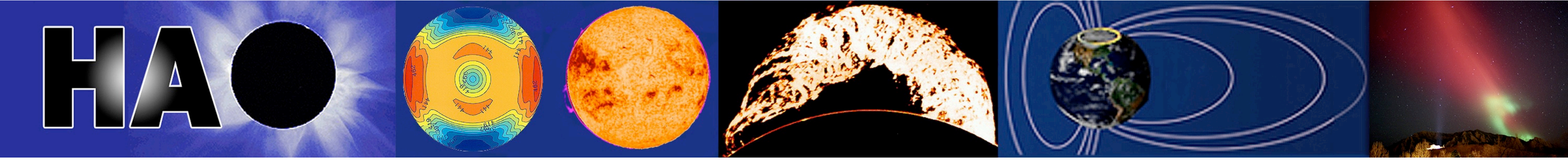


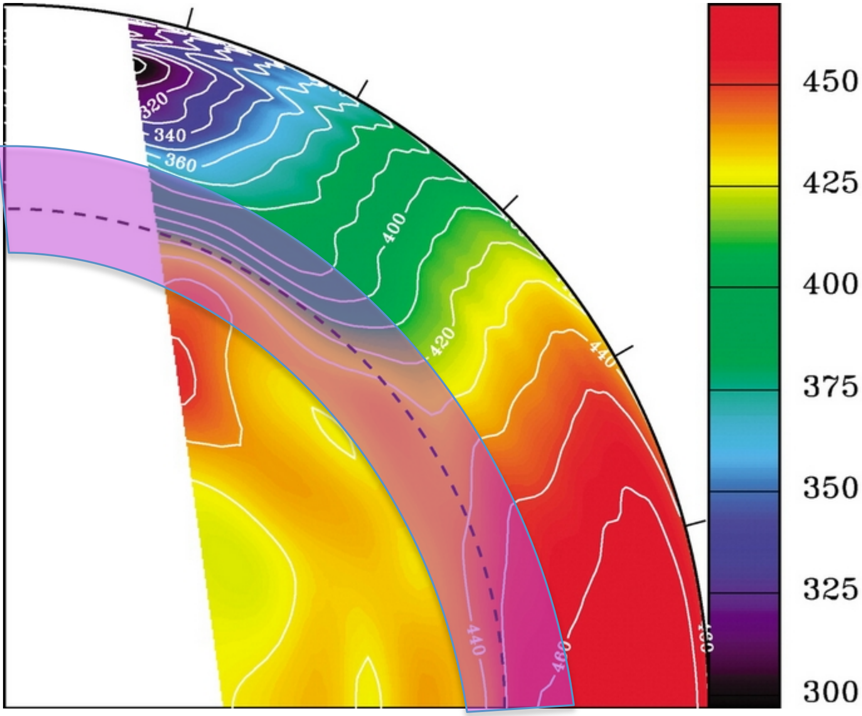
Nonlinear Evolution of "Shallow-water" Instability in Solar Tachocline

Mausumi Dikpati (HAO/NCAR)

NCAR

The National Center for Atmospheric Research is sponsored by the National Science Foundation.

The Solar Tachocline



Thompson et al. (2003)

Brown et al. 1989

Goode et al. 1991

Tomczyk, Chou & Thompson 1995

Kosovichev 1996

Basu 1997

Charbonneau et al. 1997

Corbard et al. 1998

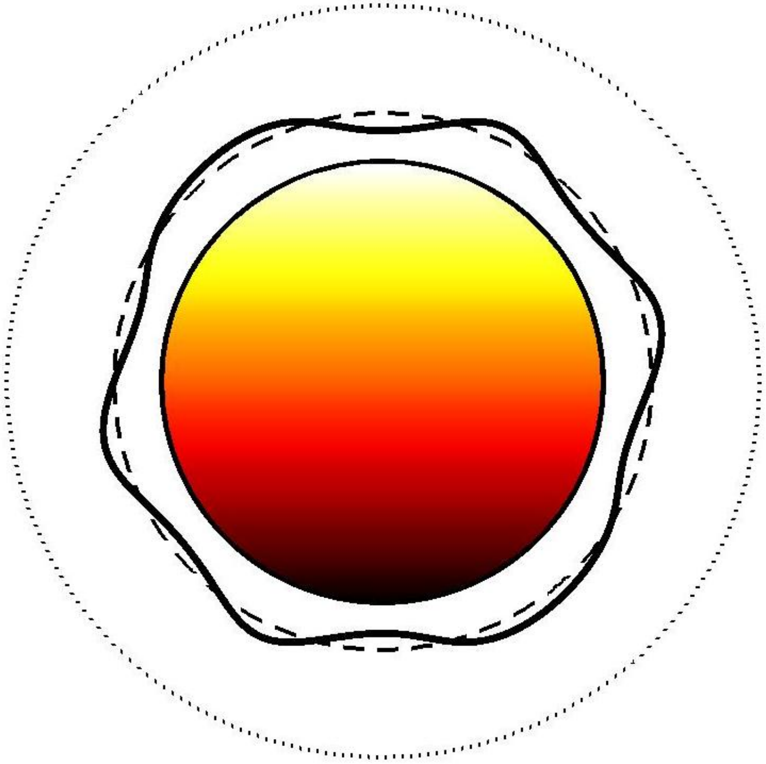
- There exists a thin layer (<0.04R), called "tachocline", which straddles convection zone base (0.7R)
- Tachocline contains strong radial differential rotation along with latitudinal differential rotation at the top that declines to solid rotation at the bottom of tachocline

Global HD/MHD Instabilities in Solar Tachocline

Extensive studies in 2D, 3D and Shallow-water models of solar tachocline indicate the existence of global HD/MHD instabilities in tachocline with low longitudinal wave numbers

(Arlt, Cally, Charbonneau, Dikpati, Dziembowski, Garaud, Gilman, Kosovichev, Miesch, Rempel, Ruediger)

What is MHD Shallow Water System?

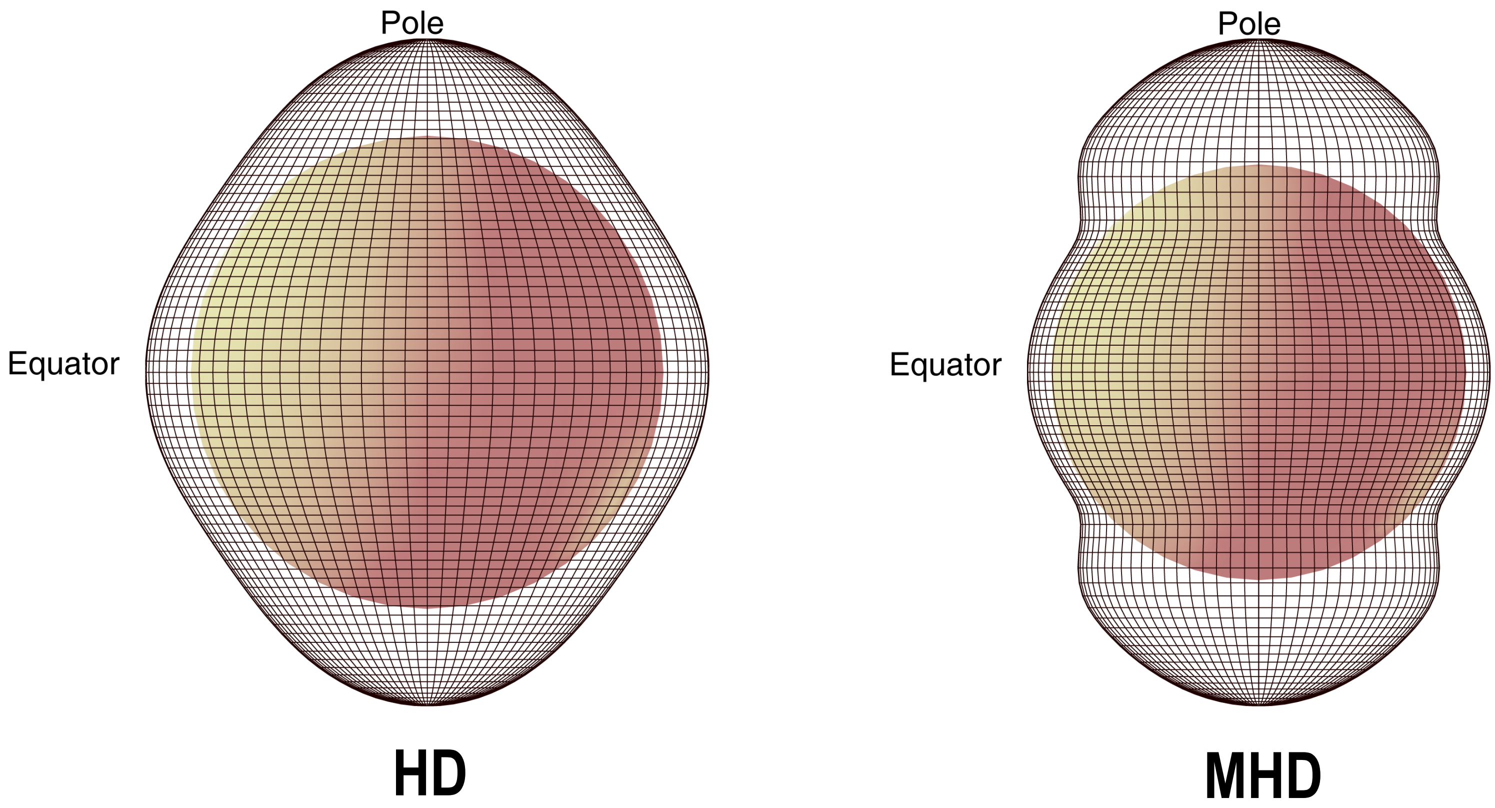


Pedlosky , Longuet & Higgins 1970; Gilman, 2000; Dikpati & Gilman 2001; Dikpati 2012

- Spherical Shell of fluid with outer boundary that can deform
- Horizontal flow, fields in shell are independent of radius
- Vertical flow, field linear functions of radius, zero at inner boundary
- Magnetohydrostatic force balance
- Horizontal gradient of total pressure is proportional to the horizontal gradient of shell thickness
- Horizontal divergence of magnetic flux in a radial column is zero

Equilibrium in Global MHD Tachocline

In general, is a balance among three latitudinal forces, including hydrostatic pressure gradient, magnetic curvature stress, and coriolis forces



Dikpati, Gilman, and Rempel 2003

Effective Gravity Parameter (G)

$$G = \frac{1}{2} \frac{g_t \left| \nabla - \nabla_{ad} \right| H^2}{\left(r_t \omega_c \right)^2 H_p}$$

in which:

g_t	gravity at tachocline depth
$\left\| \nabla - \nabla_{ad} \right\|$	fractional departure from adiabatic temperature gradient
H	thickness of tachocline "shell"
H_p	pressure scale height
r_t	solar radius at tachocline depth
ω_c	rotation of solar interior

G ~ 10^{-1} for Overshoot Tachocline

G ~ 10^{2} for Radiative Tachocline

Shallow Water Equations of Motion and Mass Continuity

$$\frac{\partial u}{\partial t} = -G\frac{1}{\cos\phi}\frac{\partial h}{\partial\lambda} + \frac{v}{\cos\phi}\left[\frac{\partial v}{\partial\lambda} - \frac{\partial}{\partial\phi}(u\cos\phi)\right] - \frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{u^2+v^2}{2}\right)$$

$$-\frac{b}{\cos\phi}\left[\frac{\partial b}{\partial\lambda} - \frac{\partial}{\partial\phi}(a\cos\phi)\right] + \frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{a^2+b^2}{2}\right),$$

$$\frac{\partial v}{\partial t} = -G\frac{\partial h}{\partial\phi} - \frac{u}{\cos\phi}\left[\frac{\partial v}{\partial\lambda} - \frac{\partial}{\partial\phi}(u\cos\phi)\right] - \frac{\partial}{\partial\phi}\left(\frac{u^2+v^2}{2}\right)$$

$$+\frac{a}{\cos\phi}\left[\frac{\partial b}{\partial\lambda} - \frac{\partial}{\partial\phi}(a\cos\phi)\right] + \frac{\partial}{\partial\phi}\left(\frac{a^2+b^2}{2}\right),$$

$$w = \frac{\partial}{\partial t}(1+h) = -\frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left[(1+h)u\right] - \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left[(1+h)v\cos\phi\right],$$

Pedlosky , Longuet & Higgins 1970;

Shallow Water Induction and Flux Continuity Equations

$$\frac{\partial a}{\partial t}=\frac{\partial}{\partial\phi}\left(ub-va\right)+\frac{a}{\cos\phi}\left[\frac{\partial u}{\partial\lambda}+\frac{\partial}{\partial\phi}\left(v\cos\phi\right)\right]-\frac{u}{\cos\phi}\left[\frac{\partial a}{\partial\lambda}+\frac{\partial}{\partial\phi}\left(b\cos\phi\right)\right],$$

$$\frac{\partial b}{\partial t}=-\frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left(ub-va\right)+\frac{b}{\cos\phi}\left[\frac{\partial u}{\partial\lambda}+\frac{\partial}{\partial\phi}\left(v\cos\phi\right)\right]-\frac{v}{\cos\phi}\left[\frac{\partial a}{\partial\lambda}+\frac{\partial}{\partial\phi}\left(b\cos\phi\right)\right],$$

$$\frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left[\left(1+h\right)a\right]+\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left[\left(1+h\right)b\cos\phi\right]=0.$$

(Gilman, 2000; Gilman & Dikpati 2002)

Singular Points

Occur at latitudes where:

$$S_r = \omega_o - c_r = 0$$

$$S_m = \left(\omega_o - c_r\right)^2 - \alpha_o^2 = 0$$

$$S_g = \left(1 - \mu^2\right)\left[\left(\omega_o - c_r\right)^2 - \alpha_o^2\right] - G(1 + h_o)$$

h_σ is departure of shell thickness from uniform thickness

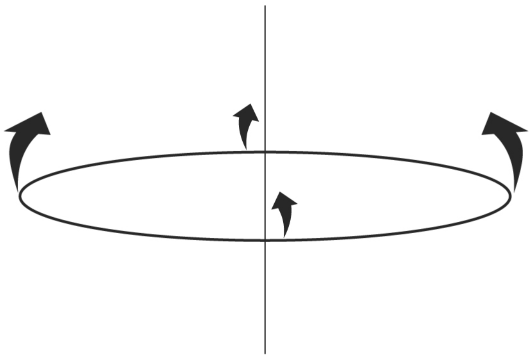
For cases of solar interest:

S_r , $S_m = 0$ are important, $S_g = 0$ is not

- Singular points define places of rapid phase shifts with latitude in unstable modes
- Therefore much of disturbance structure, as well as energy conversion processes, can occur in this neighborhood
- Singular points play major role in interpreting how this instability is driven

