At least 3 kinds of scaling in the solar wind (including turbulence)

S. C. Chapman,

With thanks to:

G. Gogoberidze, B. Hnat, E. Leonardis, R. M. Nicol,, A Turner, C. Foullon, (CFSA, Warwick),

K. Kiyani, R. Wicks, (CFSA, Warwick, now at ICSTM),

W. –C. Mueller (MPI Garching), Y. Khotyaintsev (IRF Uppsala), F. Sahraoui (CNRS), M. W. Dunlop (RAL)

- some things we see in the solar wind
- questions we can ask [with statistics]
- Turbulence, and other kinds of scaling
- Finite range nature of turbulence in corona and solar wind- generalized similarity- is it universal?

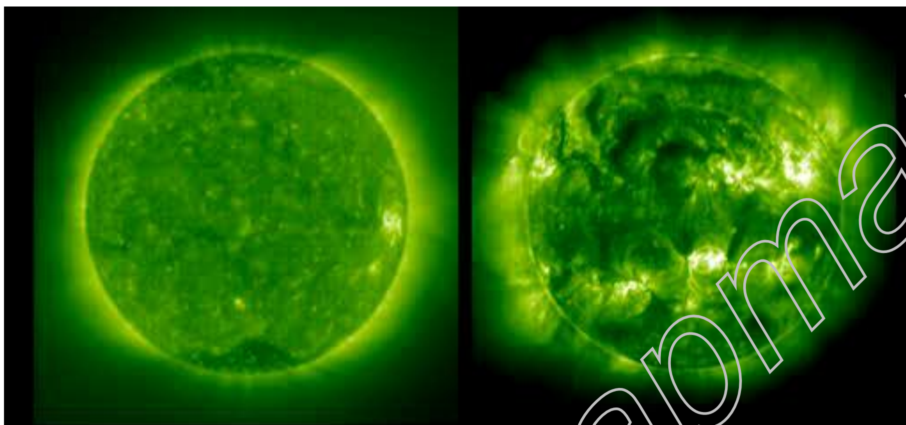
Aim- to spark a discussion on what processes can be responsible for the observed scaling…

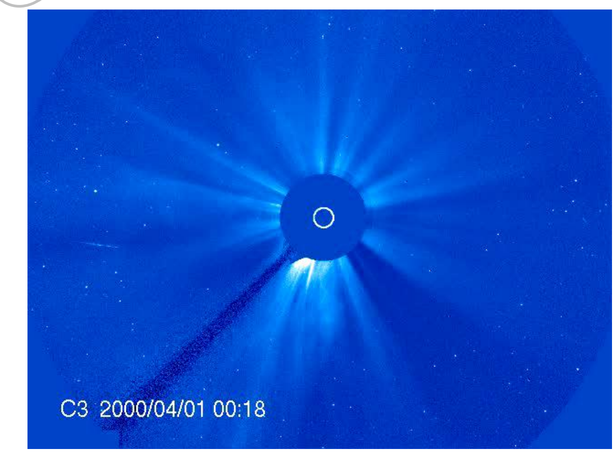
- ***Data thanks to Hinode, CLUSTER, WIND, ACE, ULYSSES teams***

Overview: the solar wind as a turbulence laboratory

SOHO-EIT image of the corona
at solar minimum and solar maximum

SOHO- LASCO image
of the outer corona
near solar maximum





I: coronal signature has scaling properties
II: solar wind has intermittent (multifractal) inertial range of turbulence
III: in-situ observations span inertial range,
dissipation/dispersion range and lower k

Solar wind at 1AU power spectra- suggests inertial range of (anisotropic MHD) turbulence.

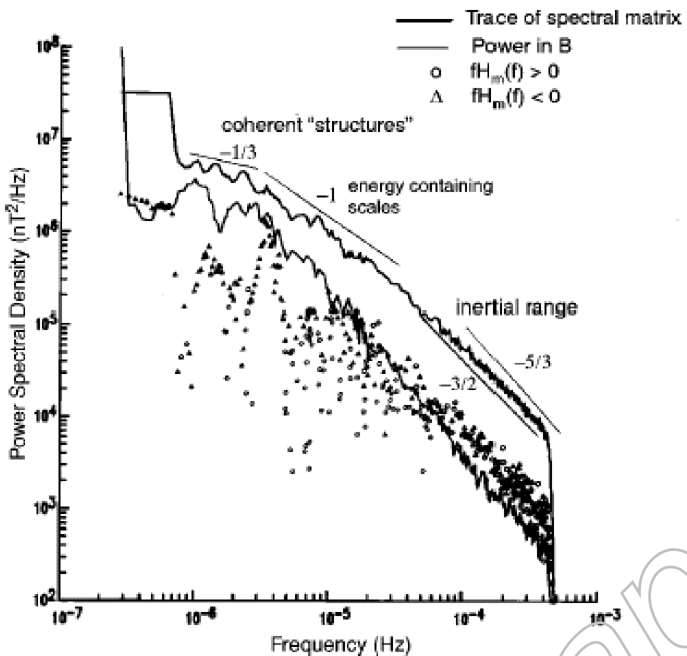


FIG. 1. A power spectrum of the solar wind magnetic field from a time series spanning more than a year. The upper curve is the trace of the power spectral matrix of the three components of B, the lower solid curve is the power in |B|, and the circles and triangles are the positive and negative values of the [reduced (Ref. 91)] magnetic helicity (Refs. 39 and 92) respectively.

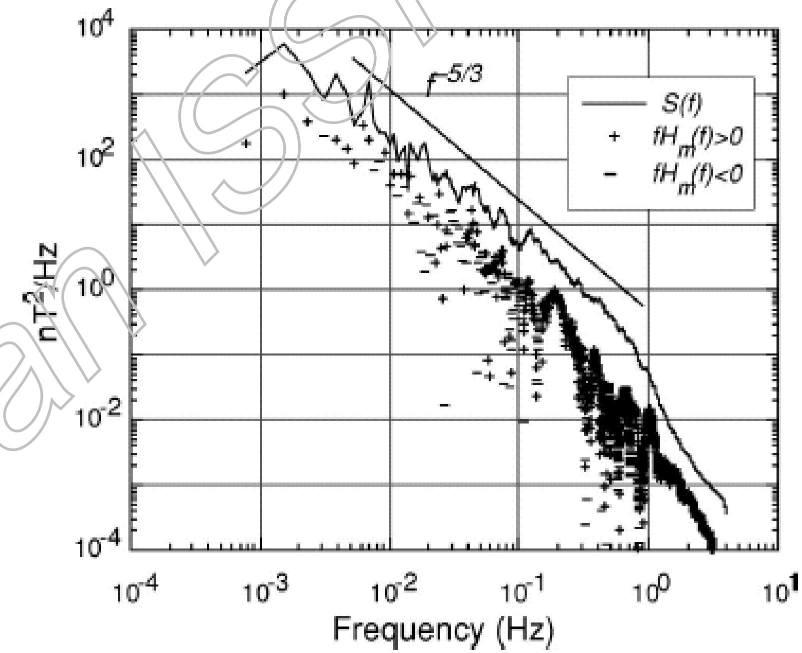


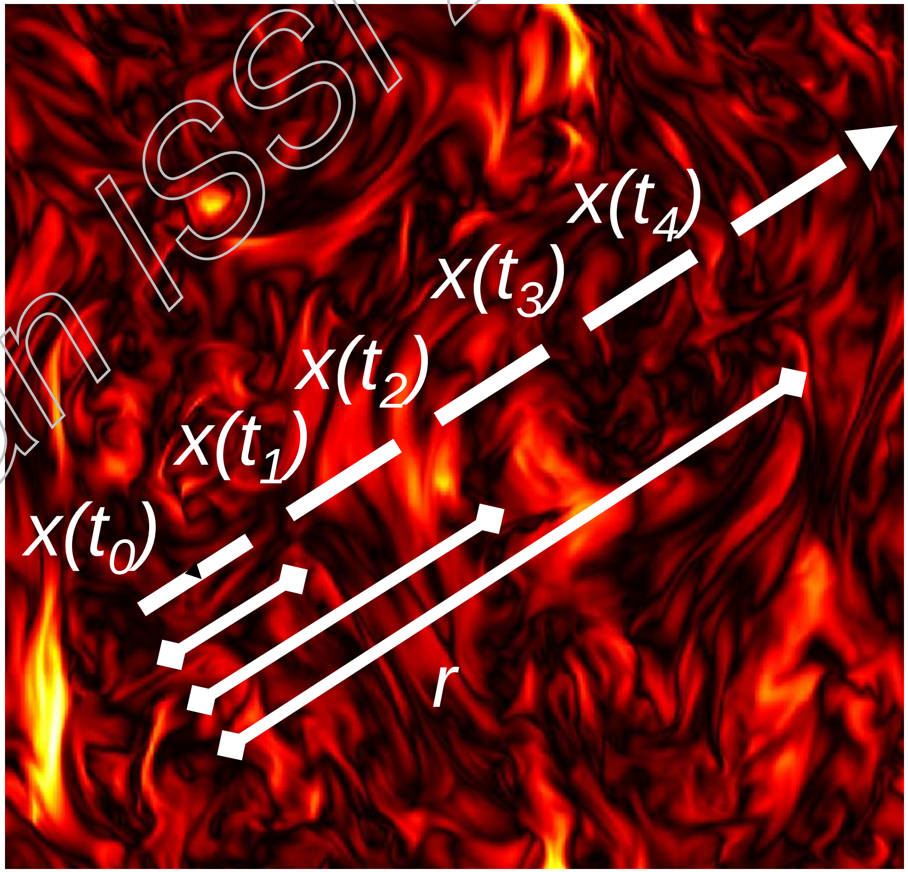
FIG. 2. A power spectrum of Mariner 10 data from 0.5 AU showing the dissipation range of magnetic fluctuations. Also shown are positive and negative values of $fH_m(f)$.

Goldstein and Roberts, POP 1999, See also Tu and Marsch, SSR, 1995

Will talk mainly about GSF analysis in space and time
either as below, or using wavelets… this gives us exponents..
(big industry, is it multifractal? What kind of turbulence is it?)

images- have space and time directly

single spacecraft- time interval τ a proxy for space



look at (time-space) differences:

$y(t,\tau) = x(t+\tau) - x(t)$

for all available t_k of the timeseries $x(t_k)$

test for statistical scaling i.e

structure functions $S_p(\tau) = \langle |y(t,\tau)|^p \rangle \propto \tau^{\zeta(p)}$

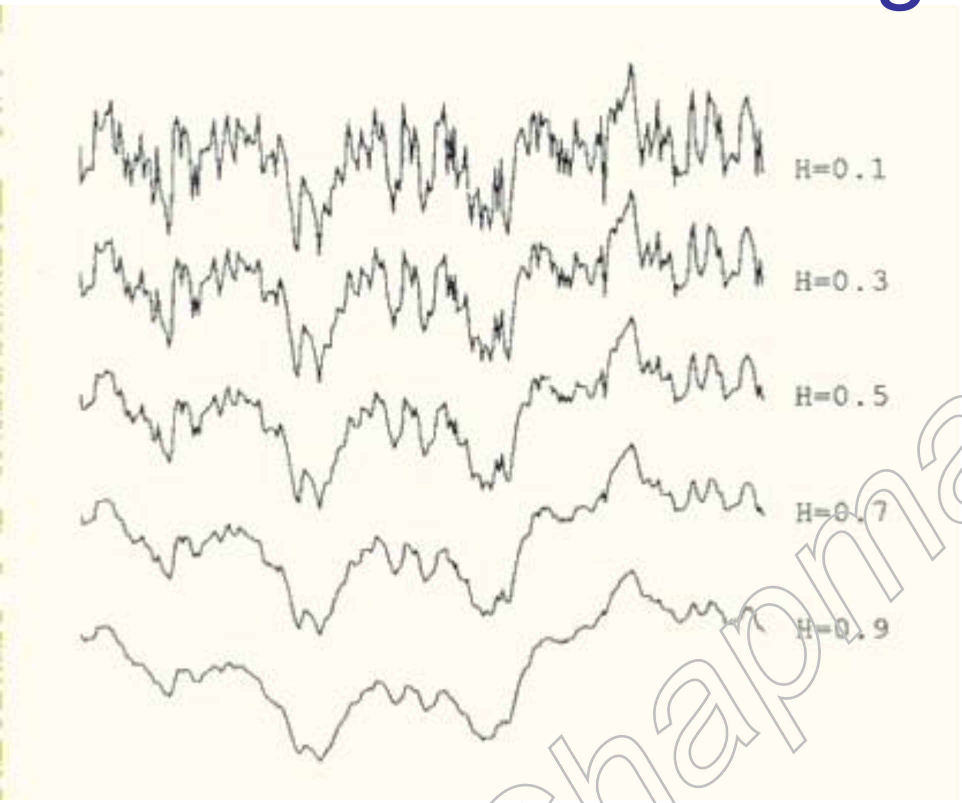
we want to measure the (scaling exponent) $\zeta(p)$

fractal (self- affine) $\zeta(p) \sim \alpha p$

multifractal $\zeta(p) \sim \alpha p - \beta p^2 + ...$

DNS of 2D compressible MHD turbulence
Merrifield, SCC et al, POP 2006,2007

Random fractals and H- characterize 'roughness'



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$y(t,\tau) = x(t+\tau) - x(t)$

for all available t_k of the timeseries $x(t_k)$

structure functions $S_p(\tau) = <|y(t,\tau)|^p> \propto \tau^{\zeta(p)}$

fractal (self- affine) $\zeta(p) \sim \alpha p = Hp$

$\zeta(2) \to \beta$ the PSD exponent

H is Hurst exponent

$H = \frac{1}{2}$ Brownian walk PSD $\sim \frac{1}{f^2}$

('mostly' $\beta = 2H+1$, and PSD $\sim \frac{1}{f^\beta}$

think- low pass filter of PSD='smoothing')

one exponent H, or $\xi(p) = \alpha p$

In solar wind usually assumed
multifractal, non Gaussian=turbulence

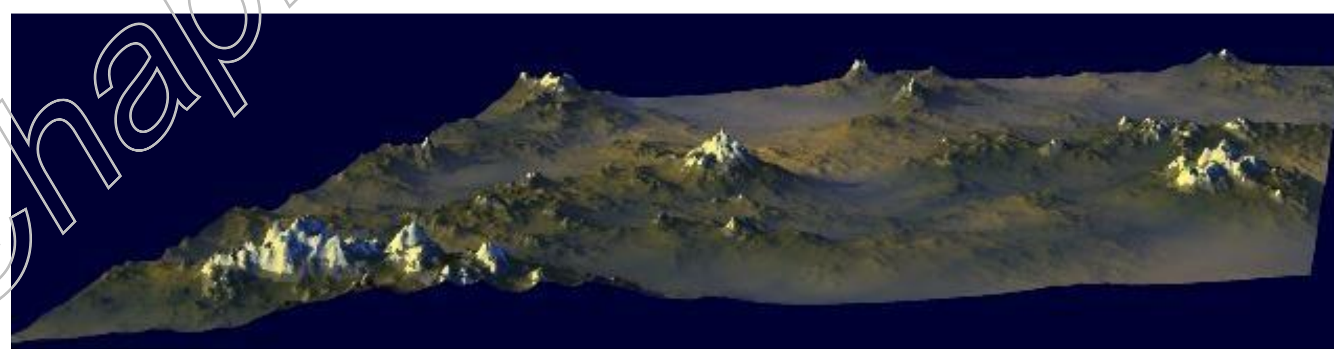
Fractal- fBm D~2.2 pointy mountains everywhere

Multifractal- pointy mountains intermittently between smooth plains

Courtesy K Musgrave

exponent is local

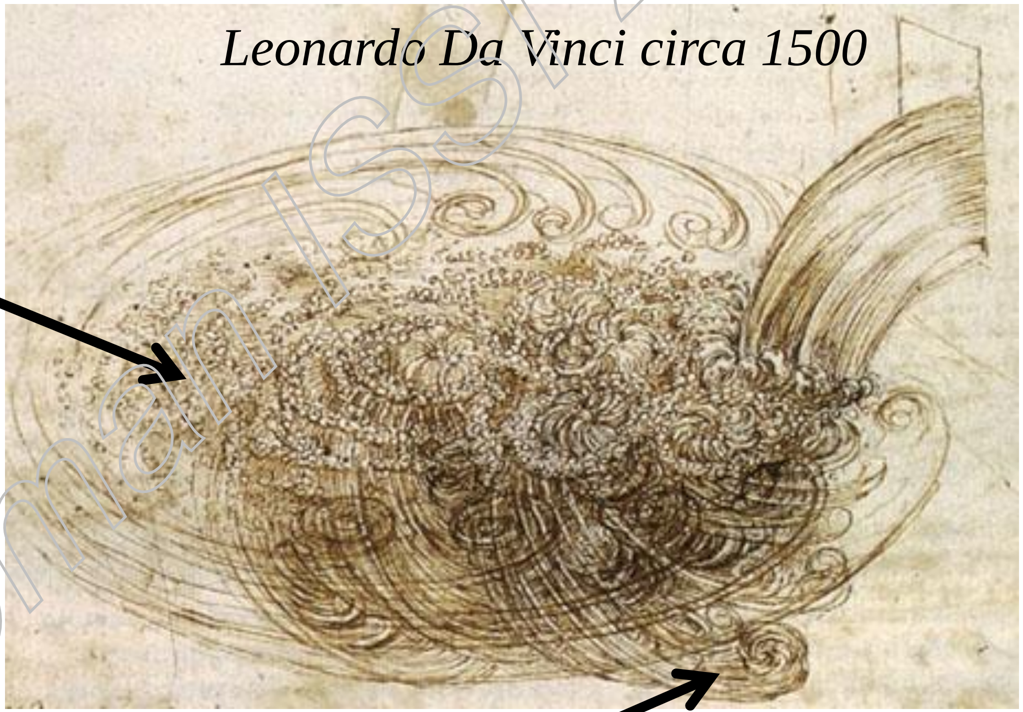
$$\xi(p) = \alpha p + \beta p^2 + \ldots$$



Physics of real finite sized turbulence- generalized similarity

Smallest scales..
Energy dissipation and crossover to different physics (here, bubbles in the fluid)

Intermediate scales..
Cascade- small structures are 'scaled down' versions of big structures



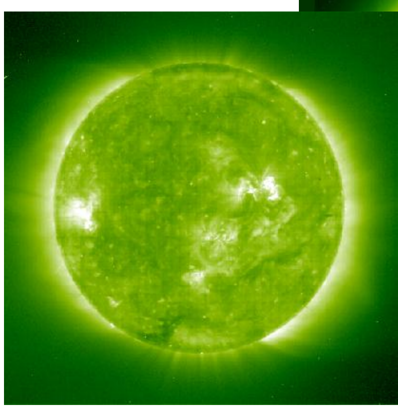
Largest scales..
eddies of different sizes- scale onto each other

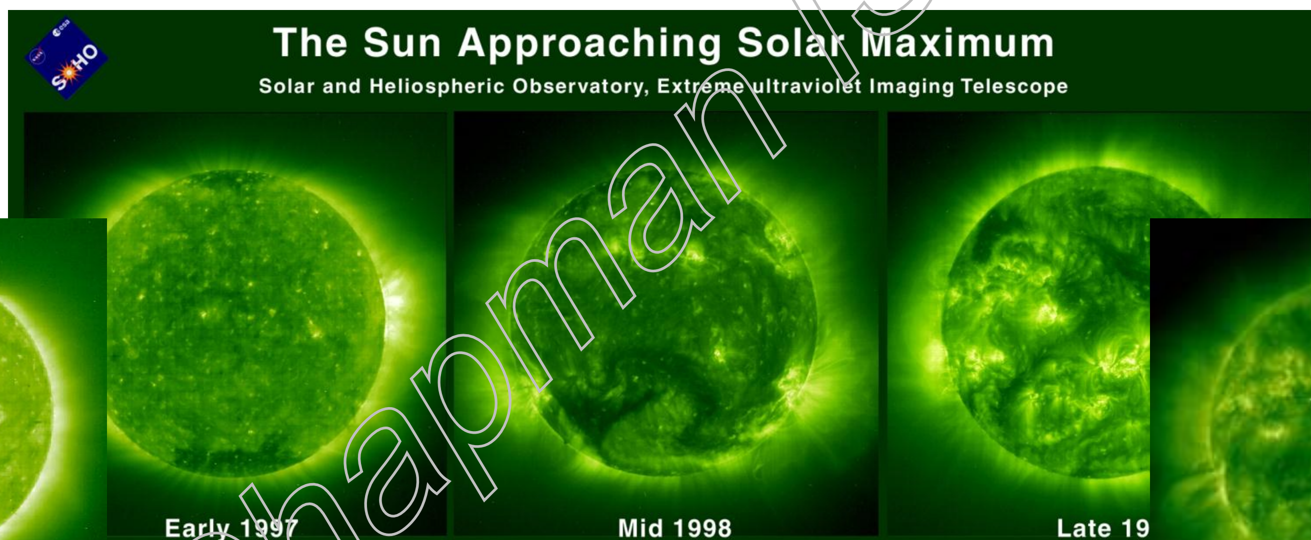
Inertial range MHD turbulence and its outer scale- in time

