Tides and inertial waves in late-type stellar and exoplanet binary systems

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Tides in close binary systems

Going beyond the simple two-body problem with point-like masses, we assume that one of the bodies is extended with mass M_1 and radiu
R_1, while the other is assumed point-like with a mass M_2 ;

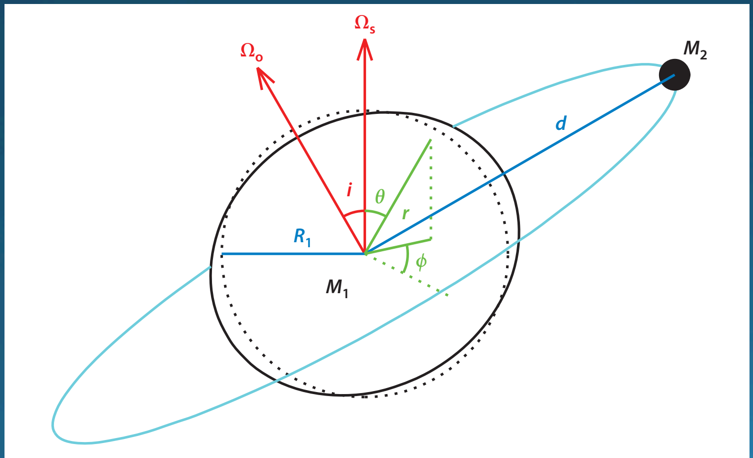
The gravitational potential of the mass M_2 and the centrifugal potential of the body of mass M_1 produce a distortion of the latter;

We assume a uniform rotation of the body of mass M_1 with an angular velocity Ω_s;

The mean orbital motion is indicated with Ω_o;

We choose a spherical polar reference frame (r, θ, ϕ) with the polar axis along the spin axis of M_1 and the origin at its barycentre of M_1;

The semimajor axis of the orbit is *a*, its eccentricity is *e*, and the inclination of the orbital plane with respect to the equatorial plane of M_1 is



Tidal potential

- The potential of the body M_2 can be developed in a series in the reference frame associated with the body M_1; the terms of the series are:
  - A uniform potential (=> no associated force);
  - A potential depending on 1/*d*, where *d* is the instantaneous separation between the two bodies (=> Keplerian motion);
  - A potential depending on higher powers of 1/*d*, that is called the ***tidal potential*** Ψ; it produces a non-uniform acceleration that deforms body M_1;
- In a non-rotating reference frame, we can develop the tidal potential as (see Ogilvie 2014):

$$\Psi = \mathrm{Re} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=-\infty}^{\infty} \frac{GM_2}{a} A_{l,m,n}(e, i) \left(\frac{r}{a}\right)^l Y_l^m(\theta, \phi) \mathrm{e}^{-\mathrm{i}n\Omega_0 t}.$$

Table 1 Quadrupolar components of the tidal potential, correct to first order in eccentricity and obliquity

l	m	n	$\lvert A \rvert$	Description
2	0	0	$\sqrt{\frac{\pi}{5}}$	Static tide
2	2	2	$\sqrt{\frac{6\pi}{5}}$	Asynchronous tide
2	0	1	$3e\sqrt{\frac{\pi}{5}}$	Eccentricity tides
2	2	1	$\frac{1}{2}e\sqrt{\frac{6\pi}{5}}$	
2	2	3	$\frac{7}{2}e\sqrt{\frac{6\pi}{5}}$	
2	1	0	$i\sqrt{\frac{6\pi}{5}}$	Obliquity tides
2	1	2	$i\sqrt{\frac{6\pi}{5}}$	

If d $/R_1$ is sufficiently large, only the quadrupolar components (l=2) are important (see table above). For a circular orbit, the static tide and the asynchronous tide are the important components.

Tidal frequency

Because $Y_l^m(\theta,\phi) \propto \mathrm{e}^{\mathrm{i}m\phi}$, the phase of each tidal component is $\arg A_{l,m,n} + m\phi - n\Omega_\mathrm{o} t$. When $m \neq 0$, the phase rotates with angular velocity $n\Omega_\mathrm{o}/m$. The angular frequency of each component measured in a nonrotating frame is $\omega = n\Omega_\mathrm{o}$, which may be called the tidal frequency in the inertial frame. Of greater importance for the physical response of the fluid is the angular frequency measured in a frame that rotates with the spin angular velocity Ω_s (spin frequency) of body 1, $\hat{\omega} = n\Omega_\mathrm{o} - m\Omega_\mathrm{s}$, which may be called the tidal frequency in the fluid frame. When $m \neq 0$, the difference between ω and $\hat{\omega}$ is due to an angular Doppler shift. (In general, body 1 may rotate differentially, in which case $\hat{\omega}$ depends on position.)

