Multifractal analysis of simulated solar flares

Laura F. Morales[(1)], Paul Charbonneau[(1)] & Shaun Lovejoy[(2)]
(1) Département de Physique, Université de Montréal
(2) Physics Department, McGill University

Outline

- Motivation: last year's ISSI meeting
  understudied numerical model
- Multifractality and solar flares
- A first attempt to make calculations
- One application

Fractal and multifractal analysis:

→ a way to treat complex systems exhibiting fluctuations on a wide range of time scales.

Multifractal studies had been successfully applied to a variety of natural phenomena such as:

- precipitation
- ozone levels
- wind speed
- seismic events
- climate dynamics
- astrophysical time series **(Solar Flares)**

Multifractal analysis of geomagnetic storm and solar flare indices and their class dependence

Zu-Guo Yu,[1,2] Vo Anh,[1,3] and Richard Eastes[3]

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 114, A05214,

Are Solar Active Regions with Major Flares More Fractal, Multifractal, or Turbulent Than Others?

Manolis K. Georgoulis

Solar Phys (2012) 276:161–181
DOI 10.1007/s11207-010-9705-2

Multifractality as a Measure of Complexity in Solar Flare Activity

Asok K. Sen

Solar Phys (2007) 241: 67–76
DOI 10.1007/s11207-006-0254-7

THE BURSTY NATURE OF SOLAR FLARE X-RAY EMISSION

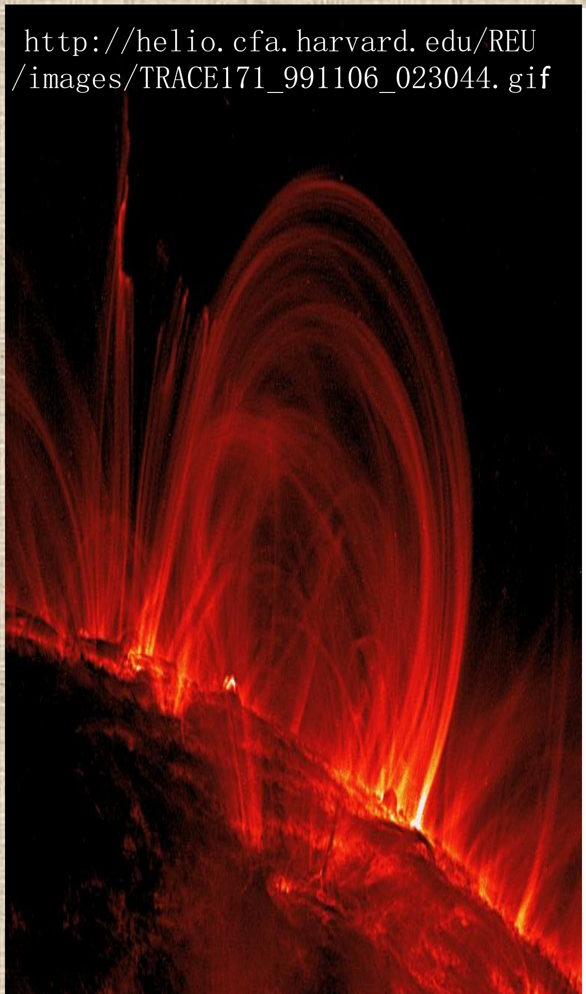
R. T. James McAteer,[1,2] C. Alex Young,[2,3] Jack Ireland,[2,3] and Peter T. Gallagher[4]