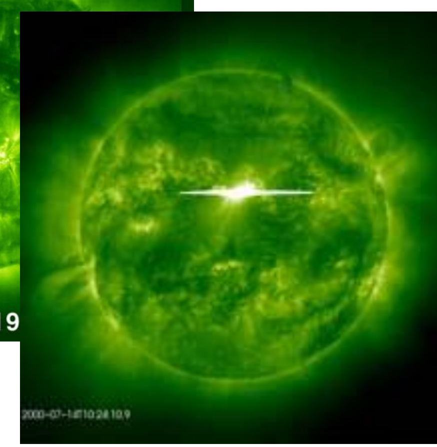
-See *Nicol, SCC et al, ApJ (2008), SCC et al ApJL (2009), SCC, Nicol PRL (2009)*

ULYSSES- north and south polar passes at solar minimum and the unusually quiet recent solar min

ULYSSES 60s averages B field components
-Multifractal
-[NB this is a rough measure]

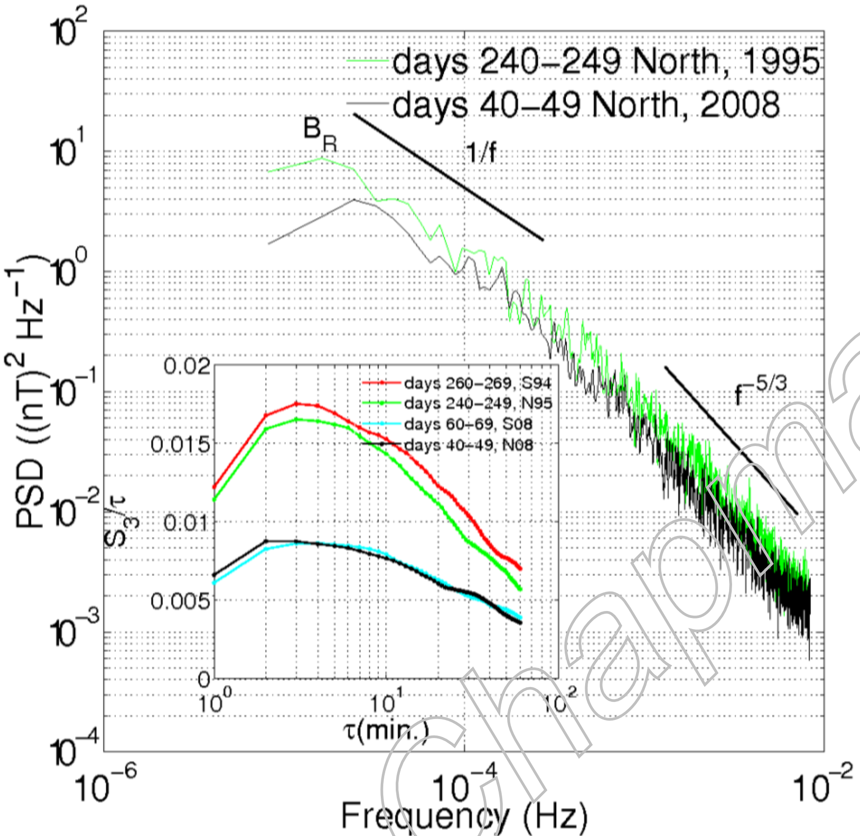






Solar wind turbulence at the two solar cycle minima

ULYSSES- 10 day interval from each of the last two minima, power spectra



Density- 17% lower
B field- 15% lower
Fluctuations- ~ factor of 2 lower

McComas et al GRL (2008)
Smith and Balogh GRL (2008)
Issautier et al GRL (2008)

Finite range turbulence-
Quiet, fast polar solar wind: 2008 North polar pass, solar min, ULYSSES

GSF analysis in time

$$S_p = \langle B(t+\tau) - B(t) \rangle_t$$

Inertial range turbulence- expect

$$S_3 \sim \tau^{\zeta(3)}$$

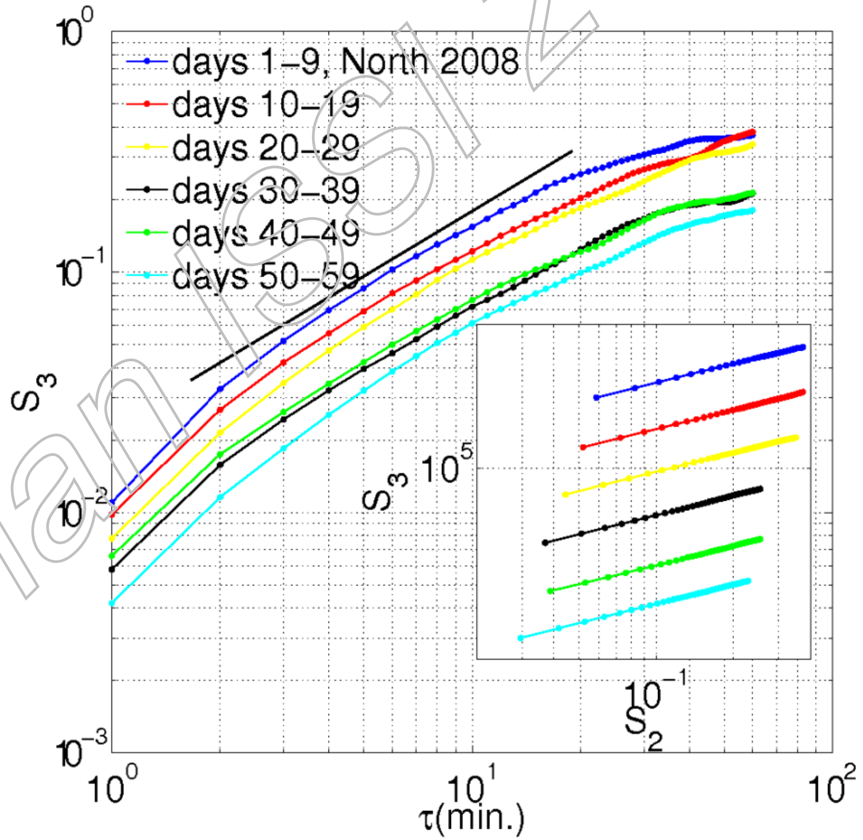
i.e. straight line on log-log plot

not quite seen here! instead

$$S_3 \sim g(\tau)^{\zeta(3)}$$

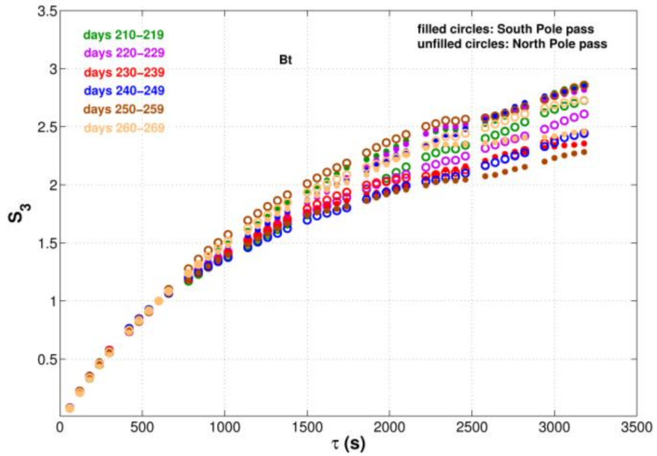
$$S_p \sim S_q^{\zeta(p)/\zeta(q)}$$

Extended Self Similarity (ESS)

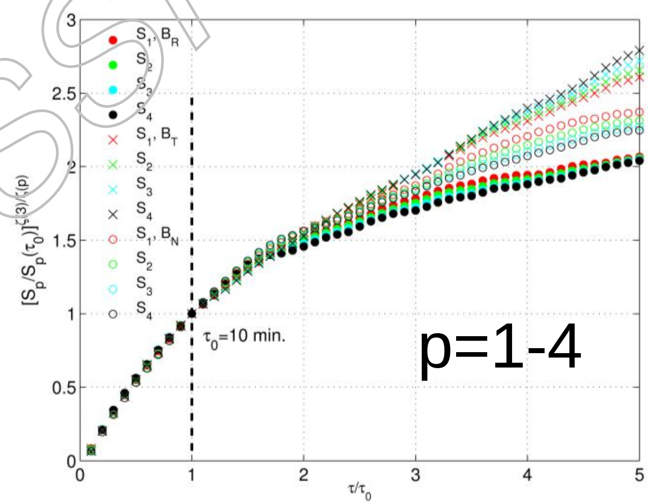


Generalized similarity (scaling)- turbulence at the outer scale- universal behaviour?

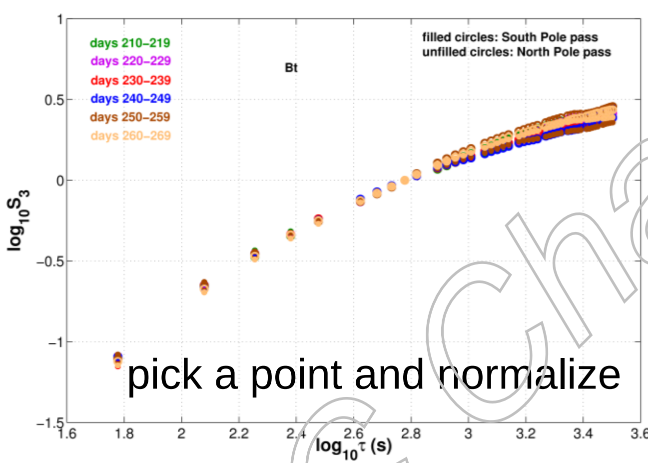
All intervals South pass 1994
North pass 1995, solar min



All components one interval



p=1-4



pick a point and normalize

$$S_p \sim g(\tau)^{\xi(p)}$$

invert to obtain $g(\tau)$

same $g(\tau)$ seen

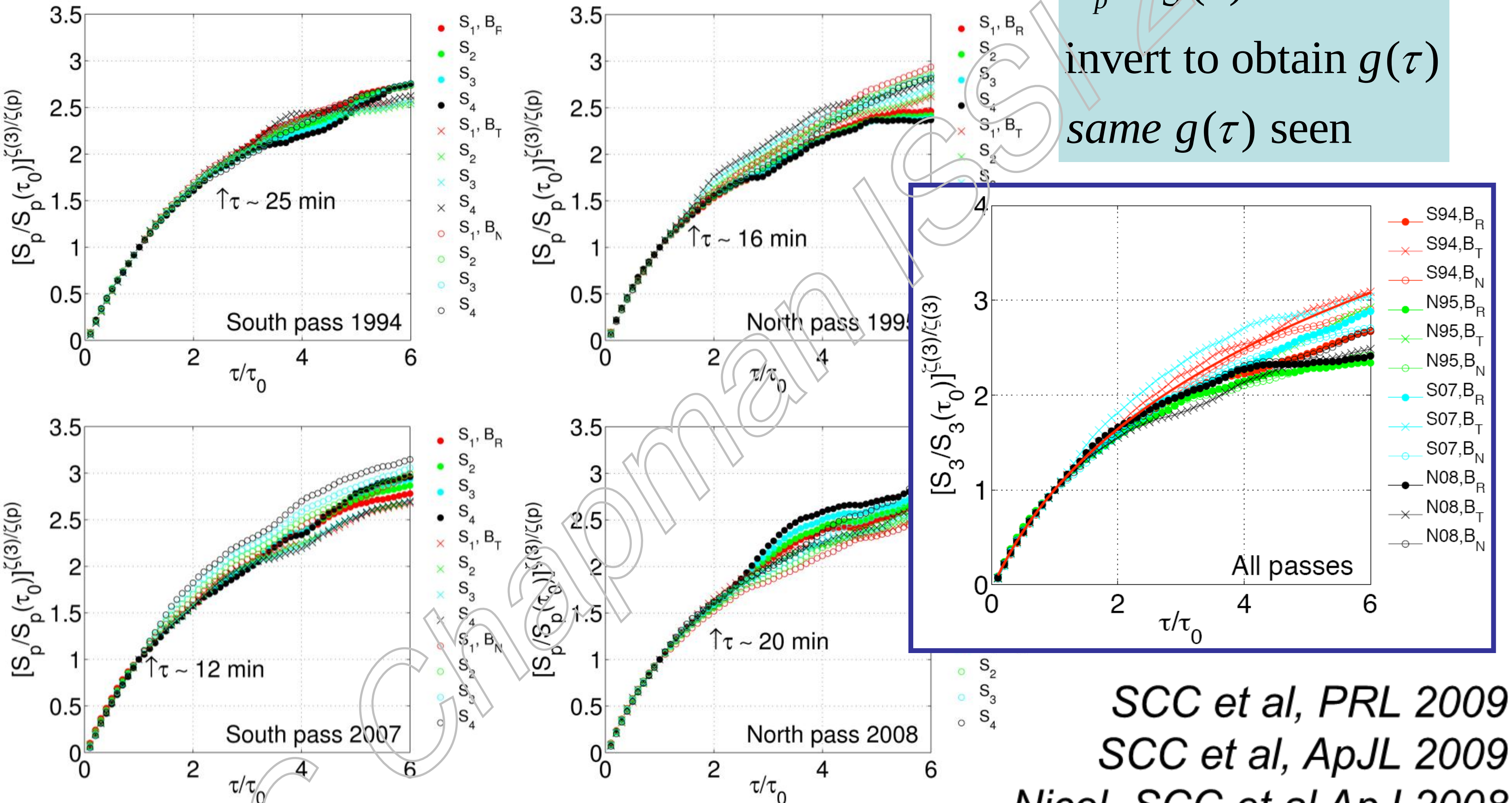
SCC et al, PRL 2009
SCC et al, ApJL 2009
Nicol, SCC et al ApJ 2008

g(τ) all four passes all B components

$S_p \sim g(\tau)^{\xi(p)}$

invert to obtain $g(\tau)$

same $g(\tau)$ seen

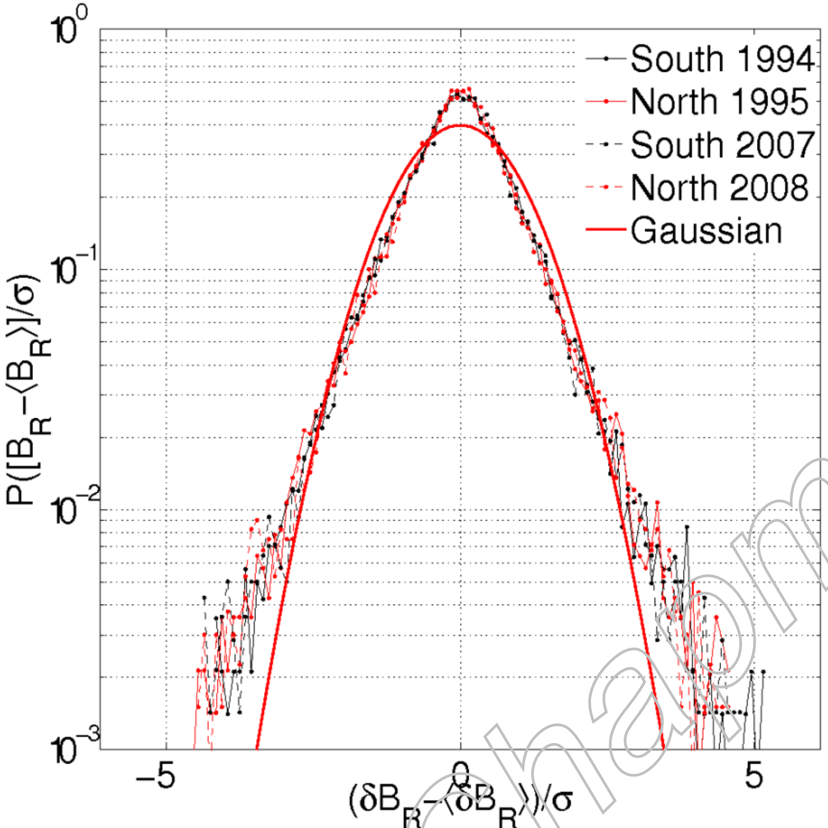


SCC et al, PRL 2009

SCC et al, ApJL 2009

Nicol, SCC et al ApJ 2008

Fluctuation PDF, the two solar minima compared



Bottom line- a robust underlying fluctuation pdf, scaling and G function

Inertial range MHD turbulence and its ‘outer scale- in space

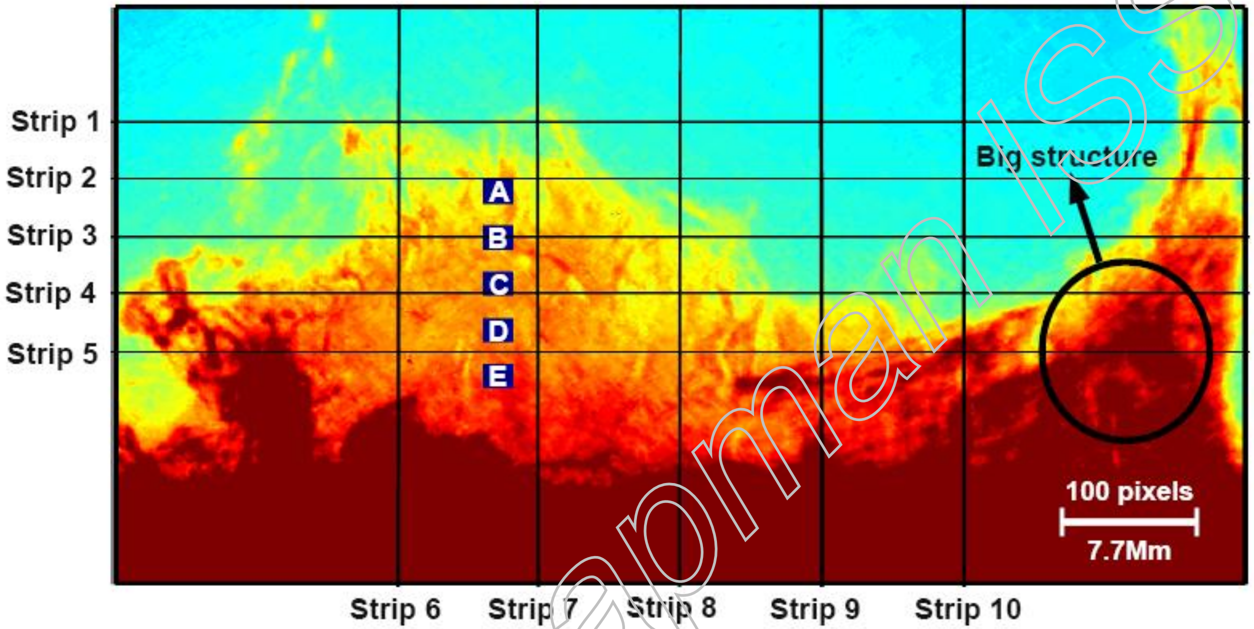
-See *Leonardis, SCC et al, ApJ (2012)*

HINODE SOT optical observations- is this finite sized turbulence?

