Instability of magnetic Rossby waves in the solar tachocline

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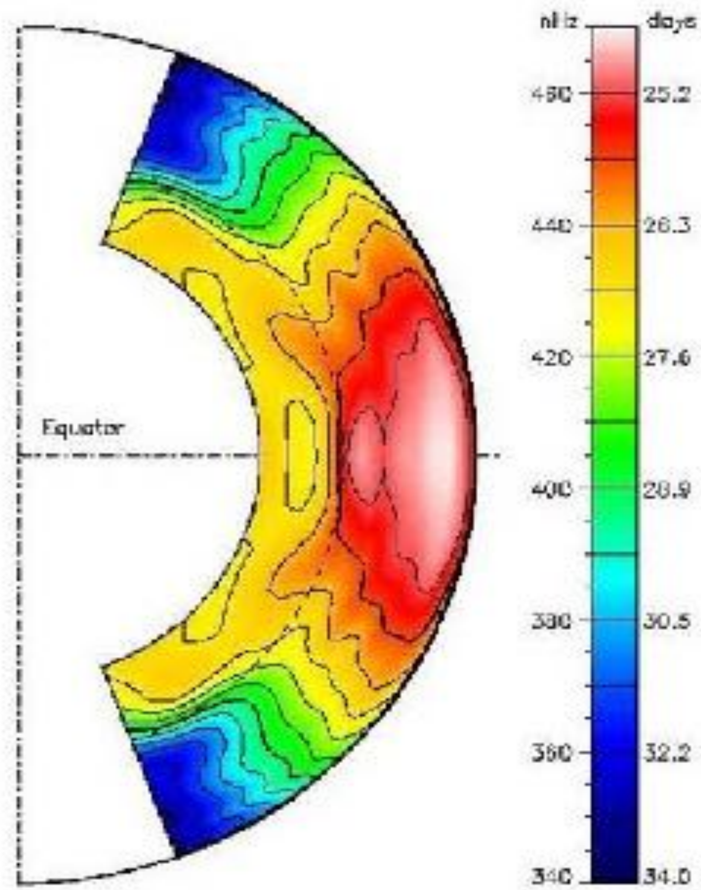
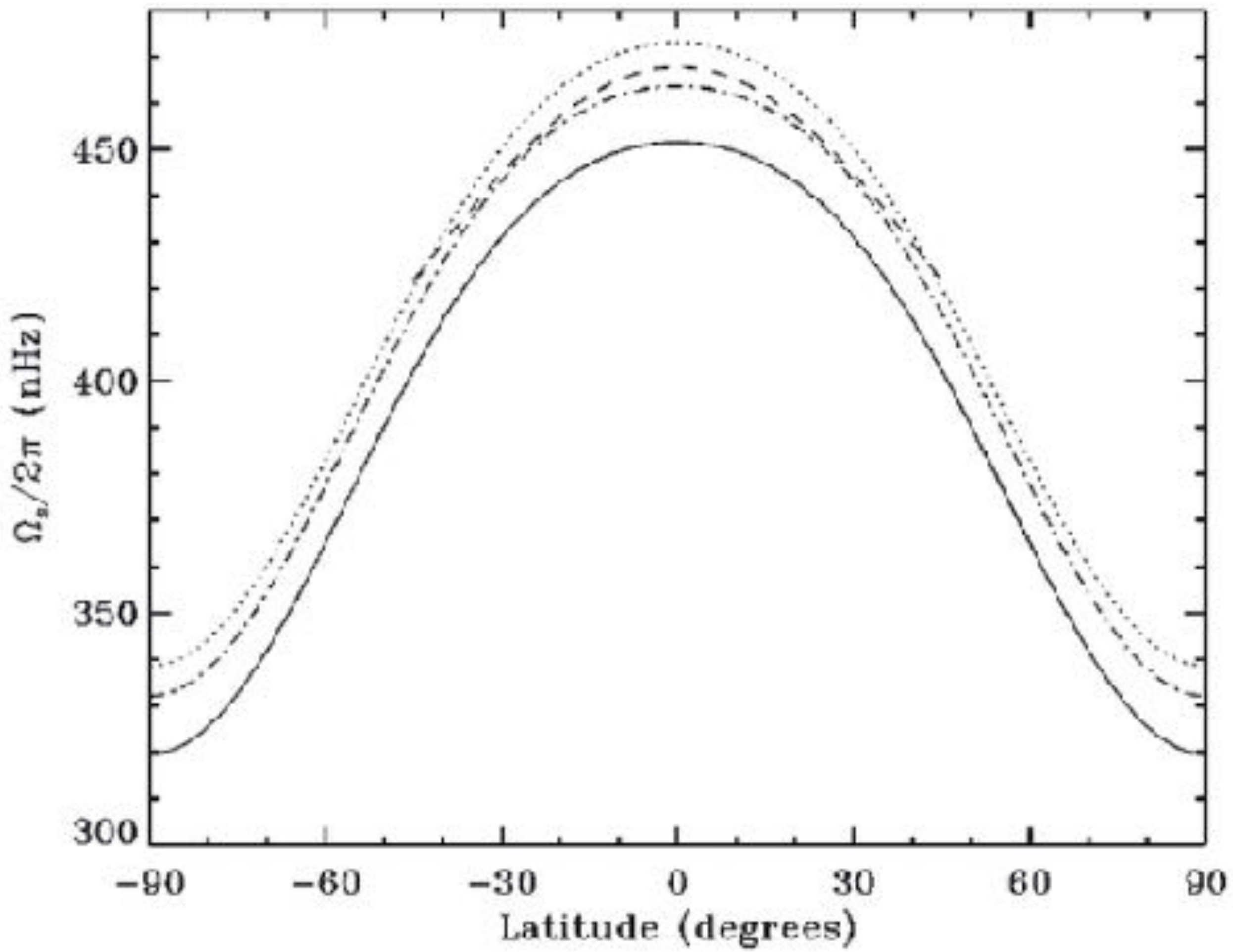
- Helioseismic observations show that the radiative zone rotates uniformly with both latitude and radius.
- The convection zone has strong differential rotation with latitudes and almost uniform rotation with radius.