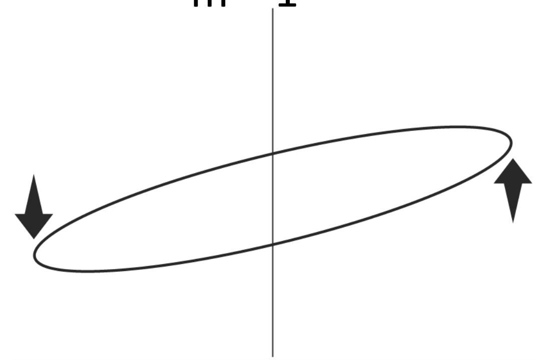
Schematic of Possible Modes of Instability in MHD "Shallow Water" Shell

m = 0



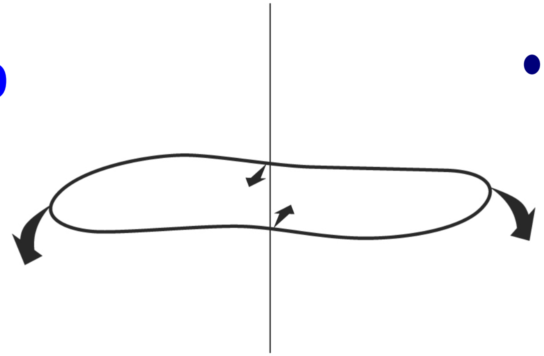
- $\bar{h}$ increases poleward
- Toroidal ring shrinks
- Fluid in ring spins up

m = 1



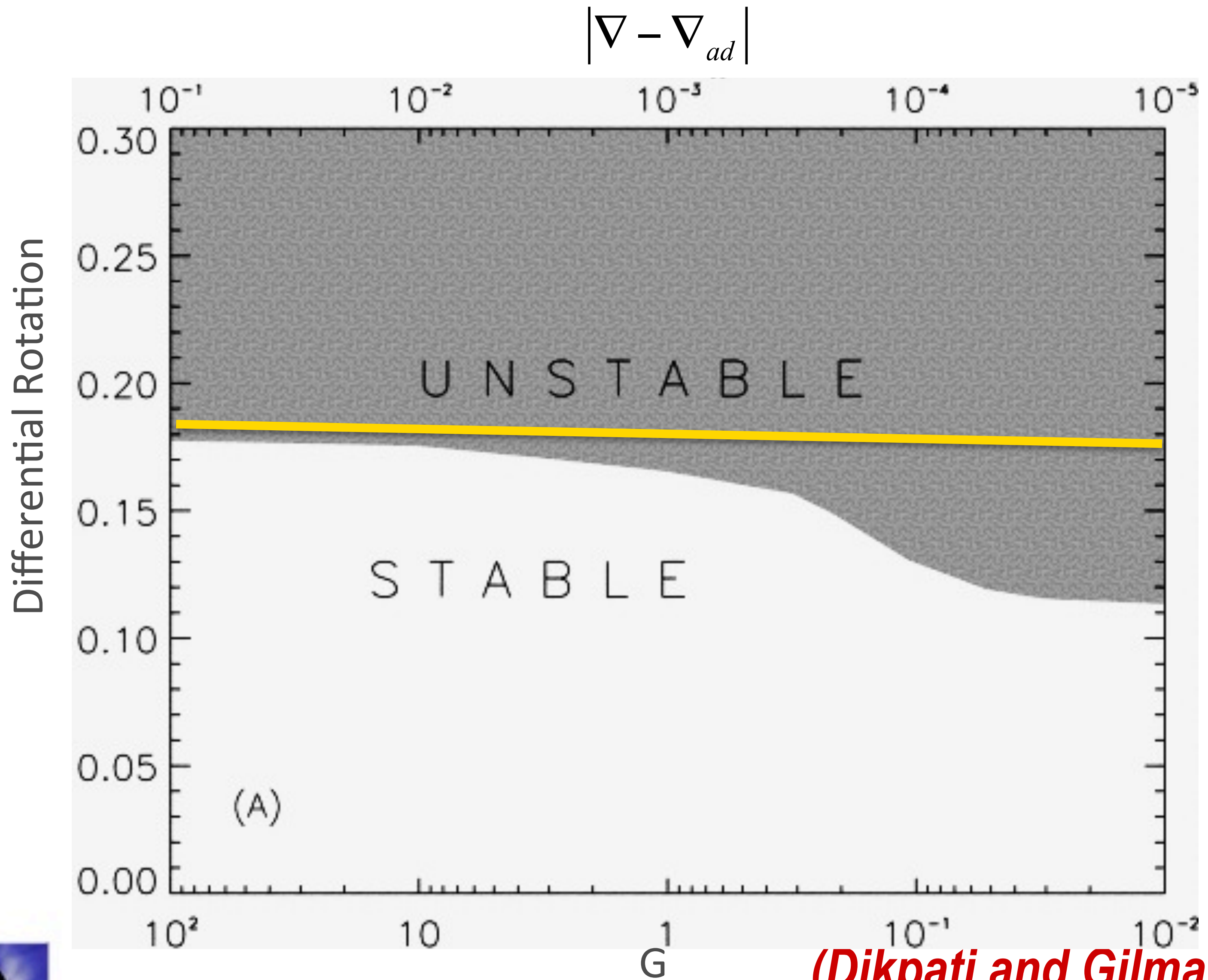
- $\bar{h}$ redistributed but no net rise
- Toroidal ring tips but remains same circumference
- Fluid in ring keeps same speed but flow tips

m = 2



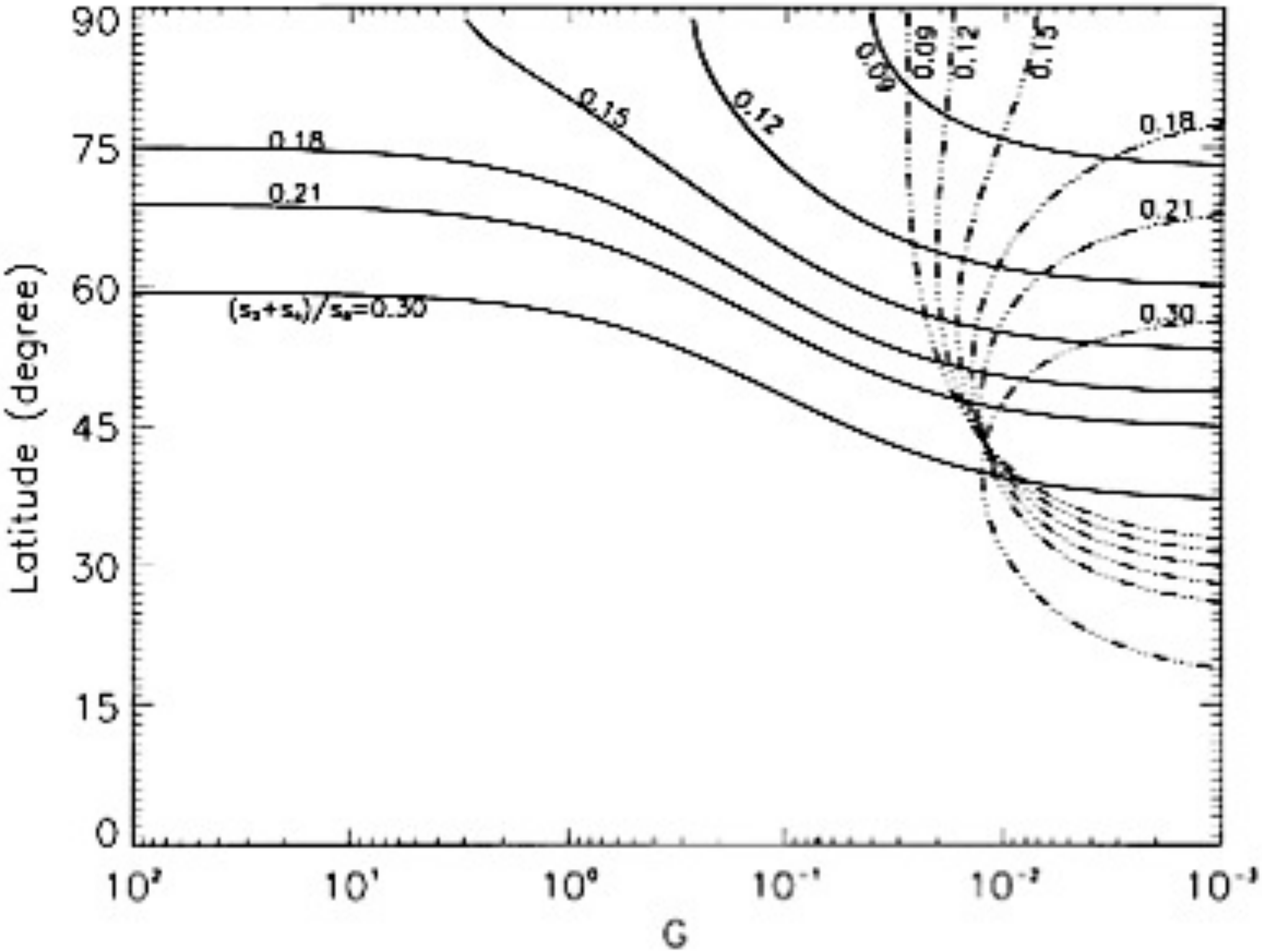
- $\bar{h}$ redistributes but no net poleward rise
- Toroidal ring deforms, creating Maxwell Stress
- Fluid flow inside ring deforms but does not spin up

Stability Diagrams for HD Shallow Water System

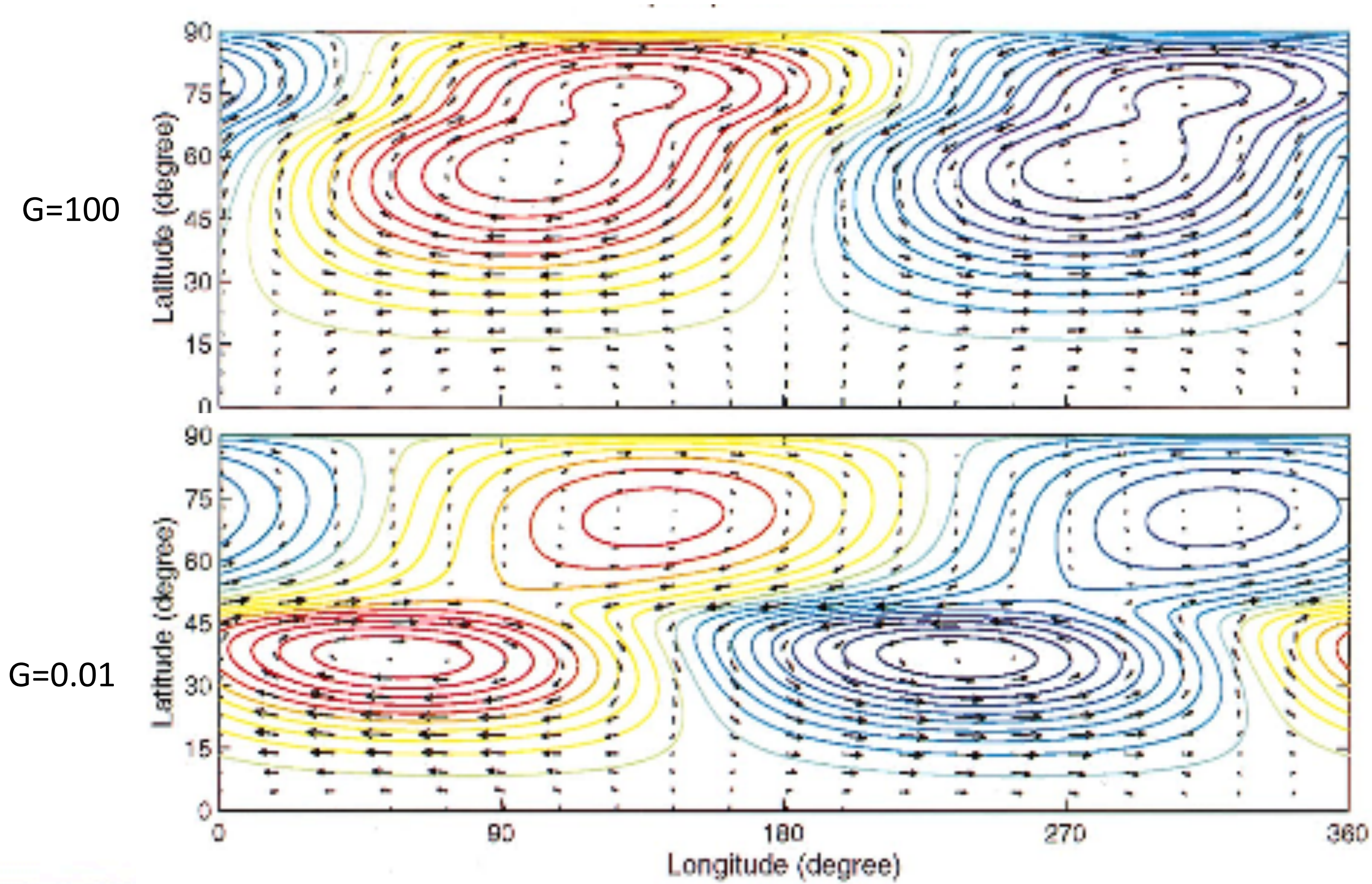


(Dikpati and Gilman, 2001)

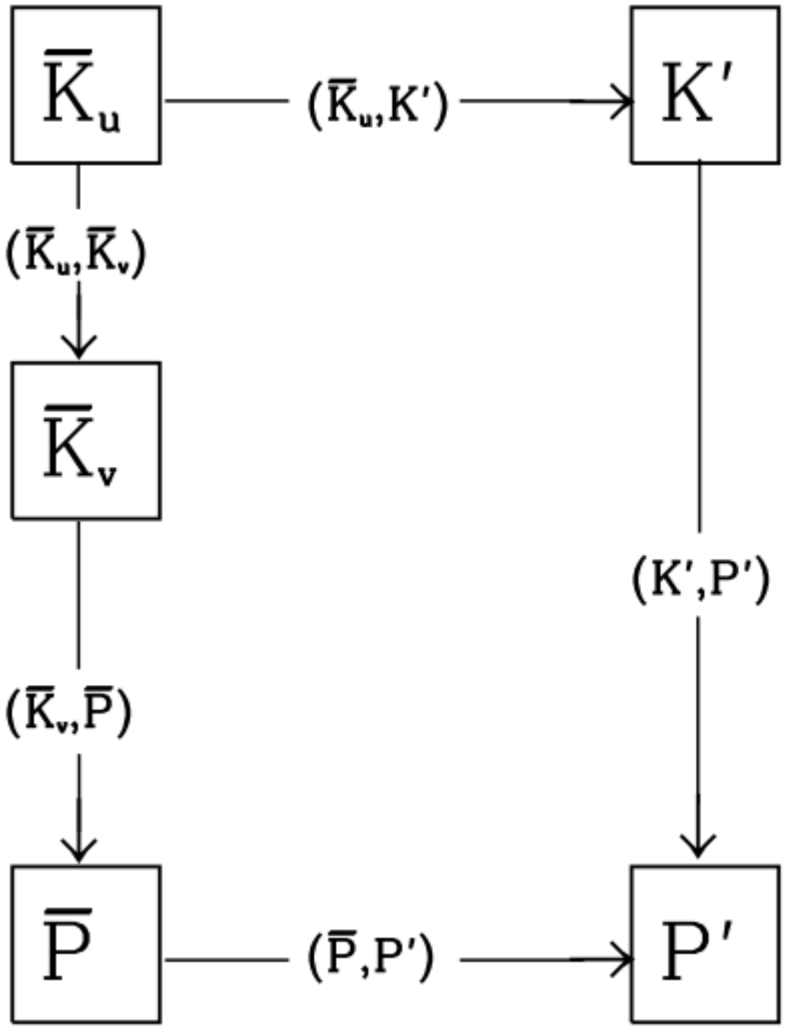
Location of inflection point in potential vorticity