Received 2006 November 14; accepted 2007 March 6

The Astrophysical Journal, 662:691–700, 2007 June 10

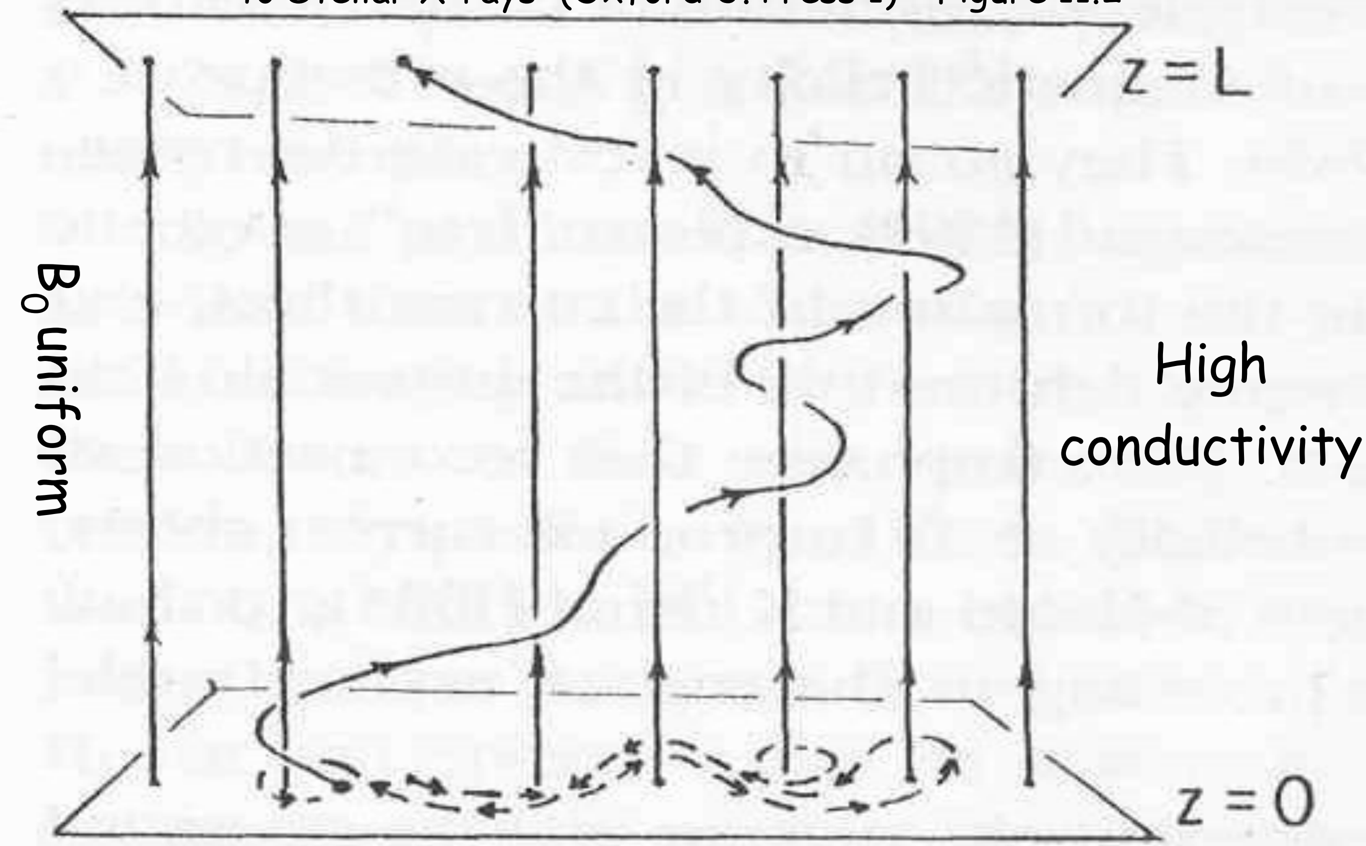
A direct comparison of the multifractal spectra with SOC models (e.g., Vlahos et al. 1995) may provide a direct comparison with the underlying physics of the highly nonlinear physics of magnetic energy dissipation in solar flares.

Parker's Model for solar flares



http://helio.cfa.harvard.edu/REU/images/TRACE171_991106_023044.gif

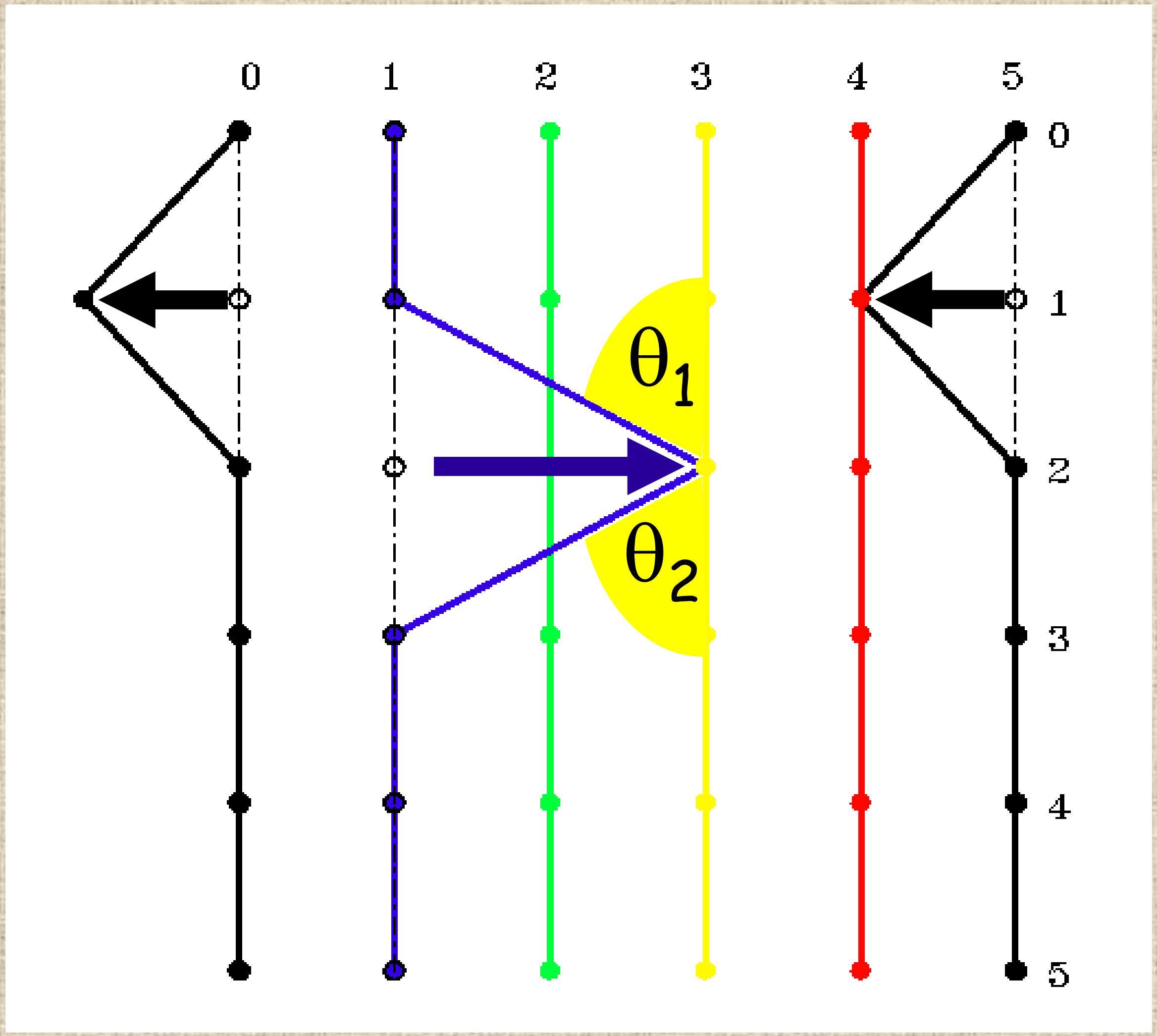
Spontaneous Current Sheets in Magnetic Fields: with applications to Stellar X-rays (Oxford U. Press 1) – Figure 11.2