In space: HINODE SOT optical observations

- **Data Set:** about 1000 images (2084 x 1024 pixels with a resolution of 0.10896 arcsec/pixel) of the solar corona in the Ca II H-line provided by the Broadband Filter Imager (BFI) on the SOT;

- **Time interval considered:** observations from 01.00 UT to 06.00 UT on November 30, 2006 with a cadence of ~16.5 sec on average.

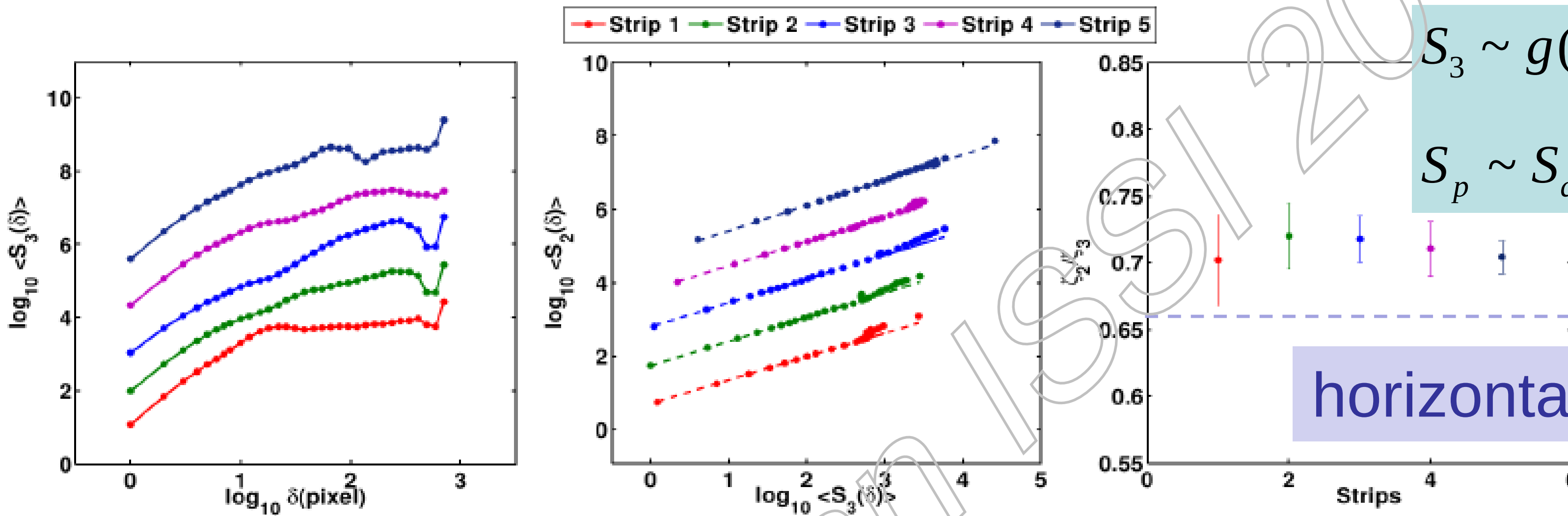


Do all the strips show the same fluctuation properties?

can look at structure functions

in space δ $S_p(\delta) = <| y(t,\delta) |^p > \propto g(\delta)^{\zeta(p)}$

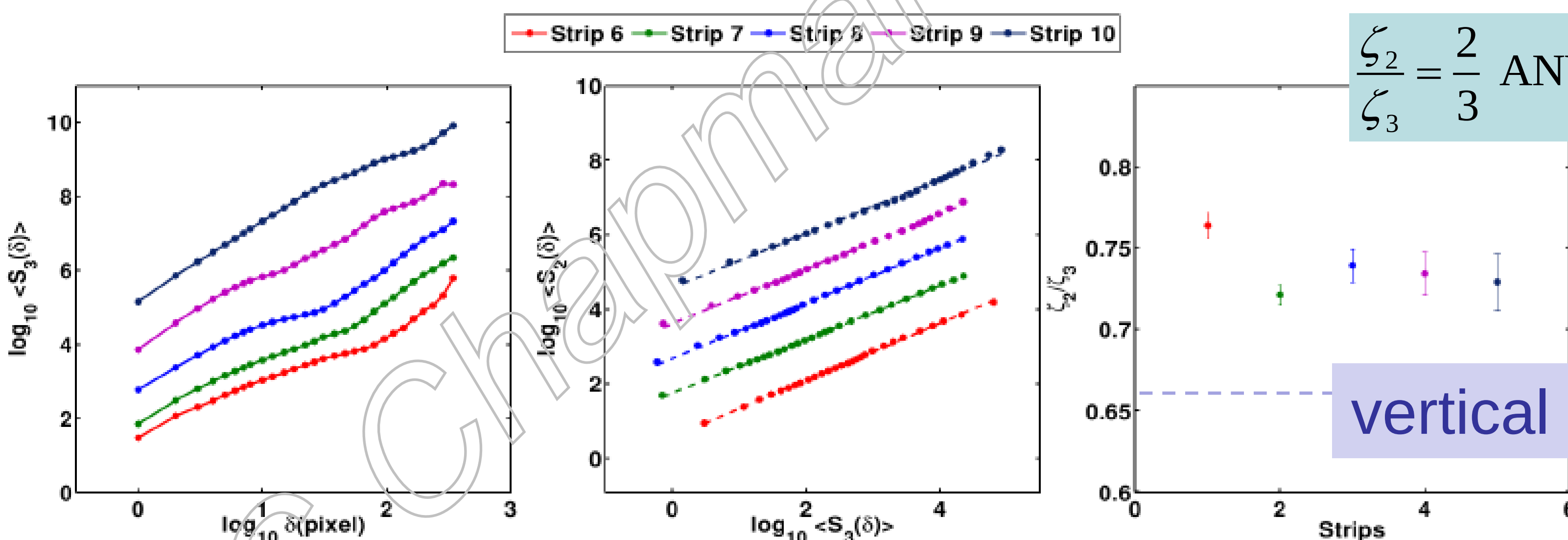
Looking in space- ESS a signature of finite range turbulence



$$S_3 \sim g(\delta)^{\zeta(3)}$$

$$S_p \sim S_q^{\zeta(p)/\zeta(q)}$$

horizontal strips



$$\frac{\zeta_2}{\zeta_3} = \frac{2}{3} \text{ ANY fractal}$$

vertical strips

Leonardis, SCC et al, Ap J 2012

Beyond inertial range MHD turbulence – things from the sun?

-See *Hnat, SCC et al PRE 2011*

In situ turbulence is not the only mechanism for scaling

Solar wind at 1AU power spectra- suggests inertial range of (anisotropic MHD) turbulence in components, ‘1/f’ … and a single power law range in |*B*|

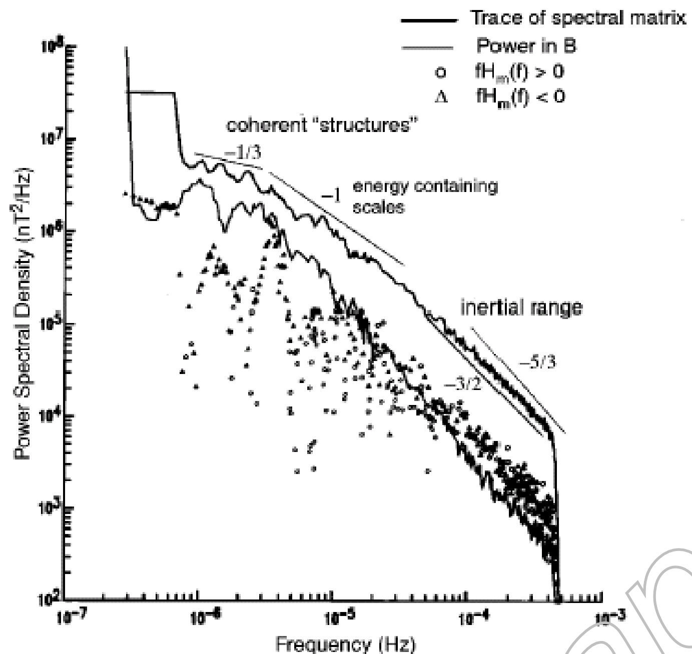


FIG. 1. A power spectrum of the solar wind magnetic field from a time series spanning more than a year. The upper curve is the trace of the power spectral matrix of the three components of B, the lower solid curve is the power in |B|, and the circles and triangles are the positive and negative values of the [reduced (Ref. 91)] magnetic helicity (Refs. 39 and 92) respectively.

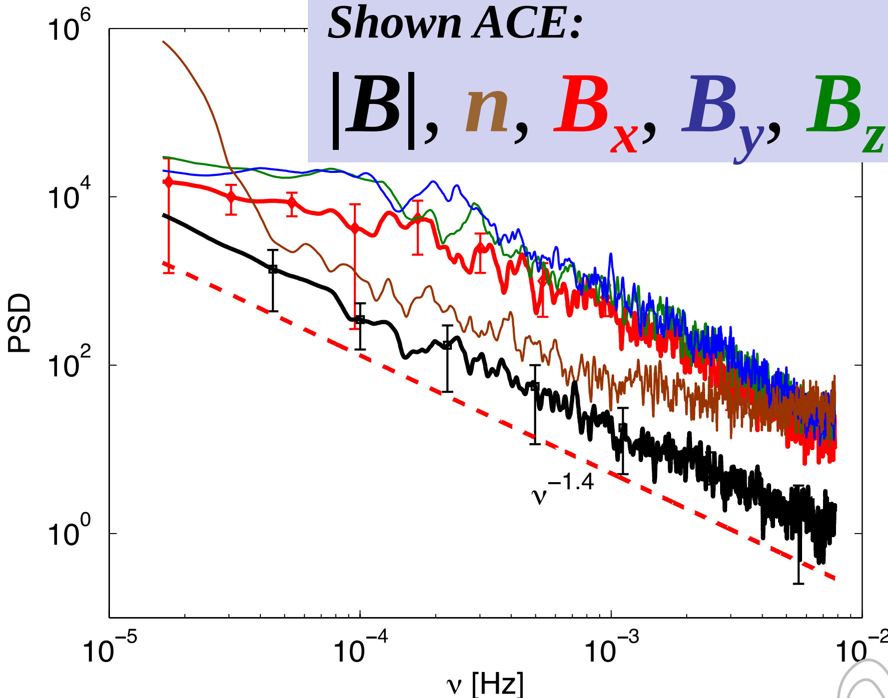
Goldstein and Roberts, POP 1999,
see also Tu and Marsch, SSR, 1995

‘compressive’ fluctuations on mins to hrs at 1AU:

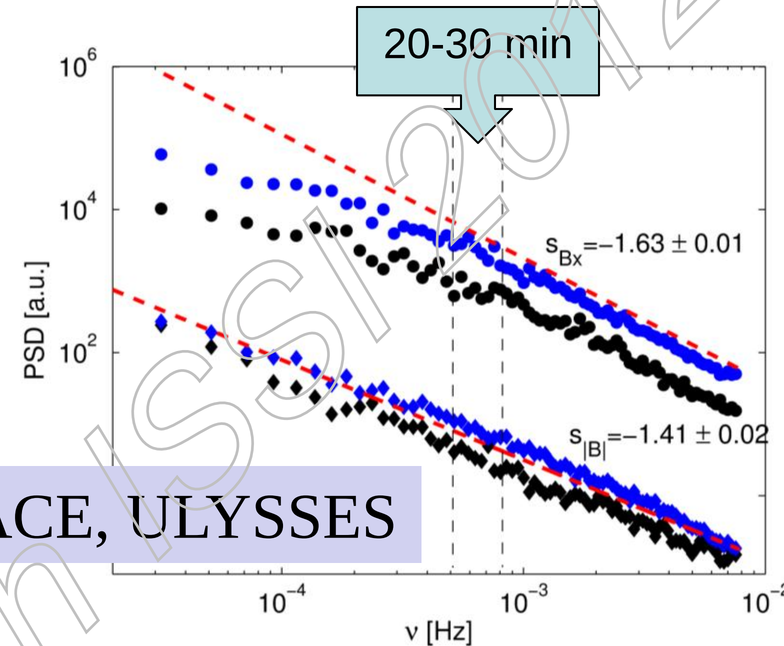
- |*B*| PSD suggests a single scaling range
  - -distinct from components
  - -distinct from *n*
- Evolution of PSD with distance (HELIOS)
  (Denskat & Neubauer 1982; Bavassano et al. 1982; Marsch & Tu 1990)
  - -overall power decreases, power ratio $|B|/B_k$ increases.
  - -PSD exponent invariant with radial distance below *10−2Hz*; ’flattening’ at higher frequencies seen in the inner heliosphere evolves away by the time fast solar wind reaches 1AU.
- scaling in long time bulk hourly averages *(Burlaga 1992)*
- scaling in statistics *(Hnat et al. 2002; Hnat et al. 2003)* up to 10s of hrs.
- admixture of compressive fluctuations and pressure balanced structures *(Tu & Marsch 1994)*
- |*B*| and *n* do not simply advect together as passive scalars *(Hnat et al. 2005).*
- Solar cycle dependence in the scaling of B^2 *(Kiyani et al 2007)*

Shown ACE:

|B|, n, B_x, B_y, B_z



ACE, ULYSSES

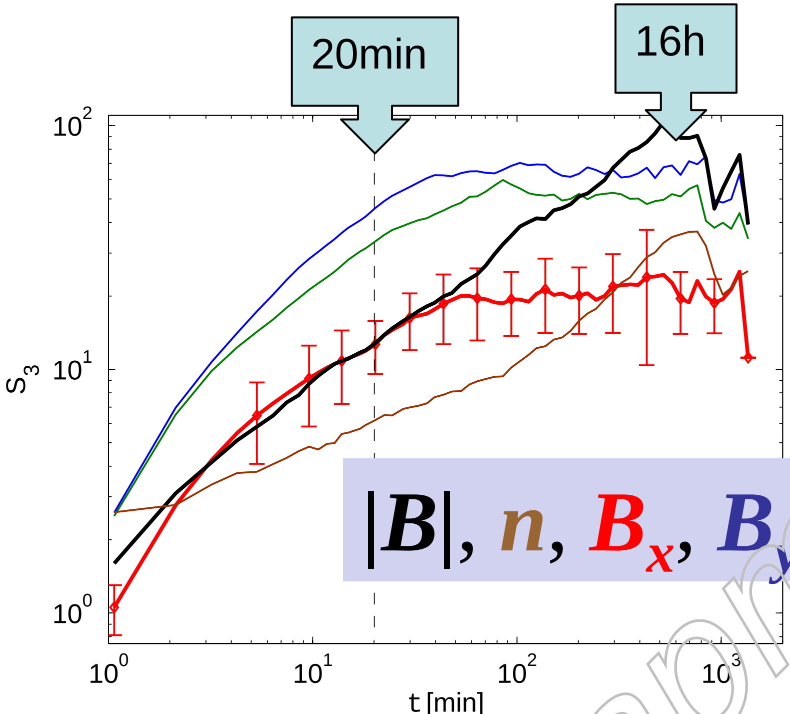


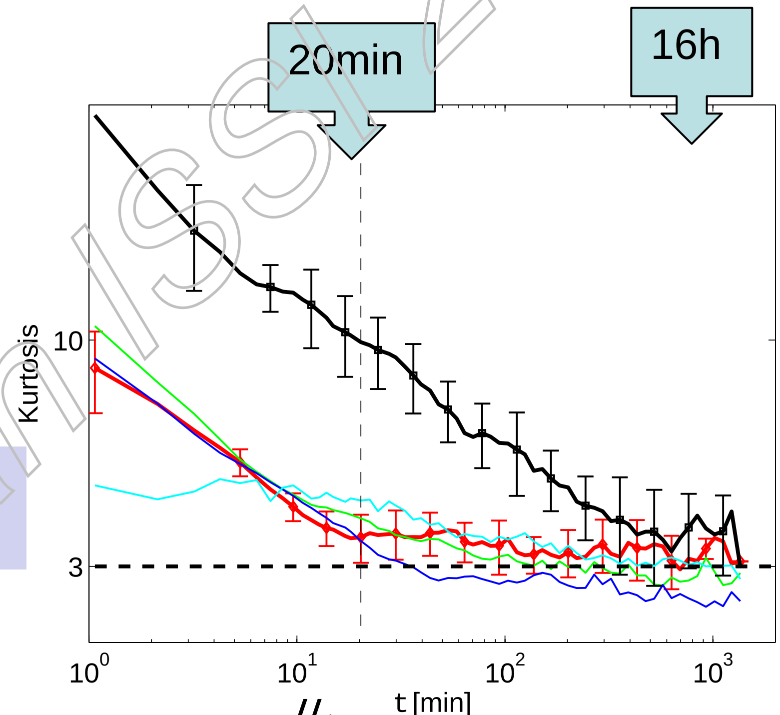
Hnat, SCC et al, 2011 arXiv

The datasets: fast quiet solar wind speed $< v > > 550$ *km/s*
standard deviation $\sigma(v) < 50$ *km/s*.
ACE: 64 s average magnetic field and plasma observations
Continuous >20 hr intervals of quiet fast solar wind at solar minimum which did not contain large coherent structures or secular trends in P
5 fast solar wind intervals were identified in 2007 and 2008 with wind
Ulysses: 5 intervals of 3 days duration, 1995(solar min) north polar pass.
1 min average B field, 4 min plasma

For each statistical quantity
1) Compute
2) Average
3) Error bars indicate variation on the average

Statistical measures with scale: ACE





$$S_3 = \left\langle \left| x(t+\tau) - x(t) \right|^3 \right\rangle$$

$$x \equiv B_x, B_y, B_z, |B|, n$$

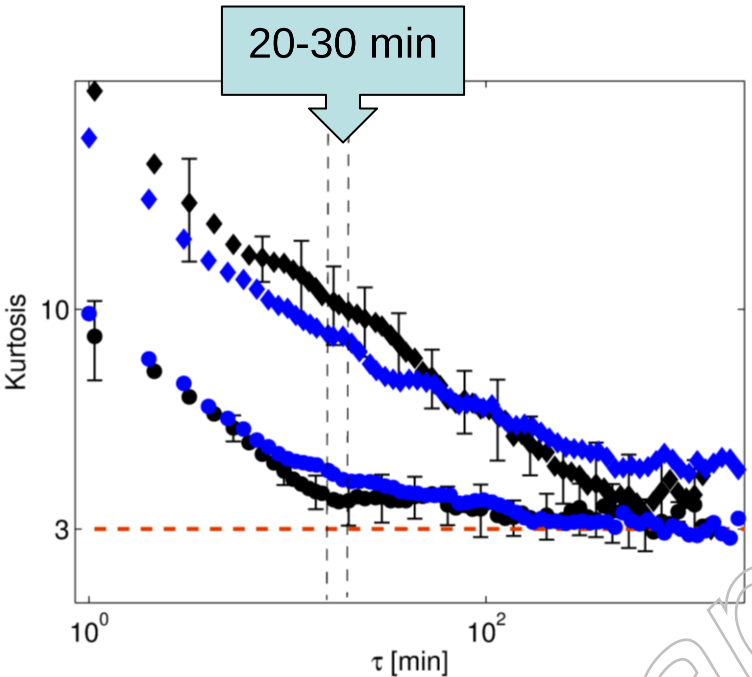
$$Kurtosis\ K = \frac{\mu_4}{\mu_2^2}$$

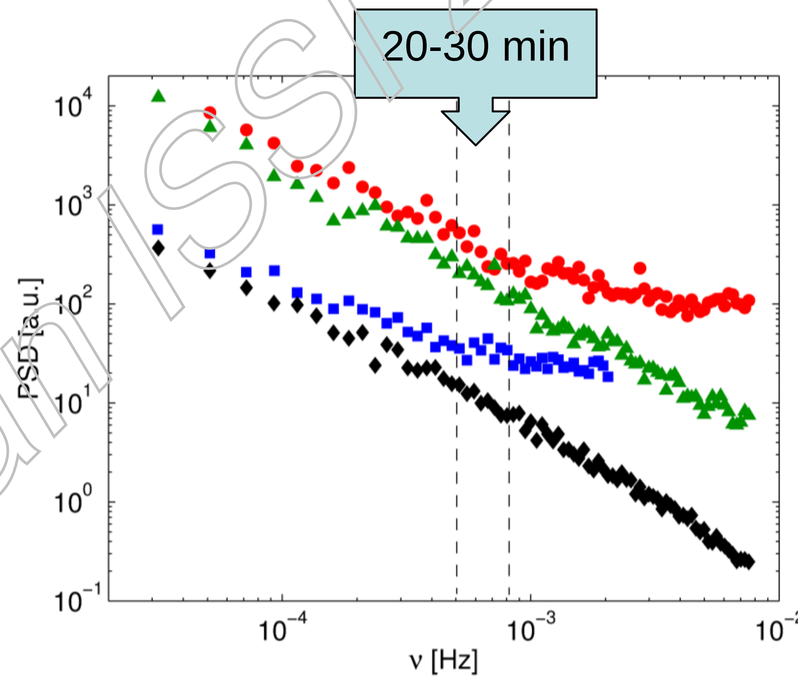
$$\mu_k = \left\langle \left[(x(t+\tau) - x(t)) - \langle x(t+\tau) - x(t) \rangle \right]^k \right\rangle$$

$K = 3$ Gaussian

Kurtosis μ_4 / μ_2^2

$$\mu_k = <(\delta x(t,\tau) - <\delta x(t,\tau)>)^k>$$





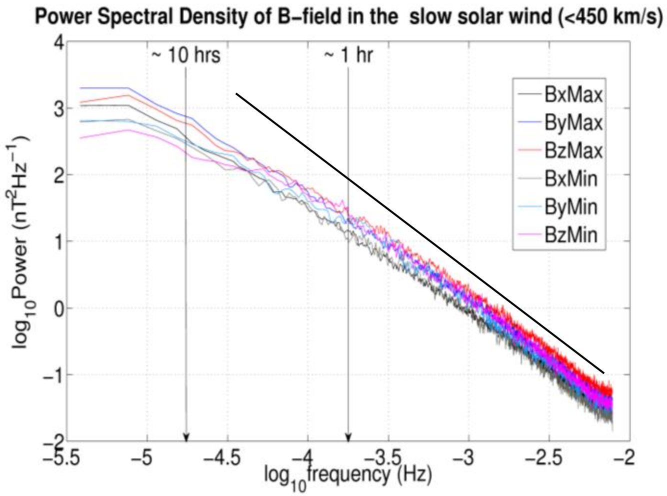
Left: Kurtosis of B_x from ACE (black circle), B_R from Ulysses (blue circle) and |B|: ACE-black diamond, Ulysses-blue diamond.