HD Shallow water Instability



Energetics

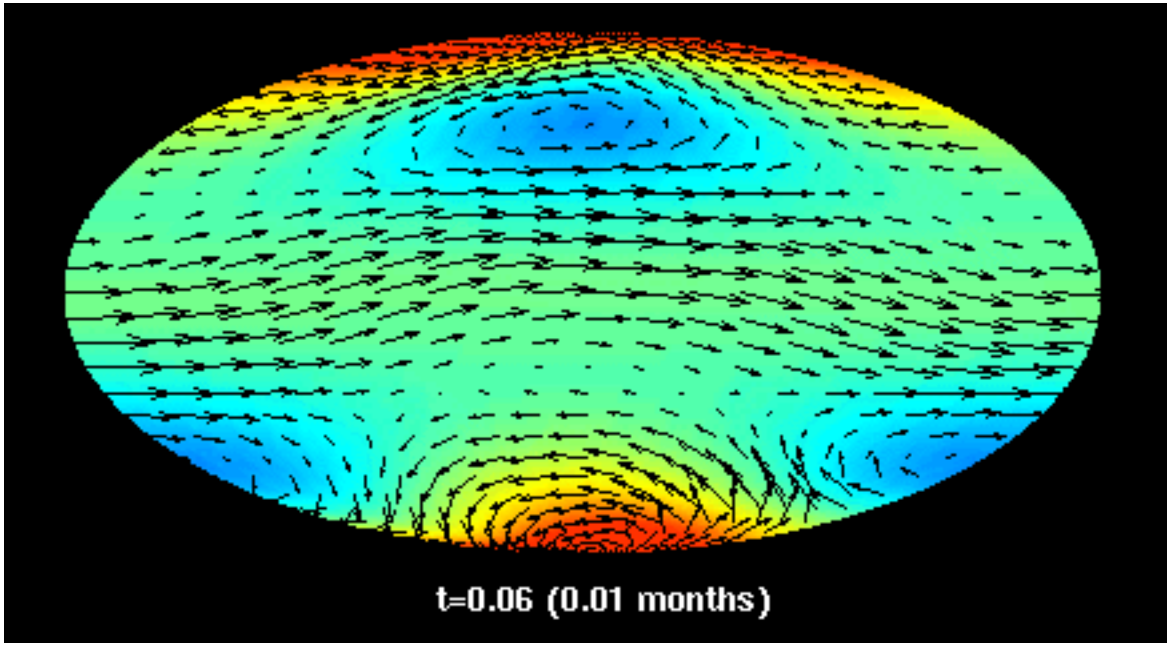

$\overline{K}_u$
$(\overline{K}_u, K')$
K'
$(\overline{K}_u, \overline{K}_v)$
$\overline{K}_v$
(K', P')
$(\overline{K}_v, \overline{P})$
$\overline{P}$
$(\overline{P}, P')$
P'

Solving Nonlinear Shallow-water Equations in Spherical Coordinate

- Start with real space first order shallow-water equations
- Decompose h, u and v in scalar and vector spherical harmonics to deal with the *pole problem*
- Evaluate nonlinear terms by implementing pseudo-spectral scheme (*Swarztrauber 1996*)
- Implement semi-implicit scheme to take the advantage of larger time-step so that high-frequency gravity waves are integrated out *(Hack & Jacob 1992)*
- Implement fourth order Runge-Kutta time integration scheme
- Parallelize forward and backward transforms in shared memory parallelization scheme *(Dikpati 2012)*

Nonlinear evolution of shallow-water instability

Arrow vectors: Global flow (clockwise in swelling, and anticlockwise in depression)
Color shades: Tachocline thickness (red represents swelling, blue depression)
Flow is nearly geostrophic in a hydrodynamic or weakly magnetized tachocline



Features to note in simulation:

- Quasi-periodic oscillations in thickness and velocities
- Slow retrograde propagation relative to rotating frame
- Flow nearly geostrophic in hydrodynamic tachocline, and magnetostrophic in magnetized tachocline
- Occasional appearance of gravity waves

Dikpati 2012

Nonlinear evolution of shallow-water instability

Arrow vectors: Global flow
Color shades: Tachocline thickness (red swelling, blue depression)

Flow is nearly geostrophic in a hydrodynamic or weakly magnetized tachocline

Features to note in simulation:

- Quasi-periodic oscillations in thickness and velocities
- Slow retrograde propagation relative to rotating frame
- Flow nearly geostrophic in hydrodynamic tachocline, and magnetostrophic in magnetized tachocline
- Occasional appearance of gravity waves

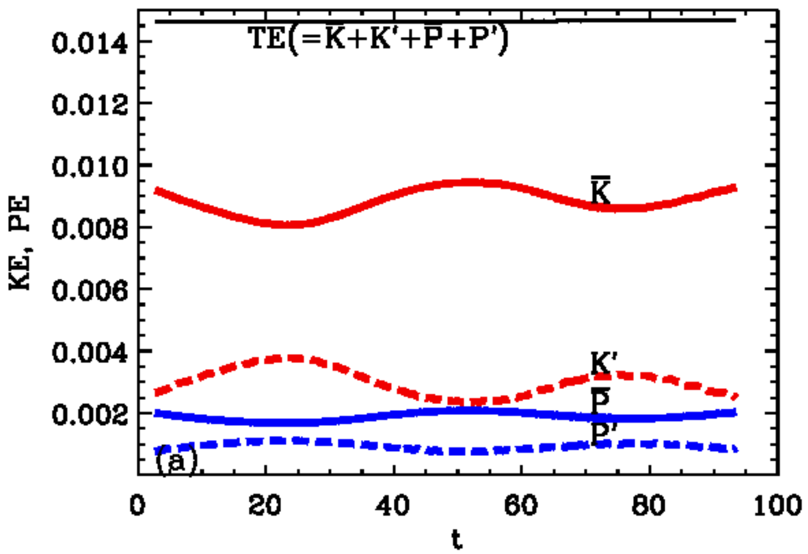
Dikpati 2012

Evolution of kinetic and potential energies

- Interaction between reference state and perturbation kinetic energies is oscillatory

- These oscillations are related to oscillatory interaction between Rossby waves and differential rotation

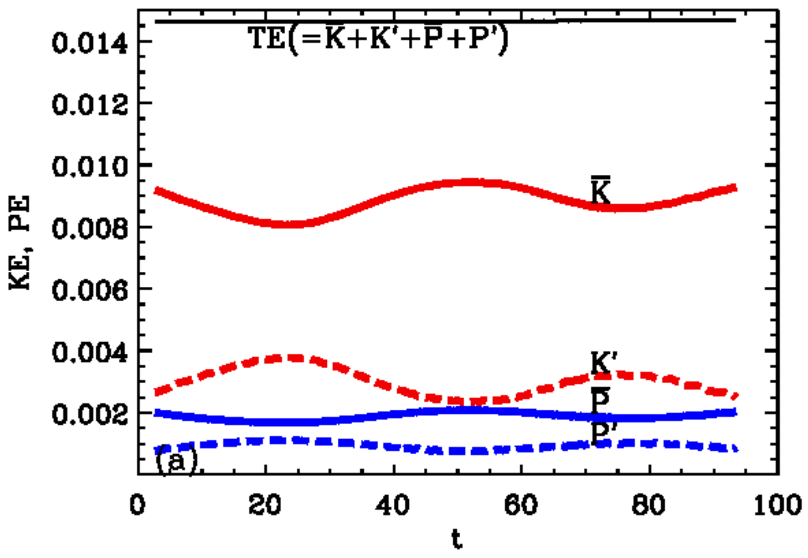
- These periodicities organize space weather that influences the earth

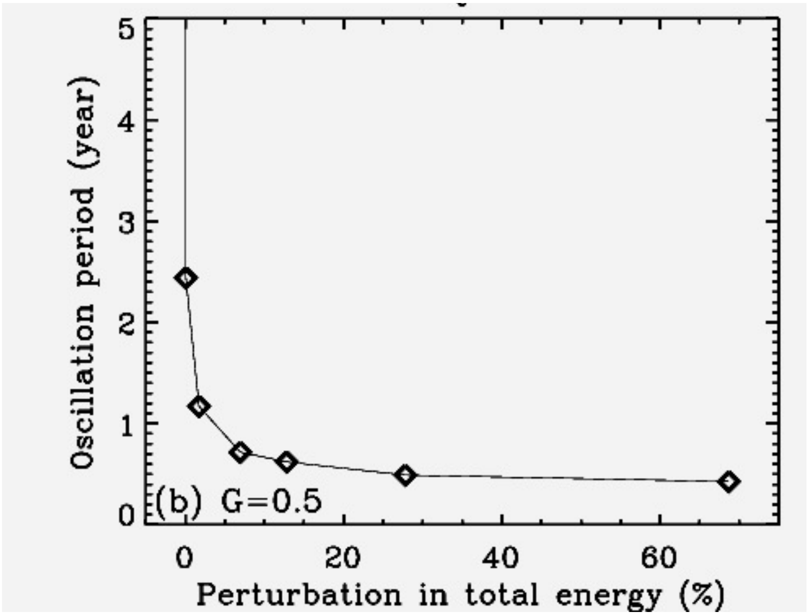


(Dikpati 2012)

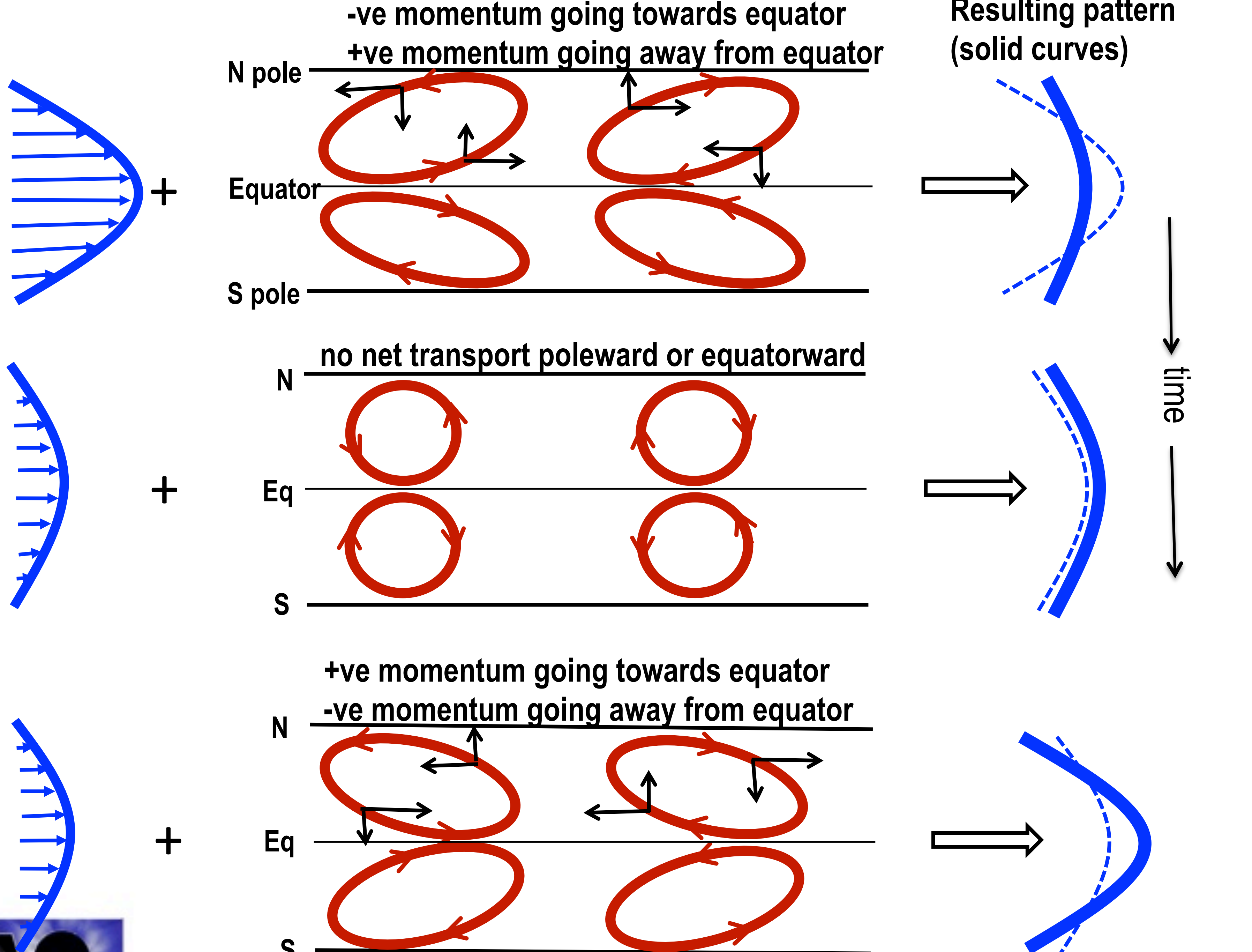
Evolution of kinetic and potential energies

- Interaction between reference state and perturbation kinetic energies is oscillatory
- These oscillations are related to oscillatory interaction between Rossby waves and differential rotation
- These periodicities organize space weather that influences the earth
- Oscillation frequency is a function of perturbation in total energy