Equilibrium tides and dynamic tides

- The *equilibrium tide* is an approximation to the tidal response of a fluid body considering only the large-scale deformation of the body under the tidal potential and assuming a *phase lag* between the tidal potential and the hydrostatic (equilibrium) deformation;

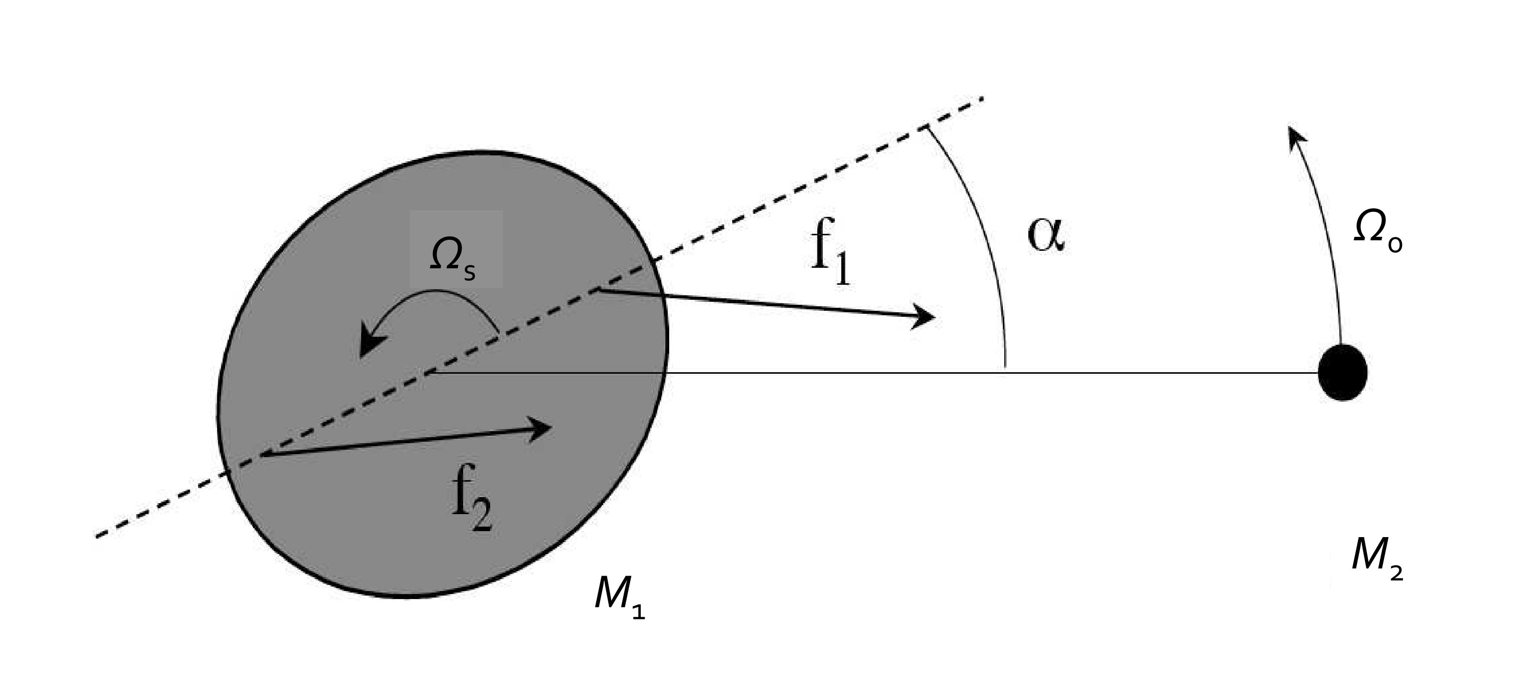
- The dynamic tide includes corrections to the equilibrium tide, generally in the form of internal waves as well as nonwave-like components with the purpose of satisfying the equation of motion. The frequencies of the waves are the tidal frequencies in the fluid frame, i.e.:

$$\hat{\omega} = n\Omega_o - m\Omega_s,$$

the case of a coplanar and circular orbit (e=0, i=0), we have n=m and l-m must be eve

or (R_1/a) << 1, the dominating component is the one with $l = m = n = 2$.

Equilibrium tide and tidal lag



Owing to the dissipation of the kinetic energy of the equilibrium tide inside the body M_1, the longitude of the tidal bulge lags the longitude of the orbiting body by a *phase lag angle* α. The tidal lag can also be expressed as a *time lag* $\tau \equiv \alpha/|\hat{\omega}|$.

The direction of tidal evolution

- If the body M_1 is rotating faster than the orbital motion ($\Omega_s > \Omega_o$), the tidal lag is positive and the angular momentum is transferred from the rotation of M_1 to the orbital motion, thus increasing the orbital separation;

- On the other hand, if M_1 is rotating slower than the orbital motion ($\Omega_s < \Omega_o$), the tidal lag is negative and the angular momentum is drawn from the orbit to accelerate the spin of M_1; in this case, the orbital separation decreases.