- Observed differential rotation profile

$$\Omega = \Omega_0(1 - s_2 \cos^2\theta - s_4 \cos^4\theta) = \Omega_0 + \Omega_1(\theta),$$



Thompson et al. 2003

Linear shallow water MHD equations in rotating frame

$$\frac{\partial u_\theta}{\partial t} + \Omega_1 \frac{\partial u_\theta}{\partial \phi} - 2\Omega \cos\theta u_\phi = -\frac{g}{R_0}\frac{\partial h}{\partial \theta} + \frac{B_\phi}{4\pi\rho R_0}\frac{\partial b_\theta}{\partial \phi} - 2\frac{B_\phi \cos\theta}{4\pi\rho R_0} b_\phi,$$

$$\frac{\partial u_\phi}{\partial t} + \Omega_1 \frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta}{\sin\theta}\frac{\partial[\Omega \sin^2\theta]}{\partial \theta} = -\frac{g}{R_0 \sin\theta}\frac{\partial h}{\partial \phi} + \frac{B_\phi}{4\pi\rho R_0}\frac{\partial b_\phi}{\partial \phi} + \frac{b_\theta}{4\pi\rho R_0 \sin\theta}\frac{\partial[B_\phi \sin^2\theta]}{\partial \theta},$$

$$\frac{\partial b_\theta}{\partial t} + \Omega_1 \frac{\partial b_\theta}{\partial \phi} = \frac{B_\phi}{R_0}\frac{\partial u_\theta}{\partial \phi},$$

$$\frac{\partial}{\partial \theta}(\sin\theta b_\theta) + \frac{\partial b_\phi}{\partial \phi} + \frac{B_\phi \sin\theta}{H_0}\frac{\partial h}{\partial \phi} = 0,$$

$$\frac{\partial h}{\partial t} + \Omega_1 \frac{\partial h}{\partial \phi} + \frac{H_0}{R_0 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta u_\theta) + \frac{H_0}{R_0 \sin\theta}\frac{\partial u_\phi}{\partial \phi} = 0,$$

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MAGNETIC ROSSBY WAVES IN THE SOLAR TACHOCLINE AND RIEGER-TYPE PERIODICITIES

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ABSTRACT

Apart from the eleven-year solar cycle, another periodicity around 155–160 days was discovered during solar cycle 21 in high-energy solar flares, and its presence in sunspot areas and strong magnetic flux has been also reported. This periodicity has an elusive and enigmatic character, since it usually appears only near the maxima of solar cycles, and seems to be related with a periodic emergence of strong magnetic flux at the solar surface. Therefore, it is probably connected with the tachocline, a thin layer located near the base of the solar convection zone, where a strong dynamo magnetic field is stored. We study the dynamics of Rossby waves in the tachocline in the presence of a toroidal magnetic field and latitudinal differential rotation. Our analysis shows that the magnetic Rossby waves are generally unstable and that the growth rates are sensitive to the magnetic field strength and to the latitudinal differential rotation parameters. Variation of the differential rotation and the magnetic field strength throughout the solar cycle enhance the growth rate of a particular harmonic in the upper part of the tachocline around the maximum of the solar cycle. This harmonic is symmetric with respect to the equator and has a period of 155–160 days. A rapid increase of the wave amplitude could give rise to a magnetic flux emergence leading to observed periodicities in solar activity indicators related to magnetic flux.

Key words: Sun: oscillations – magnetic fields – magnetohydrodynamics (MHD) – waves

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Let us first consider 2D case neglecting h

$$u_\theta = \frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\phi},\; u_\phi = -\frac{\partial\Psi}{\partial\theta},\; b_\theta = \frac{1}{\sin\theta}\frac{\partial\Phi}{\partial\phi},\; b_\phi = -\frac{\partial\Phi}{\partial\theta}.$$

$\exp[im(\phi - ct)]$

$$(\Omega_d - \omega)L\Psi + \left(2 - \frac{d^2}{d\mu^2}[\Omega_d(1-\mu^2)]\right)\Psi - \beta^2 BL\Phi$$
$$+ \beta^2\frac{d^2}{d\mu^2}[B(1-\mu^2)]\Phi = 0 \qquad (10)$$

$$(\Omega_d - \omega)\Phi = B\Psi, \qquad (11)$$

where $\mu = \cos\theta$,

$$L = \frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} - \frac{m^2}{1-\mu^2}$$

$$\Omega_d(\mu) = \frac{\Omega_1(\mu)}{\Omega_0},\; \omega = \frac{c}{\Omega_0},\; \beta^2 = \frac{B_0^2}{4\pi\rho\Omega_0^2R_0^2},\; B(\mu) = \frac{B_\phi(\mu)}{B_0}.$$

In this section, we derive the analytical instability bounds using a well-known technique (Howard 1961; Drazin & Reid 1981; Watson 1981; Gilman & Fox 1997; Dahlburg et al. 1998; Hughes & Tobias 2001).

We define a new function

$$\Psi = (\Omega_d - \omega)H, \quad \Phi = BH.$$

$$\frac{\partial}{\partial \mu}(1-\mu^2)P(\mu)\frac{\partial H}{\partial \mu} - \frac{m^2}{1-\mu^2}P(\mu)H + 2(\Omega_d - \omega) \times [1 + (\mu\Omega_d)']H - 2\beta^2 B(\mu B)'H = 0,$$

where

$$P(\mu) = (\Omega_d - \omega)^2 - \beta^2 B^2$$

Now, multiplying Equation (12) by H^*, integrating from -1 to 1 and using the boundary conditions $H(\mu = \pm 1) = 0$, we get

$$\int_{-1}^{1} P(\mu) Q d\mu - \int_{-1}^{1} 2(\Omega_d - \omega)[1 + (\mu\Omega_d)'] |H|^2 d\mu + \int_{-1}^{1} 2\beta^2 B(\mu B)' |H|^2 d\mu = 0, \tag{13}$$

where

$$Q = (1 - \mu^2) \left| \frac{\partial H}{\partial \mu} \right|^2 + \frac{m^2}{1 - \mu^2} |H|^2 > 0.$$

The real and imaginary parts of Equation (13) with $\omega = \omega_r + i\omega_i$ are

$$\int_{-1}^{1} [(\Omega_d - \omega_r)^2 - \omega_i^2 - \beta^2 B^2] Q d\mu - \int_{-1}^{1} 2(\Omega_d - \omega_r) \times [1 + (\mu\Omega_d)'] |H|^2 d\mu + \int_{-1}^{1} 2\beta^2 B(\mu B)' |H|^2 d\mu = 0 \tag{B1}$$

and

$$2i\omega_i \left[\int_{-1}^{1} (\Omega_d - \omega_r) Q d\mu - \int_{-1}^{1} [1 + (\mu\Omega_d)'] |H|^2 d\mu \right] = 0. \tag{B2}$$

Unstable harmonics should have non-zero ω_i, therefore, Equation (B2) requires

$$\int_{-1}^{1} (\Omega_d - \omega_r) Q d\mu = \int_{-1}^{1} [1 + (\mu\Omega_d)'] |H|^2 d\mu.$$