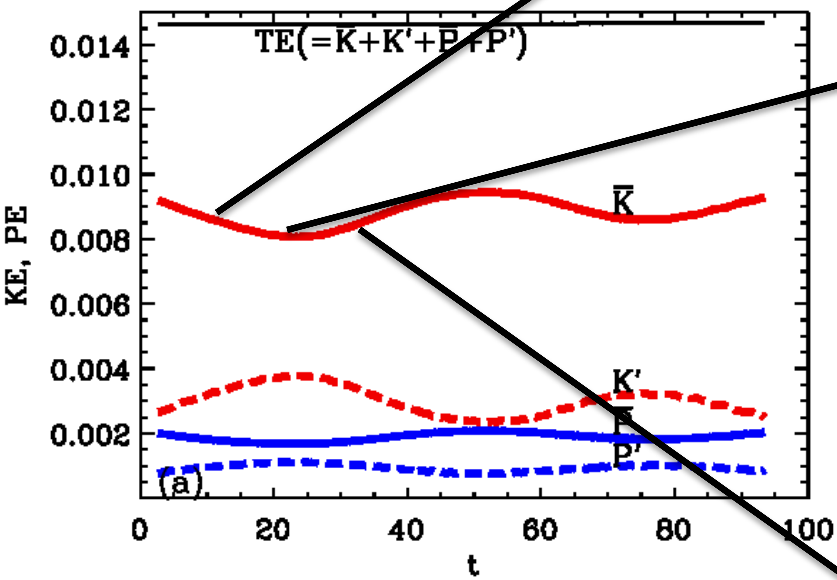


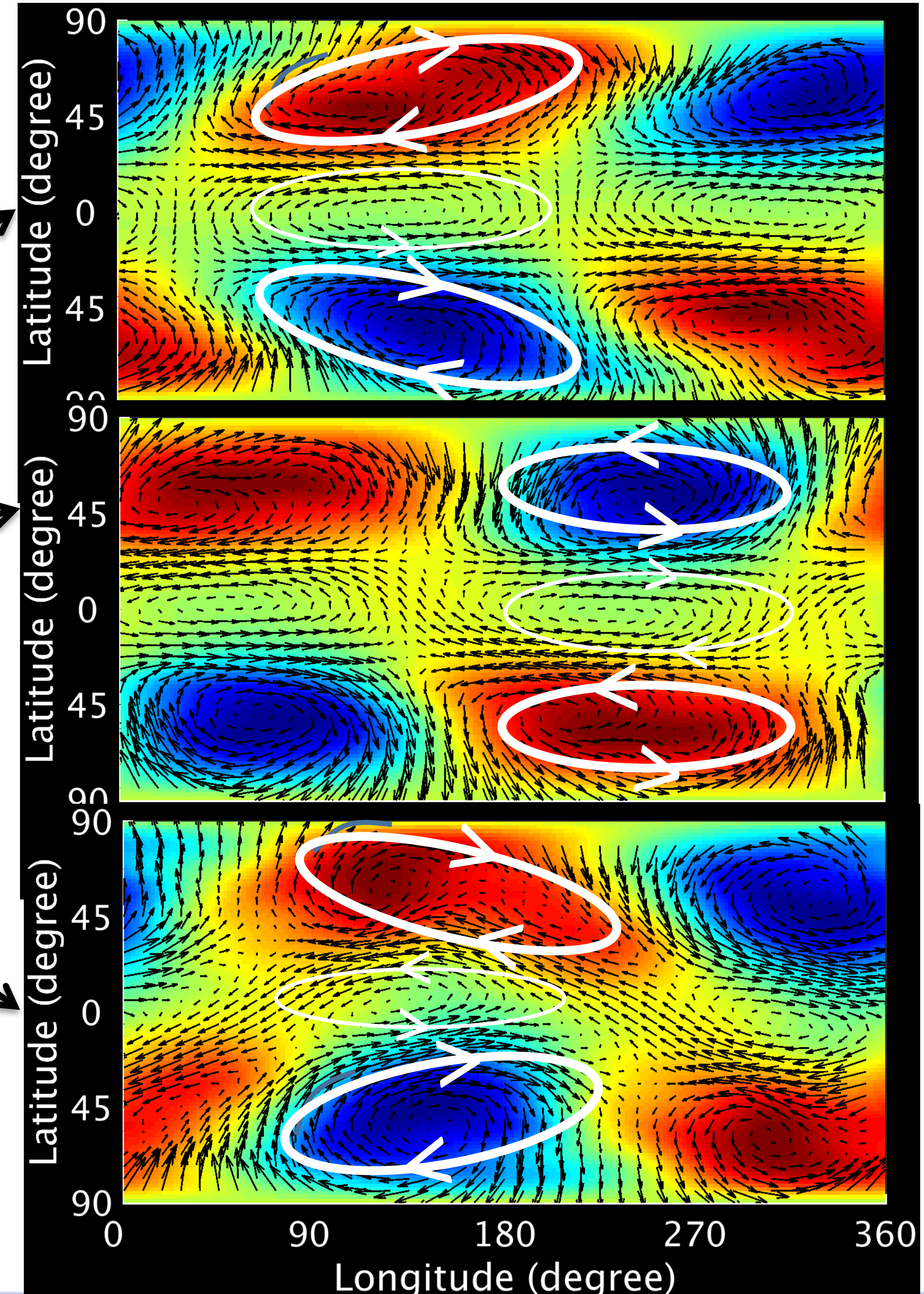
(Dikpati 2012)



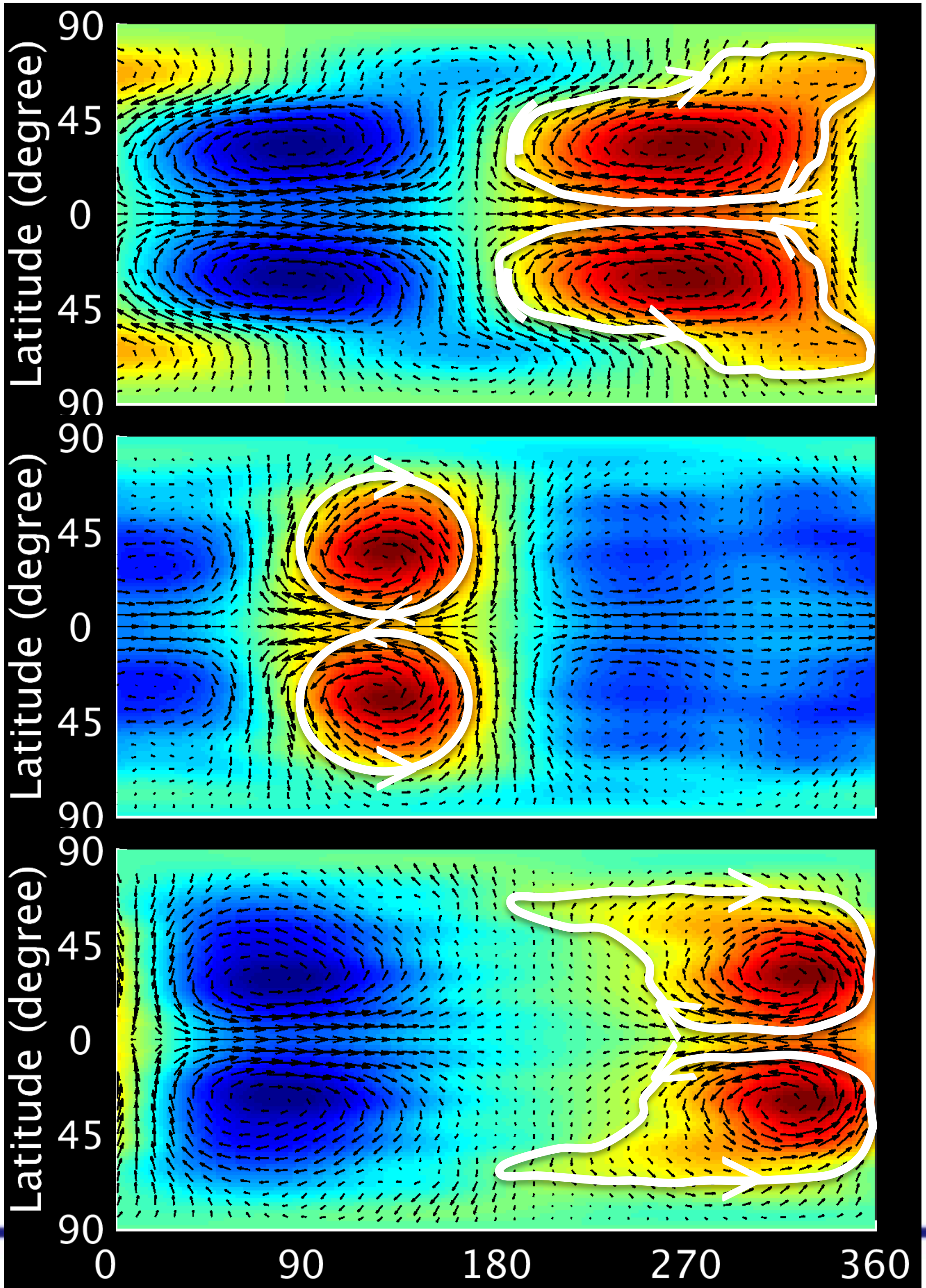
Schematic of interaction between differential rotation and Rossby waves

Solutions from hydrodynamic tachocline

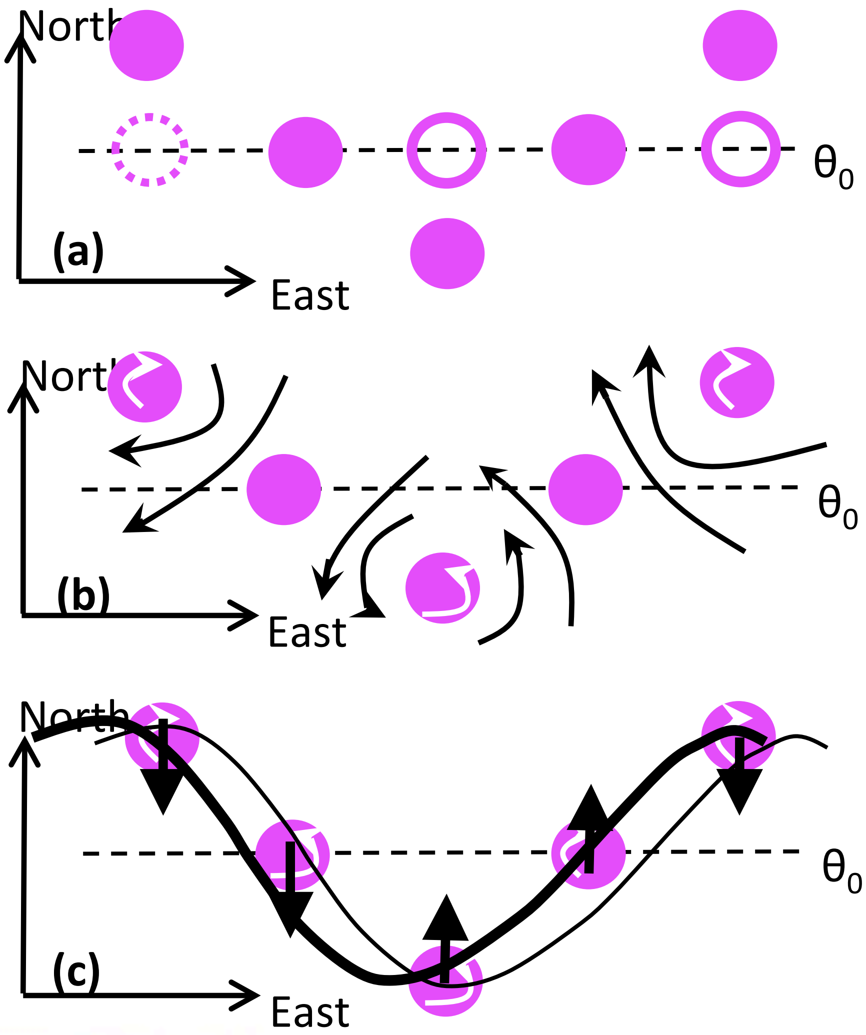


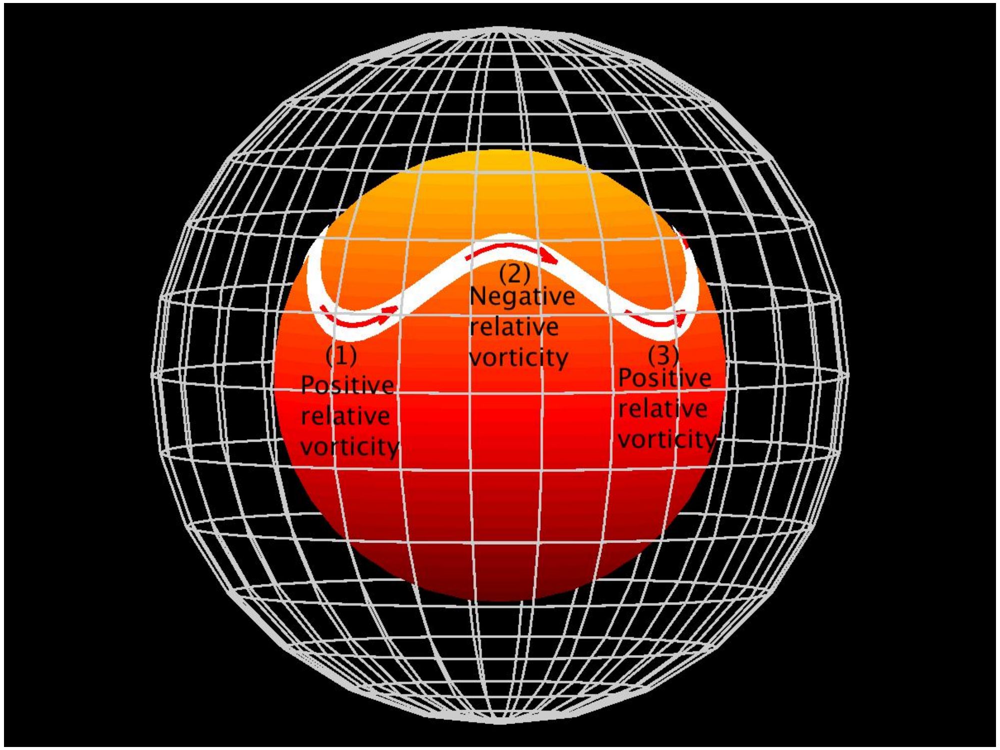


Another case (G=0.1)

