INTERNATIONAL SPACE SCIENCE INSTITUTE

DEPARTMENT OF EARTH SCIENCES,
RAYMOND AND BEVERLY SACKLER FACULTY OF EXACT SCIENCES

Interfacial Dynamics in Protoplanetary Accretion Disks

BERN 2017

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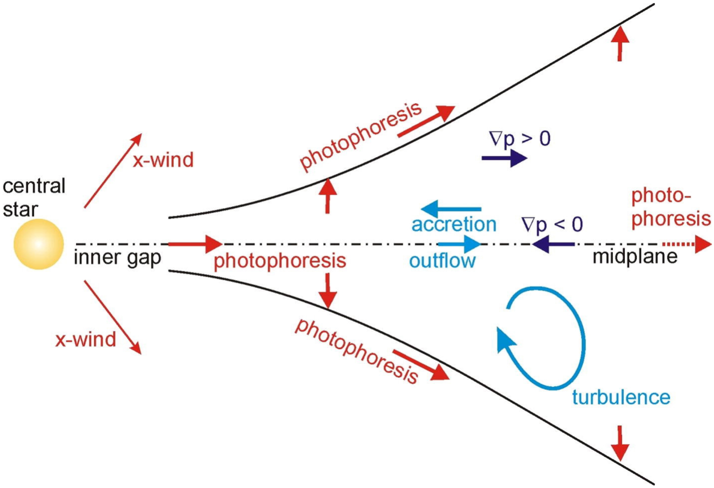
Outline

Introduction



The angular momentum problem

- Observations shows that new stars accrete material.
- $v_\theta \approx v_{kep} = \sqrt{\frac{GM_*}{r}} \Rightarrow L = mvr = m\sqrt{GM_* r}$.
- $\nu_{kinematic}$ is insufficient $\Rightarrow \nu = \alpha C_s H \approx v_{turb} l_{turb}$ (Shakura and Sunyaev,1973).
- Cold disks do not support the magnetorotational instability (MRI).
- Disks Keplerian shear profile is linear stable (i.e., the Rayleigh (Rayleigh, 1880) and the Fjørtoft (Fjørtoft, 1953) conditions), so where does the turbulence come from?
- Many different instabilities with different degree transport and some connection to planet formation .

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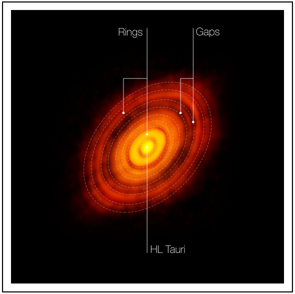
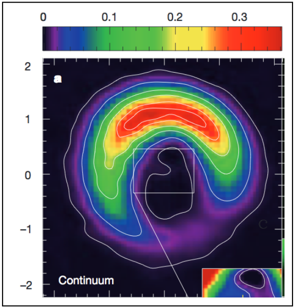
Observations

Atacama Large Millimeter/submillimeter Array (ALMA)

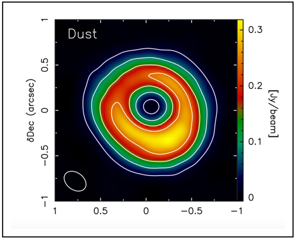
Observational data



(a) HL Touri system, band 6(1mm) dust emissions (ALMA website, 2015).



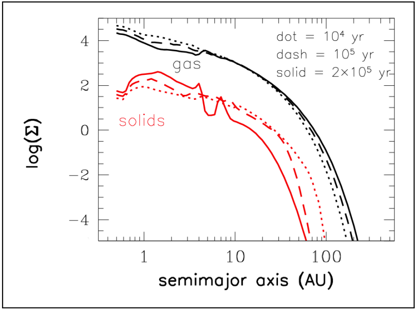
(b) HD 142527 system emission, band7(0.85mm) dust emissions (Casassus et al., 2013).