Photospheric motions shuffle the footpoints of magnetic coronal loops

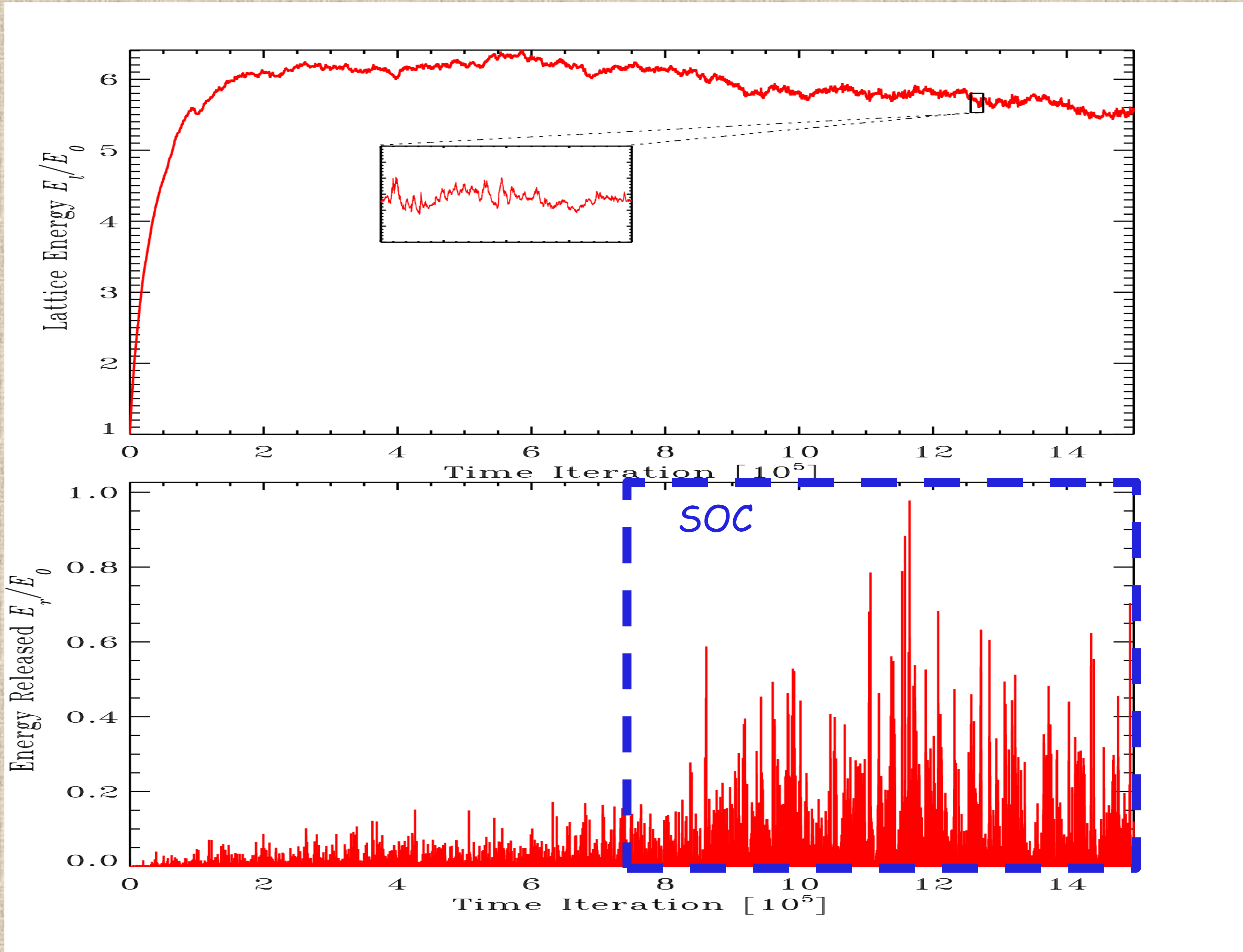
FIELD LINE MODEL

Lattice +
perturbation

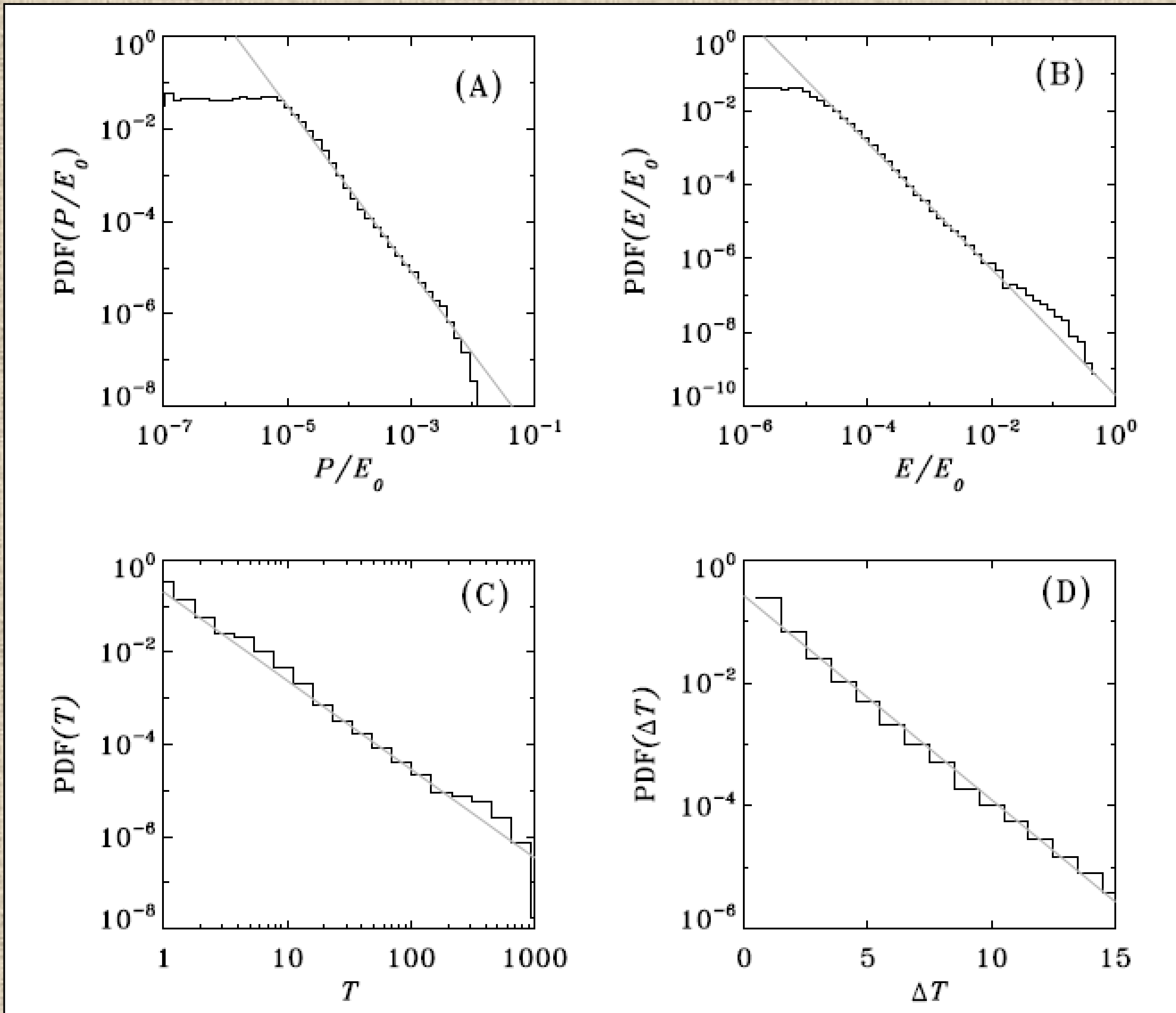


Threshold
$\theta = \theta_1 + \theta_2$
angle
formed by
2 fieldlines

$$\text{Lattice Energy} \sim \sum_i L_i(t)^2$$



Morales, L. & Charbonneau, P. ApJ. 682,(1), 654-666. 2008



PDF'S

Morales, L. & Charbonneau, P. ApJ. 682,(1), 654-666. 2008

	α_E	α_P	α_T
	1.63 ± 0.03	1.79 ± 0.03	1.82 ± 0.09
	1.63 ± 0.03	1.78 ± 0.02	1.95 ± 0.09
A3	1.64 ± 0.04	1.73 ± 0.05	1.93 ± 0.05
B1	1.70 ± 0.02	1.74 ± 0.03	1.80 ± 0.09
B2	1.70 ± 0.03	1.76 ± 0.01	1.89 ± 0.09
B3	1.65 ± 0.02	1.81 ± 0.02	1.90 ± 0.07
C1	1.66 ± 0.05	1.84 ± 0.04	1.79 ± 0.08
C2	1.72 ± 0.05	1.84 ± 0.04	1.85 ± 0.07
C3	1.71 ± 0.05	1.78 ± 0.05	1.92 ± 0.02