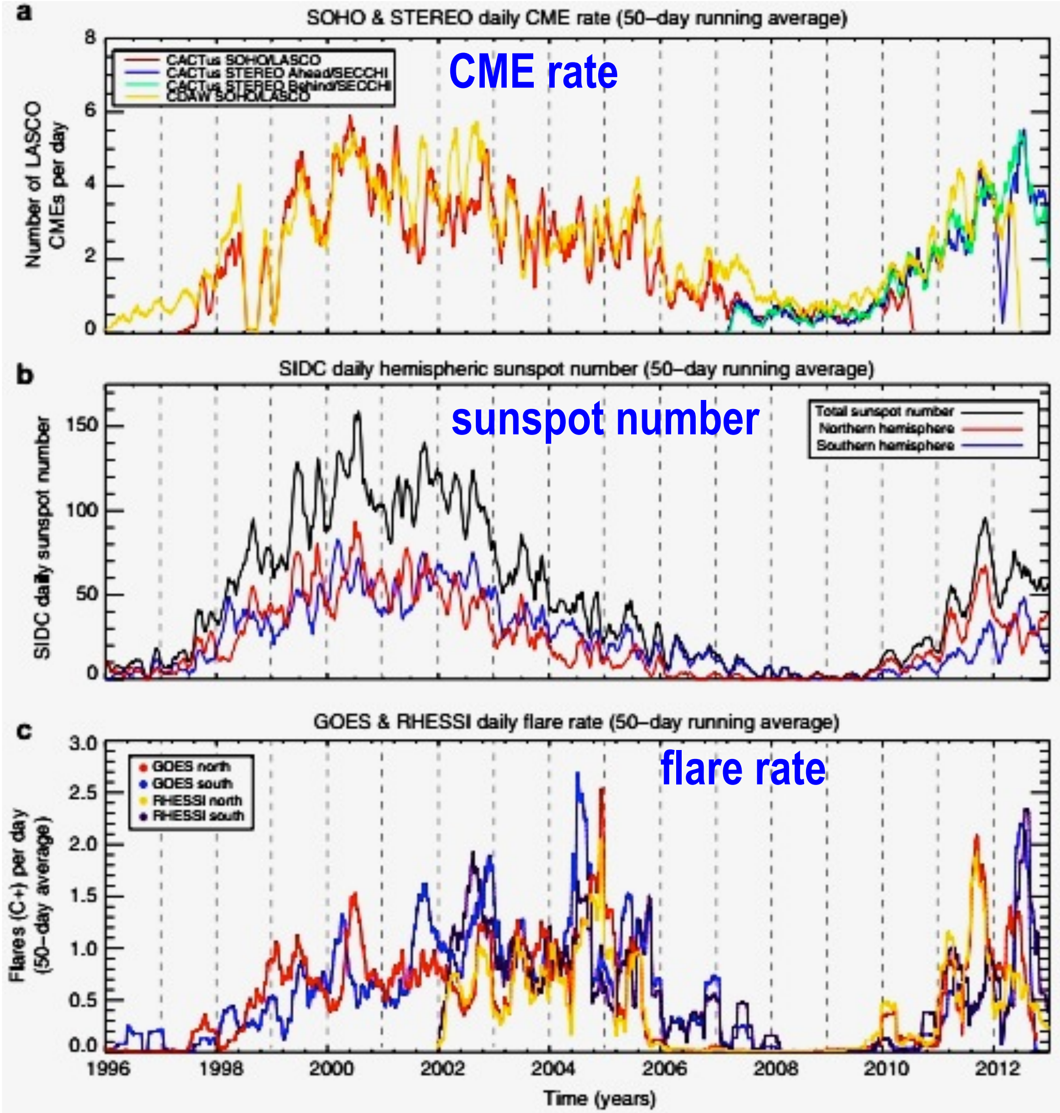


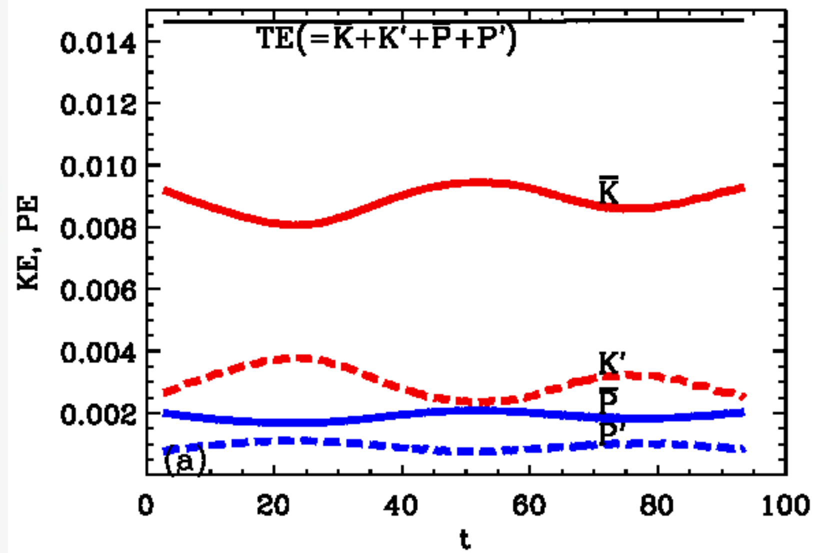
Rossby waves

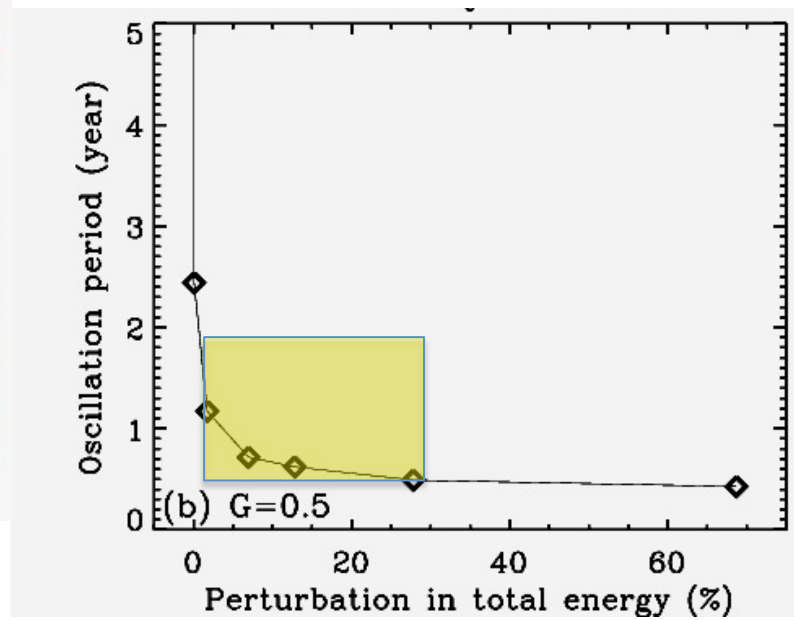




Evolution of kinetic and potential energies



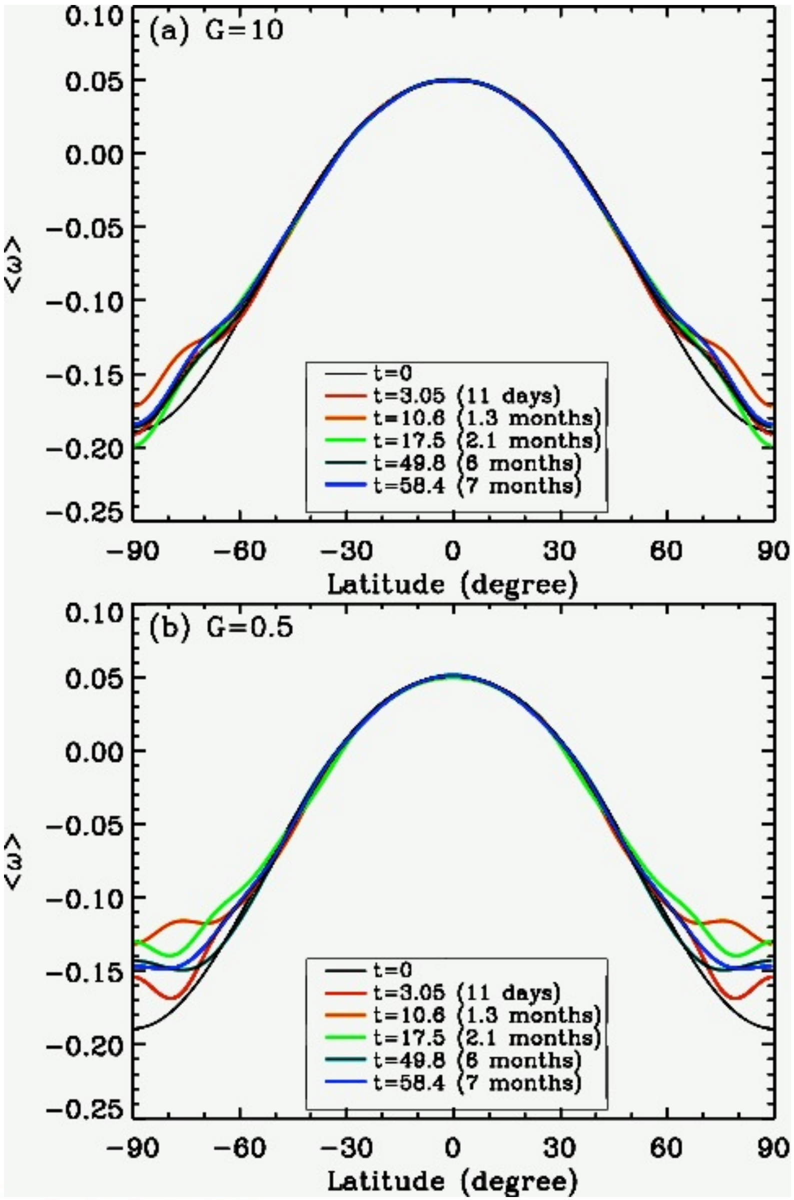


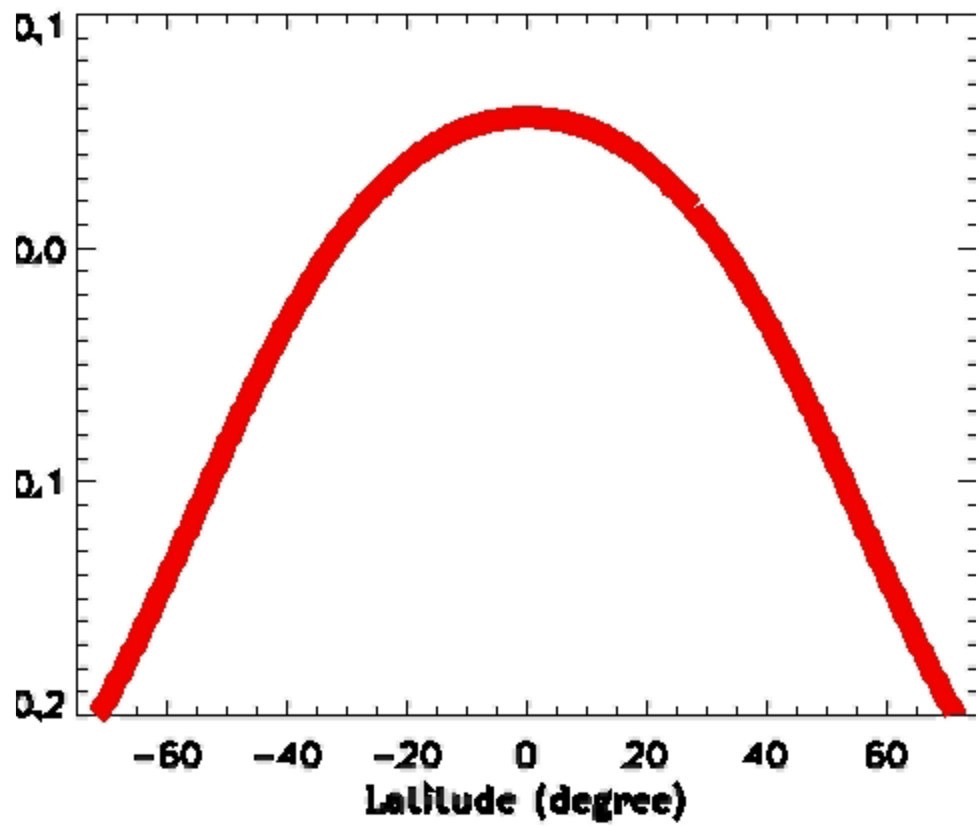


McIntosh et al. (2015)

High latitude jets

Reynolds stress transports angular momentum towards the poles

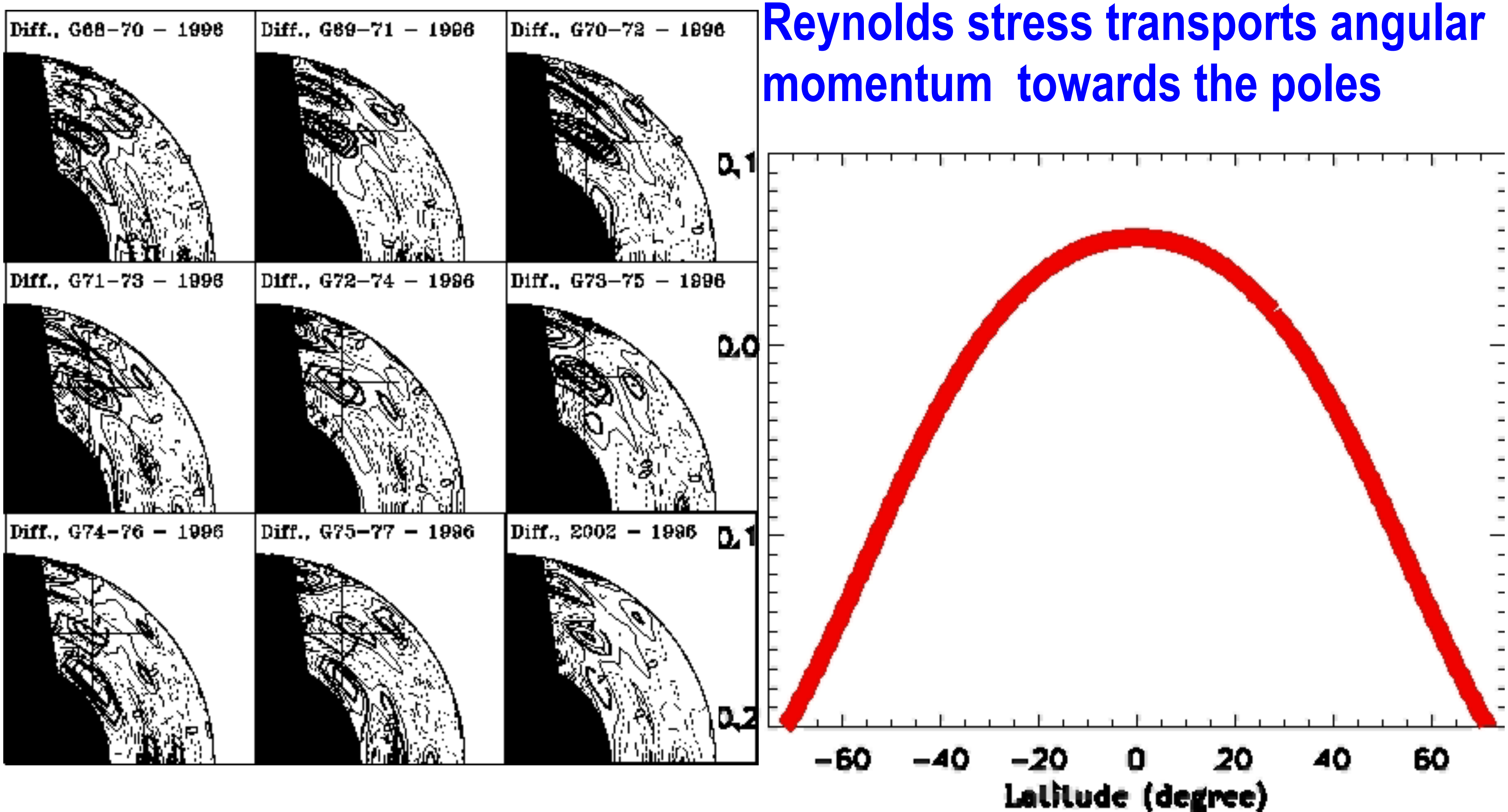




(Dikpati, 2012)

