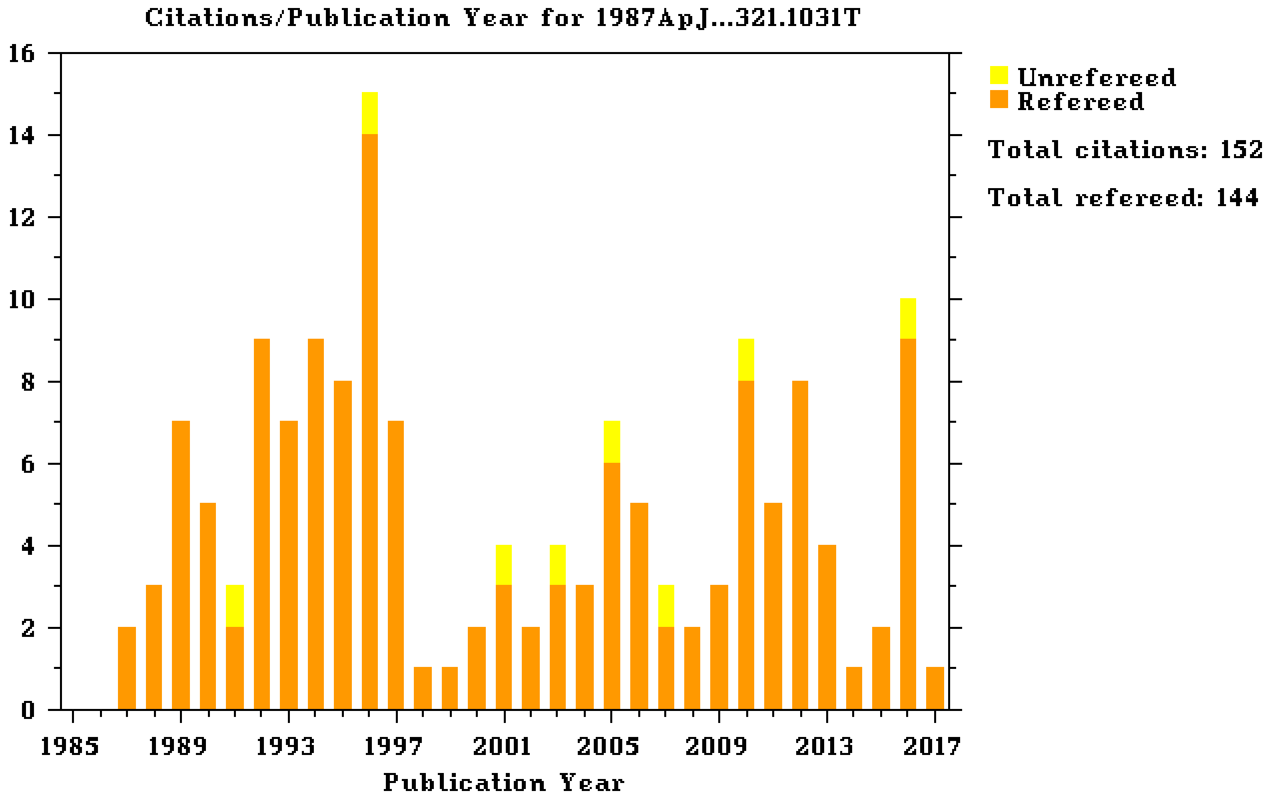
$$\ddot{a} = -\frac{v_A^2}{\lambda^2 a^2} + \frac{c_s^2}{\lambda^2 a^\gamma},$$

$a \propto n^{-1}$

Periods are of about 1 s and longer

Citations history for 1987ApJ...321.1031T from the ADS Databases

The Citation database in the ADS is **NOT** complete. Please keep this in mind when using the ADS Citation lists.