The substitution of $\int_{-1}^{1} \Omega_d Q d\mu$ from this equation into Equation (B1) leads to the equation

$$\int_{-1}^{1} [\Omega_d^2 - \omega_r^2 - \omega_i^2 - \beta^2 B^2] Q d\mu - \int_{-1}^{1} 2\Omega_d [1 + (\mu\Omega_d)'] \times |H|^2 d\mu + \int_{-1}^{1} 2\beta^2 B(\mu B)' |H|^2 d\mu = 0, \tag{B3}$$

$$\int_{-1}^{1}[\Omega_d^2-\omega_r^2-\omega_i^2-\beta^2B^2](1-\mu^2)\left|\frac{\partial H}{\partial\mu}\right|^2 d\mu$$
$$+\int_{-1}^{1}\left[\Omega_d^2-\omega_r^2-\omega_i^2-\beta^2B^2-2\Omega_d[1+(\mu\Omega_d)']\frac{1-\mu^2}{m^2}\right.$$
$$\left.+2\beta^2B(\mu B)'\frac{1-\mu^2}{m^2}\right]\frac{m^2}{1-\mu^2}|H|^2 d\mu=0.$$

This equation will be satisfied if both integrals are zero, which requires

$$\left(\Omega_d^2-\beta^2B^2\right)_{\min}\leqslant\omega_r^2+\omega_i^2\leqslant(\Omega_d^2-\beta^2B^2)_{\max} \quad \text{(B4)}$$

and

$$\left(\Omega_d^2-\beta^2B^2-2\Omega_d[1+(\mu\Omega_d)']\frac{1-\mu^2}{m^2}+2\beta^2B(\mu B)'\frac{1-\mu^2}{m^2}\right)_{\min}$$
$$\leqslant\omega_r^2+\omega_i^2\leqslant\left(\Omega_d^2-\beta^2B^2-2\Omega_d[1+(\mu\Omega_d)']\frac{1-\mu^2}{m^2}\right.$$
$$\left.+2\beta^2B(\mu B)'\frac{1-\mu^2}{m^2}\right)_{\max}. \quad \text{(B5)}$$

The first condition states that the instability takes place when

$$\omega_r^2 + \omega_i^2 \leqslant R_1^2,$$

$$R_1^2 = [(s_2\mu^2 + s_4\mu^4)^2 - \beta^2\mu^2]_{\max}.$$

This means that the frequencies of unstable harmonics (actually phase speeds, while frequencies can be obtained by multiplying by m) lay inside the upper semicircle of complex ω-plane with center at the origin and radius R_1 (see Figure 1).

The second condition implies Howard semicircle theorem

$$\int_{-1}^{1} (\Omega_d - \Omega_{d\min})(\Omega_d - \Omega_{d\max}) Q \, d\mu \leqslant 0.$$

After some algebra

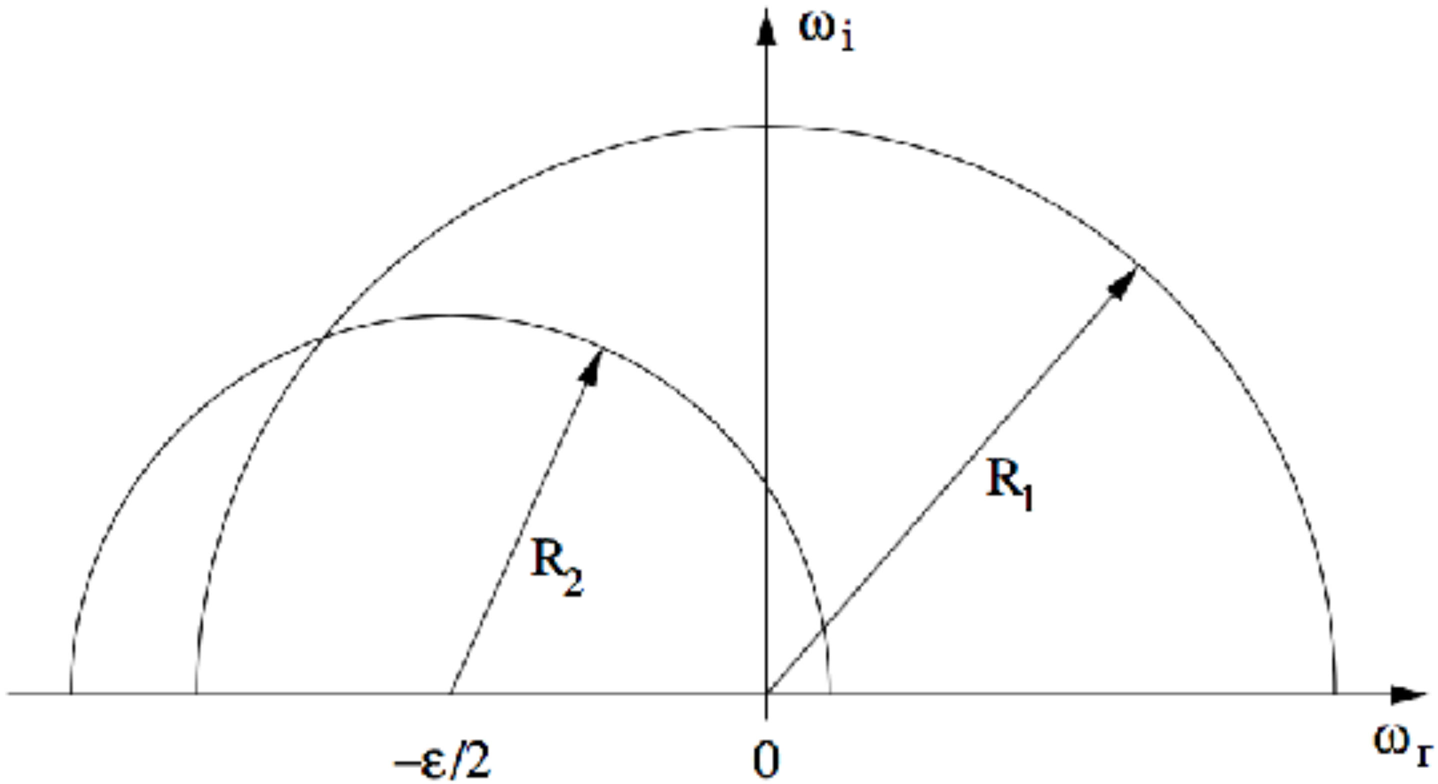
$$\left(\omega_r - \frac{\Omega_{d\min} + \Omega_{d\max}}{2}\right)^2 + \omega_i^2 - \left(\frac{\Omega_{d\min} + \Omega_{d\max}}{2}\right)^2 + \Omega_{d\min}\Omega_{d\max} - A_{\max} \leqslant 0,$$

$$A(\mu) = \frac{1-\mu^2}{m^2}(\Omega_{d\min} + \Omega_{d\max} - 2\Omega_d)[1 + (\mu\Omega_d)'] + \frac{1-\mu^2}{m^2} 2\beta^2 B(\mu B)' - \beta^2 B^2.$$

$\Omega_{d\max} = 0$ and $\Omega_{d\min} = -\epsilon$, where $\epsilon = s_2 + s_4$,

$$\left(\omega_r + \frac{\epsilon}{2}\right)^2 + \omega_i^2 \leqslant \frac{\epsilon^2}{4} + A_{\max}.$$

$$\left(-\frac{\epsilon}{2}, 0\right) \qquad R_2 = \sqrt{\frac{\epsilon^2}{4} + A_{\max}}.$$



The Equations are two necessary conditions of instability. They define two different semicircles in the complex ω-plane, and the instability occurs when the two semicircles overlap (see Hughes & Tobias 2001 for the same statement in the rectangular case). If the radius of one semicircle tends to zero, the instability disappears.