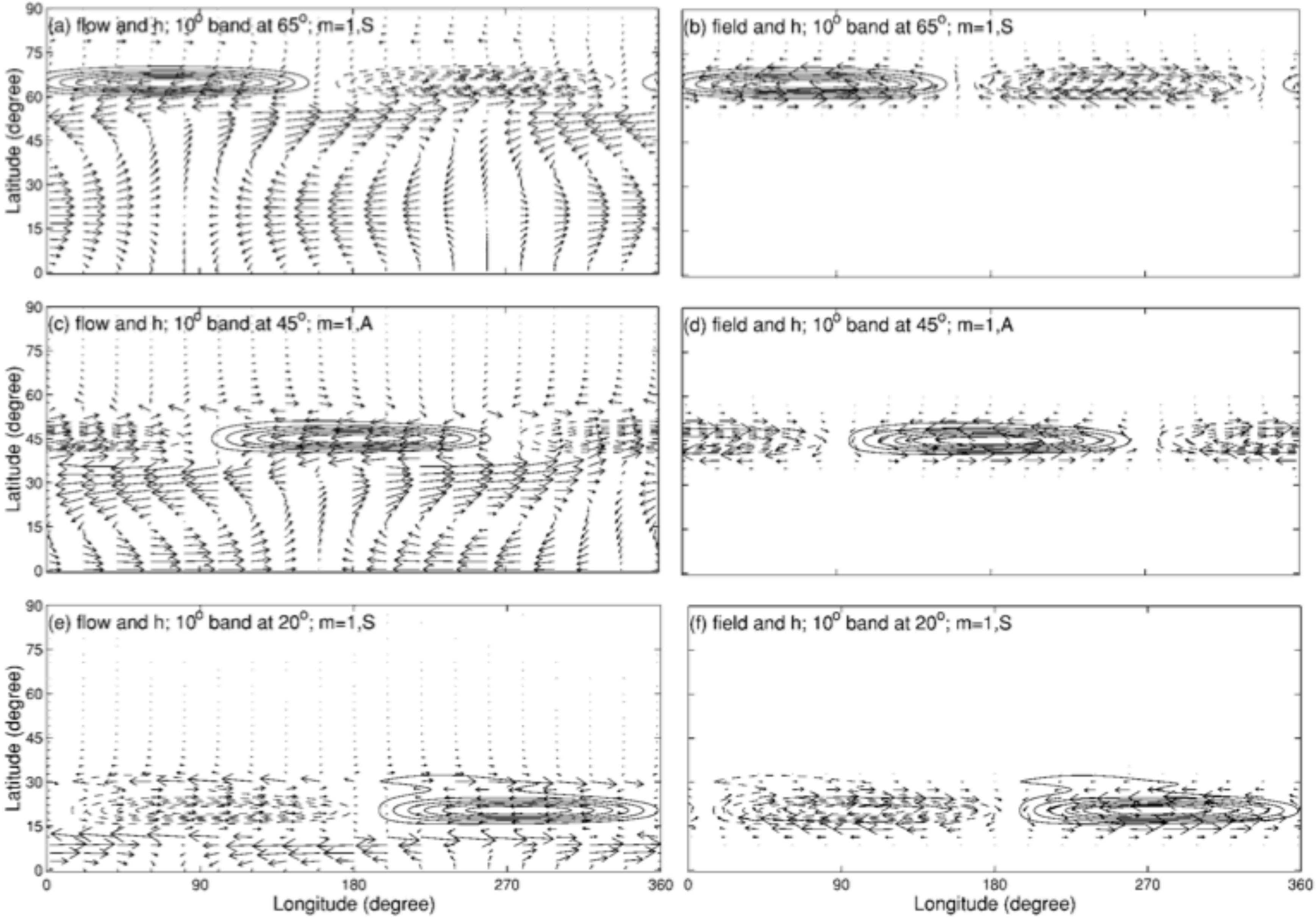
High latitude jets

Reynolds stress transports angular momentum towards the poles

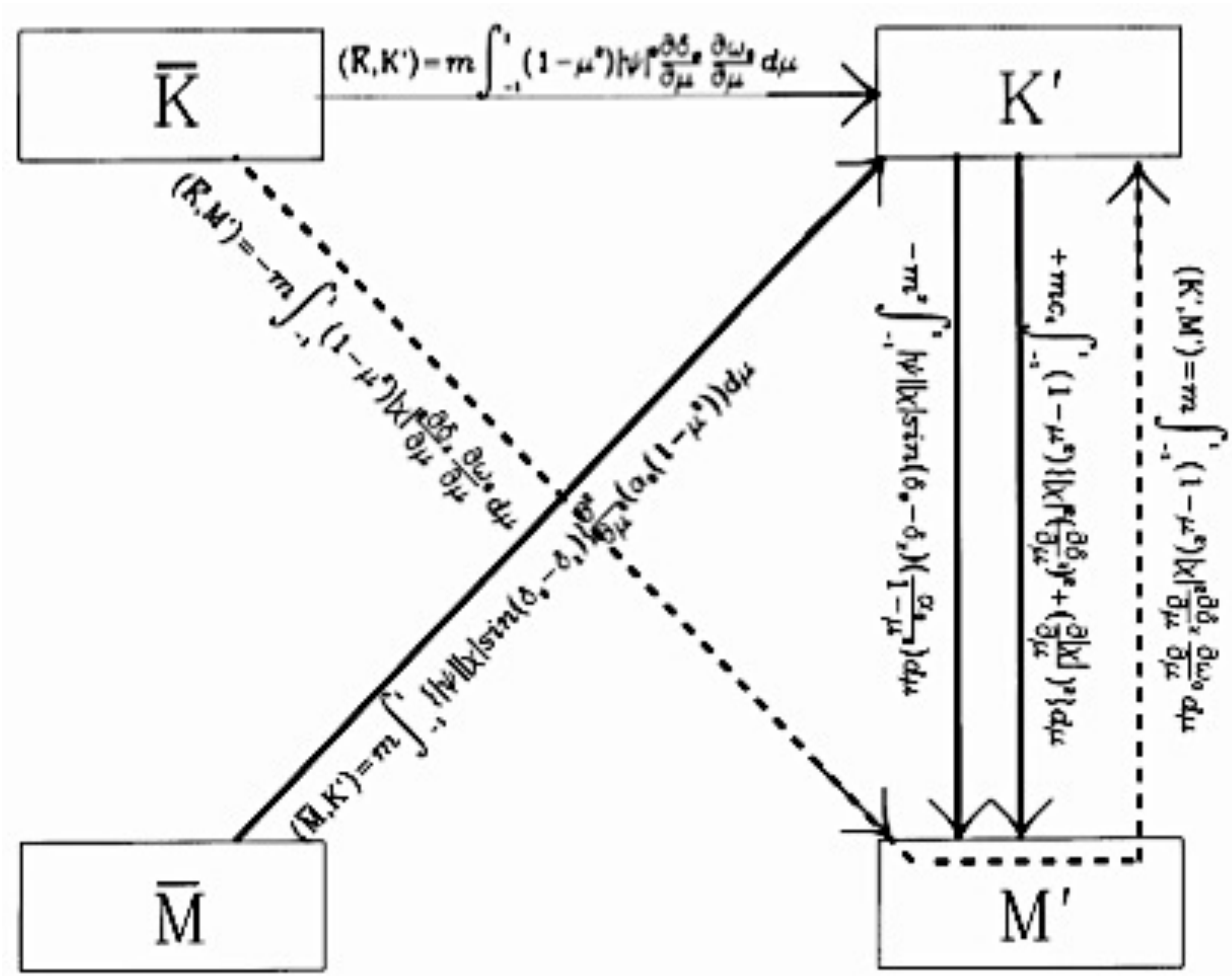


(Christensen-Dalsgaard et al. 2003)

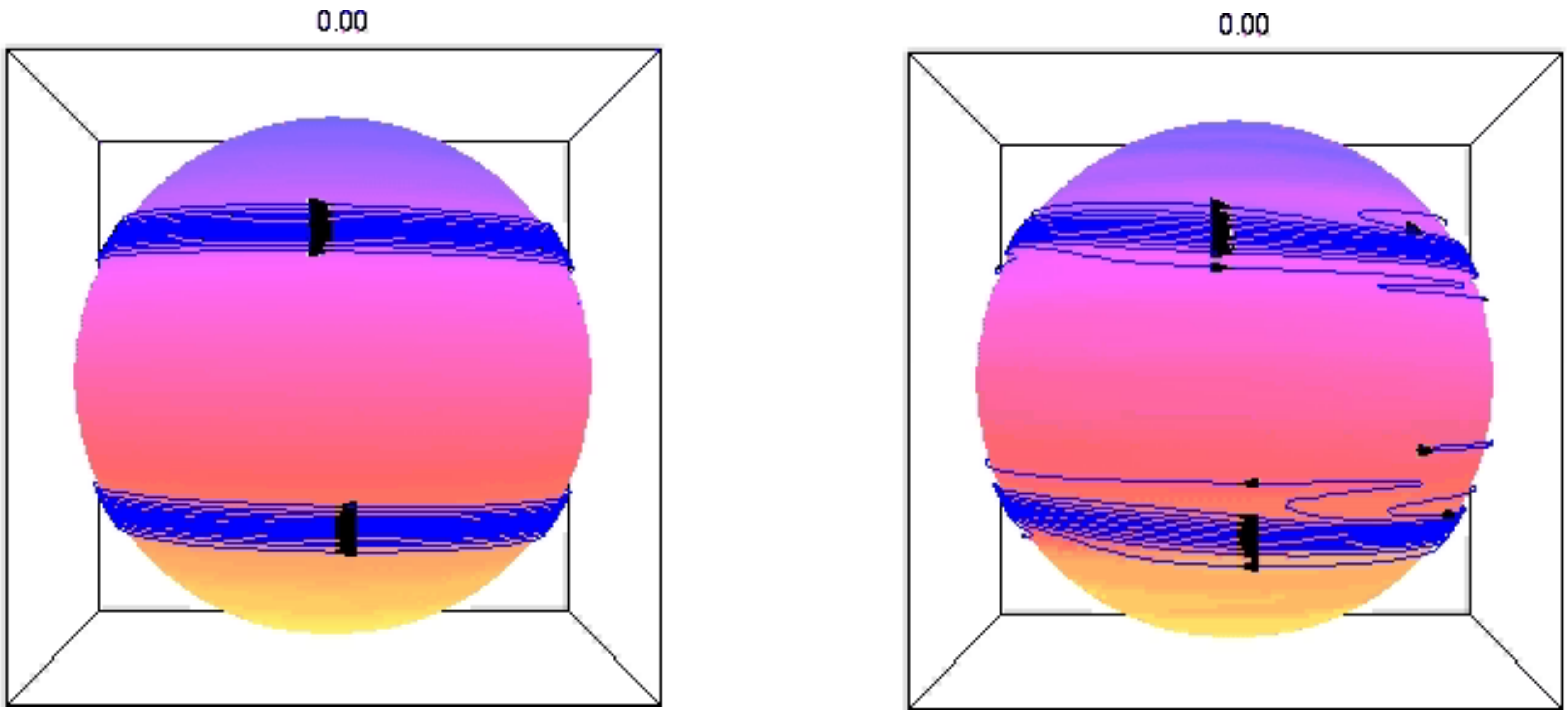
MHD Shallow-water Instability



Energetics in magnetically dominated case

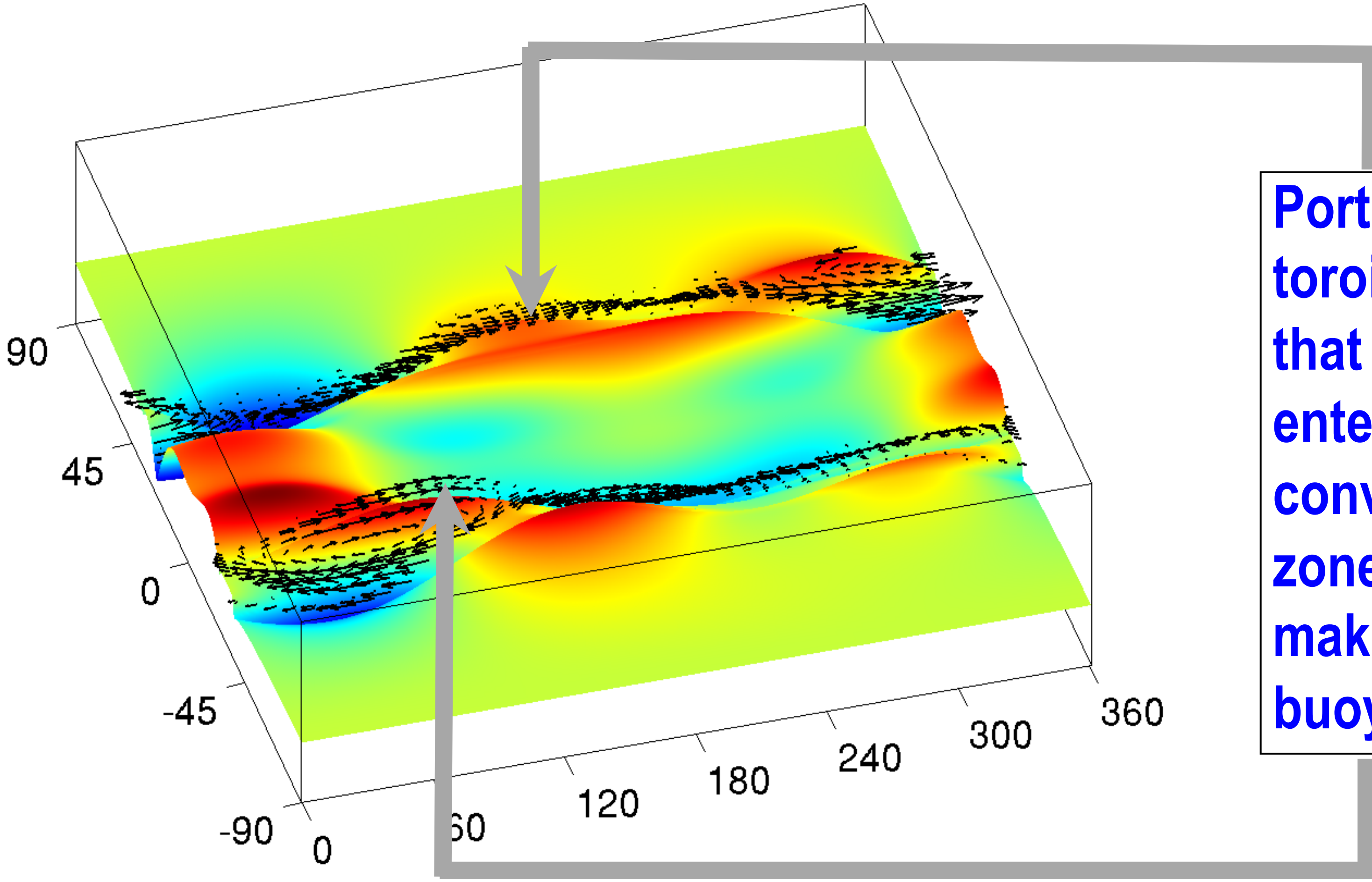


Nonlinear Evolution of MHD Instability



(Cally, Dikpati and Gilman, 2003)