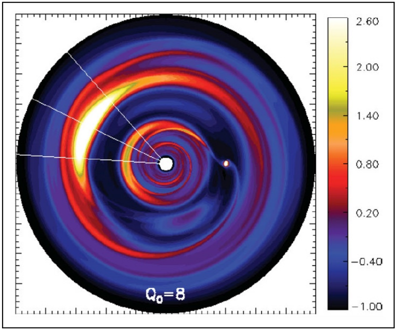
(c) SAO 206462 system, band9(0.45mm) dust emission (Perez et al., 2014).

Figure: Observational images of PPD systems

Numerical young disk density profiles



Estrada et al., 2015



Lin 2012b

density gradients profiles

The potential vorticity (PV) perspective

So what is missing?

What is the basic mechanism/s of the instabilities?

Additional questions

- Which profiles go unstable? and why?
- What is the role of stratification and shear? how does self gravity (SG) contribute to the dynamics?

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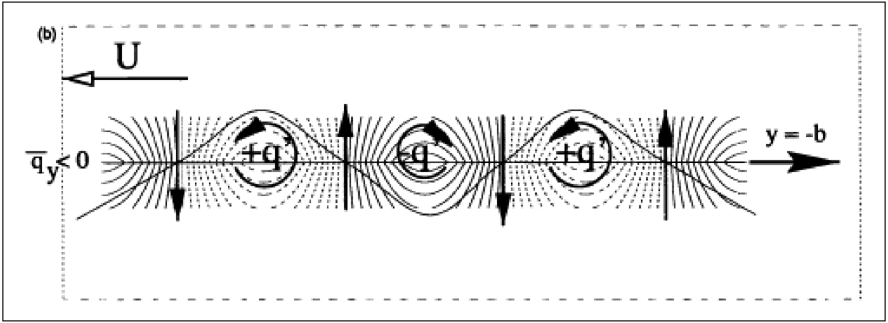
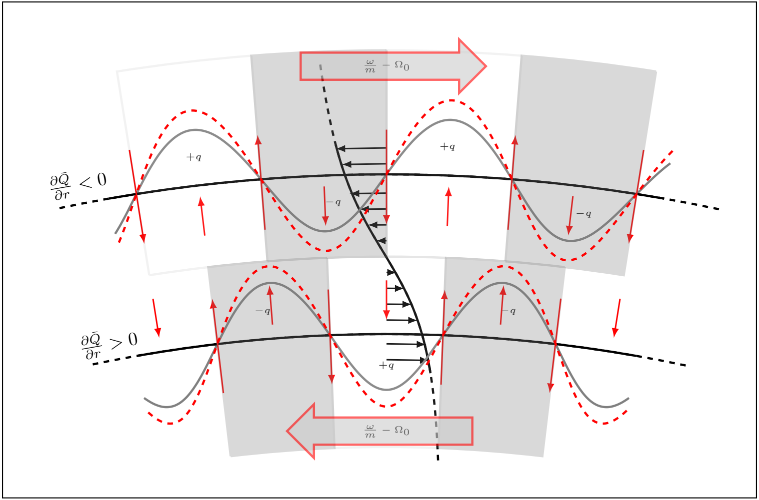
To get answers we look at analogous systems in GFD which also include shear, rotation and stratification, in which the PV perspective was used in order to understand the instability as emerging from interacting Rossby waves