General case, $n_{\rm e} \neq n_{\rm i}$

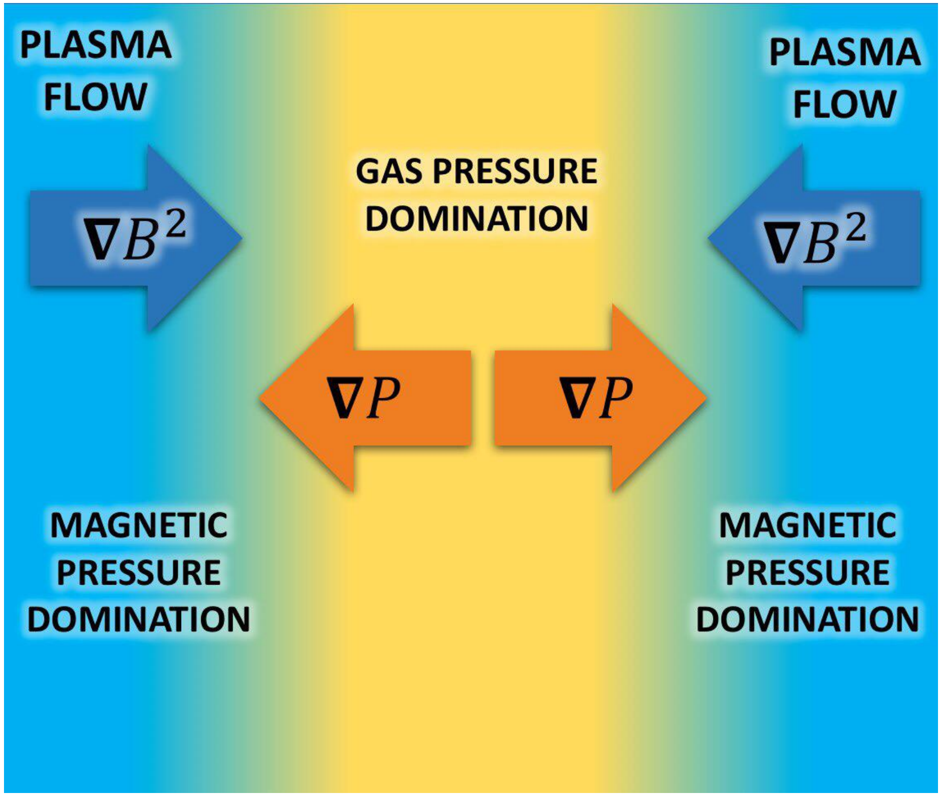
$$\frac{d^2 a}{d\,t^2} = -\omega_{\rm e}^2 \left(\frac{a}{b} - 1\right) - \frac{m_{\rm i}}{m_{\rm e}} \frac{V_{\rm A}^2}{\lambda^2 a^2} + \frac{m_{\rm i}}{m_{\rm e}} \frac{V_{\rm s}^2}{\lambda^2 a^\gamma},$$

$$\frac{d^2 b}{d\,t^2} = \omega_{\rm i}^2 \left(1 - \frac{b}{a}\right),$$

$a \propto n_e^{-1}$ and $b \propto n_i^{-1}$

Can be analysed analytically in the assumption of inertialess electrons ($\omega_e \to \infty$) and massive ions (ω_i is finite)

Static solution



Taking $d/dt = 0, \bar{a}_0 = \bar{b}_0 = \left(\frac{V_A^2}{V_s^2}\right)^{\frac{1}{2-\gamma}} = \left(\frac{B_0^2}{4\pi P_0}\right)^{\frac{1}{2-\gamma}} \approx \beta^{\frac{1}{\gamma-2}}$

Static state of the current sheet is determined by

the magnetic and thermodynamical pressure balance

B(A) dependence. Two regimes of CS oscillations

$$B\left(A\right)=\frac{A^{\gamma+3}}{A^{\gamma+2}-\bar{\phi}(A^{\gamma}-A^{2})},$$

$A \propto n_e^{-1}$
$B \propto n_i^{-1}$

which reduces to the Tajima's ideal MHD limit $A = B$, $n_{\rm e} = n_{\rm i}$ for small values of $\bar{\phi} = (V_{\rm A}/V_{\rm s})^6(\lambda_{\rm D}/\lambda)^2 \approx \beta^{-3}(\lambda_{\rm D}/\lambda)^2$