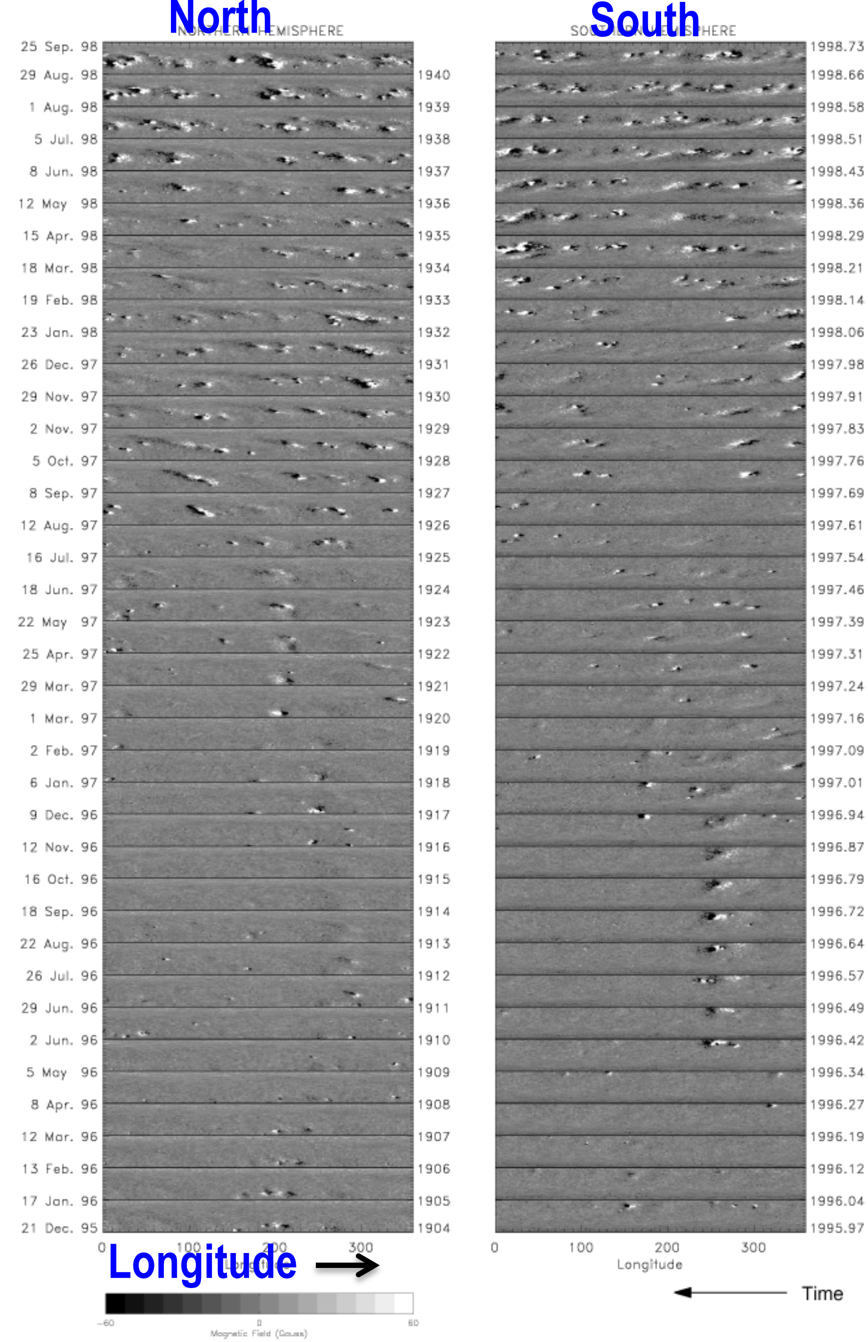
MHD Shallow-water Instability :A Theory of Active Longitudes



Portion of toroidal band that starts entering convection zone and making its buoyant rise

Active longitudes

- Active regions persistently appear in one longitude location or two longitudes ~180-degree apart
- Longitude location persists for ~15-20 solar rotation
- Rotation of active longitudes vary, strongest ones rotate rigidly with core rotation rate, but both faster and slower rotations have been found
- Modulation over several months to a few years has been observed in amplitude



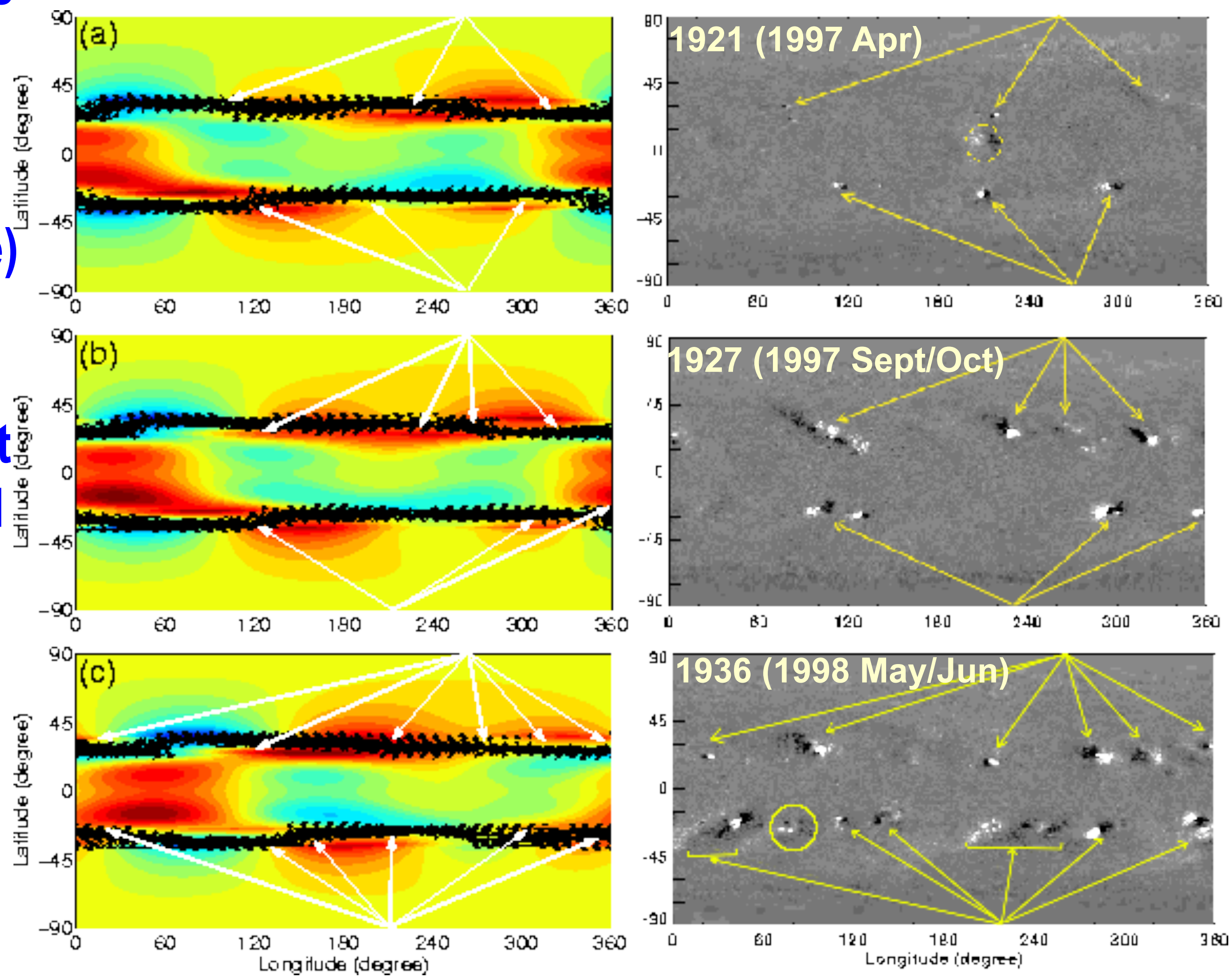
de Toma, White & Harvey 2000

Evolution of theoretical and observed active longitudes

Initial toroidal field is the unperturbed toroidal band with 20 kG peak field plus perturbations caused by all plausible unstable longitudinal modes (m=1S, m=1A, m=2A here)

Identify longitude locations of the band that coincide with the swelled fluid

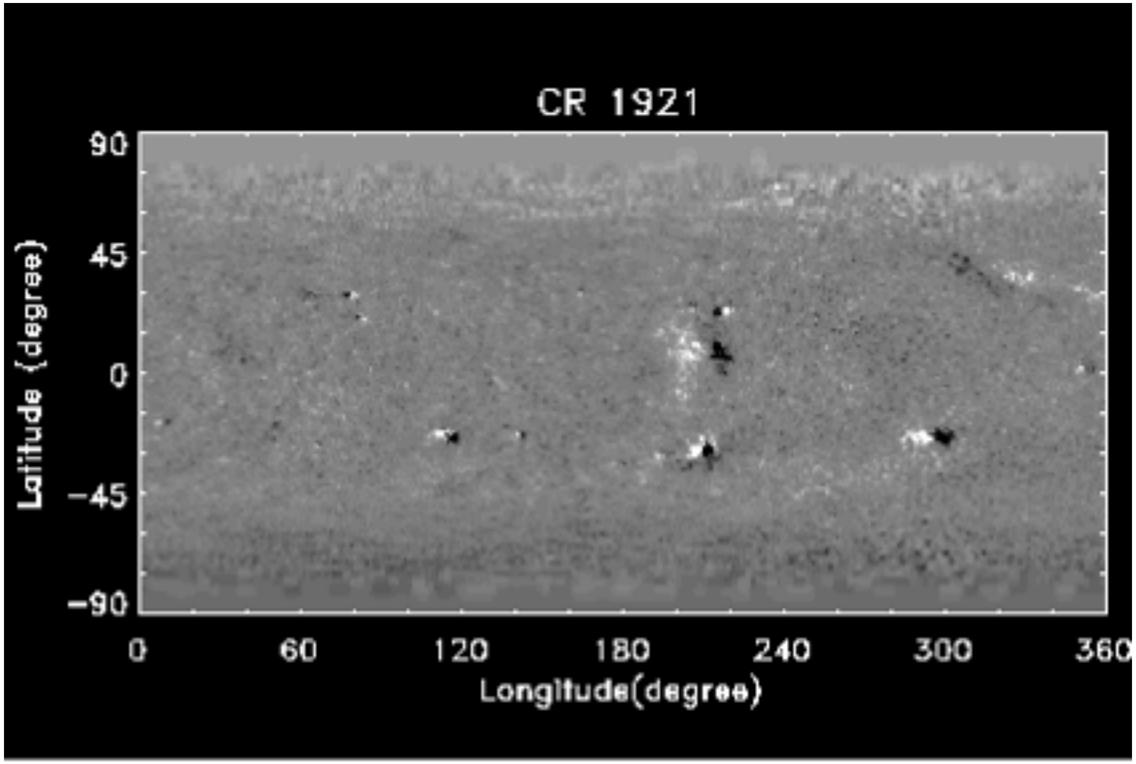
Active longitudes evolve according to the propagation of unstable MHD shallow-water modes

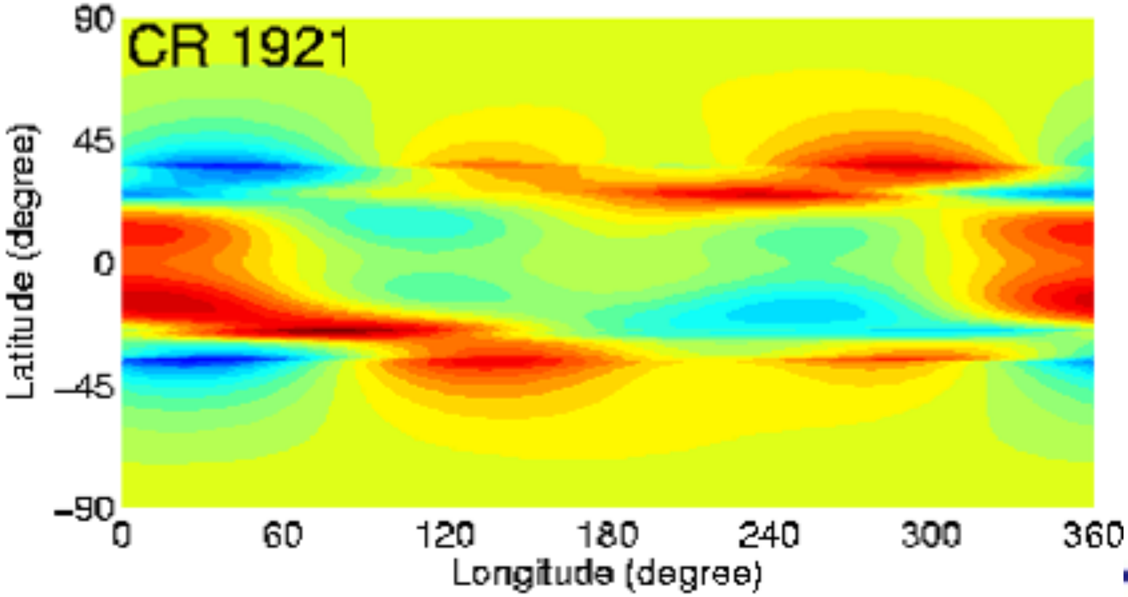


Dikpati and Gilman, 2005

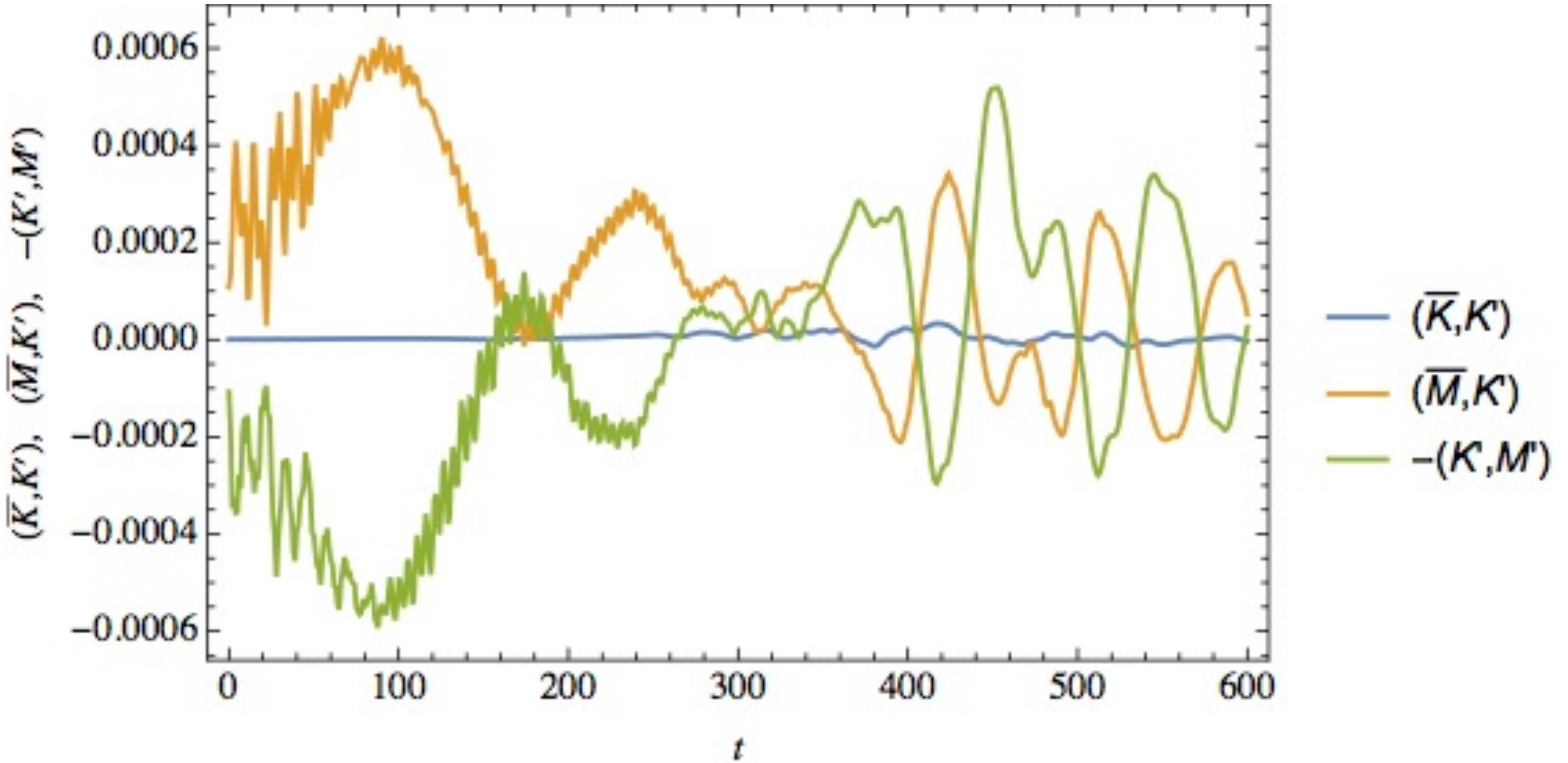
Evolution of swelling/ depression of tachocline fluid and observed active longitudes