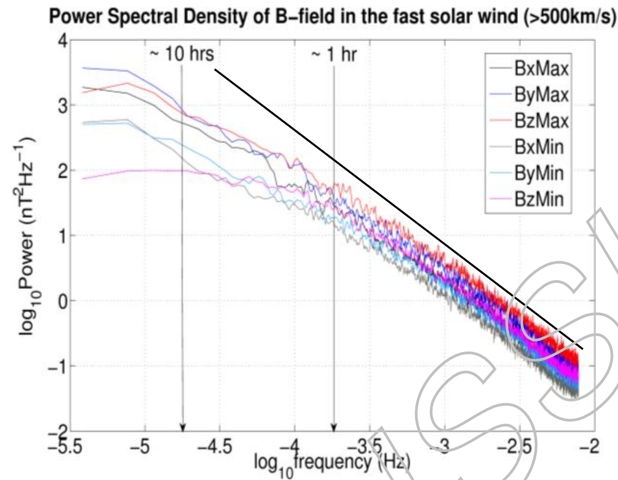
Right:|B| and density

Not passive scalar, see also Hnat, SCC et al PRL 2005

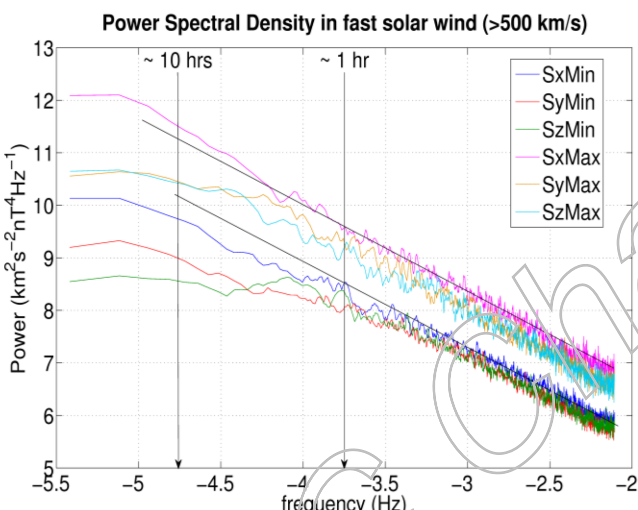
Components show 2 regions inertial range and '1/f'

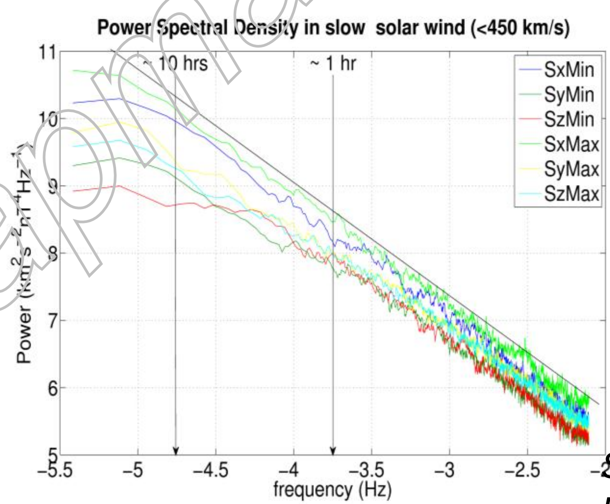
x- component of Poynting flux B magnitude one single region

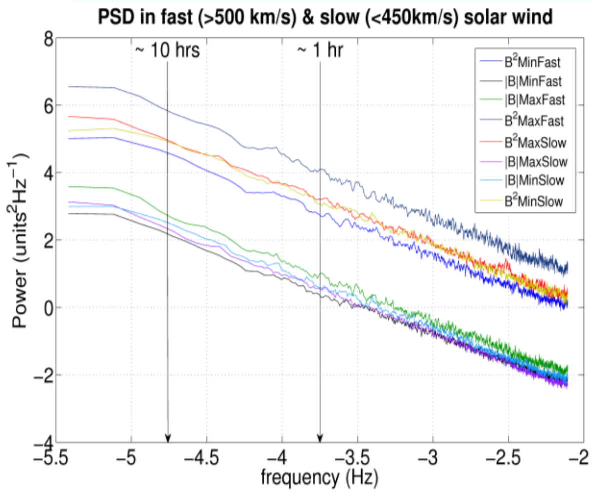




Shown: *log-log* plots of PSD of 3 day intervals averaged over 1 year
ACE solar max (2000); solar min (2007)
Plotted: **|B|**, B^2 and normalized **S**=-[**B**(**v.B**)-**v**B^2]
Fast $v>500kms^{-1}$ and slow $v<450kms^{-1}$







Signature of coronal fields within IR- Kiyani, SCC et al PRL, 2007

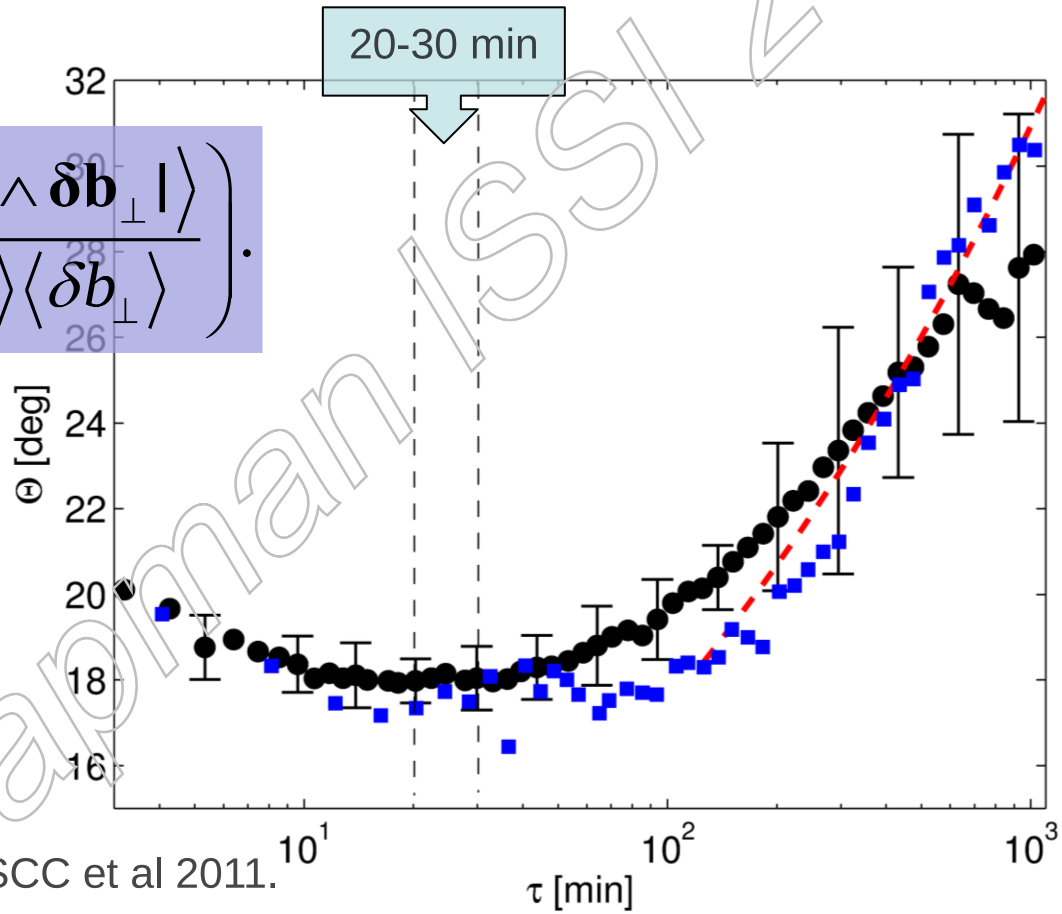
Scale dependent dynamic alignment in the 1/f range

$$\Theta(\tau) = arcsin\left(\frac{\langle | \boldsymbol{\delta v}_\perp \wedge \boldsymbol{\delta b}_\perp | \rangle}{\langle \delta v_\perp \rangle \langle \delta b_\perp \rangle} \right).$$

$$\boldsymbol{\delta v}_\perp = \boldsymbol{\delta v} - (\boldsymbol{\delta v} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}}$$

$$\boldsymbol{\delta b}_\perp = \boldsymbol{\delta b} - (\boldsymbol{\delta b} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}}$$

$$\text{where } \hat{\mathbf{b}} = \boldsymbol{\delta B} / \mathbf{B}$$

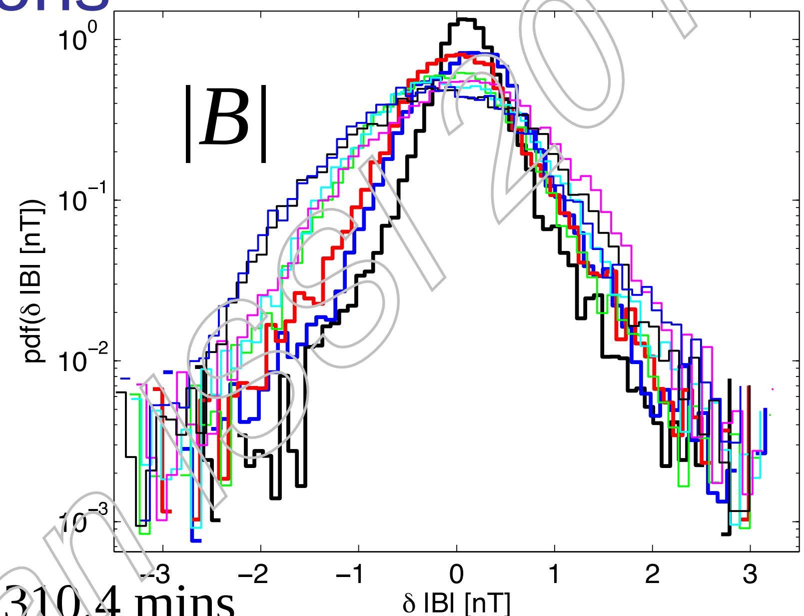


ACE-black, Ulysses-blue: Hnat, SCC et al 2011.

Boldyrev (1995) – dynamic alignment with scale, see also Podesta (2009); Gogoberidze, SCC et al, MNRAS (2012)

PDFs of temporal fluctuations as a function of temporal scale

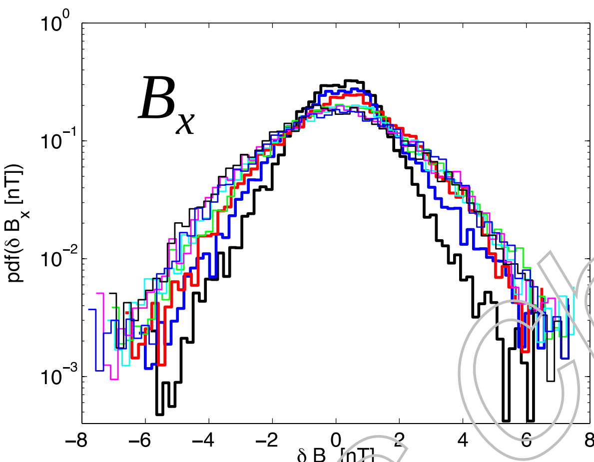
last thick line is 18min
first thin line is 40 min

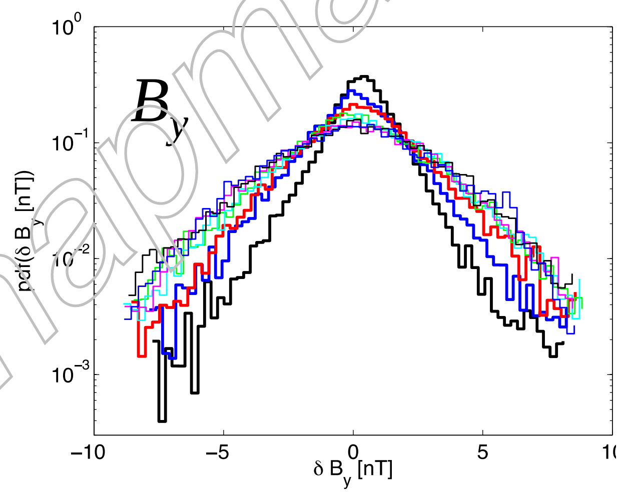


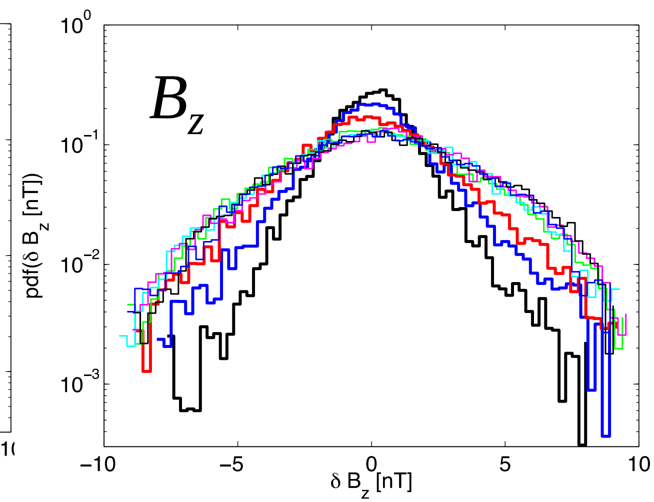
PDFs of $x(t+t) - x(t)$, x ° $B_x, B_y, B_z, |B|$

as we vary t

t =3.2, 7.5, 18.1, 40.5, 60.8, 137.6, 206.9, 310.4 mins

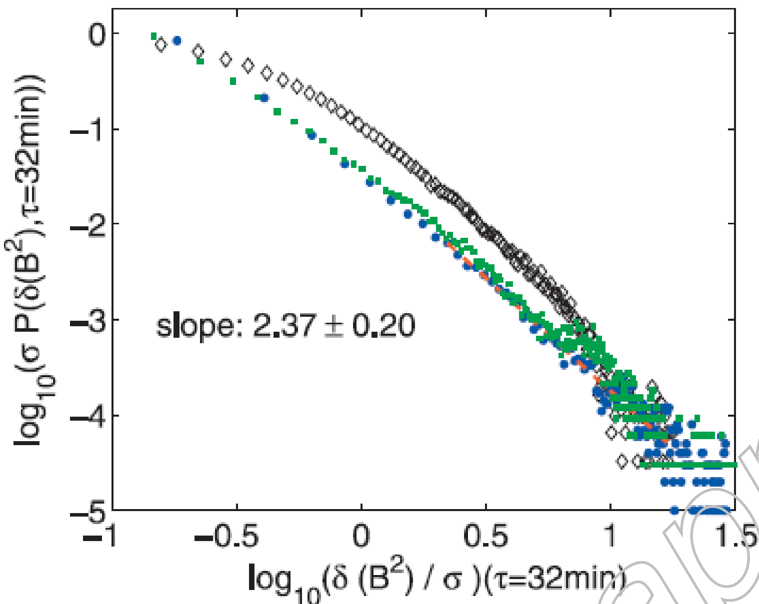


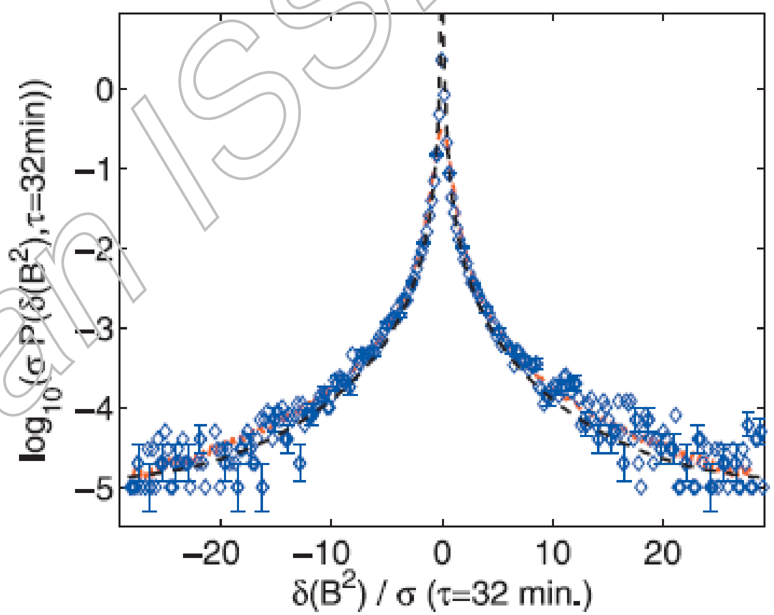




Solar cycle dependence:

Left: B² fluctuation PDF solar max and solar min Right: solar max a signature of fractal scaling within the inertial range of turbulence…