MHD regime

small $\overline{\phi}$, $\lambda \gg \lambda_D, \beta \leq 1$,

$n_e = n_i$, low frequencies

High-frequency (HF) regime

large $\overline{\phi}$, $\lambda \sim \lambda_D, \beta \ll 1$,

$n_e \neq n_i$, frequencies $\sim \omega_i$

Small-amplitude limit

Expanding $A(\tau) = 1 + \eta x(\tau)$, **harmonic oscillator equation**:

$$\frac{d^2 x}{d\tau^2} + \frac{\bar{\phi}(2-\gamma)}{\bar{\phi}(2-\gamma)-1}x = 0$$

with period $P = 2\pi(1+\bar{\phi}^{-1})^{1/2}$

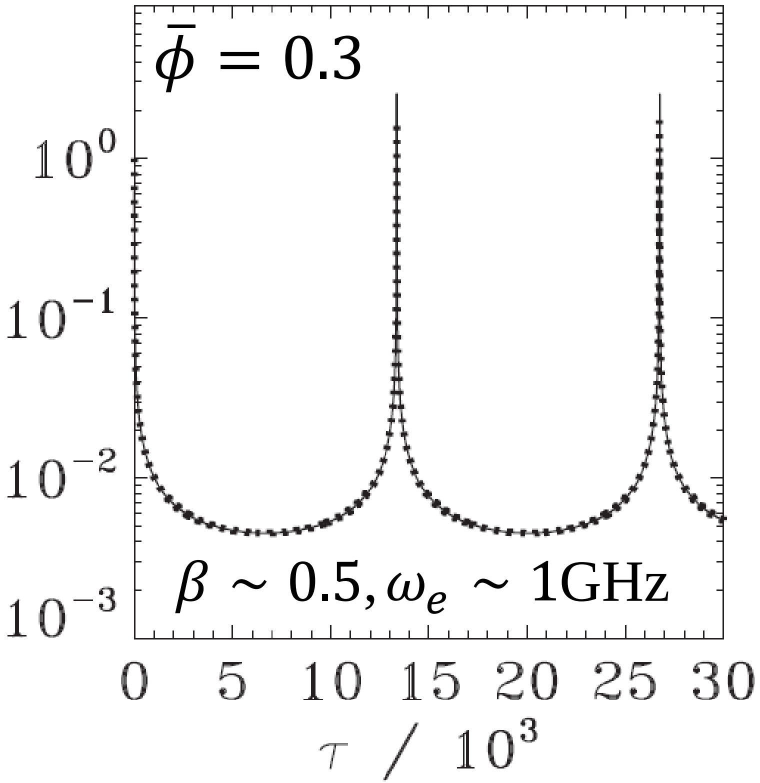
For small $\bar{\phi}$ (MHD regime), P may have arbitrarily long values

For large $\bar{\phi}$ (HF regime), $P \rightarrow 2\pi(V_s/V_A)^{1/2}\omega_i^{-1}$

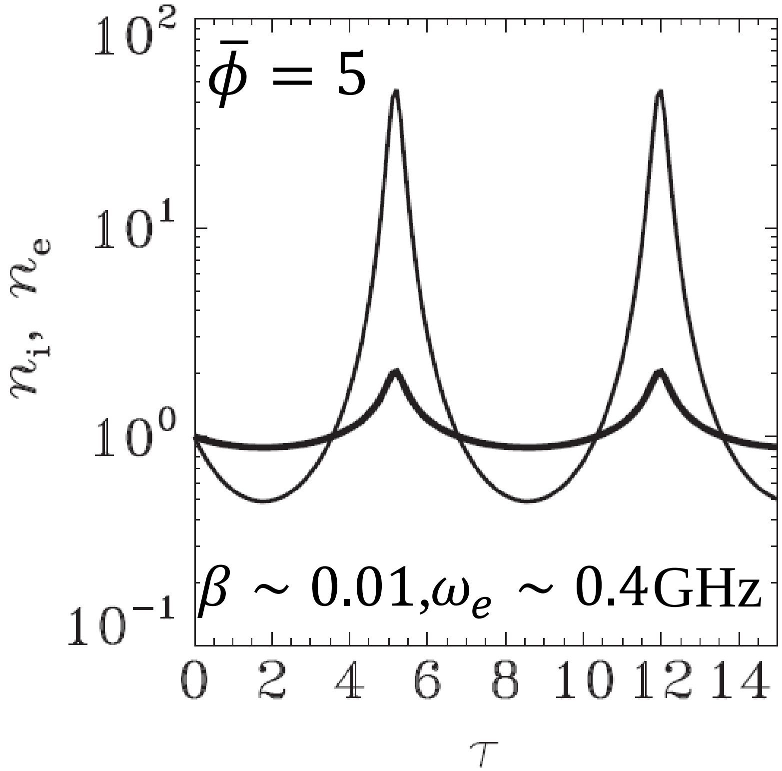
Nonlinear governing ODE and oscillations