Vorticity and cross stream displacement- positive correlation



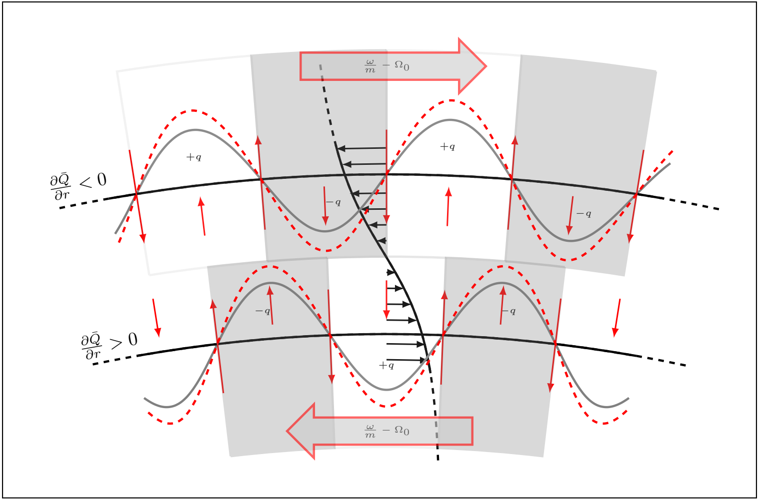
Heifetz et al. 1999

Rossby wave resonance type instability



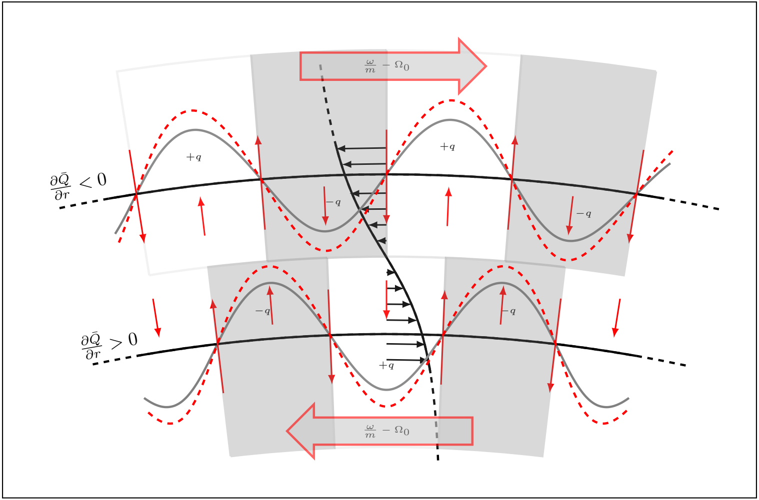
Shear

Rossby wave resonance type instability



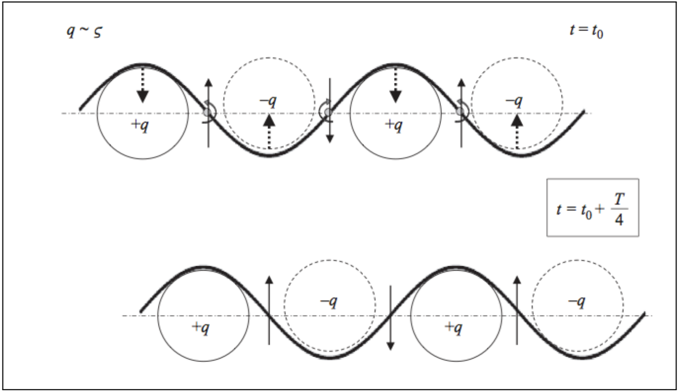
Shear stable!! action at a distance

Rossby wave resonance type instability



Shear stable!! action at a distance

Density interfaces and vorticity



Gravity vorticity waves (Rabinovich et al. 2011)

Interfaces and vorticity waves resonance type instability

Also in:

- Buoyancy and Rossby-Gravity vorticity waves (Harnik et al. 2008; Rabinovich et al. 2011).
- MHD and vorticity Alfven vorticity waves (Heifetz et al. 2015).
- Surface tension and capillary vorticity waves (Heifetz et al. 2014).