Multifractals: Basis

In a cascade over a scale ratio: $\lambda = L/l$

The statistical moments have the scaling behaviour:

$$M_q \sim \lambda^{K(q)} \quad q \geq 0$$

Schertzer and Lovejoy, J. Geophys. Res. 92, 9693-9714, (1987)

Multifractals (Schertzer et al. 2013)

$$H = -K(1)$$

Hurst scaling exponent of the mean field

$$C_1 = dK(q)/dq|_{q=1} + H$$

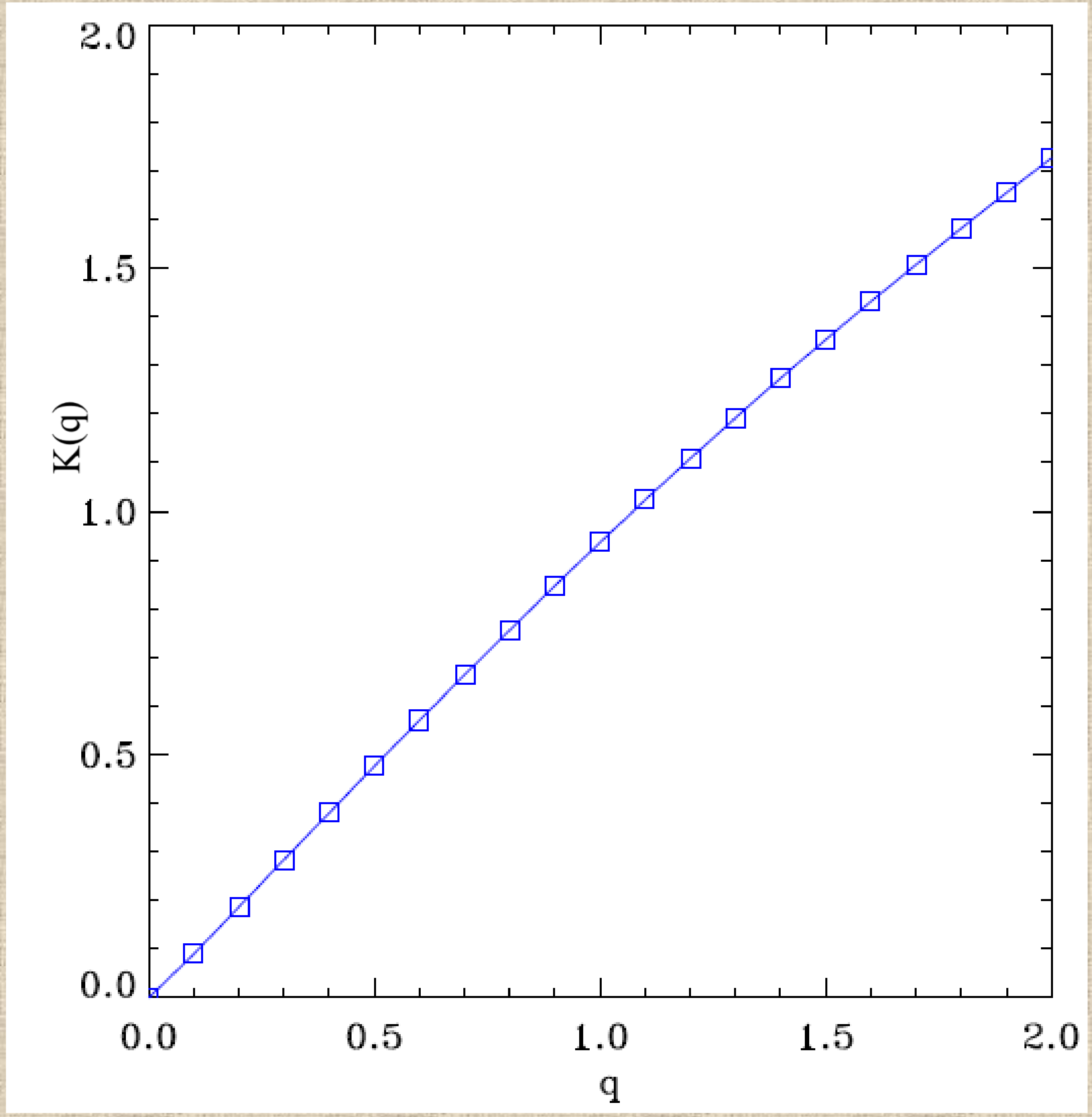
The mean intermittency

$$\alpha = d^2K(q)/dq^2|_{q=1}/C_1$$

Levy Index (multifractality)