We use an unperturbed toroidal magnetic field, which changes sign at the equator and vanish at poles (Gilman and Fox 1997)

$$B_\varphi = B_0 \cos\theta \sin\theta$$

For $s_2 = s_4 = 0.13$ and the typical values of the tachocline

$\Omega_0 = 2.7\times10^{-6}\ s^{-1}$, $R_0 = 5\times10^{10}$ cm, r=0.2 g cm^{-3}, $B_0 = 10^4$ G

the minimum period of the m=1 unstable mode is

$$T_{min} \approx 105 \text{ days.}$$

Therefore only the magnetic Rossby modes with periods longer than 105 days may grow in time.

However, it only gives lower bound on the periods of unstable modes.

A more detailed analysis is required to reveal the spectrum of unstable harmonics.

We expand Ψ and Φ in infinite series of associated Legendre polynomials

$$\Psi = \sum_{n=m}^{\infty} a_n P_n^m(\mu), \quad \Phi = \sum_{n=m}^{\infty} b_n P_n^m(\mu),$$

which satisfy the boundary conditions Ψ=Φ=0 at μ=±1.

Using a recurrence relation of Legendre polynomials,

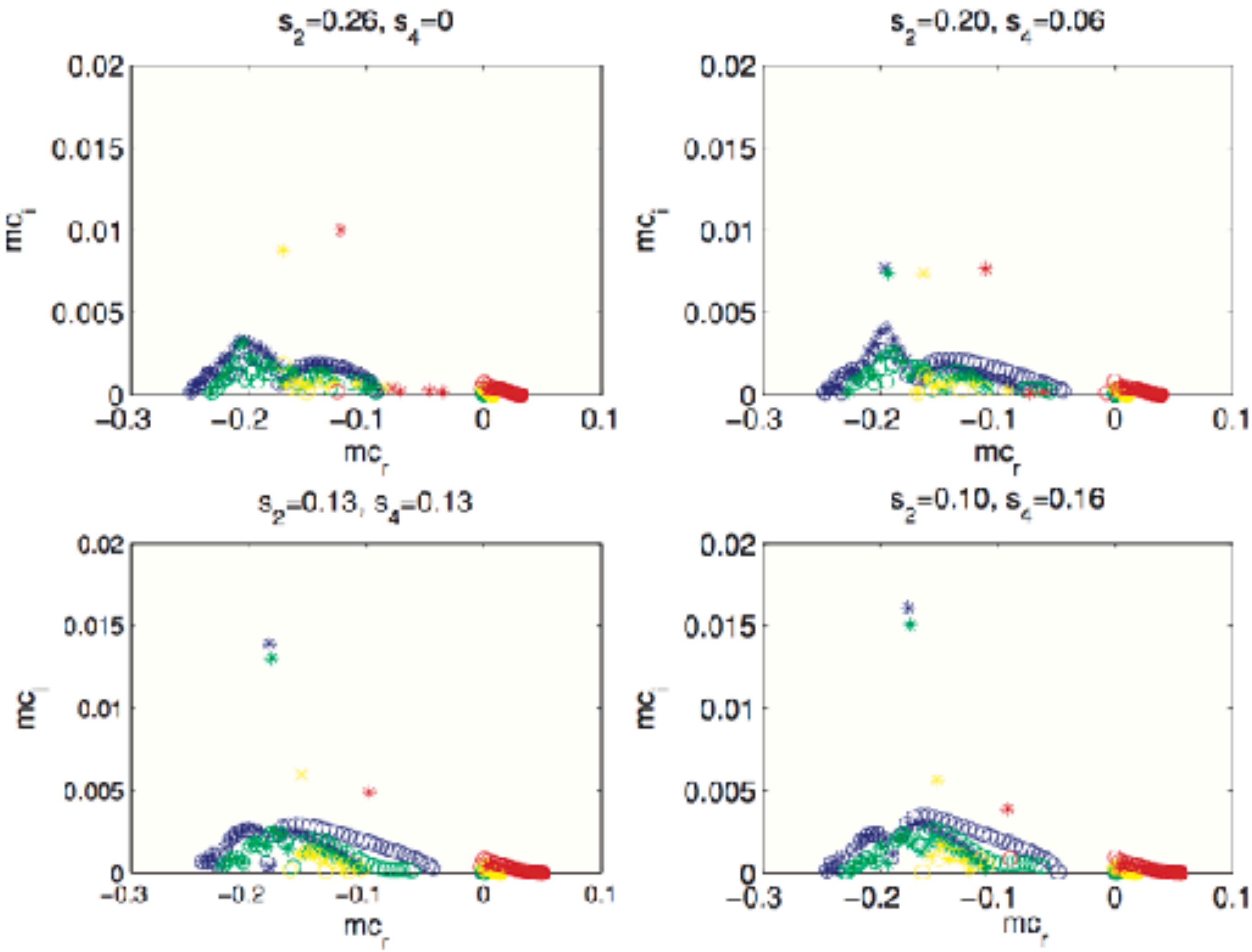
$$\mu^2 P_n^m = A_n P_{n-2}^m + B_n P_n^m + C_n P_{n+2}^m,$$

$$\mu P_n^m = D_n P_{n-1}^m + E_n P_{n+1}^m,$$

we obtain algebraic equations as infinite series.

The dispersion relation for the infinite number of harmonics can be obtained when the infinite determinant of the system is set to zero.

In order to solve the determinant, we truncate the series at n = 75 and solve the resulting polynomial in ω numerically. The frequencies of different harmonics can be real or complex giving the stable or unstable character of a particular harmonic.



Blue – 2×10^3 G, green - 6×10^3 G, yellow - 2×10^4 G, red - 4×10^4 G.

mc_r =0.18 corresponds to the period of 150 days.