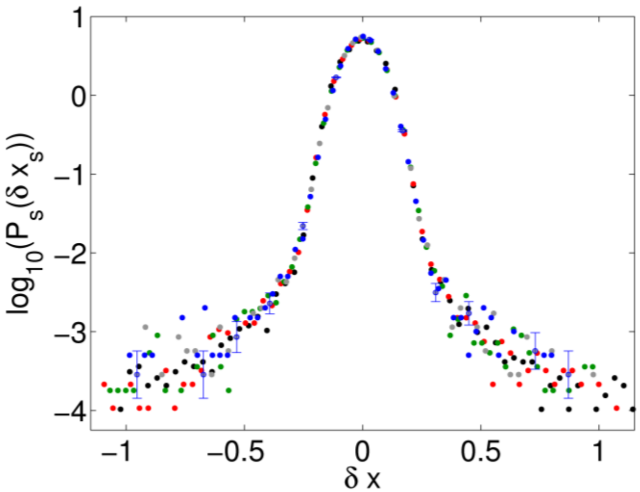


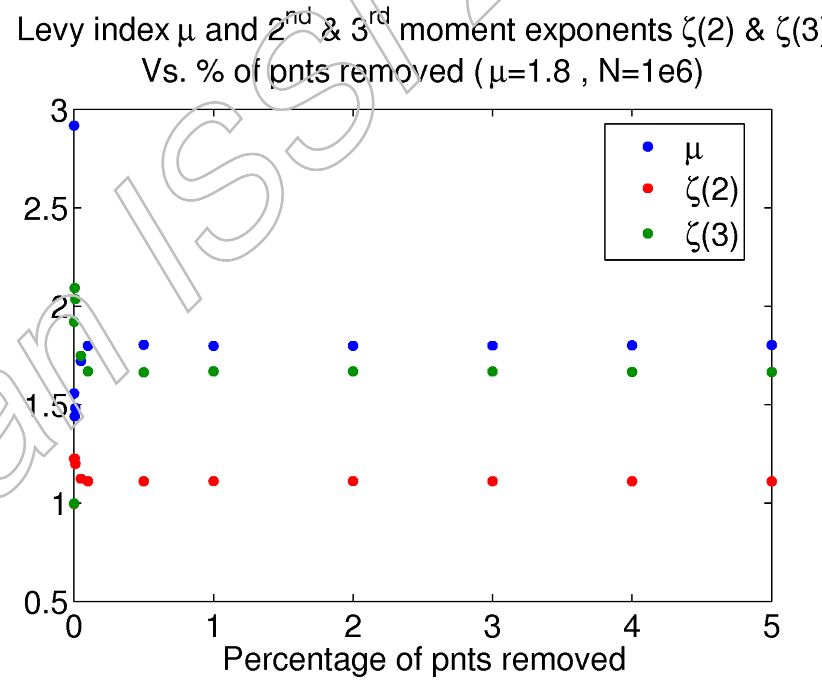


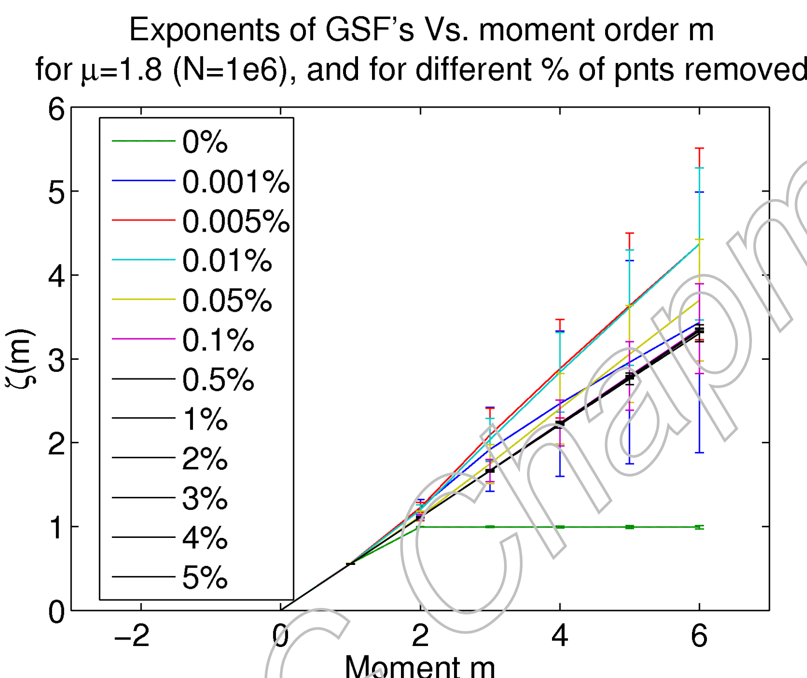
WIND 1996 min (◊), 2000 max (◦), ACE 2000 max (□)

Hnat, SCC et al, GRL, (2007), Kiyani, SCC et al PRL (2007)

More sensitive test for self- similarity
Conditioning by successively removing extremal points
Shown for Lévy flight μ=1.8





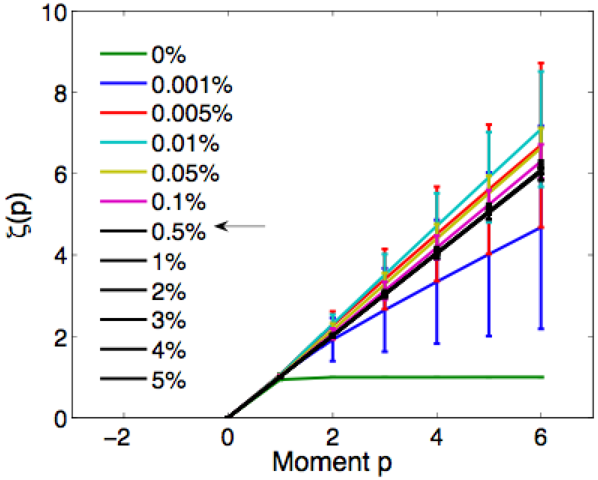


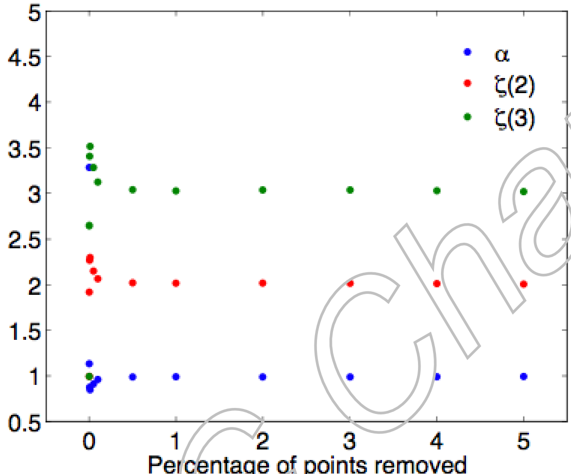
All points on the PDF have the same scaling exponent

Kiyani, SCC et al PRE (2006,09)

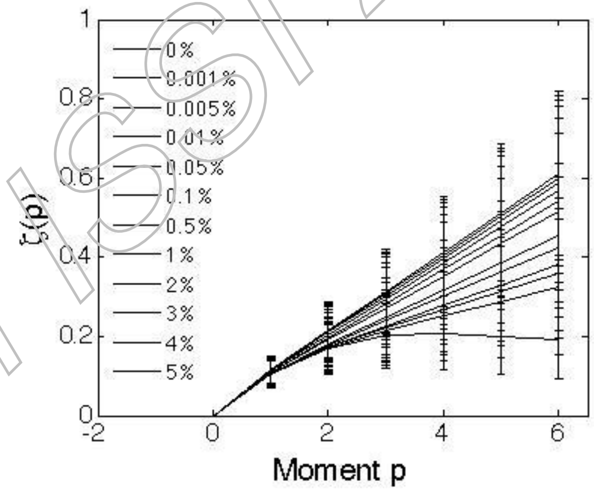
Kiyani, SCC et al, PRL (2007)

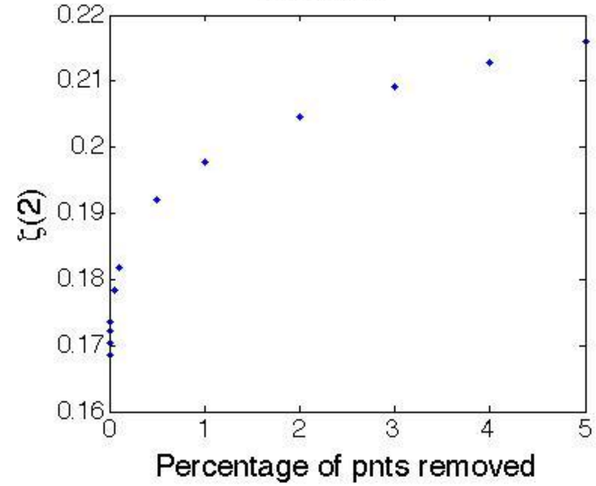
Levy flight -- Fractal





P-model -- Multifractal

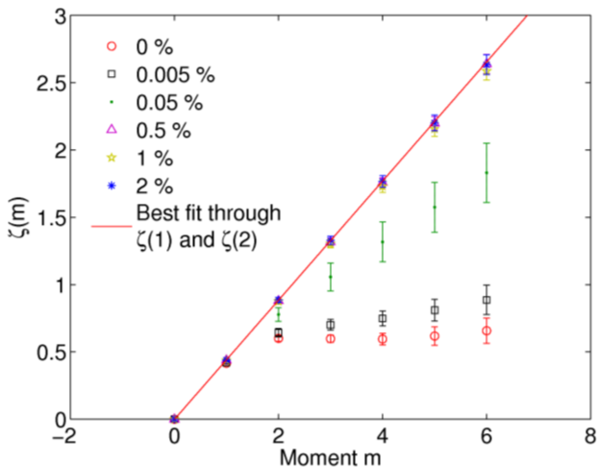


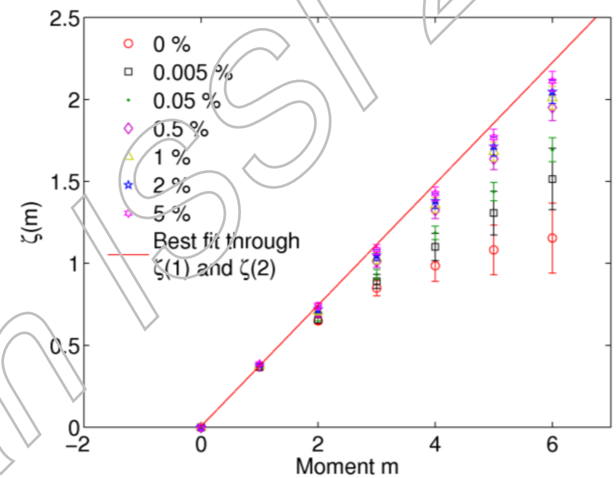


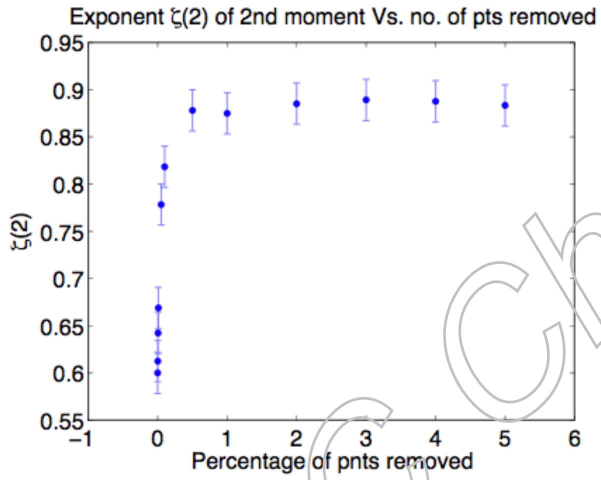
Solar cycle variation WIND -- $|B|^2$

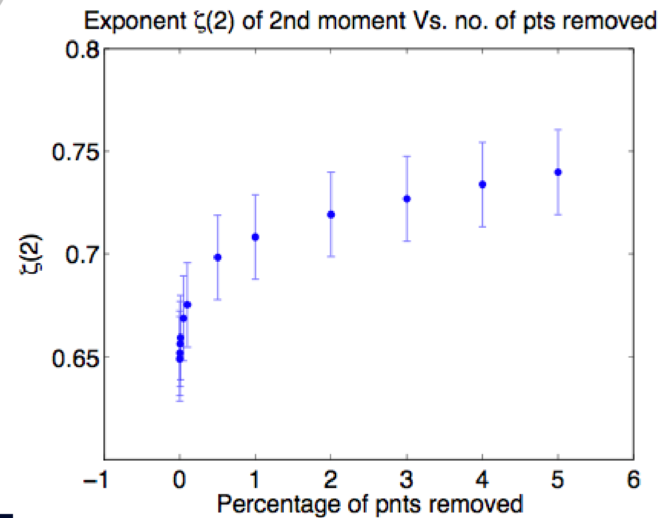
2000 - Solar max fractal

1996 - Solar min multifractal







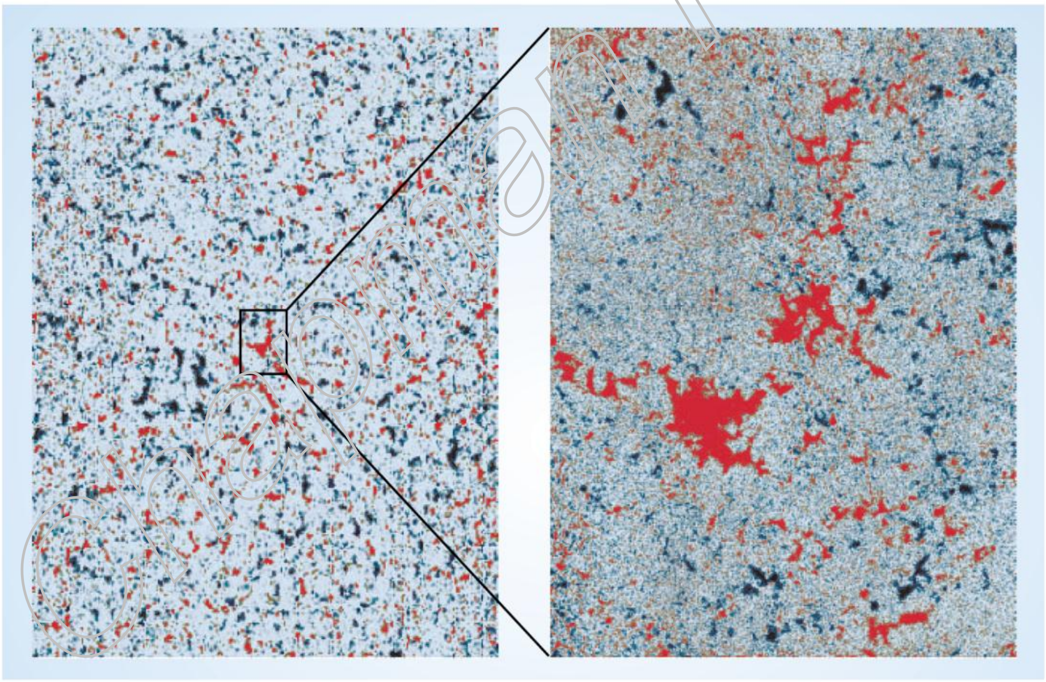


Scaling from the sun: Fractal patches of magnetic polarity on the quiet sun

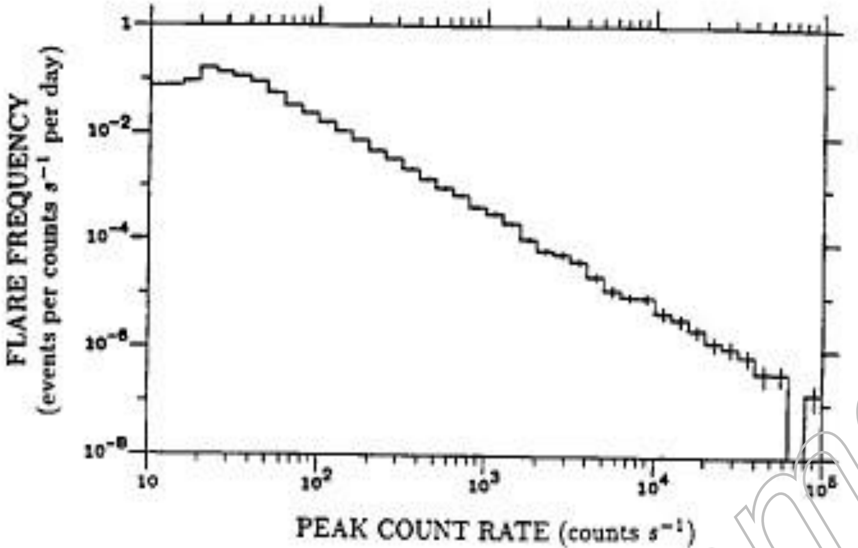
Patches of opposing polarity –

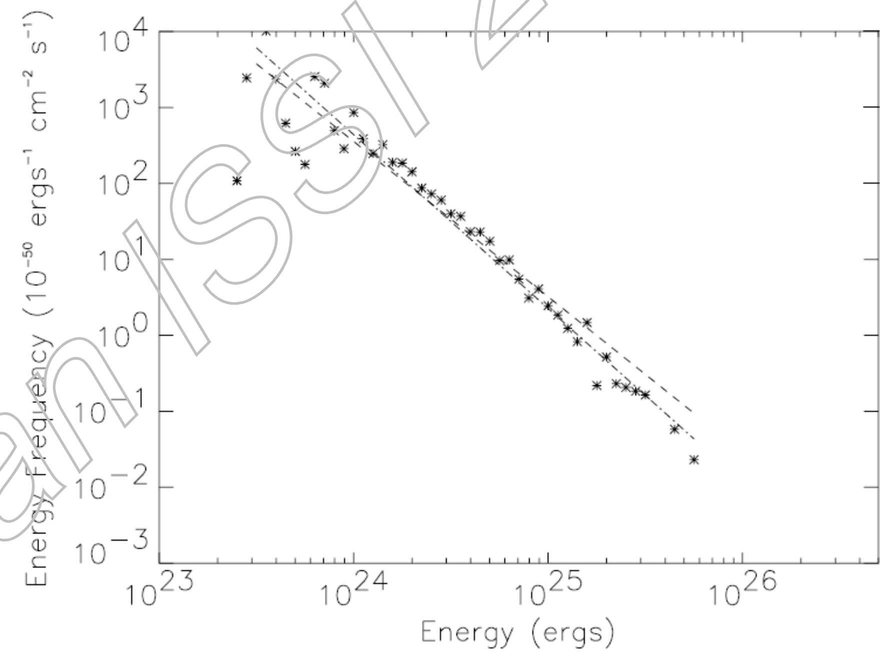
Zeeman effect photosphere, quiet sun,

(Stenflo, Nature 2004, See eg Janssen et al A&A 2003, Bueno et al Nature 2004+..) - ***spatial***



Scaling from the sun: power law flare statistics





Peak flare count rate *Lu&Hamilton ApJ 1991*

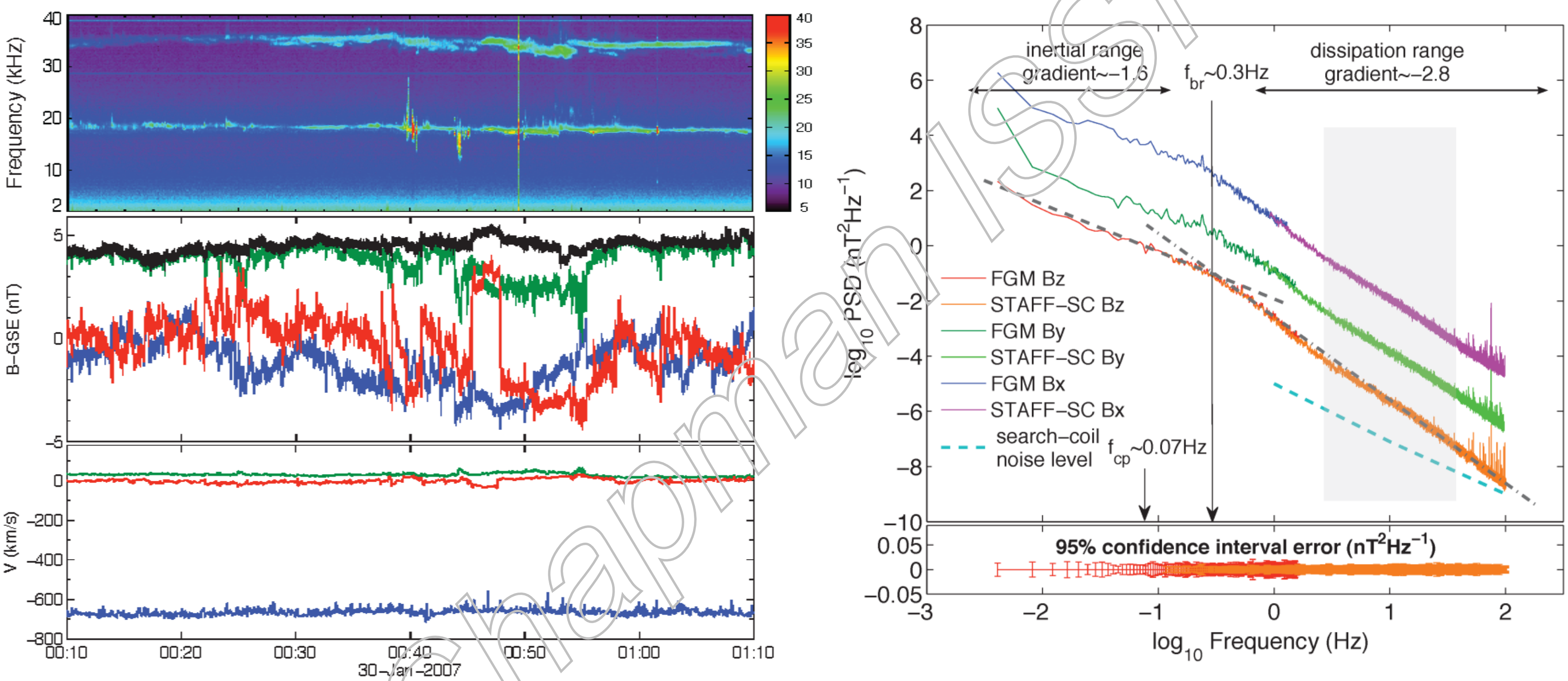
TRACE nanoflare events *Parnell&Judd ApJ 2000*