The end point of tidal evolution

- If the total angular momentum (orbit + spin) L is *constant* and larger than a critical value L_c , then the system can reach a stable equilibrium characterized by:
  - A circular orbit (e = 0);
  - Spin-orbit synchronization ($\Omega_s = \Omega_o$);
  - Alignment of the spin and orbital angular momenta (i = o);

- If $L < L_c$, no stable equilibrium is possible and the body of mass M_2 will eventually collide with that of mass M_1 (Hut 1980);

- If the total angular momentum L is *decreasing* in time because the body M_1 is a late-type star losing angular momentum through a magnetized wind, the endpoint will be a collision of the two bodies (see, e.g., Damiani & Lanza 2015 for details).

Physical processes of tidal dissipation

- According to Zahn (1977; 1989), in late-type stars, the dissipation of the kinetic energy of the equilibrium tide is mainly produced by the turbulent convection that acts on the tidal flow;

- The reason is that the frequencies of the p-mode ν_p in their convective envelopes or of the g-modes ν_g in their radiative interiors are too different from the typical tidal frequencies to excite a comparable dynamic tide;

- Typically: $1/\nu_p \approx$ 1-10 minutes; $1/\nu_g \approx$ 1-10 hours; while $1/\omega \approx$ 1-10 days.

Love number

If a tidal potential component

$$\mathrm{Re}[\mathcal{A}(r/R)^l Y_l^m(\theta,\phi)\exp(-i\omega t)]$$

is applied to a body of mean radius R and the resulting deformation of the body generates an external gravitational potential perturbation

$$\mathrm{Re}[\mathcal{B}(R/r)^{l+1} Y_l^m(\theta,\phi)\exp(-i\omega t)],$$

then the complex dimensionless ratio

$$k_l^m(\omega) \equiv \mathcal{B}/\mathcal{A}$$

defines the *potential Love number.*

The tidal torque

- The tidal torque is responsible for the exchange of angular momentum and kinetic energy between the two orbiting bodies, thus ruling the tidal evolution of the system. The torque acting on the body M_1 is:

$$\mathcal{T} = \frac{(2l+1)R|\mathcal{A}|^2}{8\pi G}\mathrm{Im}[k_l^m(\omega)].$$

The tidal quality factors Q and Q'

Typically $\mathrm{Re}[k_l^m(\omega)]$ is a quantity of order unity, weakly dependent on m and ω, and can be well approximated by its hydrostatic value k_l. For a homogeneous body:

$$k_l = k_l^{\mathrm{hom}} = 3/[2(l-1)].$$

Then it is common to write

$$\mathrm{Im}[k_l^m(\omega)] = k_l/Q(\omega) = k_l^{\mathrm{hom}}/Q'(\omega),$$

where Q is the *tidal quality factor* and Q' the *modified tidal quality* factor. The importance of Q is given by its relationship with α. For $\alpha \ll 1$ and $l = 2$, we have:

$$\alpha(\omega) \simeq 1/Q(\omega).$$

The generally adopted parameterizations are summarized as follows:

$$\mathrm{Im}[k_2^m(\omega)] = \sigma\frac{3}{2Q'} = \sigma\frac{k_2}{Q} = k_2\tau\hat{\omega},$$

where $\sigma = \operatorname{sgn}\hat{\omega} = \pm 1$.