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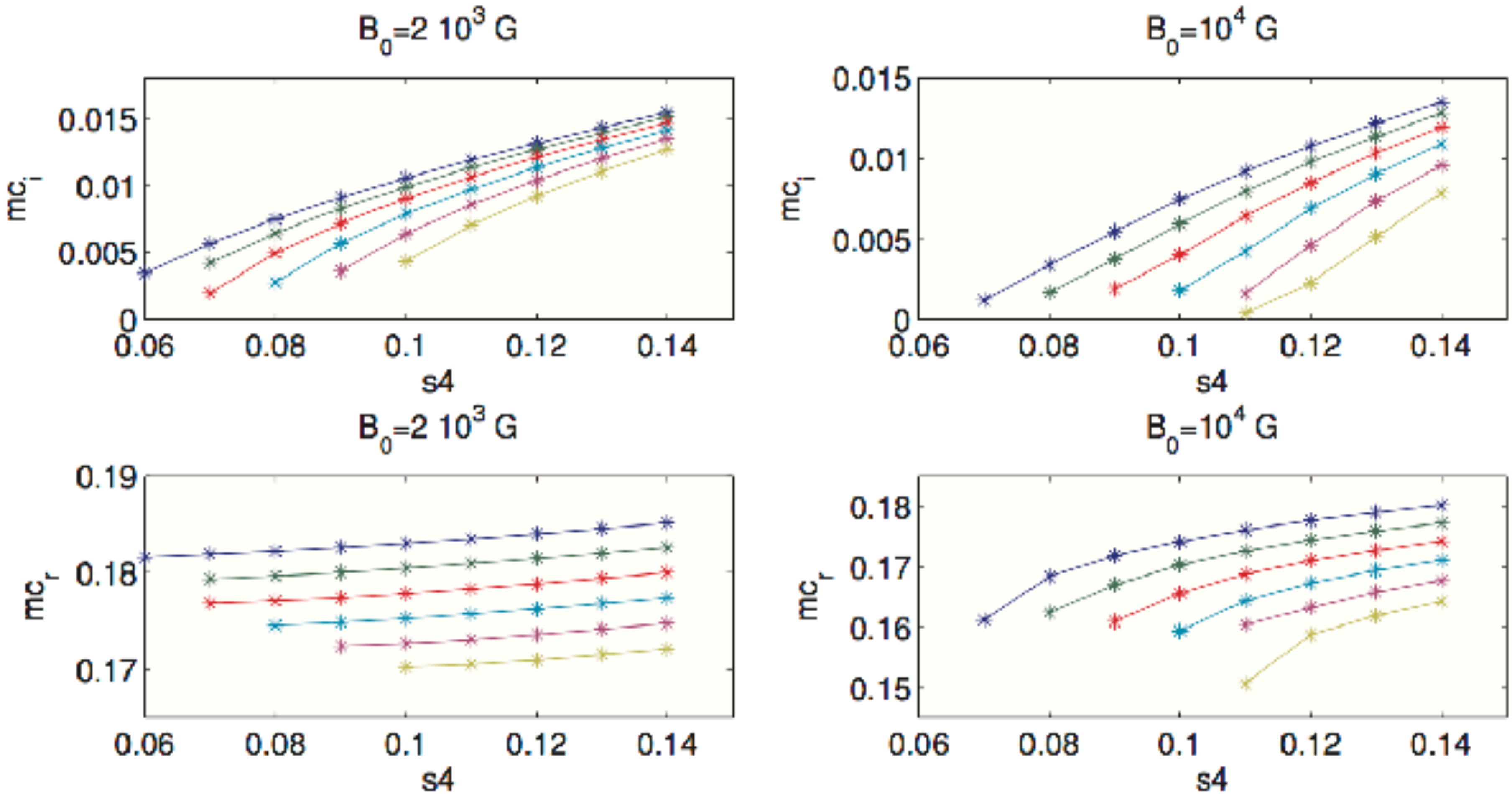

Dark blue, green, red, blue, magenta and yellow colors correspond to 0.14, 0.13, 0.12, 0.11, 0.1 and 0.09 s_2 respectively.