Outline

Basic equations

$$\frac{Du}{Dt} - \left(\frac{v^2}{r} + 2\Omega_k v\right) = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\partial \phi}{\partial r},$$

$$\frac{Dv}{Dt} + u\left(\frac{v}{r} + \frac{1}{r}\frac{\partial r^2\Omega_k}{\partial r}\right) = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{\partial \phi}{r\partial \theta},$$

where the Lagrangian time derivative is given as:

$$\frac{D}{Dt} = \left(\frac{\partial}{\partial t} + \left(\mathbf{u} + \Omega_k r\hat{\theta}\right)\cdot\nabla\right) = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + \left(\frac{v}{r} + \Omega_k\right)\frac{\partial}{\partial \theta}.$$

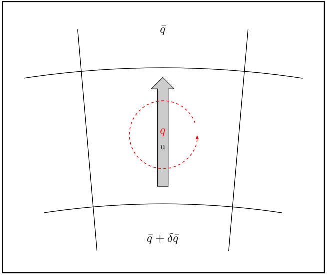
and,

$$\nabla^2\phi = 4\pi G_{grav}\rho.$$

$$\nabla\cdot\mathbf{u} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta} = 0$$

Vorticity generation

$$\frac{D_L q'}{Dt} = -u' \frac{\partial}{\partial r}\left(\bar{q} + \frac{1}{r}\frac{\partial (r^2 \Omega_k)}{\partial r}\right) - \frac{1}{r\bar{\rho}^2}\left[\frac{\partial \rho'}{\partial \theta}\frac{\partial \bar{p}}{\partial r} - \frac{\partial \bar{\rho}}{\partial r}\frac{\partial p'}{\partial \theta}\right],$$



The Rossby term

Baroclinic torque and vorticity generation

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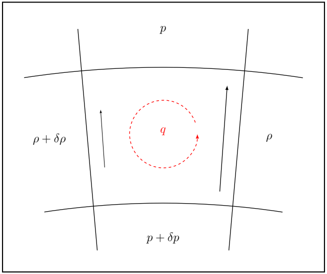

The Boussinesq term

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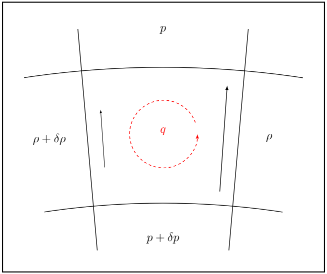

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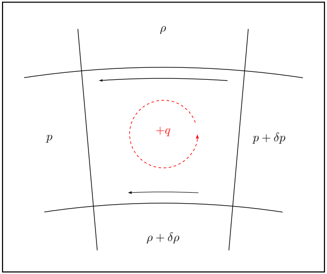
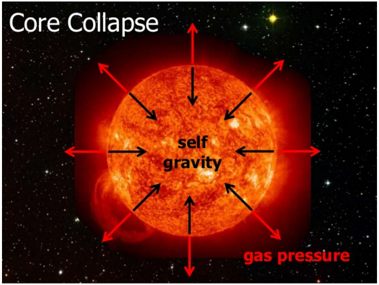


The non-Boussinesq term

What is self gravity (SG)?



basic state equation:

$$\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial r} = \frac{\partial \bar{\phi}}{\partial r} + \bar{v}\left(2\Omega_k + \frac{\bar{v}}{r}\right),$$

Vorticity waves propagation

the dispersion relation:

$$\left(\frac{\omega}{m} - \Omega_0\right) = \frac{1}{4m}\left(\Delta\bar{Q} + \frac{\Delta\bar{\rho}}{\bar{\rho}}\bar{Q}\right)_{r_0}$$

$$\pm\left\{\left[\frac{1}{4m}\left(\Delta\bar{Q} + \frac{\Delta\bar{\rho}}{\bar{\rho}}\bar{Q}\right)\right]^2 + \frac{\Delta\bar{\rho}}{\bar{\rho}}\left[\frac{1}{2mr}\left(\frac{1}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r}\right)_{r_0} + \frac{\pi G_{grav}\Delta\bar{\rho}}{m^2}\right]\right\}_{r_0}^{1/2}$$

together with the generic vorticity displacement ratio:

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wave propagation

instability

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wave propagation and the sufficient condition for instability for a single edge wave is