-temporal

The 'dissipation' range- what happens on kinetic scales -in the quiet solar wind

Kiyani, SCC et al PRL, (2009), also ApJ submitted

CLUSTER high cadence B field spanning IR and dissipation range

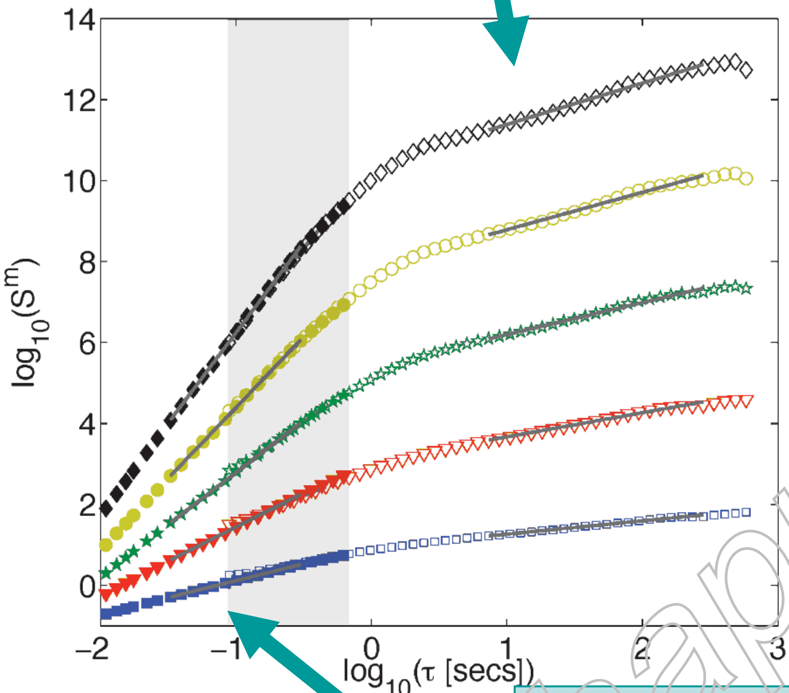


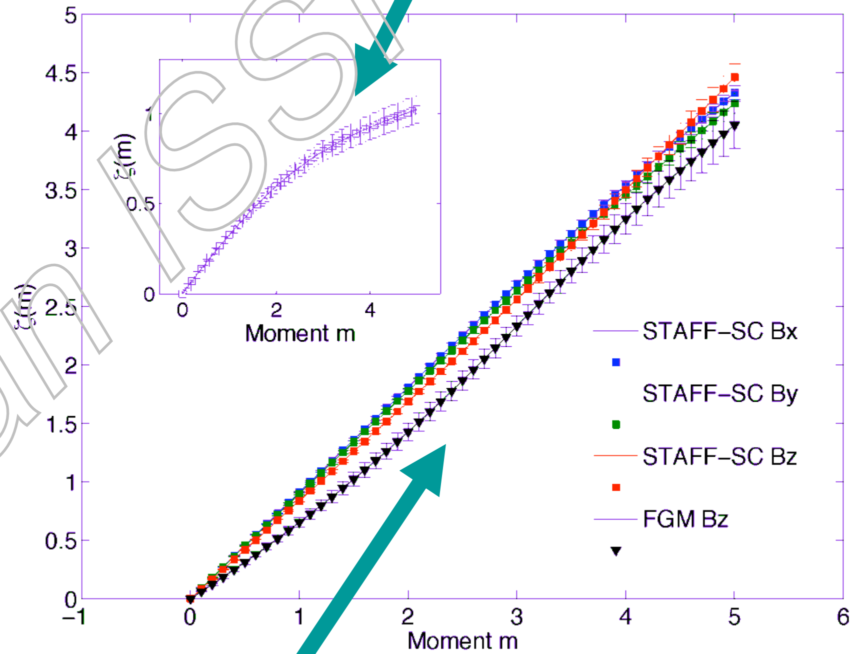
CLUSTER STAFF and FGM shown overlaid.

Kiyani, SCC et al PRL, (2009)

$S_p = \langle | x(t+\tau) - x(t) |^p \rangle \sim \tau^{\xi(p)}$, plot $\log(S_p)$ vz. $\log(\tau)$ to obtain $\xi(p)$

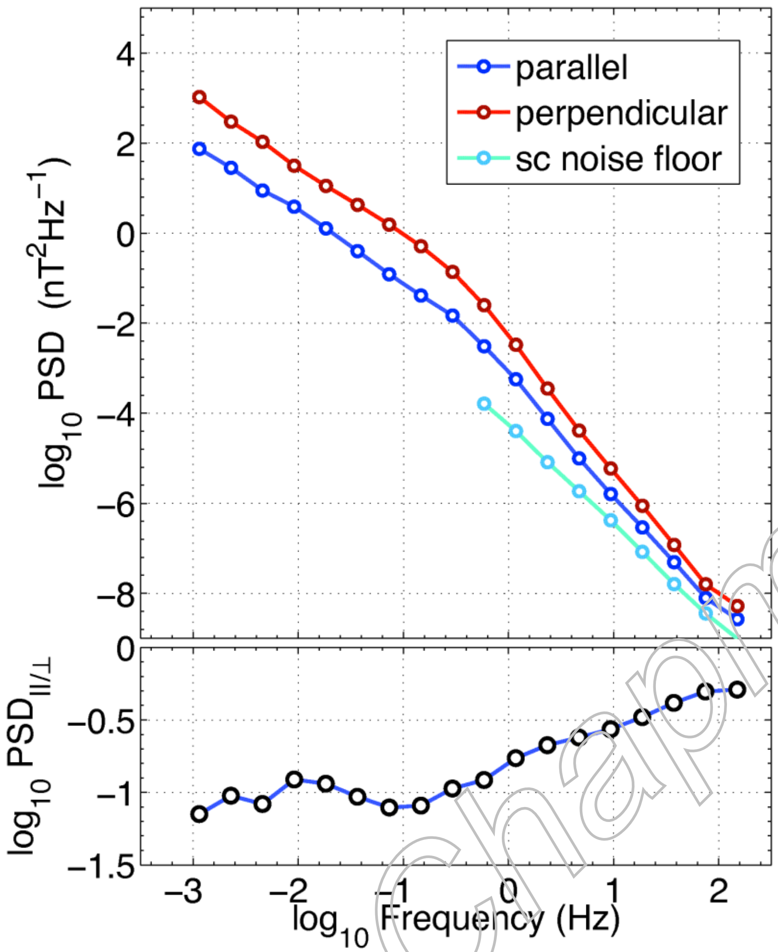
Inertial range- multifractal





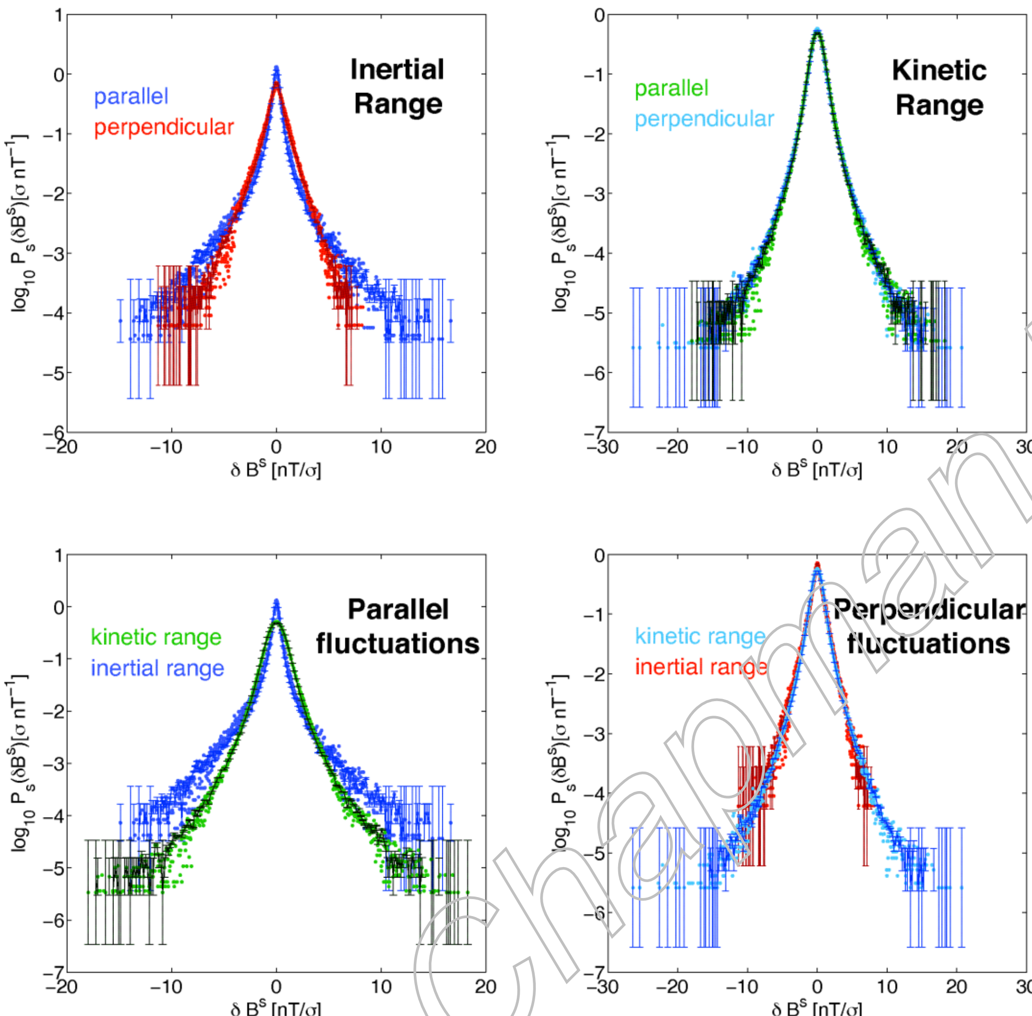
Dissipation range- fractal

CLUSTER STAFF and FGM shown overlaid.
Error estimate on exponents, see eg Kiyani, SCC et al PRE 2009

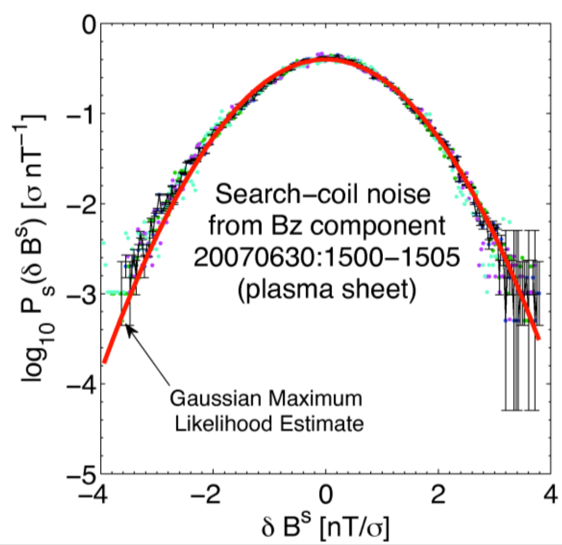


Anisotropy w.r.t. Local background B field direction

Becoming more isotropic in the kinetic range



Physics changes as we go from inertial range to kinetic range...



Non axisymmetric (w.r.t. B) anisotropy

Belcher and Davis (1971) Mariner 5 5:4:1 power ratio

$$\mathbf{e}_B(t_j, f) = \overline{\mathbf{B}}(t_j, f) / \left|\overline{\mathbf{B}}(t_j, f)\right|$$

$$\mathbf{e}_V(t_j, f) = \mathbf{e}_B \times \mathbf{e}_{BV}$$

$$\mathbf{e}_{BV}(t_j, f) = \frac{\mathbf{e}_B \times \mathbf{V}}{\left|\mathbf{e}_B \times \mathbf{V}\right|}$$

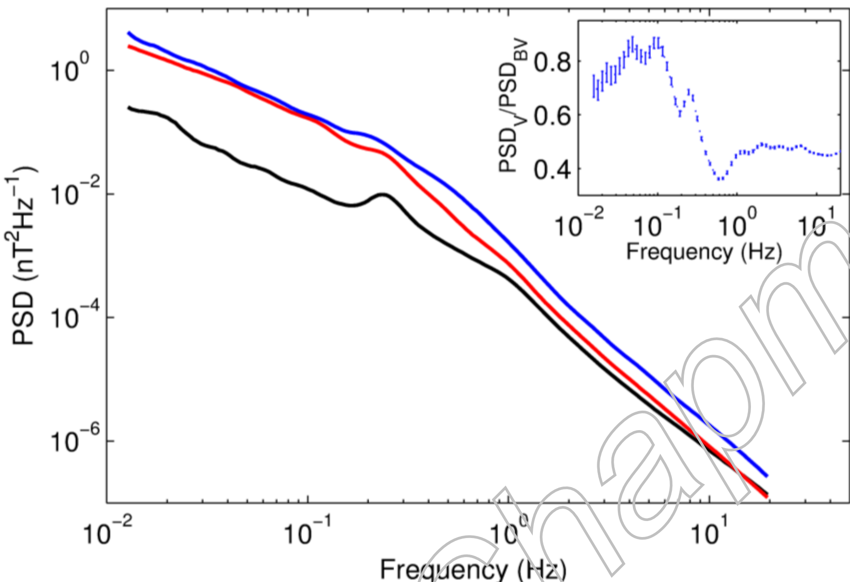
$$\mathbf{k}_\perp . \mathbf{V}_{sw} = \cos\alpha$$

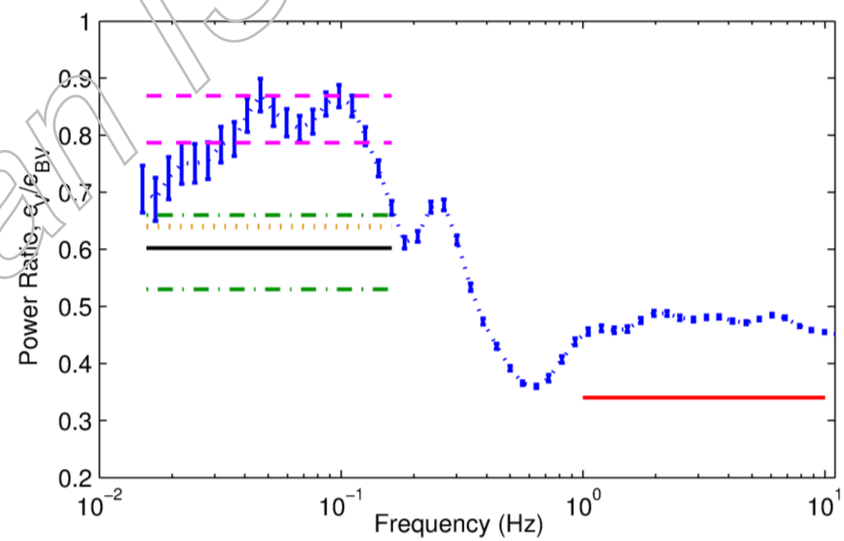
$$E_V(\omega) = \int \mathrm{d}^2\mathbf{k}_\perp E_{2D}(\mathbf{k}_\perp)\delta(\omega - k_x V_{sw})\sin^2\alpha$$

$$E_{BV}(\omega) = \int \mathrm{d}^2\mathbf{k}_\perp E_{2D}(\mathbf{k}_\perp)\delta(\omega - k_x V_{sw})\cos^2\alpha$$

and if $E_{2D}(\mathbf{k}_\perp) = E_{2D}(k_\perp) = Ck_\perp^{-\gamma}$

$$E_V(\omega) / E_{BV}(\omega) = 1/(\gamma - 1)$$





Left: PSD(B) (black), PSD(V) (red), PSD(BV) (blue)   Right: PSD(V)/PSD(BV)

arXiv:1106.2023

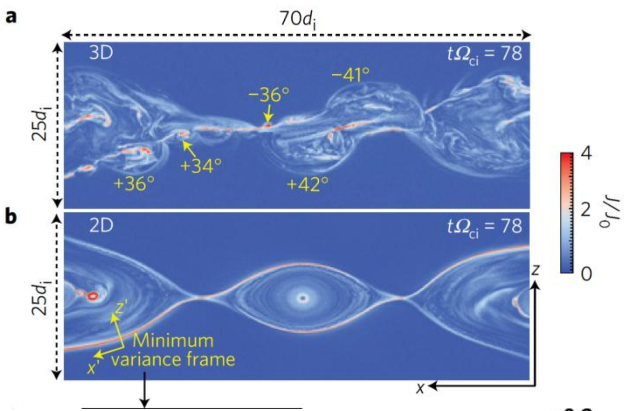
NB many recent papers: Narita, Sahraoui, Forman, Wicks, Chen, et al++

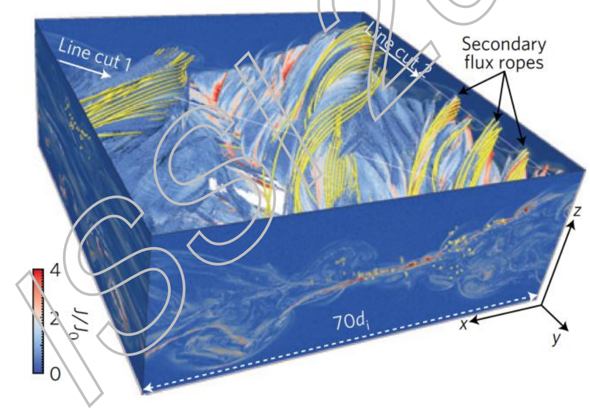
The ‘dissipation’ range- what happens on kinetic scales -in simulations of ‘turbulent’ reconnection