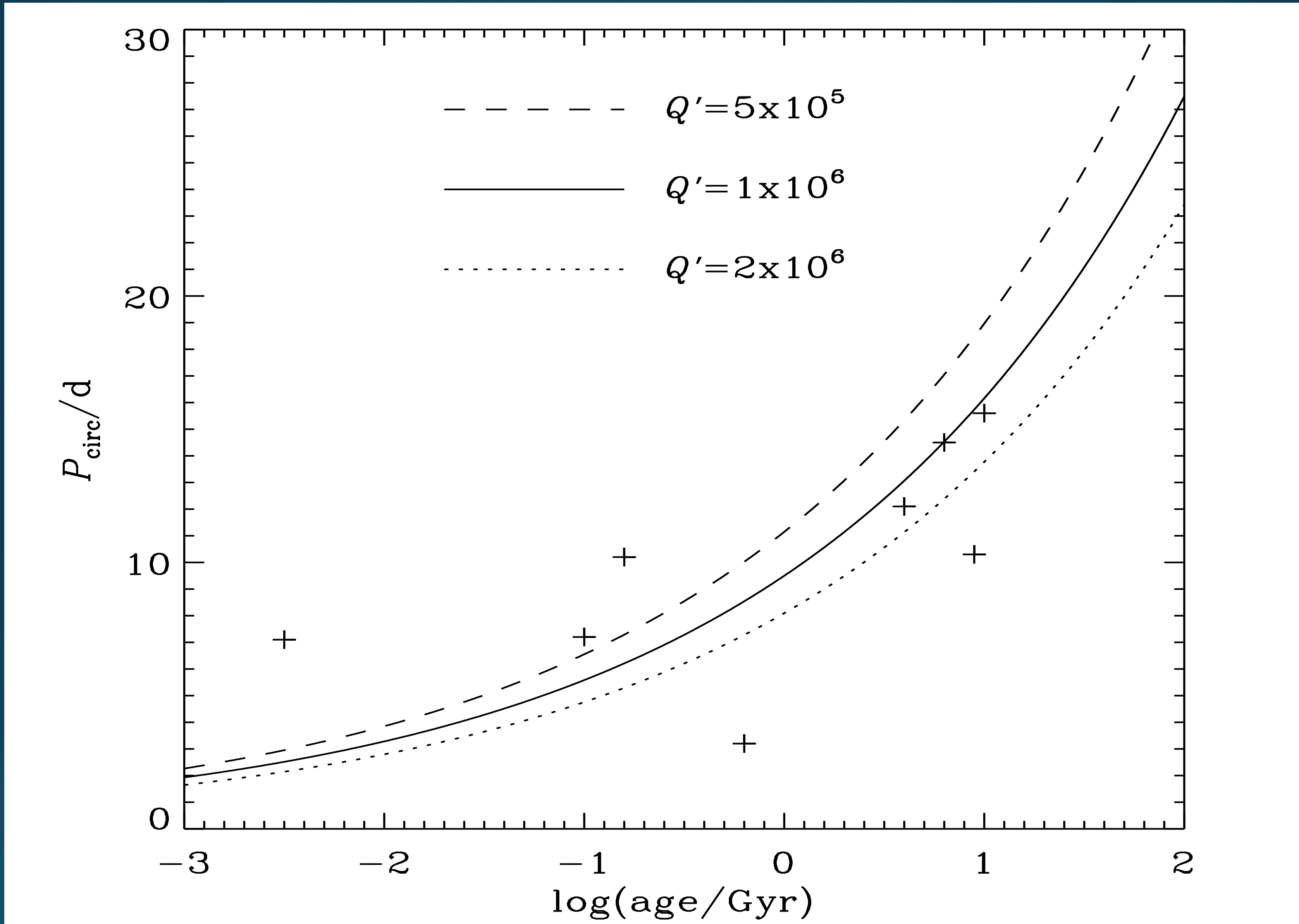
Tidal quality factor and energy dissipation

$$Q \equiv \frac{\text{energy stored in tidal distortion}}{\text{energy dissipated in one cycle}} = 2\pi E_0 \left(\oint -\dot{E} dt\right)^{-1},$$

E_o is the maximum energy associated with the tidal distortion and the integral is extended over one tidal period giving the energy lost during one complete cycle. Note that this formula contains Q, not Q′ that depends on the Love number k_2 (cf. Zahn 2008; Wilkins et al. 2017).

Tidal circularization of close stellar binary systems in clusters of different ages

- The maximum period corresponding to a circular orbit of late-type stellar binaries belonging to clusters of different ages (or a specific turnoff point in their eccentricity distribution) has been used to estimate Q′ (Meibom & Methieu 2005; Milliman et al. 2014);
- Since Q′ is a function of the tidal frequency, those values refer to some $Q'(\omega)$ averaged over system evolution;
- Ogilvie & Lin (2007) used Meibom & Mathieu's observations to estimate the average Q′ in close binaries consisting of two solar-like stars;
- They find $Q' \approx 10^6$ (see plot on the next slide).



(after Ogilvie & Lin 2007)

Application to systems with very hot Jupiters

- Considering $Q' = 10^6$ also for the systems consisting of a solar-like star and a very hot Jupiter ($M_2 \approx (1\text{-}3)\ M_{jup}$, $P_{orb} < 2$ days), we find a remaining lifetime of only 30-100 Myr;

- This is at variance with the number of detected systems and with the lower limits on Q′ derived for some systems from transit timing (e.g. WASP-18; Wilkins et al. 2017);

- The orbital period distribution of the hot Jupiters suggests $Q' \approx 10^7$ for solar-like stars (Jackson et al. 2008; 2009) giving a remaining lifetime of VHJs of about 0.3-1.0 Gyr.

The model by Ogilvie & Lin (2007)