$\alpha = 0$ corresponds to fractal

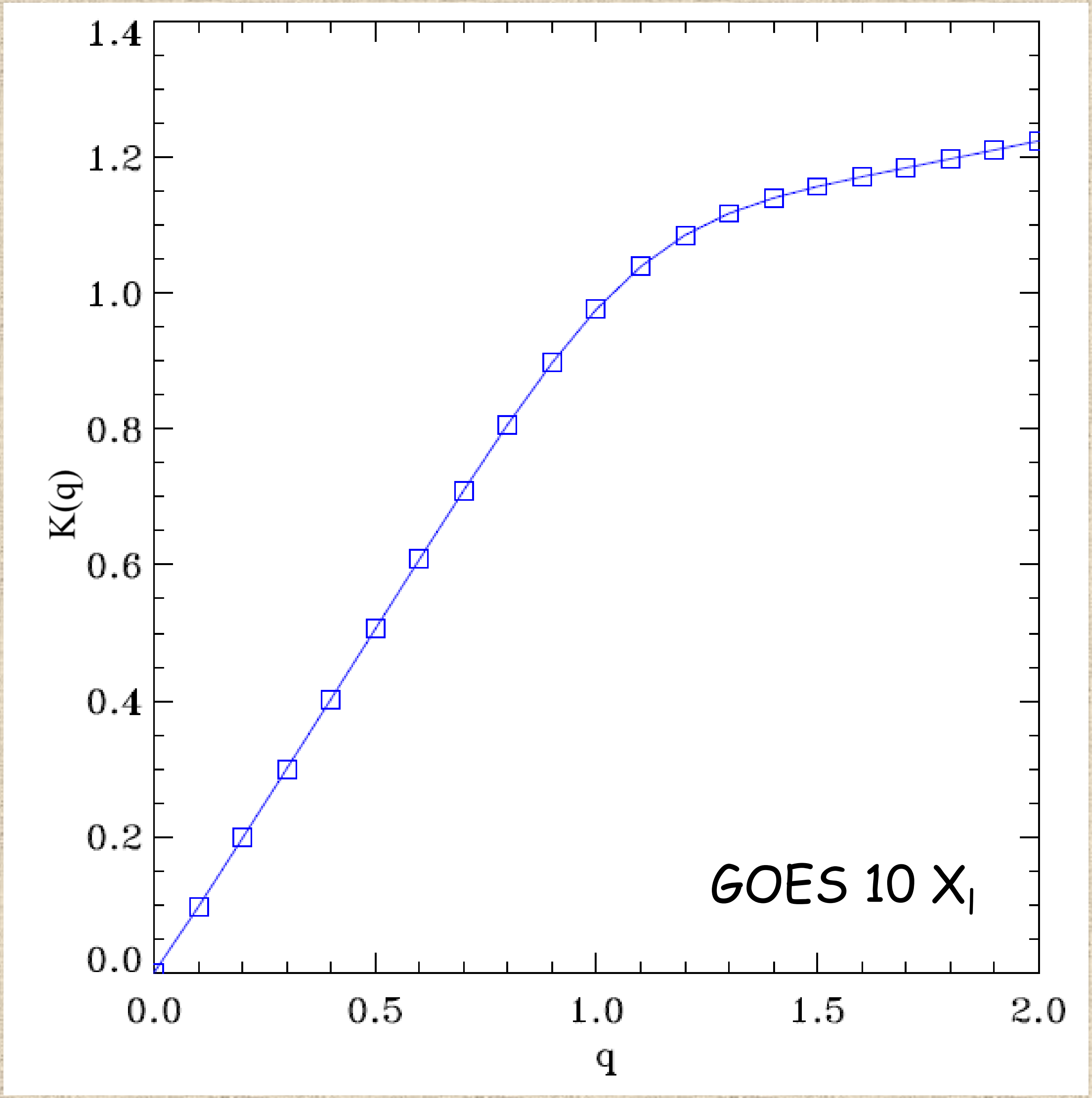
Using lattice of size N=128



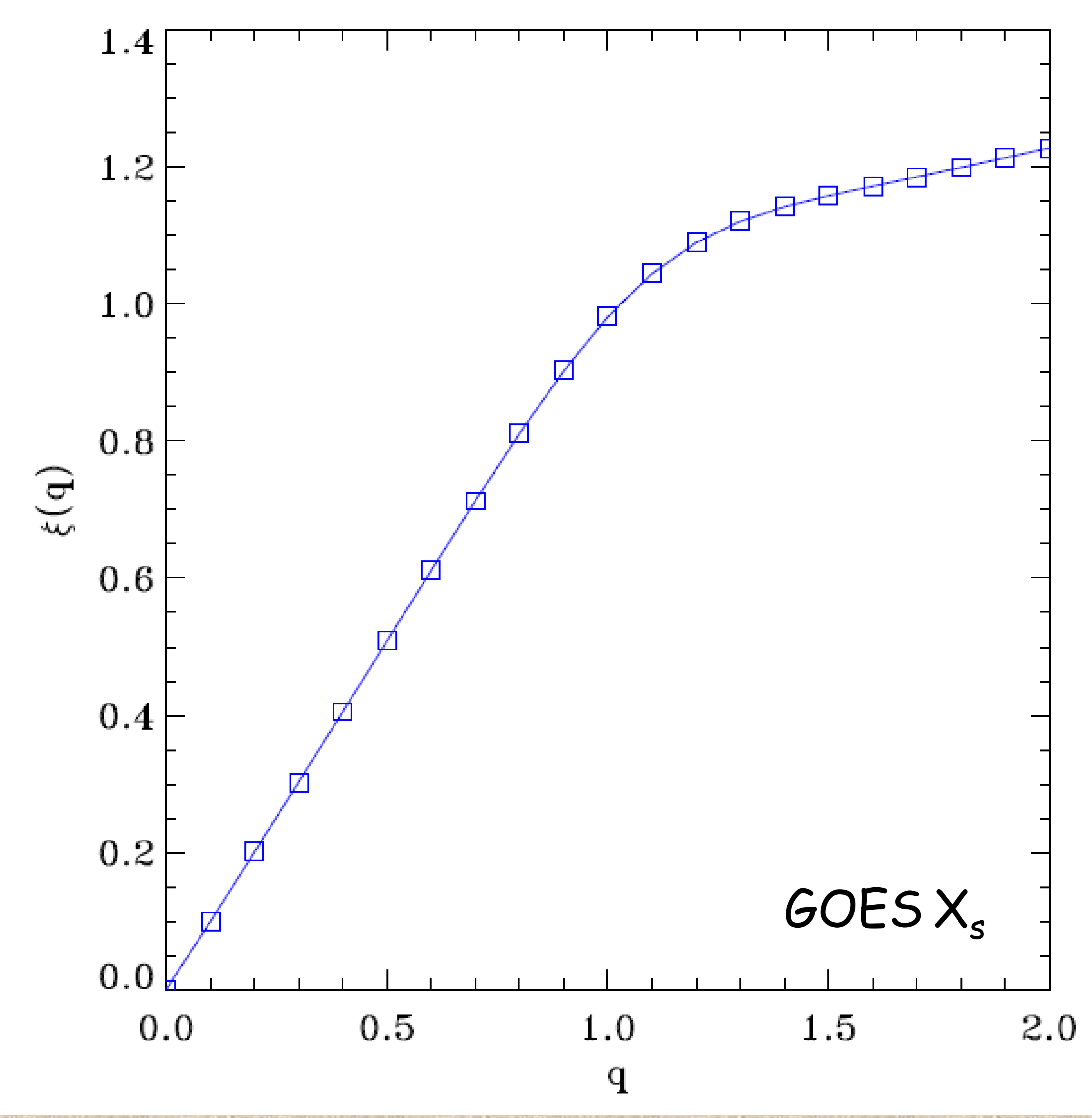
	$H \pm \Delta H$	$C1 \pm \Delta C1$	$\alpha \pm \Delta\alpha$
N = 64 θ = 2	0.86 ± 0.02	0.041 ± 0.05	2.40 ± 0.90
N = 64 θ= 2.25	0.91 ± 0.02	0.057 ± 0.045	2.43 ± 0.93
N = 64 θ= 2.5	0.76 ± 0.02	0.049 ± 0.03	2.42± 0.83
N = 128 θ = 2	0.87 ± 0.02	0.053 ± 0.043	2.47 ± 0.90
N =128 θ = 2.25	0.87 ± 0.02	0.042 ± 0.036	2.72 ± 1.00
N =128 θ = 2. 5	0.93 ± 0.02	0.053 ± 0.026	2.31 ± 0.64

Goes 10

	$H \pm \Delta H$	$C1 \pm \Delta C1$	$\alpha \pm \Delta\alpha$
1/1/2006-31/12/2006 (1 hr interval xl + xs)	0.96 ± 0.05	0.03 to 0.13	2.24 ± 0.45
1/1/2006-31/12/2006 (5 min interval - xl)	0.95 ± 0.05	0.28 to 0.13	2.24 ± 0.11
1/1/2006-31/12/2006 (5 min interval) (xl -xs double data)	0.95 ± 0.05	0.28 to 0.13	2.28 ± 0.12



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
K(q)
0.0
0.5
1.0
1.5
2.0
q
GOES 10 X_l