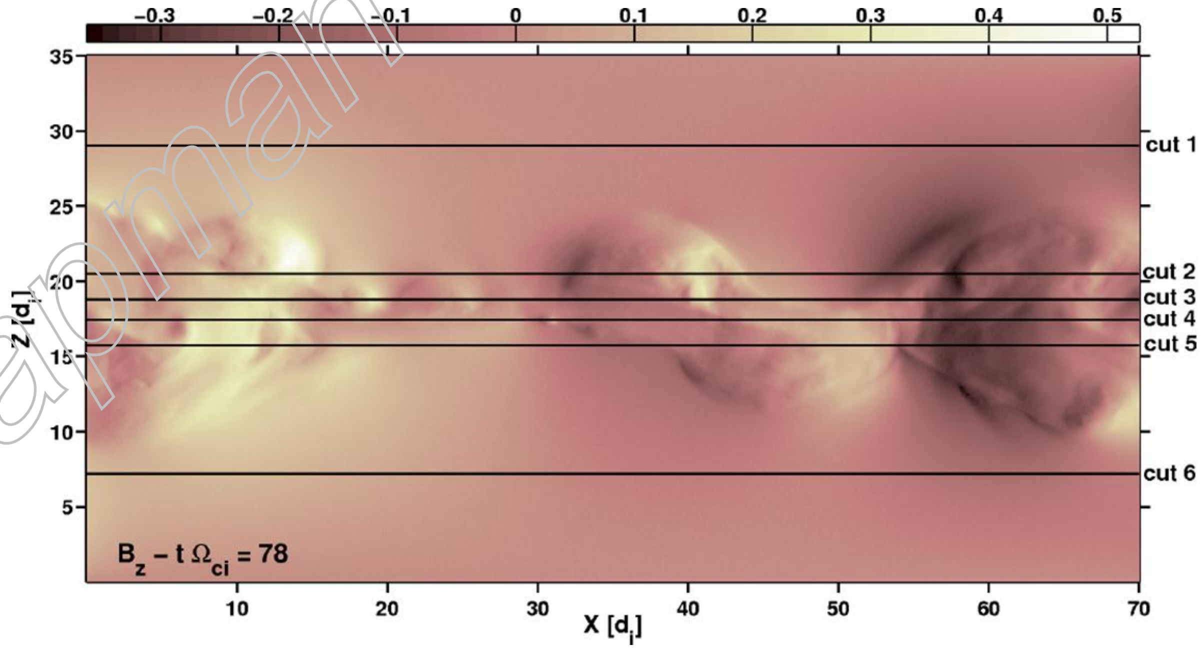
Leonardis, SCC et al in prep

3D PIC simulations of ‘turbulent’ reconnection on kinetic scales

Daughton et al, Nature Physics 2011

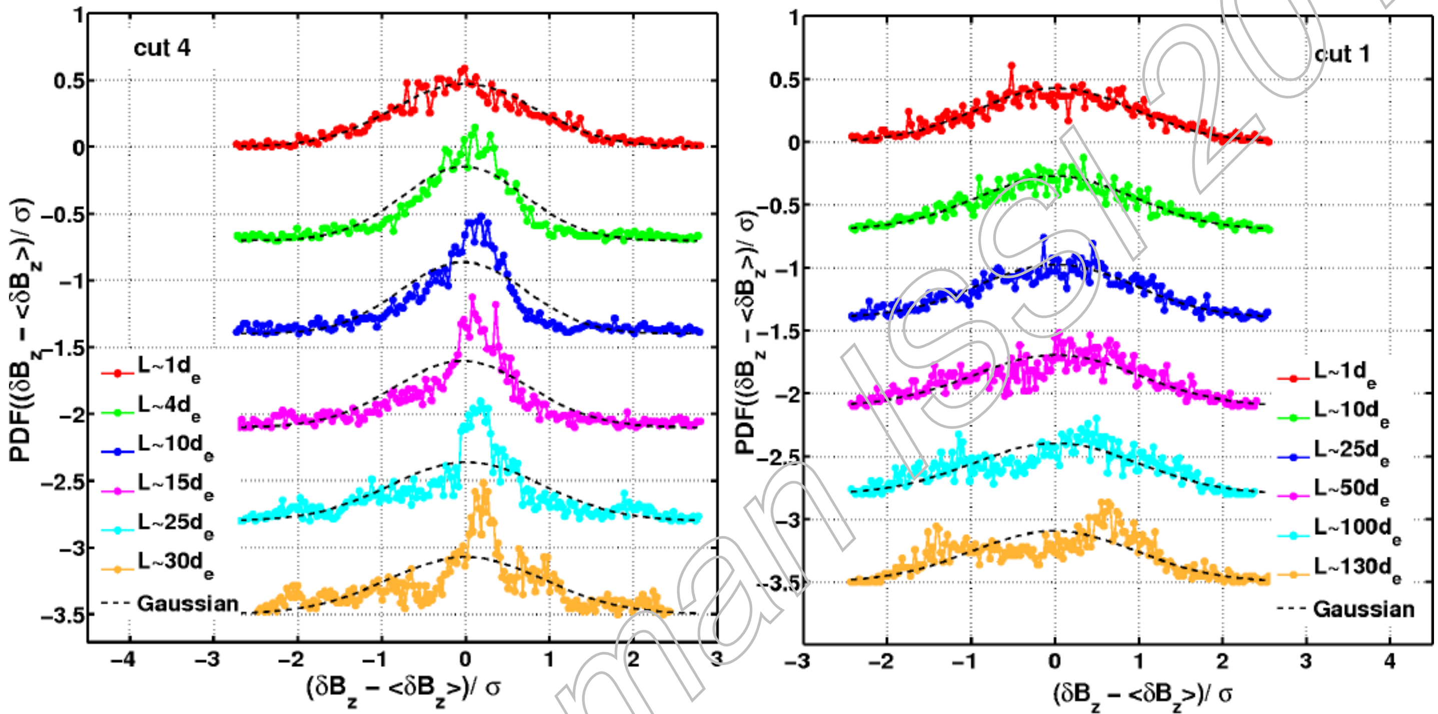




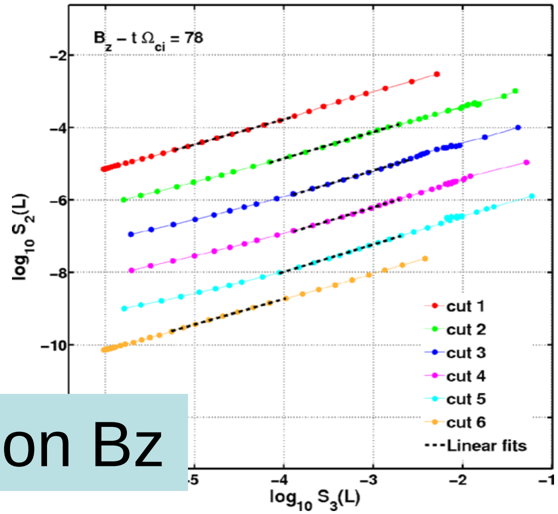


In what sense is it turbulent? Lets see…

Analyse cuts.

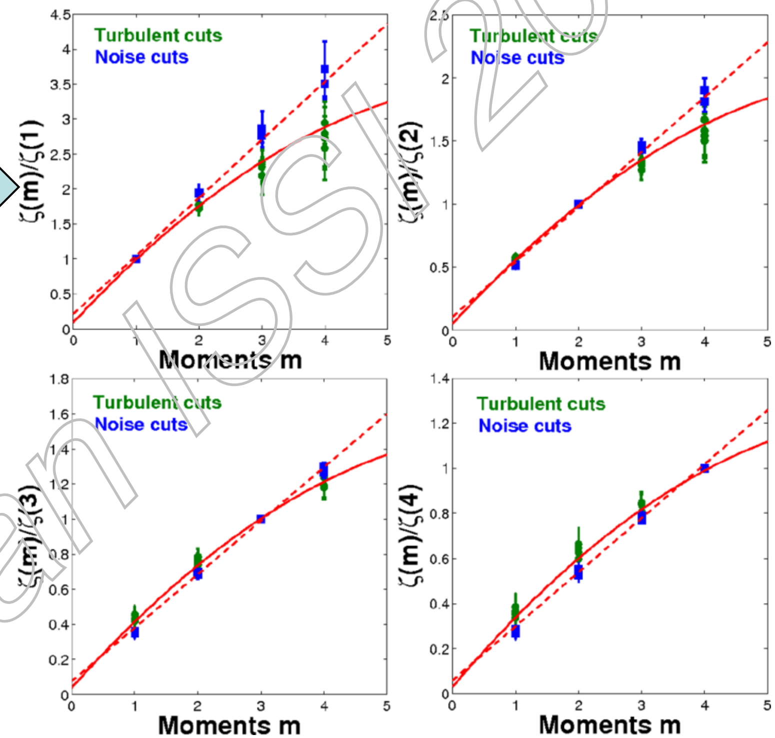


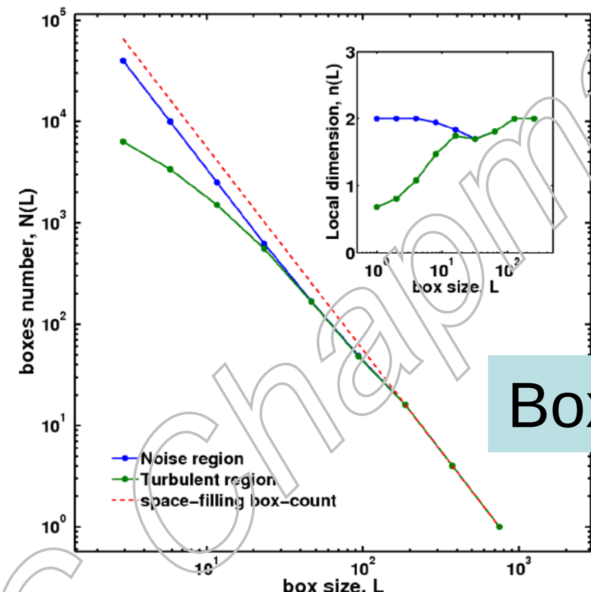
signal,….. and noise….



ESS on Bz

exponents





Box counting on Jz.Ez

Summary- lots of lovely scaling..

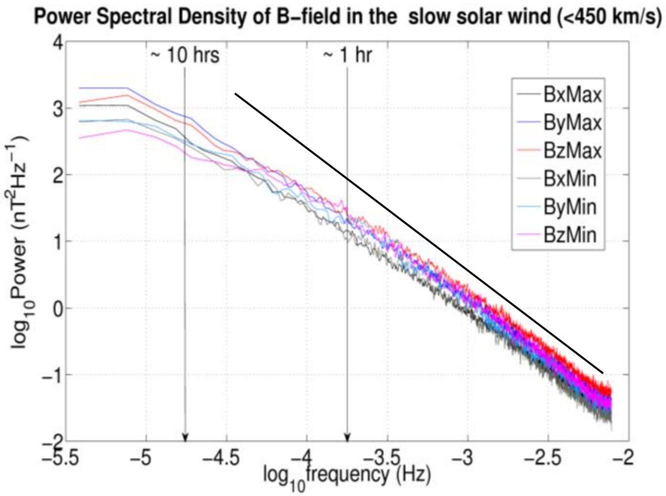
Techniques for quantifying fluctuations statistically maps to turbulence theories but not exclusively

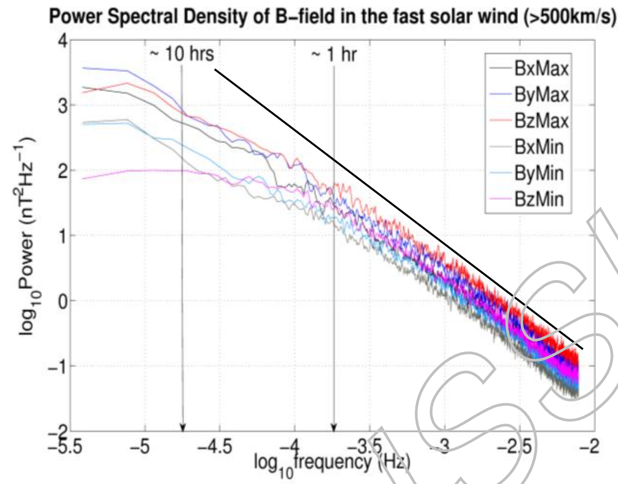
- ➢Flows in corona, in solar wind- evidence for finite range *generalized similarity-* is it universal?
- ➢The *distinctive dissipation range physics-* cascade more space filling compared to inertial range
- ➢|B| fluctuations- distinct from and coexistent with the classic 'inertial range - *solar origin*?

We measure scaling which may imply turbulence-

- ***Need to go beyond PSD exponents to distinguish processes***
- ***Our systems are manifestly finite sized***

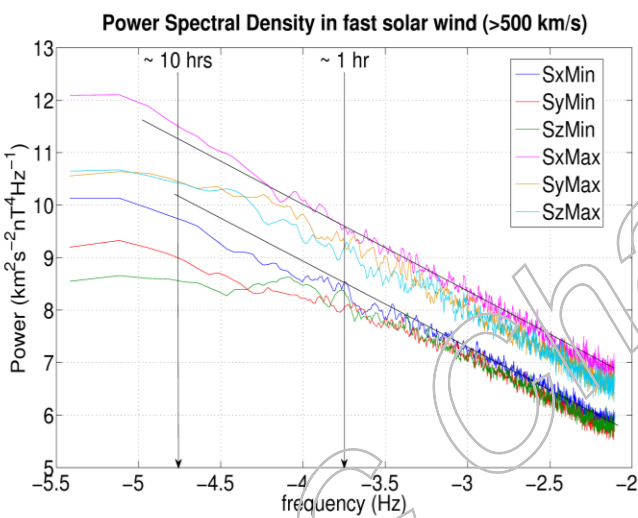
Components show 2 regions inertial range and ’ 1/f’

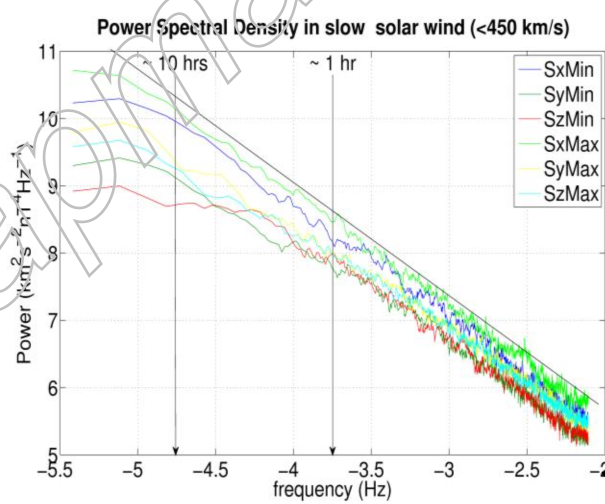


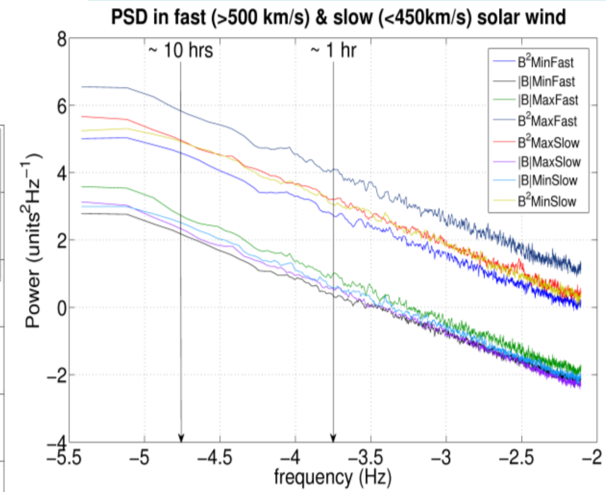


x- component of Poynting flux B magnitude one single region

Shown: *log-log* plots of PSD of 3 day intervals averaged over 1 year
ACE solar max (2000); solar min (2007)
Plotted: **|B|**, B^2 and normalized **S**=-[**B**(**v.B**)-**v**B^2]
Fast $v>500kms^{-1}$ and slow $v<450kms^{-1}$







Signature of coronal fields within IR- Kiyani, SCC et al PRL, 2007

Mutual Information

- Entropy can also be defined for joint probability distributions between 2 signals *X* and *Y*

$$H(X,Y) = -\sum_{ij}^{N} P(x_i, y_j)\log_2\left(P(x_i, y_j)\right)$$

- Mutual Information compares the information content of two signals

$$I(X,Y) = \sum_{ij}^{N} P(x_i, y_j)\log_2\left[P(x_i, y_j)/P(x_i)P(y_j)\right]$$

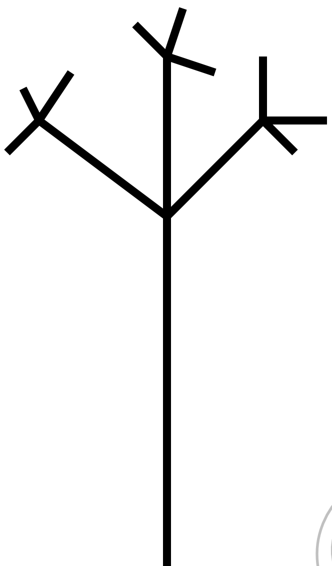
random iid or identical: $P(x_i, y_j) = P(x_i)P(y_j)$ so $I \to 0$

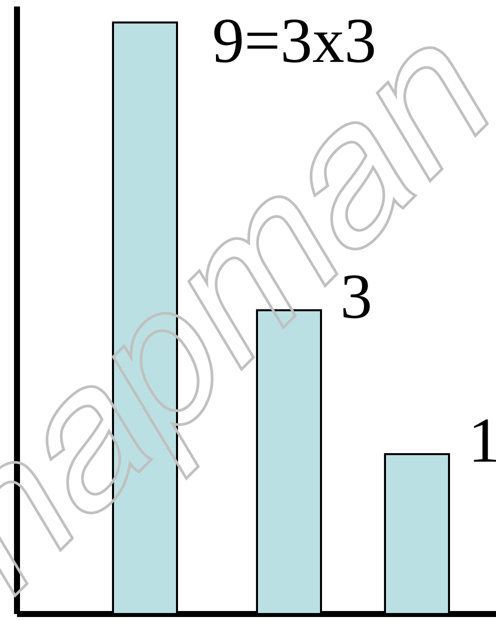
$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$

Scaling and universality-Branches on a self-similar tree

Each branch grows 3 new branches, 1/5 as long as itself..