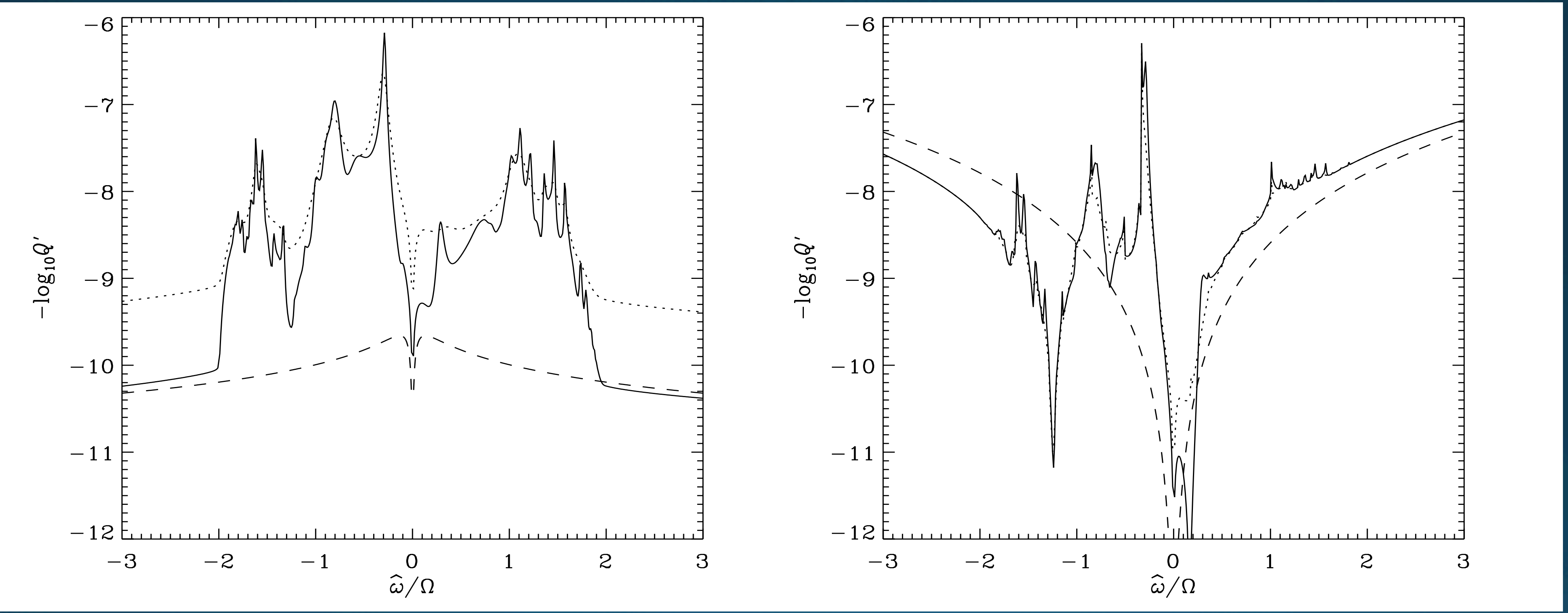
- To explain the difference between close stellar binaries and systems consisting of a star and a hot Jupiter, Ogilvie & Lin (2007) considered that the former are synchronized (i.e., $\Omega_s = \Omega_o$), while the latter are generally not;

- Therefore, for the stellar binaries, the *excitation of inertial waves* is possible in their convection zones and/or in their radiative interiors, because their tidal frequencies verify:

$$|\hat{\omega}| \leq 2\Omega_{\mathrm{s}}$$

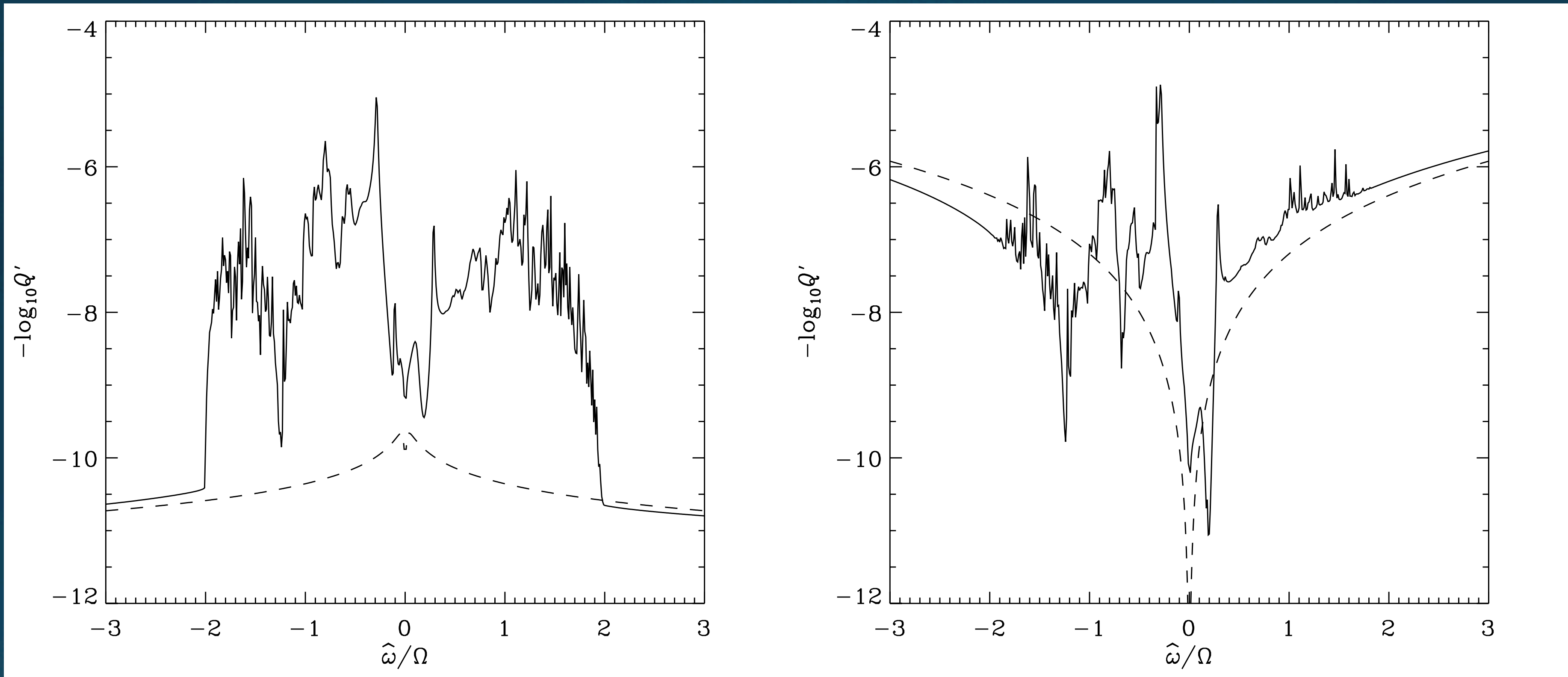
- This conditions is not verified in systems away from synchronism such as most of the star-hot Jupiter systems.