Zaqarashvili et al. 2010a

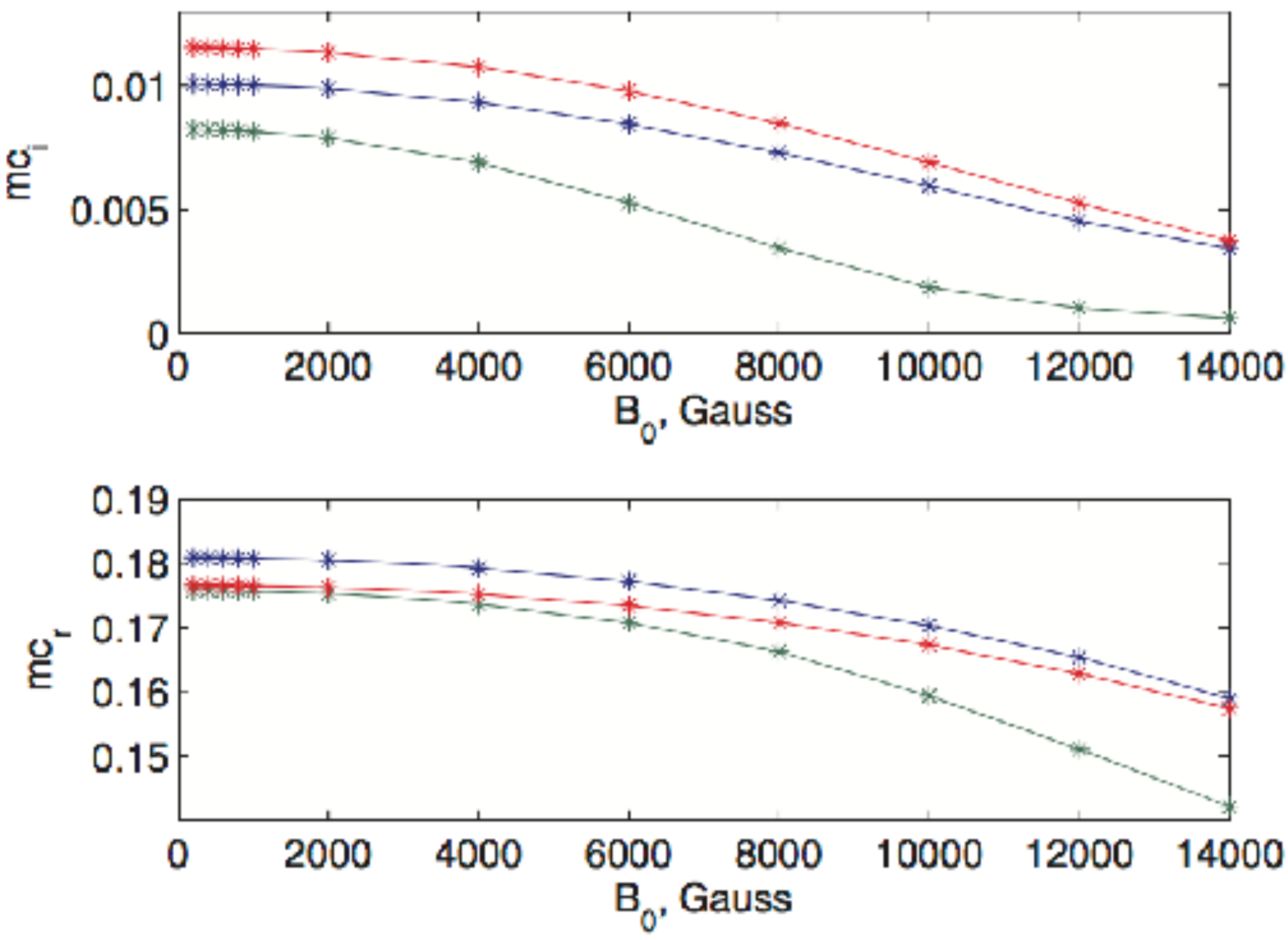

Figure 4. Dependence of real (lower panel) and imaginary (upper panel) parts of the frequency of the most unstable symmetric harmonic on the magnetic field strength for three different combinations of differential rotation parameters. The blue, green, and red lines correspond to ($s_2 = 0.13$, $s_4 = 0.1$), ($s_2 = 0.11$, $s_4 = 0.1$), and ($s_2 = 0.11$, $s_4 = 0.12$) respectively.

Zaqarashvili et al. 2010a

- Each combination of the differential rotation parameters (s_2 ,s_4) and the magnetic field strength favors a particular harmonic, which has stronger growth rate compared to other unstable harmonics.

- Therefore, this harmonic may quickly dominate over the others and may lead to a detectable oscillation.

- Frequencies of symmetric unstable modes are in the range 0.16–0.18 Ω_0, which yield the periods of 150– 170 days.

- Variation of differential rotation rate and the magnetic field strength through the solar cycle and from cycle to cycle may lead to the appearance of the periodicity only at particular times, which normally coincides to the cycle maxima.

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QUASI-BIENNIAL OSCILLATIONS IN THE SOLAR TACHOCLINE CAUSED BY MAGNETIC ROSSBY WAVE INSTABILITIES

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ABSTRACT

Quasi-biennial oscillations (QBOs) are frequently observed in solar activity indices. However, no clear physical mechanism for the observed variations has been suggested so far. Here, we study the stability of magnetic Rossby waves in the solar tachocline using the shallow water magnetohydrodynamic approximation. Our analysis shows that the combination of typical differential rotation and a toroidal magnetic field with a strength of $\geqslant 10^5$ G triggers the instability of the $m = 1$ magnetic Rossby wave harmonic with a period of ~2 years. This harmonic is antisymmetric with respect to the equator and its period (and growth rate) depends on the differential rotation parameters and magnetic field strength. The oscillations may cause a periodic magnetic flux emergence at the solar surface and consequently may lead to the observed QBO in solar activity features. The period of QBOs may change throughout a cycle, and from cycle to cycle, due to variations of the mean magnetic field and differential rotation in the tachocline.

Key words: magnetic fields – magnetohydrodynamics (MHD) – Sun: oscillations – waves

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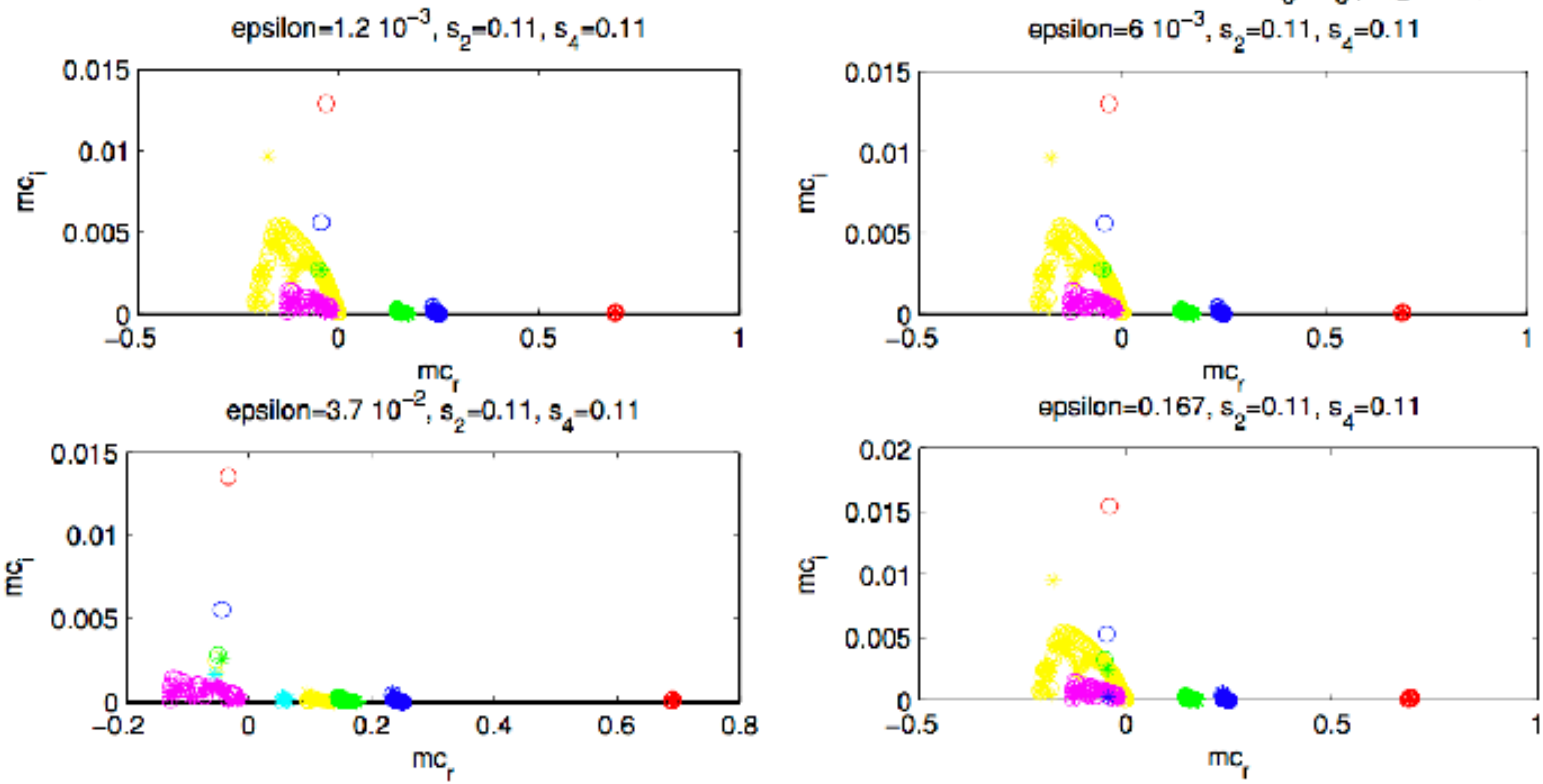
Now let us take layer thickness to be finite. Then after repeating of the same procedure we have the similar results $\epsilon = \Omega_0^2 R_0^2/(g H_0)$



Yellow-2×10³ G, magenta-2×10⁴ G, blue-6×10⁴ G, green-8×10⁴ G, dark blue-10⁵ G, red-2×10⁵ G. Asterisks (circles) denote the symmetric (antisymmetric) harmonics with respect to the equator.