Number N of branches of length L





$$N = 3^a, L = \frac{1}{5^a}$$

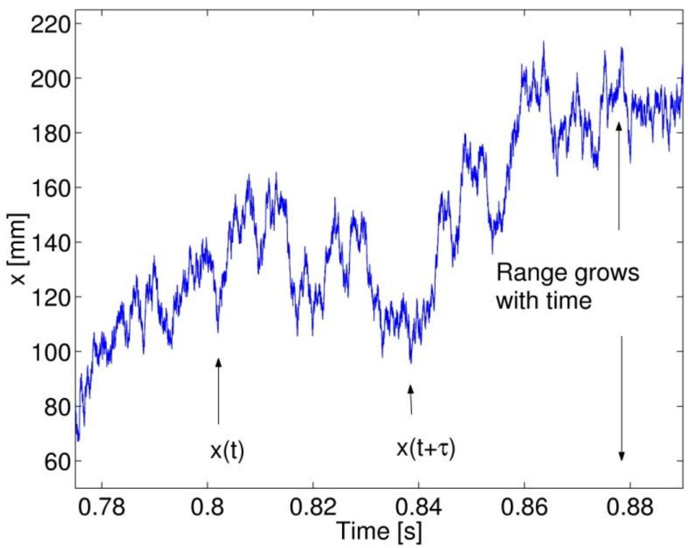
$$\log(N) = a\log(3), \log(L) = -a\log(5)$$

$$\log(N) = -\frac{\log(3)}{\log(5)}\log(L)$$

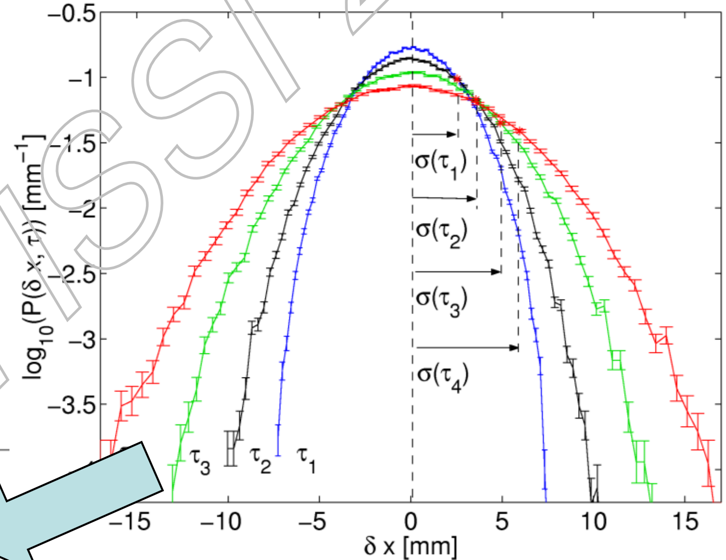
so for any tree..

$$\log(N) = -(\text{a number})\log(L)$$

Self- similar (fractal)- Brownian walk



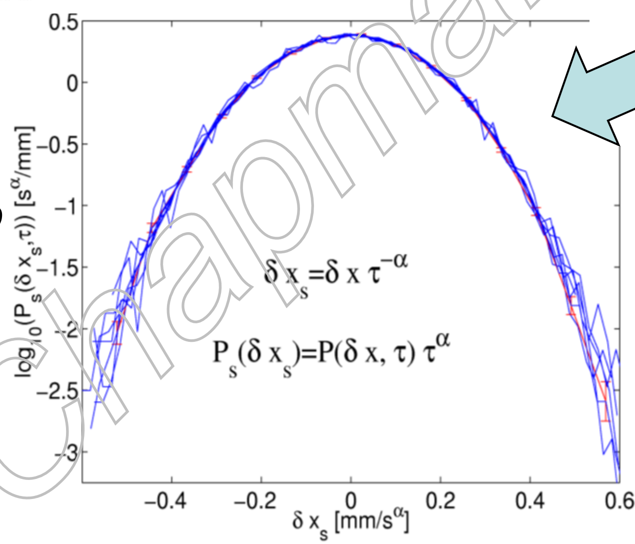
$$y(t,\tau) = x(t+\tau) - x(t)$$



measure from the data

$$\sigma(\tau) \sim \tau^{\alpha}, \zeta(p) \sim \alpha p$$

$\alpha = 1/2$ here



structure functions

$$S_p(\tau) = <| y(t,\tau) |^p> \propto \tau^{\zeta(p)}$$

fractal (self- affine) $\zeta(p) \sim \alpha p$

all moments scale the same

Basic ideas in turbulence- Kolmogorov (KO41) and intermittency

velocity difference across an eddy $d_r v = v(l+r) - v(l)$

eddy time $T(r)$ and energy transfer rate $\varepsilon_{\mathrm{r}} \propto \frac{d_r v^2}{T}$

have T as the eddy turnover time $T \propto r/d_r v$ so that $\varepsilon_{\mathrm{r}} \propto \frac{d_r v^3}{r}$

If the flow is non- intermittent $\left\langle \varepsilon_r{}^p \right\rangle = \bar{\varepsilon}^p, r$ independent for any p

$\Rightarrow \left\langle d_r v^p \right\rangle \propto r^{p/3} \bar{\varepsilon}^{p/3} \sim r^{\zeta(p)}$ -$\zeta(p) = \alpha p$ linear with p – *selfsimilar*(*fractal*) scaling

intermittency correction- r dependence $\left\langle \varepsilon_r{}^p \right\rangle \propto \bar{\varepsilon}^p \left(r/L \right)^{\tau(p)}$

$\Rightarrow \left\langle d_r v^p \right\rangle \propto r^{p/3} \bar{\varepsilon}^{p/3} \left(L/r \right)^{\tau(p/3)} \sim r^{\zeta(p)}$- $\zeta(p)$ quadratic in p

$\left\langle \varepsilon_r \right\rangle = \bar{\varepsilon}$ independent of r (steady state) so $\tau(1) = 0$,

$\Rightarrow$ $\zeta(p)$ must monotonically increase (and $\zeta(p) > 1$ for some p)

in situ solar wind observations so take $r \equiv t$: measure $\zeta(p)$ from $< d_t v^p > \sim t^{\zeta(p)}$

$p = 6$ needed to measure $\tau(2)$! predicted from phenomenology

Multifractal inertial range turbulence- examples

$S_p = \langle | x(t+\tau) - x(t) |^p \rangle \sim \tau^{\xi(p)}$, plot $\log(S_p)$ vz. $\log(\tau)$ to obtain $\xi(p)$

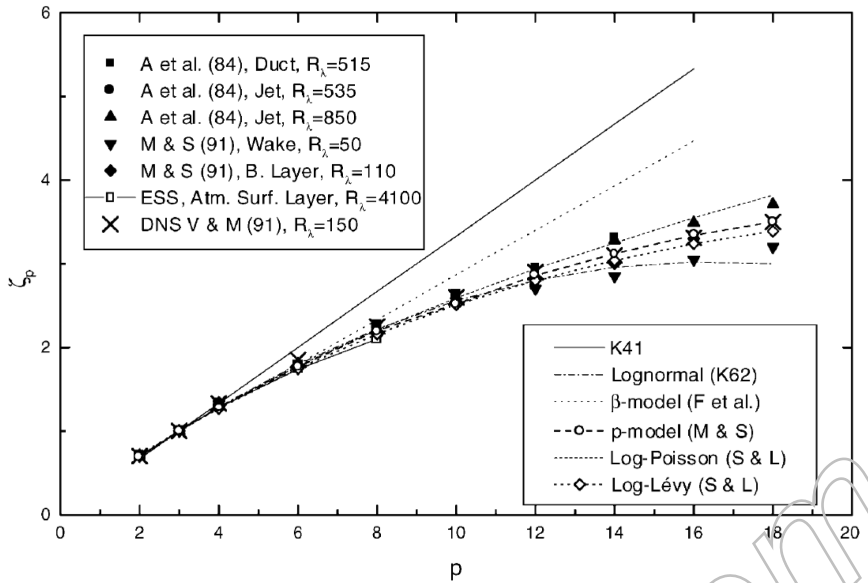
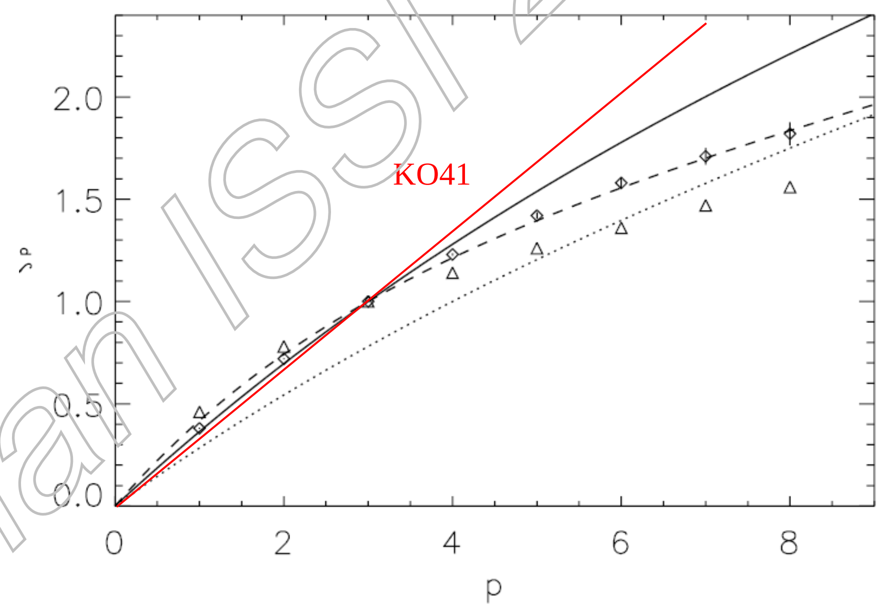


Fig. 11. Power-law exponents ζ_p of the structure functions as a function of the order p, together with the values predicted by K41 and the various intermittency models of Table 1.



IG. 4. Scaling exponents ζ_p^+ for 3D MHD turbulence (dia-ıonds) and relative exponents ζ_p^+/ζ_3^+ for 2D MHD turbulence riangles). The continuous curve is the She-Leveque model ζ_p^{SL}, the dashed curve the modified model ζ_p^{MHD} (7), and the dotted line the IK model ζ_p^{IK}.

Lab Fluid experiments,
Anselmet et al, PSS, 2001

2 and 3D MHD simulations
Muller & Biskamp PRL 2000

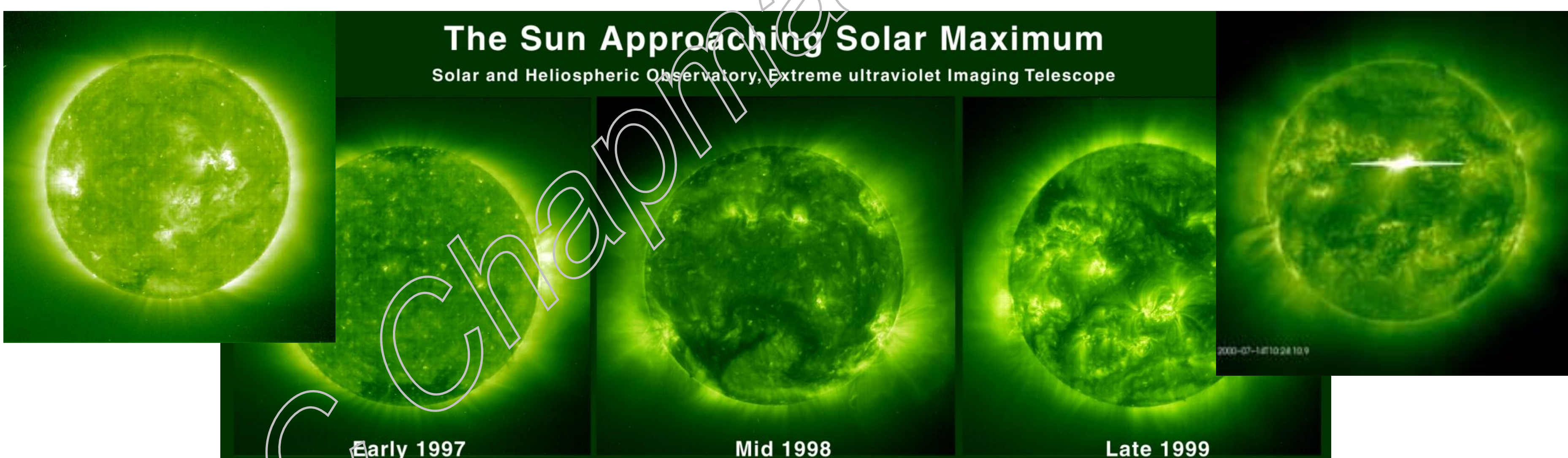
How large can we take p? *See eg Dudok De Wit, PRE, 2004*

ULYSSES- north and south polar passes at the last two solar minima

ULYSSES 60s averages B field components ~10^4 points per 10 day interval

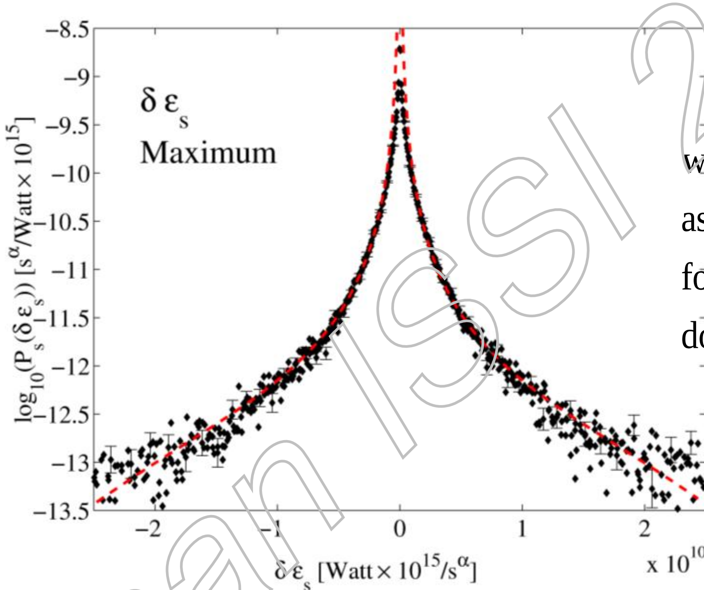
Focus on quiet 'uniform' solar wind seen at the poles at solar minimum

Recent 'unusual' solar minimum, fluctuations down by factor of 2 in power

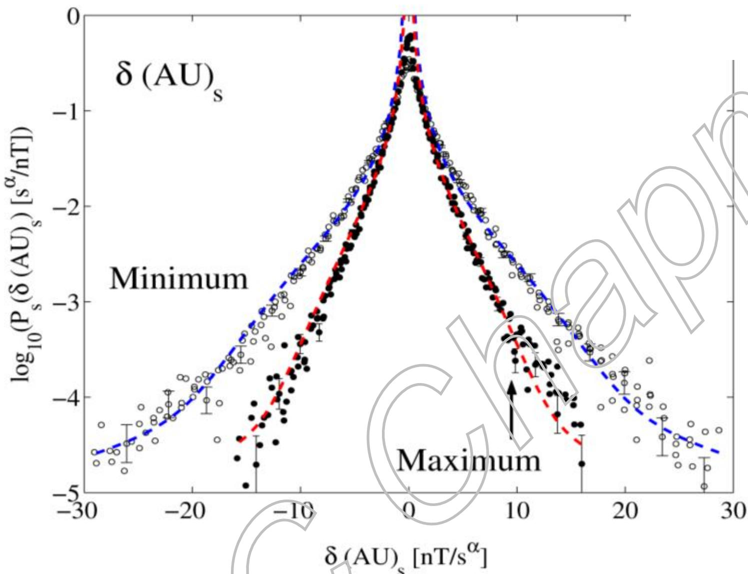


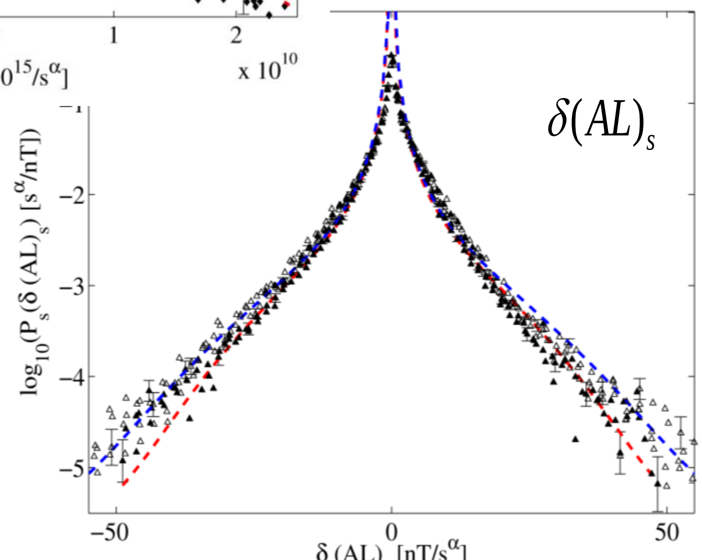
Overlaid Fokker- Planck solutions

1) measure α
2) obtain F- P equation
3)solve for PDF
4)overlay on collapsed curves
5) Good WITHIN ERRORS



will fail at $y = x(t+\tau) - x(t) \to 0$
assumed power law scaling
for all y
dominated by data uncertainties





Hnat, SCC et al JGR 2005

The inertial range- anisotropic MHD turbulence (and other things?)

Kiyani, SCC et al PRL (2007)

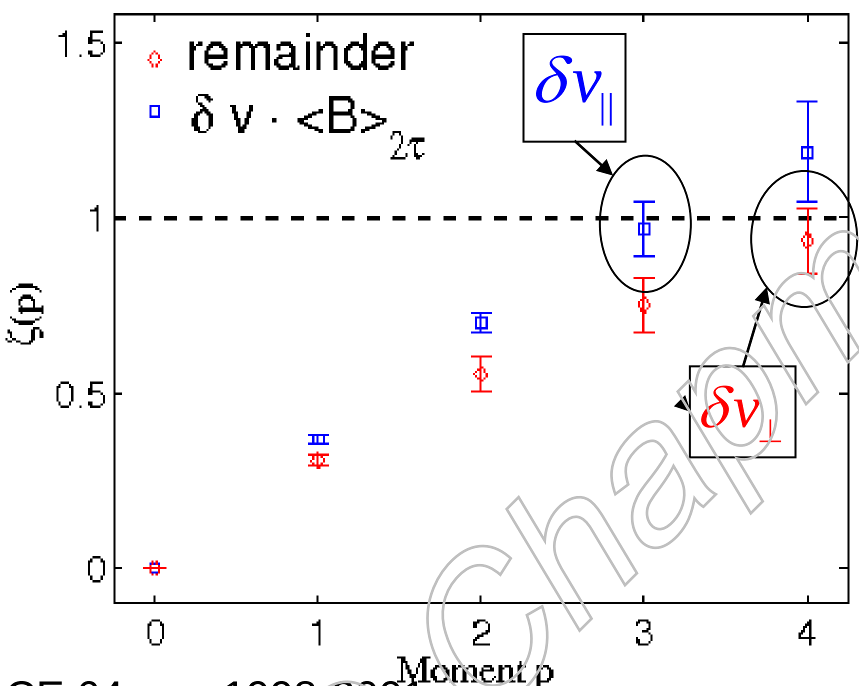
Velocity fluctuations parallel and perpendicular to the *local* B field direction

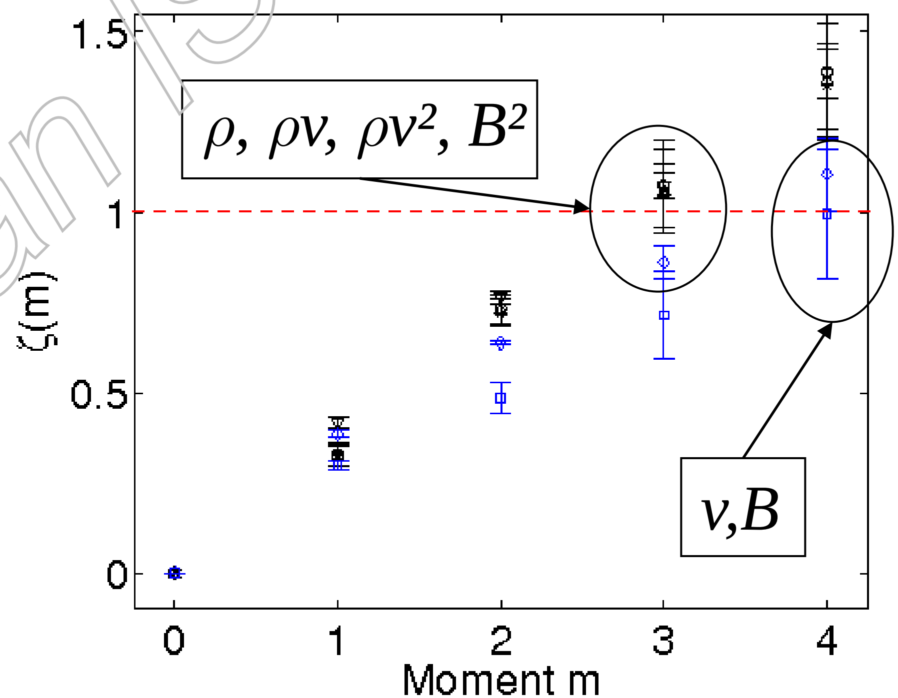
Exponents $\zeta(p)$ for $<\left|\delta v_{\parallel,\perp}\right|^p>\sim \tau^{\zeta(p)}$ for

$\delta v_{\parallel} = \delta \mathbf{v}.\hat{\mathbf{b}}$ and its remainder $\delta v_{\perp} = \sqrt{\delta\mathbf{v}\bullet\delta\mathbf{v} - \left(\delta\mathbf{v}.\hat{\mathbf{b}}\right)^2}$

$\zeta(3+\alpha) = 1$ determines phenomenology

$\bar{\mathbf{B}} = \mathbf{B}(t) + \ldots + \mathbf{B}(t+\tau'),\ \hat{\mathbf{b}} = \frac{\bar{\mathbf{B}}}{\left|\bar{\mathbf{B}}\right|}$, here $\tau' = 2\tau$ and $\delta\mathbf{v} = \mathbf{v}(t+\tau) - \mathbf{v}(t)$





ACE 64s av. 1998-2001

Chapman et al GRL (2007), see also Hnat et al PRL(2005), Kiyani et al PRL(2007)

finite range turbulence- structure and statistics of largest eddies- universal?