Q′ according to the model of O&L



Q′ for a Sun-like star with a spin period of 10 days. *Left*: Q′ from the dissipation of inertial waves in the convection zone (solid line); *Right*: the same as on the left, but for the dissipation of the Hough waves in the radiative interior. The dotted lines indicate the effect by increasing the effective viscosity by a factor of 10, while the dashed lines give the effect omitting the Coriolis force (after Ogilvie & Lin 2007).

Effect of the stellar rotation



he same as the previous figure, but for a rotation period of 3 days. Q' scal
vith the square of the stellar spin frequency (Ogilvie 2013; 2014).

Inertial waves and stellar activity

- In principle, if there is a magnetic field in the convection zone or the radiative interior of a star, inertial waves are modified by the Lorentz force (magneto-inertial waves);

- Magneto-inertial waves are characterized by oscillations of the magnetic field;

- The modulation of the field intensity could produce a modulation of the stellar activity;

- Since Rieger-like cycles have been associated with those waves, it could be of interest to compare the frequencies of Rieger cycles with the tidal frequencies in the fluid frame of their stars when they belong to binary systems.

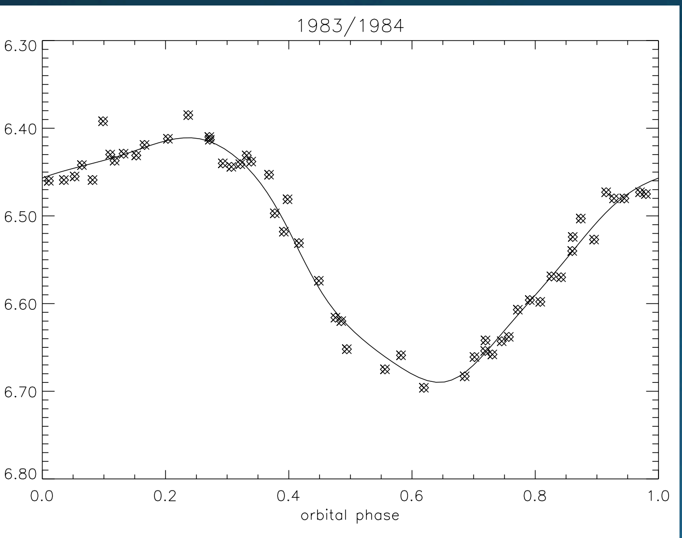
Comparison with the observations (I)

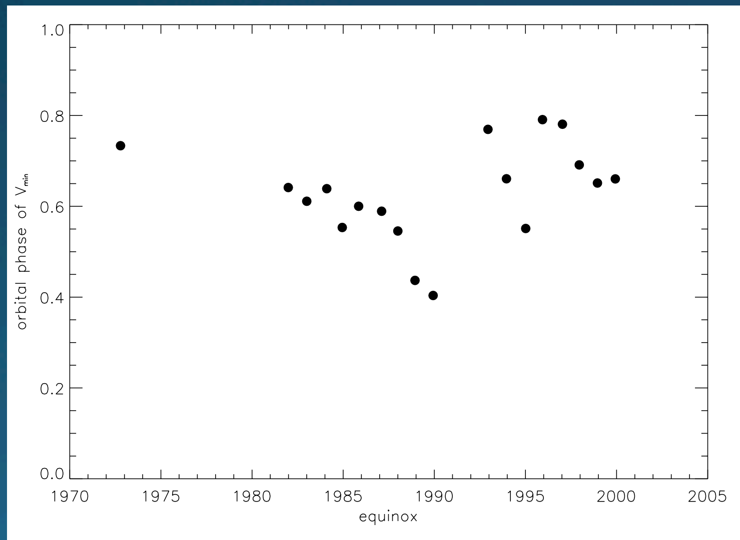
In the case of UX Ari, the deviation of the stellar rotation from the orbital period is very small and ill-determined by the migrating phase of the minima of the light curves (AarumUlvas & Henry 2003);