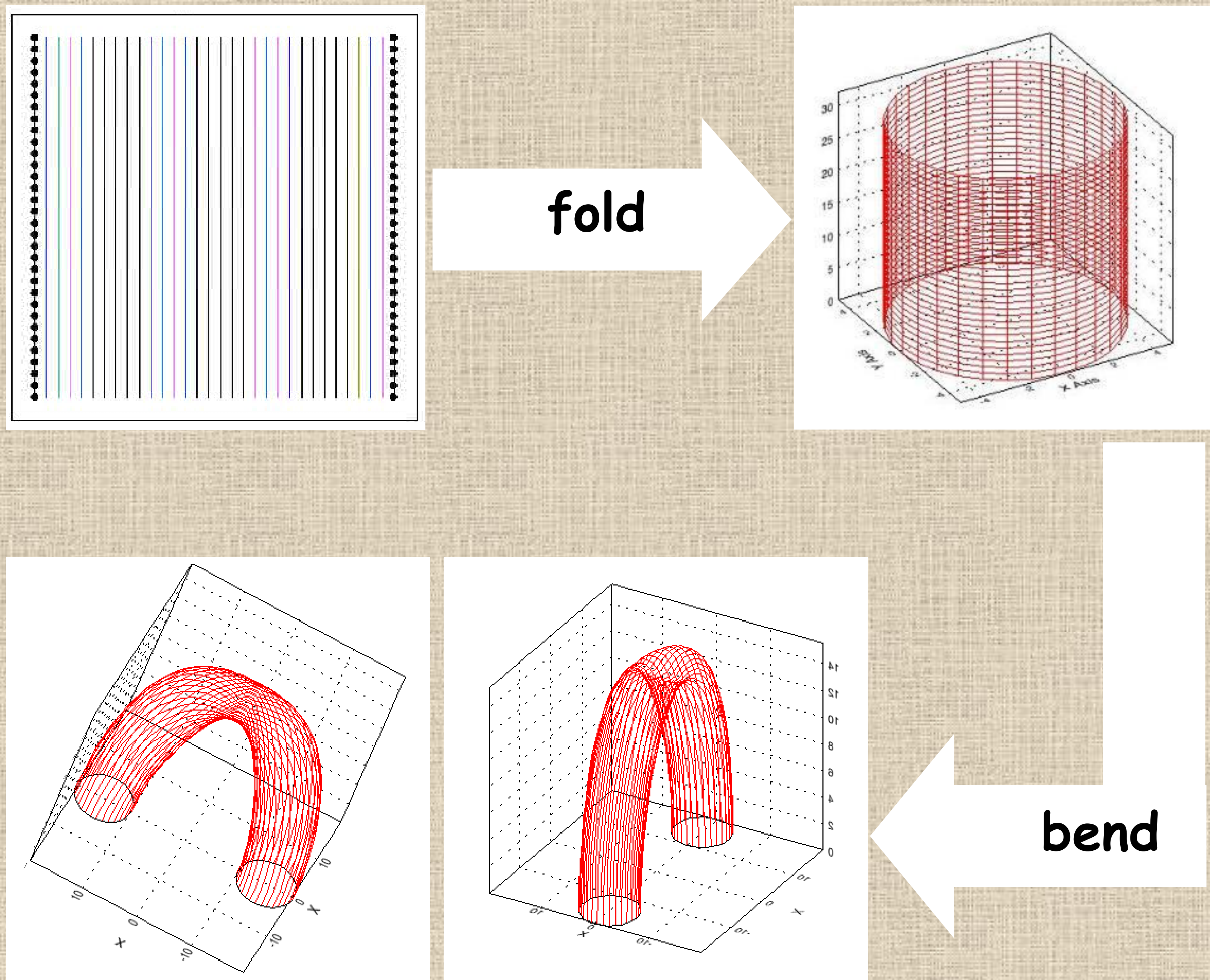

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
ξ(q)
0.0
0.5
1.0
1.5
2.0
q
GOES X_s

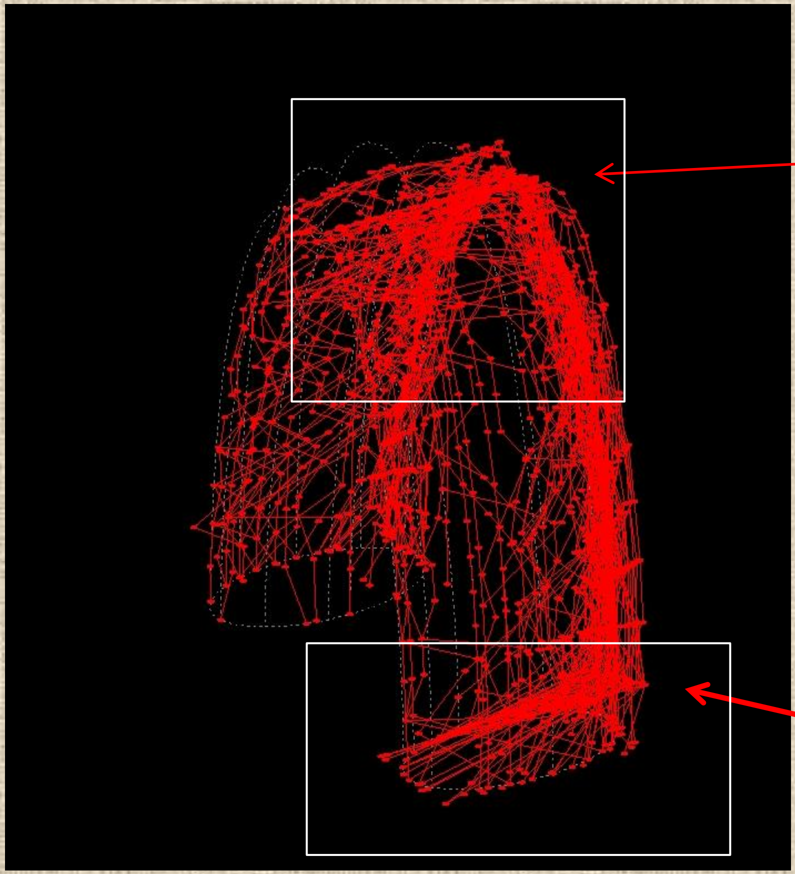
Remaining questions:

Is it possible to test Manolis claims in his SoPhys (2012) paper with this model?

Can fractal or multifractal (AR) be a predictor of solar flares?

From a 2D lattice to a loop





Calculate multifractal dimension of the corresponding flares

Calculate multifractal dimension in the "AR"

How to obtain Gunnar's description for my simulated flares?