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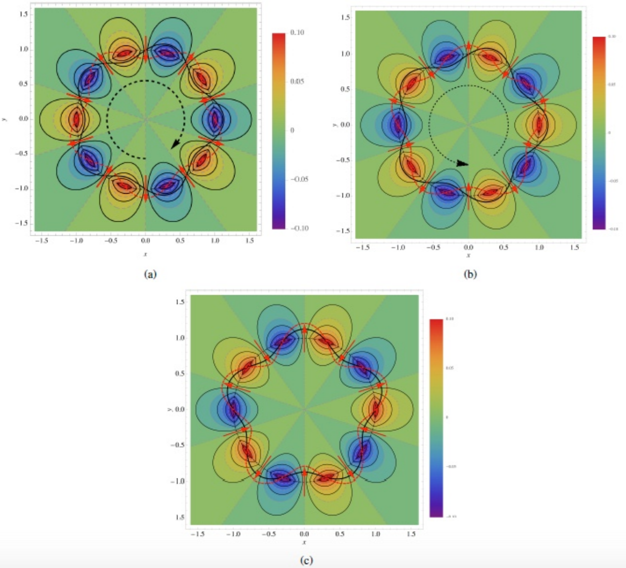
instability

Vorticity waves propagation



Results

- There are two azimuthally propagating edge gravity waves corresponding to the density interface,which move opposite to one another with respect to the local basic state rotation rate at the interface.
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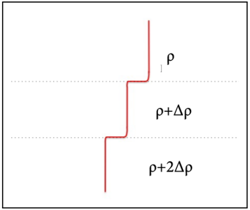
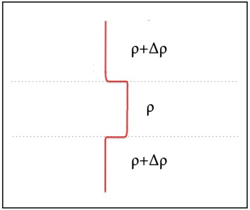


Taylor–Caulfield setup (same sign).



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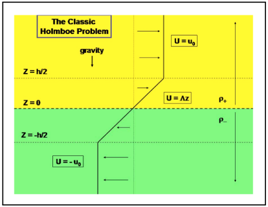

Holmboe setup (Umurhan and Heifetz, 2007).


THANK YOU!!

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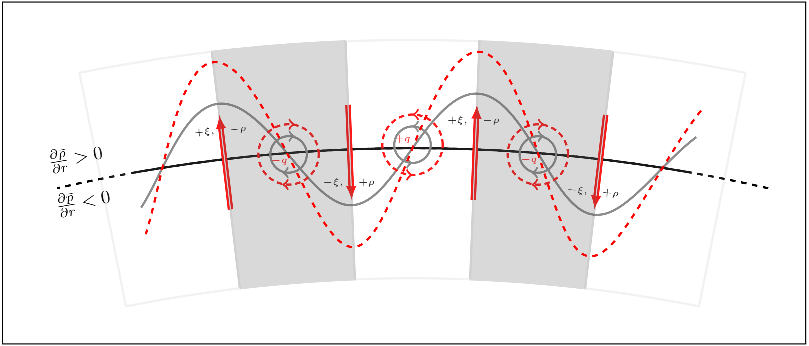
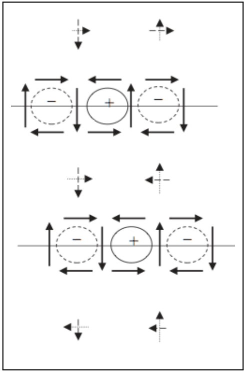
Modified Rayleigh-Taylor instability



back

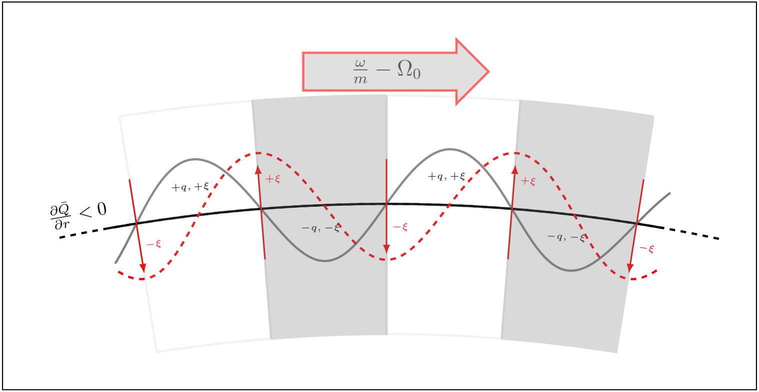
PV inversion, Greens function and the velocity field