Using the linear decrease of the phases of the minima of the light curve with respect to the orbital phase in 1986-1990, we find: $P_{tide} = P_{migr}/2 = 4.32$ yr = 1578 days for the quadrupole frequency (*l*=*m*=2; see inset below);

The Rieger cycle has a period of $P_{Rieger} = 294 \pm 9$ days;

There is no simple commensurability: $P_{tide} / P_{Rieger} = 5.37 \pm 0.16$.





$$|\hat{\omega}| = 2(\Omega_0 - \Omega_s) \Rightarrow \frac{2\pi}{P_{\rm tide}} = 2\left|\frac{2\pi}{P_{\rm orb}} - \frac{2\pi}{P_{\rm ro}}\right.$$

on the other hand

$$\frac{1}{P_{\rm migr}} = \left|\frac{1}{P_{\rm rot}} - \frac{1}{P_{\rm orb}}\right|.$$

Therefore,

$$P_{\rm tide} = P_{\rm migr}/2$$

Comparison with the observations (II)

- In the case of CoRoT-2, the Rieger period is P_{Rieger} = 28.9 ± 4.8 days; adopting the rotation period of the active longitudes of the star, P_{rot} = 4.522 days, while P_{orb} = 1.743 days; therefore, we have: P_{tide} = 1.418 days (Lanza et al. 2009);

- Therefore, in the case of CoRoT-2, we find: P_{Rieger} / P_{tide} = 20.4 ± 3.4;

- However, a physical connection would imply that the component of the tidal potential with l=20 and m=n=20 be responsible for the excitation of the waves producing the Rieger cycle.

Comparison with the observations (III)

- In the case of Kepler-17, $P_{Rieger} = 47.1 \pm 4.5$ days, while $P_{rot} = 12.01 \pm 1.9$ days and $P_{orb} = 1.486$ days; this yields $P_{tide} = 0.847 \pm 0.016$ days (Bonomo & Lanza 2012);
- Therefore, we find: $P_{Rieger} / P_{tide} = 55.6 \pm 5.3$;
- Also in this case, no simple commensurability is found.

Conclusions

- Inertial waves in the components of close binary systems are of great importance to understand their tidal evolution;
- They could account for the difference between tidal dissipation in close stellar binary systems and in systems consisting of a late-type star and a hot Jupiter (Ogilvie & Lin 2007);
- They could also account for the distribution of the obliquity of the orbits of exoplanets vs. the stellar effective temperature (Albrecht et al. 2012; Esposito et al. 2017), according to a model proposed by Lai (2012);
- The possibility of Rieger-like cycles induced by tidal inertial waves in close binary systems seems to be not supported by the presently available observations, but the number of investigated systems is very small.