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The unstable harmonics are mostly symmetric (asterisks) with respect to the equator for a magnetic field strength $<10^4$ G.

They become mostly antisymmetric (circles) for a strength $>10^5$ G.

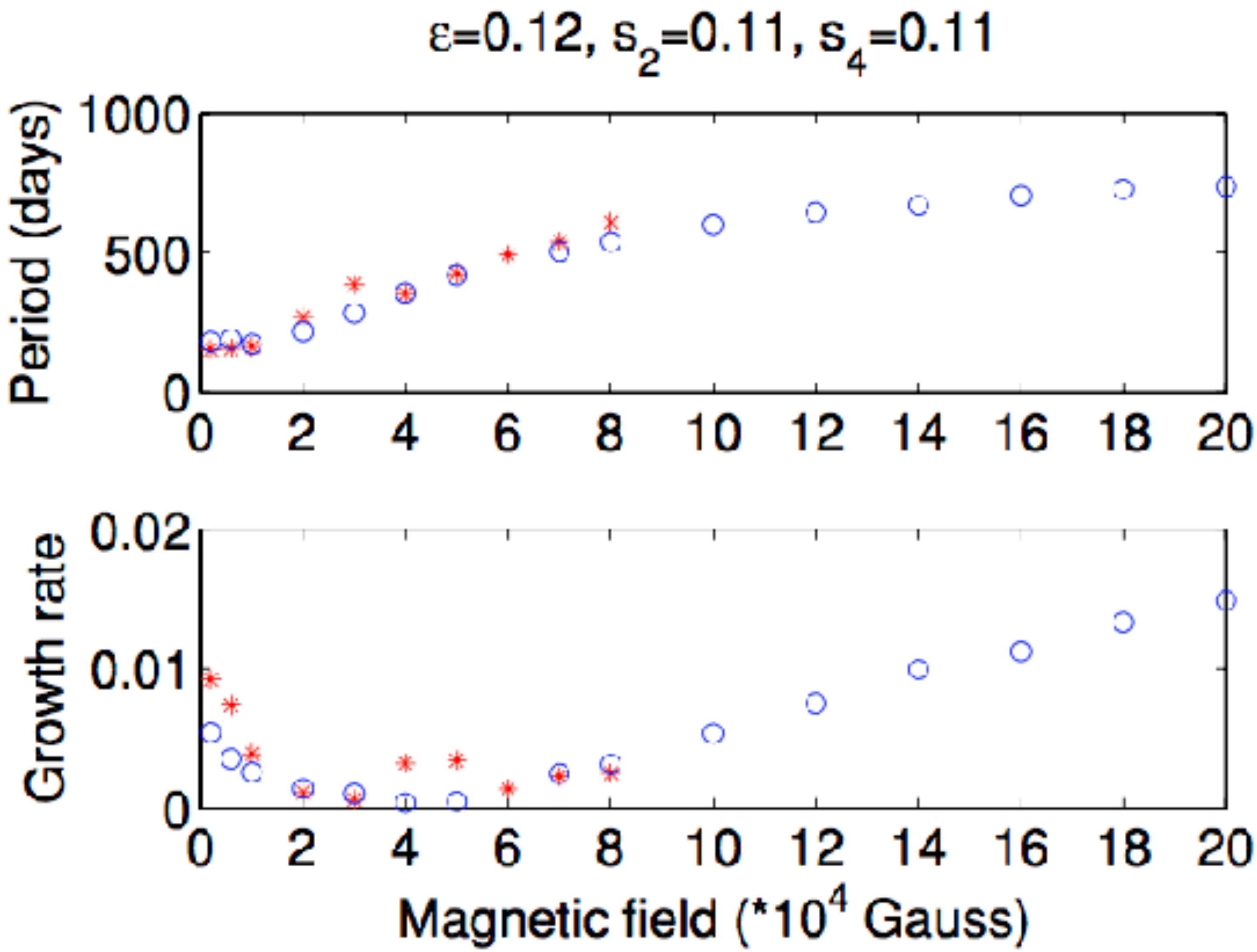
A magnetic field strength between 10^4 and 10^5 G yields unstable harmonics for both symmetries.

Equipartition between the magnetic energy and the kinetic energy of differential rotation occurs at $\sim 5 \times 10^4$ G for s2 = s4 = 0.11.

When the magnetic field strength is smaller, then the differential rotation is the main energy source for instability and this obviously yields the symmetric harmonics as the differential rotation is symmetric around the equator.

When the magnetic field is stronger, then the magnetic energy is the main source for the instability and the unstable harmonics are antisymmetric as the magnetic field is antisymmetric with respect to the equator.

Asterisks (circles) denote symmetric (antisymmetric) harmonics.