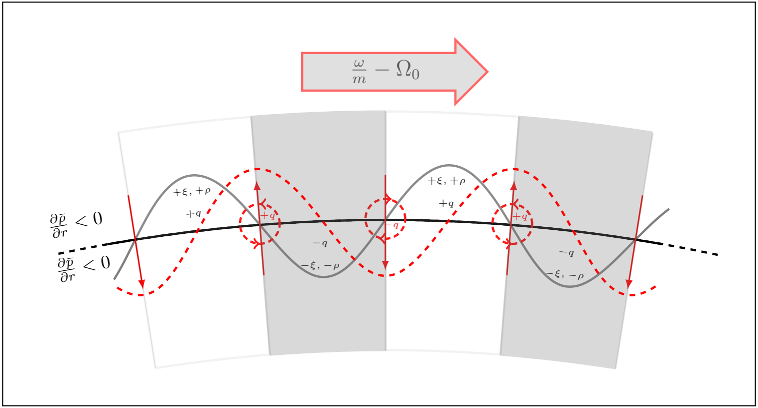
Rabinovich et al. 2011

back

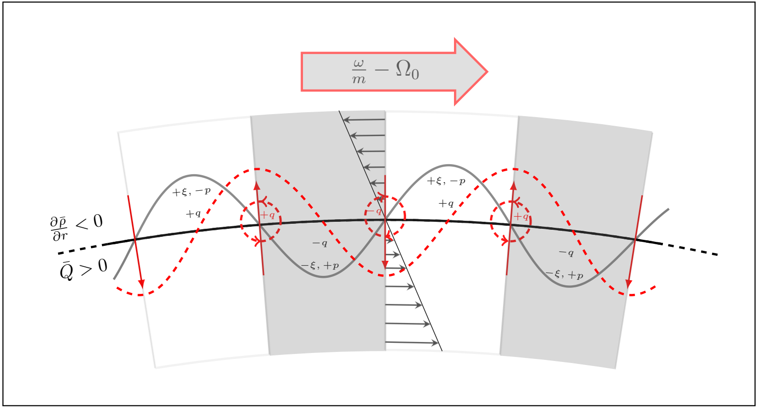
Rossby waves



Boussinesq gravity waves

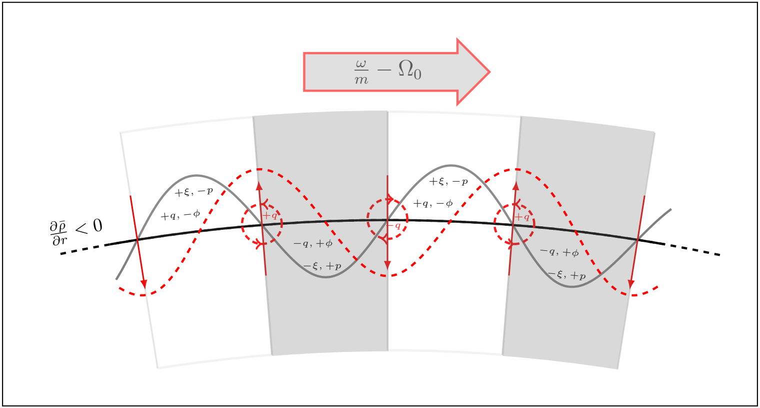


non-Boussinesq gravity waves



$$r_0 \bar{Q}_0 u_0' = -\frac{\partial}{\partial \theta}\left(\frac{p'}{\bar{\rho}}\right)$$

non-Boussinesq self gravity waves



$$p_0' = \left(\frac{2\pi}{m} r_0 G_{grav} \bar{\rho}_0 \Delta \bar{\rho}_0\right) \xi'$$

dispersion relation