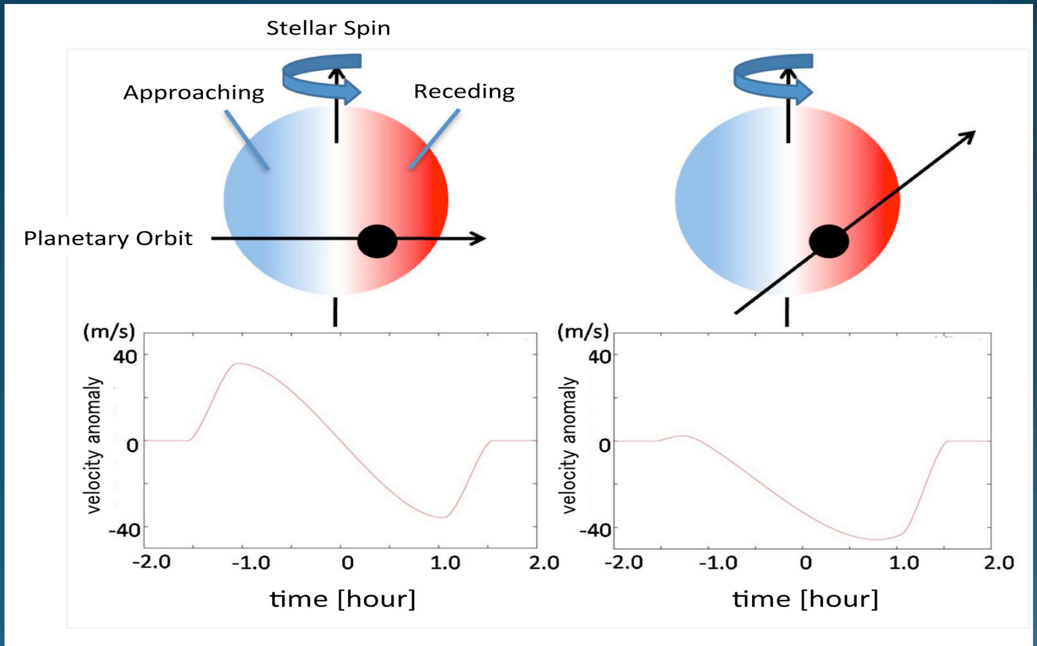
Additional material

Rossiter-McLaughlin effect

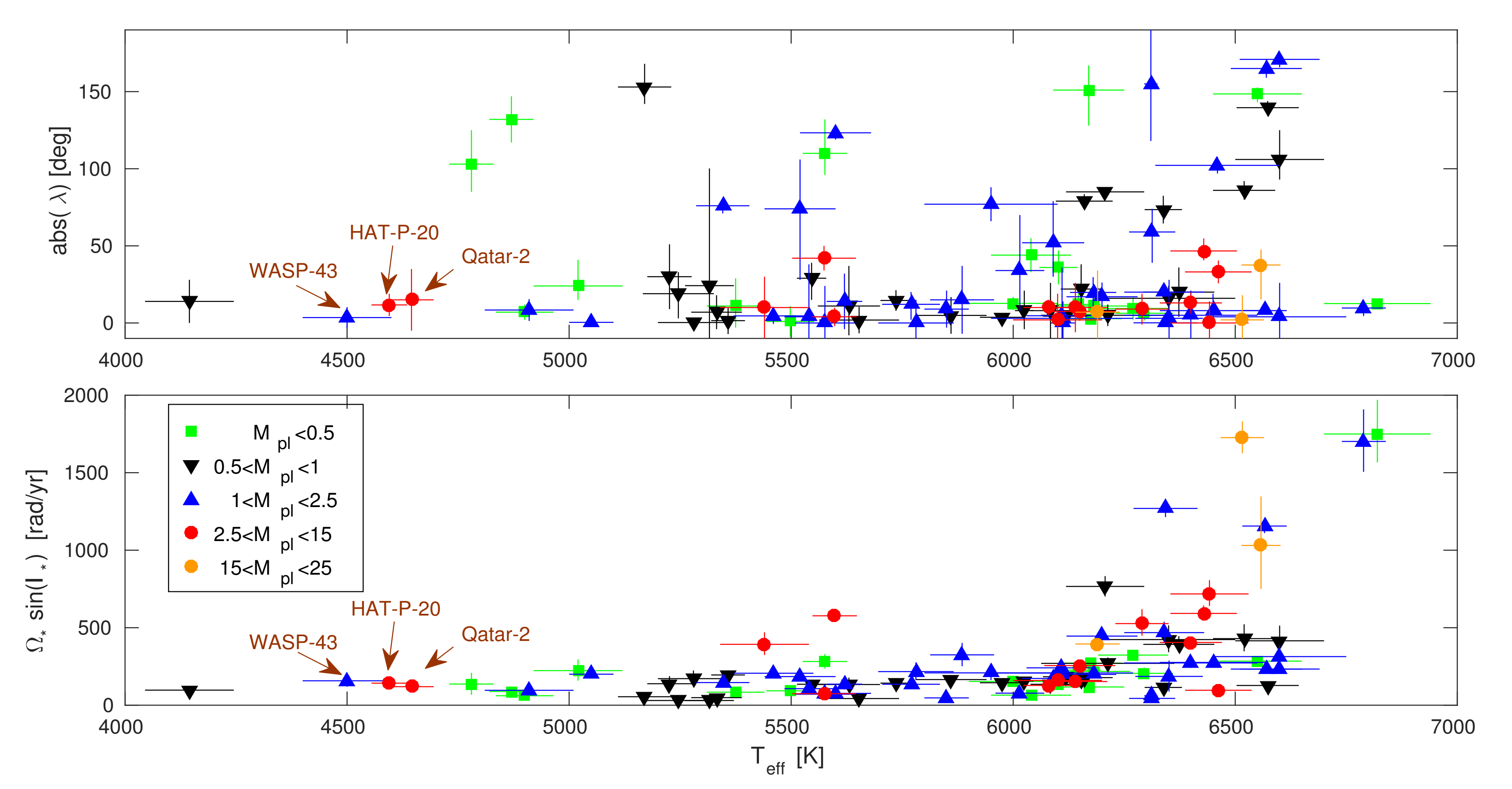
- The Rossiter-McLaughlin (RM) effect is an anomaly of the stellar radial velocity detected during transits;

- The angle between the projections on the plane of the sky of the spin axis and the normal to the orbital plane (*projecte* *obliquity* λ) can be measured by exploiting the RM effect;

- This provides complementary information on the inclination of the stellar spin axis to the line of sight in the case of transitin systems.



Obliquity in close-by planetary systems



Upper panel: Obliquity of the planetary orbit projected onto the plane of the sky λ vs. stellar effective temperature T_{ef} *lower panel*: Stellar projected angular velocity vs. effective temperature. The mass of the planet in Jupiter masses i indicated by the different symbols as specified in the inset (after Esposito et al. 2017, A&A accepted [paper XIII of th Italian GAPS collaboration]).