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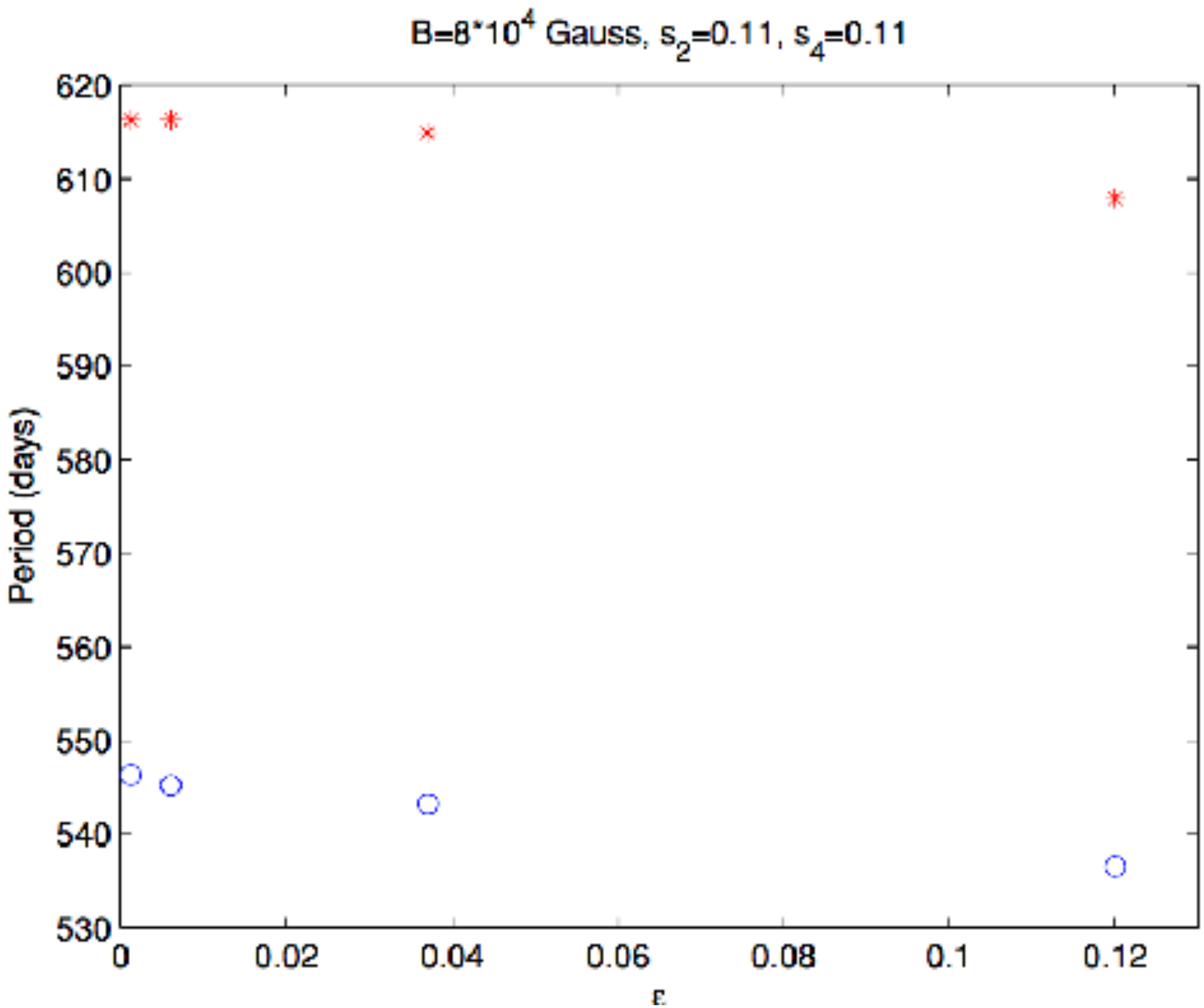

The oscillation period does not depend significantly on the reduced gravity.

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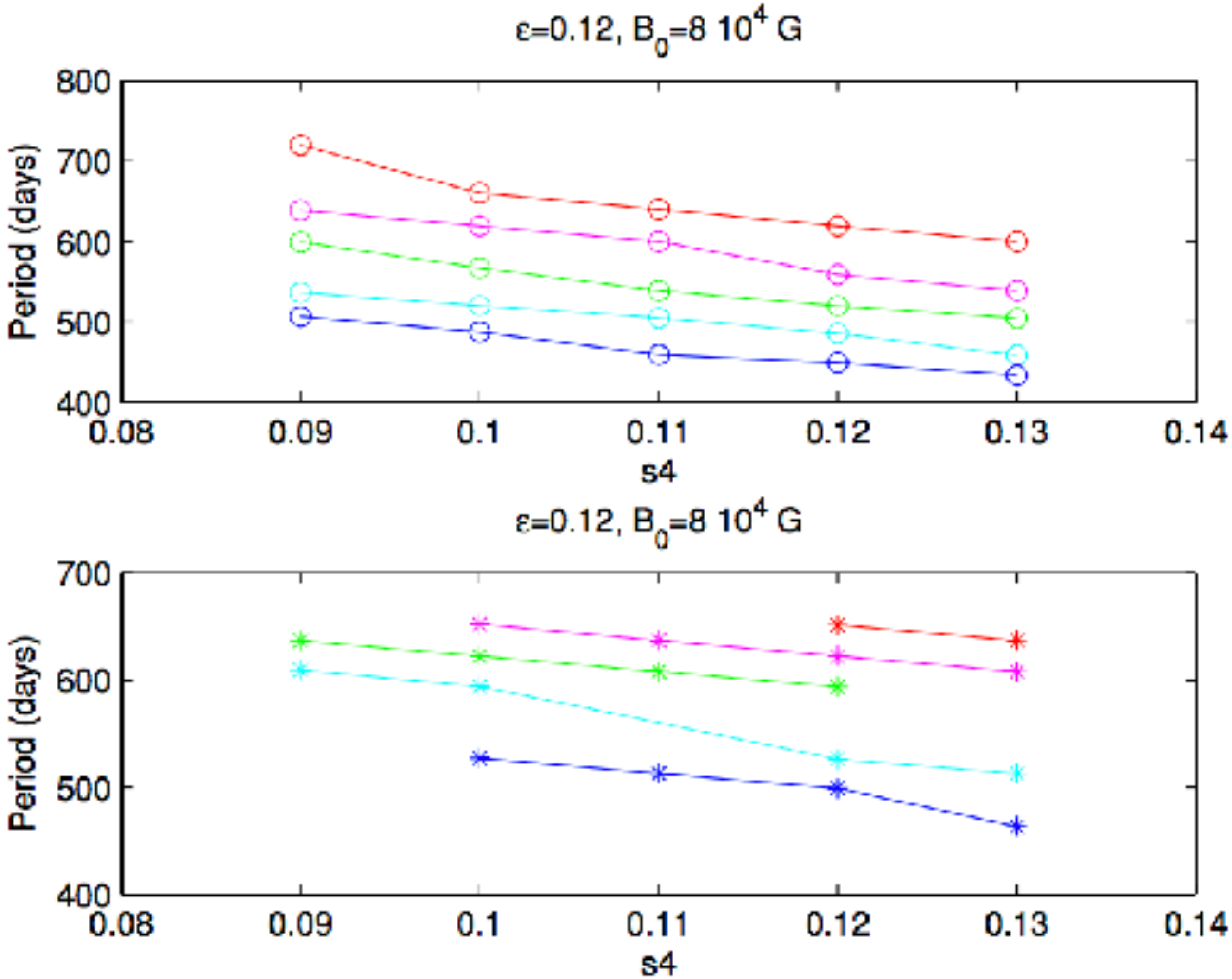

Period depends on the differential rotation parameters significantly and takes the values between 400 and 700 days. The period becomes shorter for stronger differential rotation.

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- The frequencies and growth rates of unstable harmonics depend on the combination of the differential rotation parameters and the magnetic field strength.
- The unstable harmonics are either symmetric or anti-symmetric with respect to the equator.
- The latitudinal differential rotation is mainly responsible for the growth of symmetric harmonics.
- The anti-symmetric toroidal magnetic field favors the growth of anti-symmetric harmonics.
- A magnetic field with a strength of 10^4 G leads to the oscillations with shorter periods (150-170 days).
- A stronger magnetic field of 10^5 G leads to the oscillations with longer periods (1-2.5 yrs).