$$f(A)\,\frac{\mathrm{d}^2 A}{\mathrm{d}\,\tau^2} + \frac{\mathrm{d}\,f(A)}{\mathrm{d}\,A}\left(\frac{\mathrm{d}\,A}{\mathrm{d}\,\tau}\right)^2 = g(A)\,,$$

MHD regime, $P \sim 1\,\mathrm{s}$



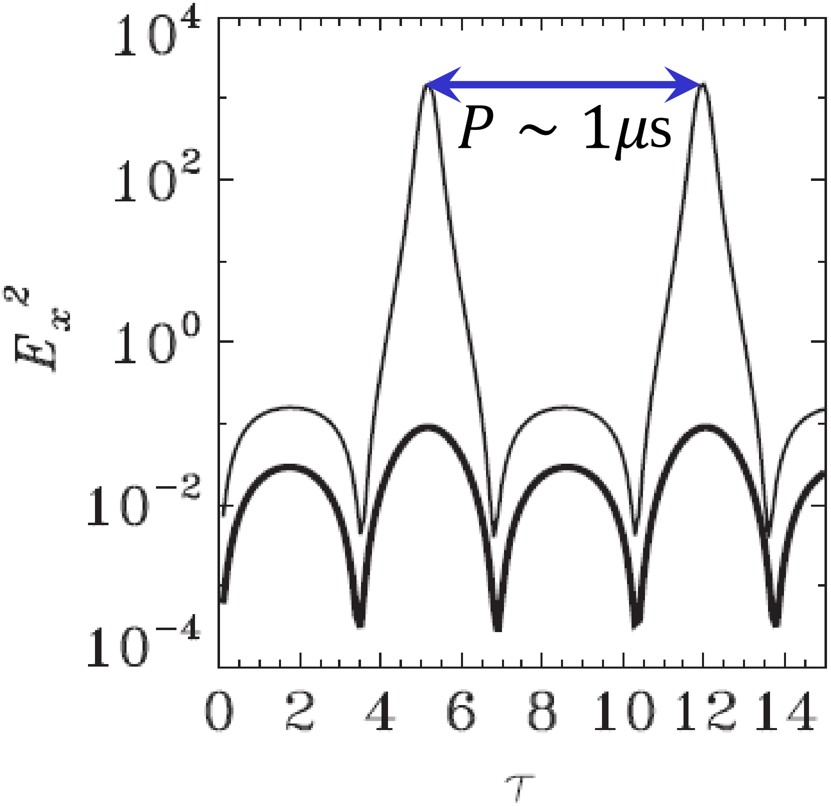
HF regime, $P \sim 1\mu\mathrm{s}$



Electrostatic field by local charge separation

$$E_x \propto n_i - n_e$$

HF regime



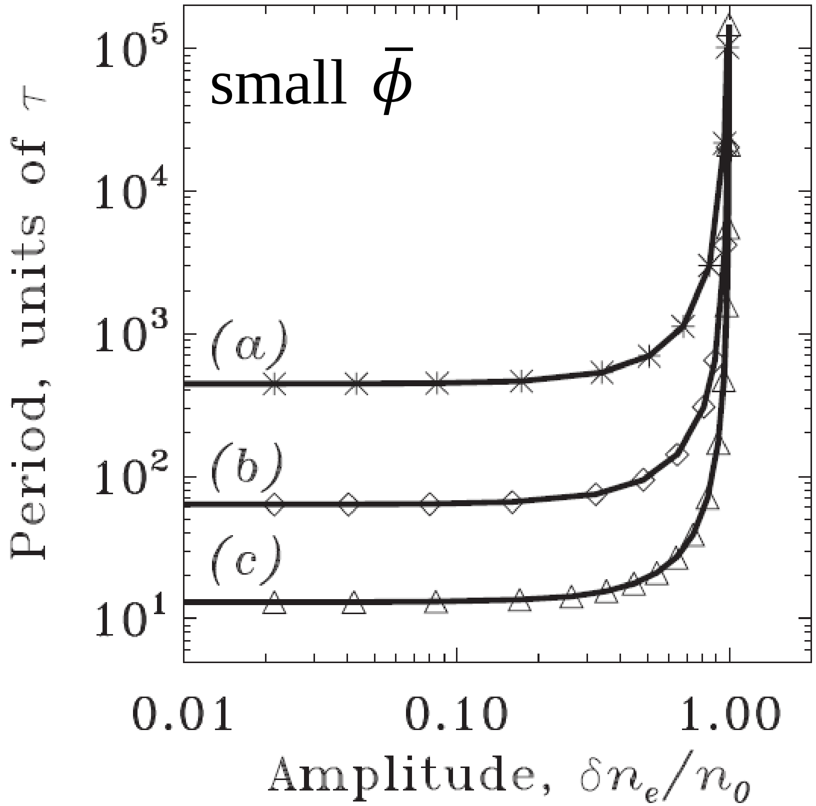
$\bar{\phi} = 5,$

$\beta \sim 0.01,$

$\omega_e \sim 0.4\,\mathrm{GHz}$

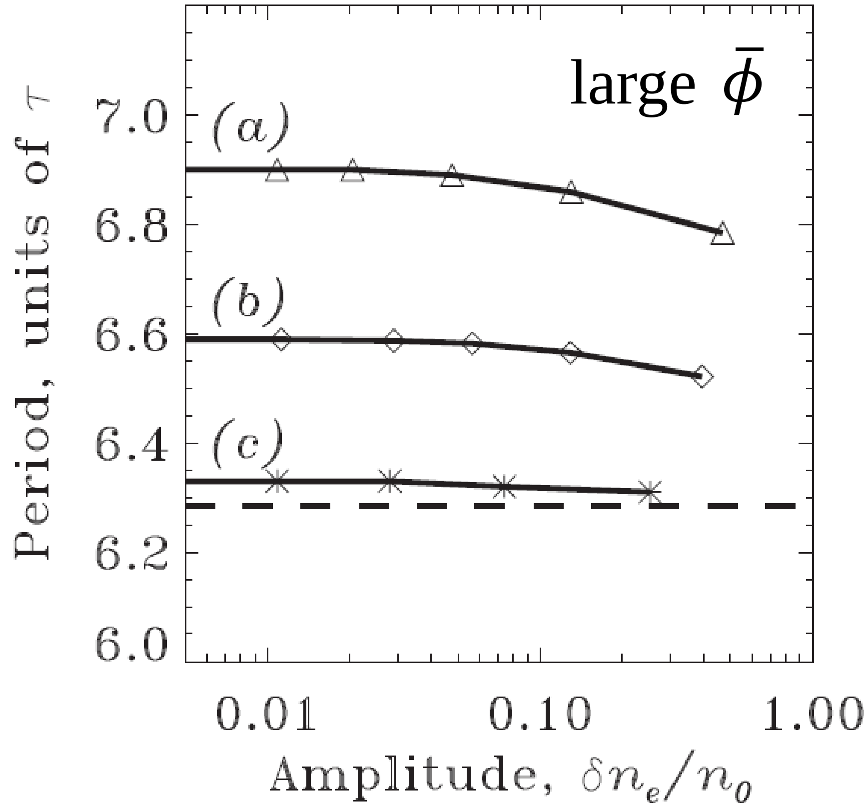
Nonlinear amplitude–period dependence

MHD regime



Strong dependence,
$P \sim 1\,\text{s}$ for $\delta n_e \approx n_0$

HF regime



Weak dependence,
$P = 2\pi(1+\bar{\phi}^{-1})^{1/2}$

Summary

An analytical model of highly nonlinear oscillations occurring during a coalescence of two magnetic flux ropes, based upon two-fluid MHD, is developed.

The model accounts for the effect of electric charge separation, and describes perpendicular oscillations of the current sheet formed by the coalescence.

The oscillation period is determined by the current sheet thickness, the plasma parameter β, and the oscillation amplitude. The oscillation periods are typically greater or about the ion plasma oscillation period.