(Dikpati and Gilman, 2005)



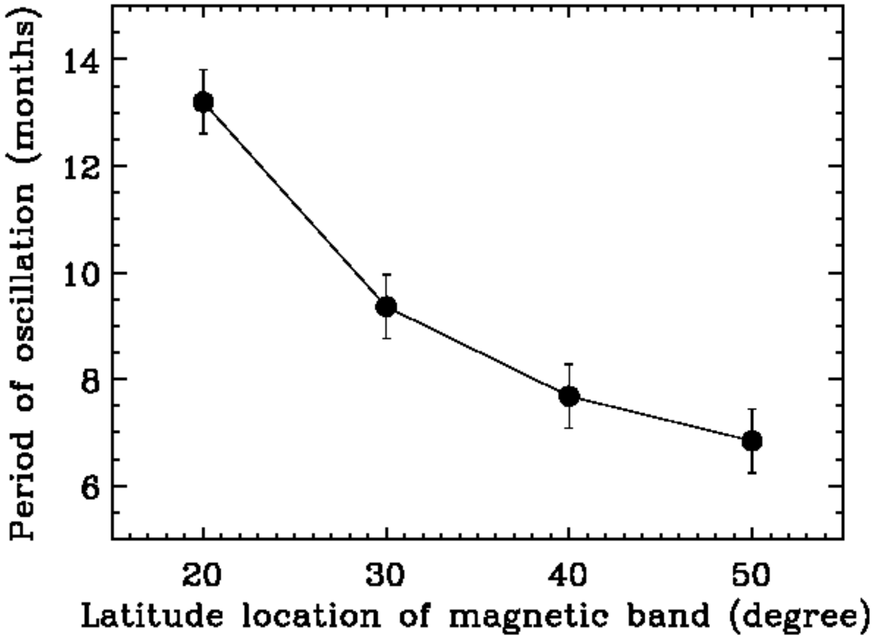


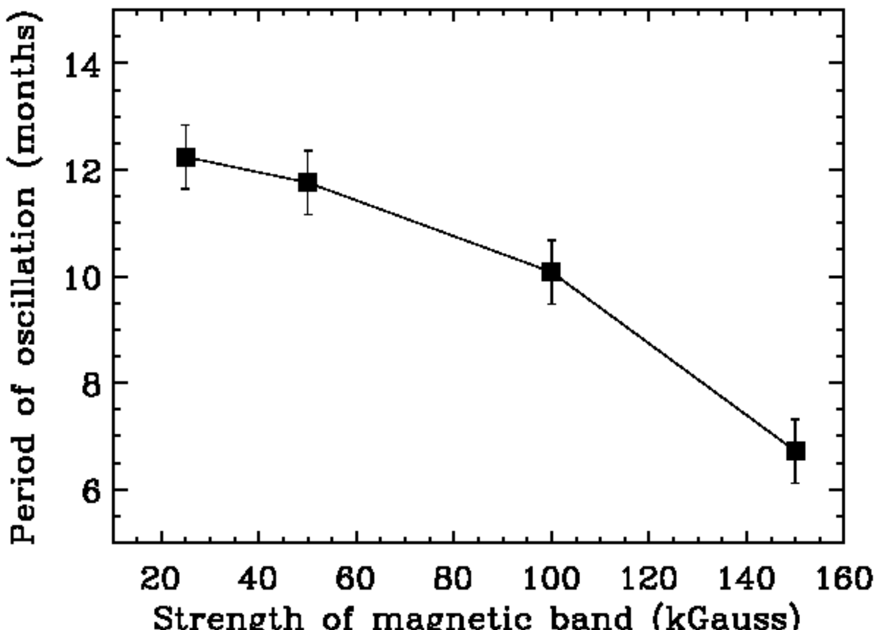
Oscillation among kinetic energy, magnetic energy and (magnetic) Rossby waves



Oscillation period as function of band location and band strength

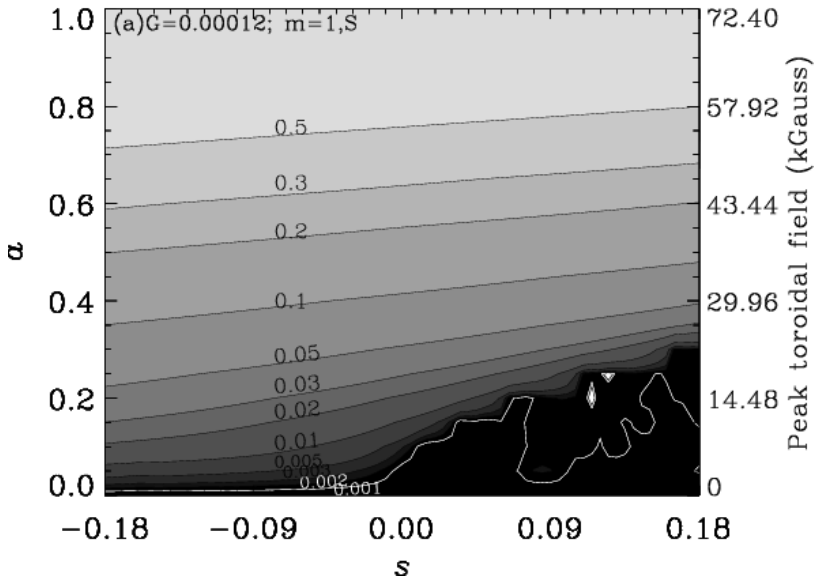
- Period increases as the band migrates towards the equator
- Period decreases as the band strength increases

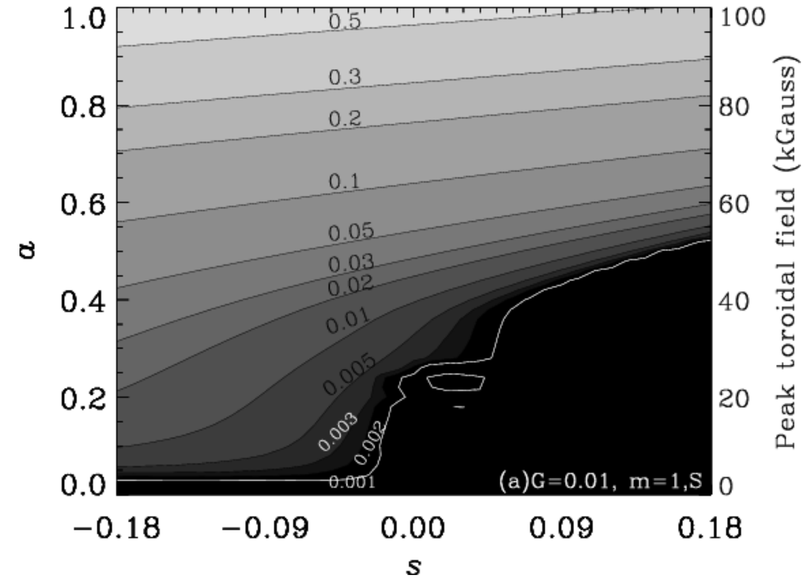




Dikpati, Cally, McIntosh & Bothun 2017 (in preparation)

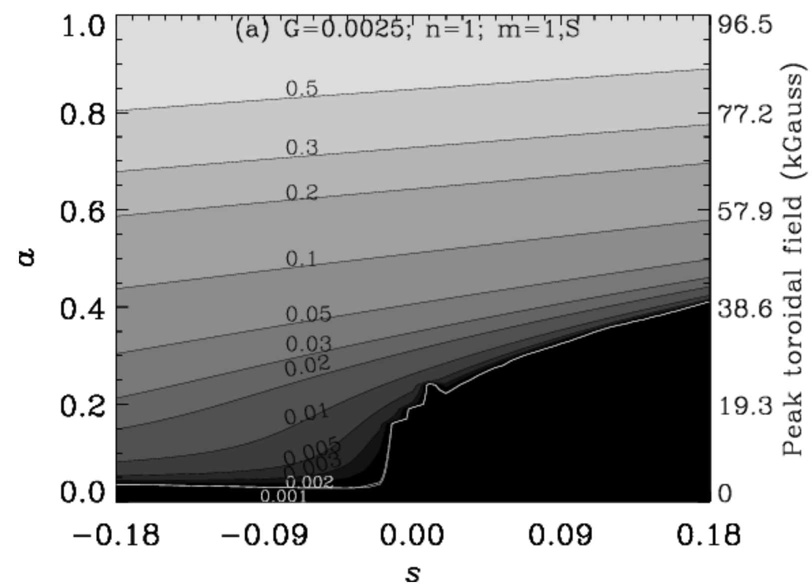
3D MHD instability of antisolar differential rotation





F-stars: faster rotators;
Heavier than the Sun

The Sun



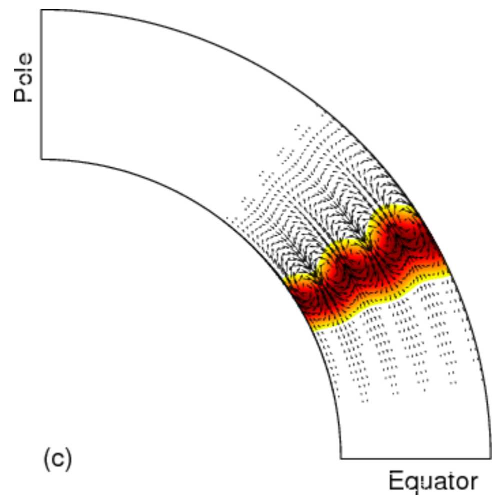
K-stars:
slower rotators;
lighter than the Sun

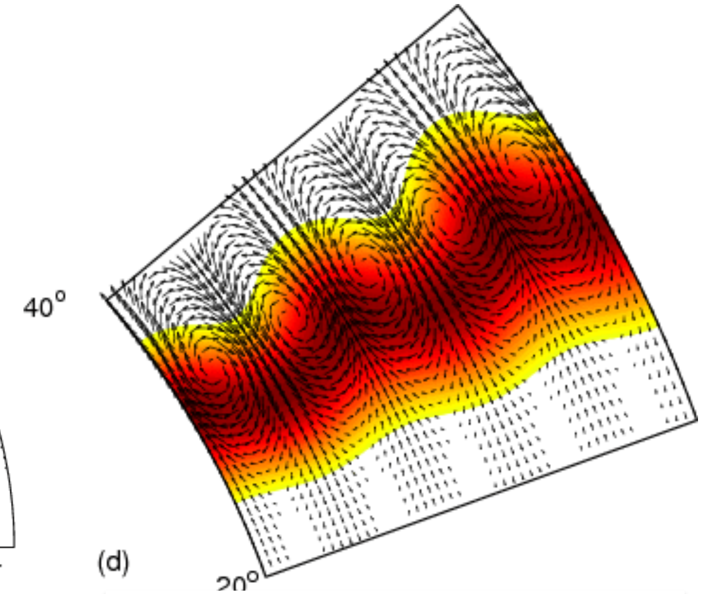
Consequences of 3D MHD axisymmetric (m=0) instability with radial wavenumber 5

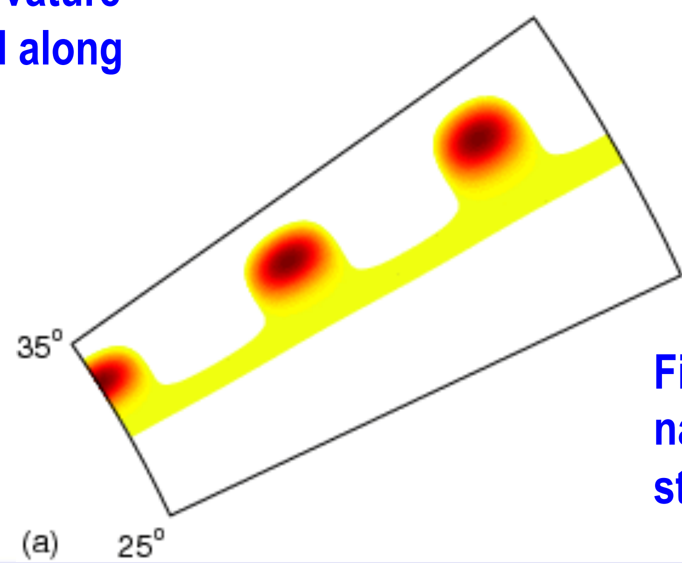
Toroidal band deforms more in its poleward side

Flow extends further in the latitude direction than the magnetic band

It seems that flows are carrying the field away from or toward the maximum field, but in fact opposite is happening; magnetic curvature forces are pushing the fluid along with the moving field







40°

Fields fragment in radial drection, narrower bands form and get stacked in depth

Summary

- HD/MHD shallow-water instability can occur in solar tachocline
- High latitude prograde jets can form due to this instability; such jets have been observed
- The Sun has “Seasons”, or characteristic time variations on a scale of 6 – 18 months . These seasons can be simulated by using MHD Shallow-water tachocline model, and can be forecast 2 years ahead when surface observations are assimilated
- A global helical flow can be generated in the tachocline and can provide a source for dynamo action there
- When a certain longitude portion of a toroidal band coincide with the swelled fluid region due to tipping of the band, that portion rises to the surface to produce spots at that longitude, creating “active longitudes”

Thank you