Does nature Self-Organise to the VICINITY of a critical state rather than to a critical state ?

Imperial College
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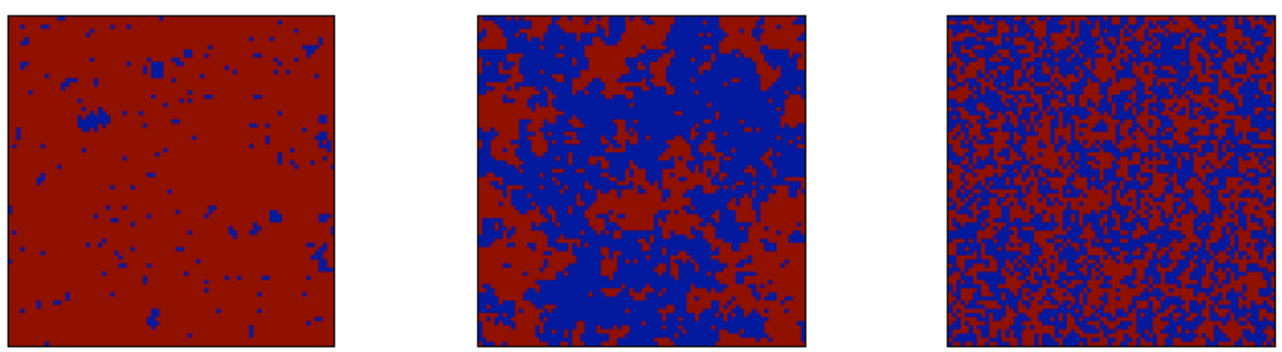
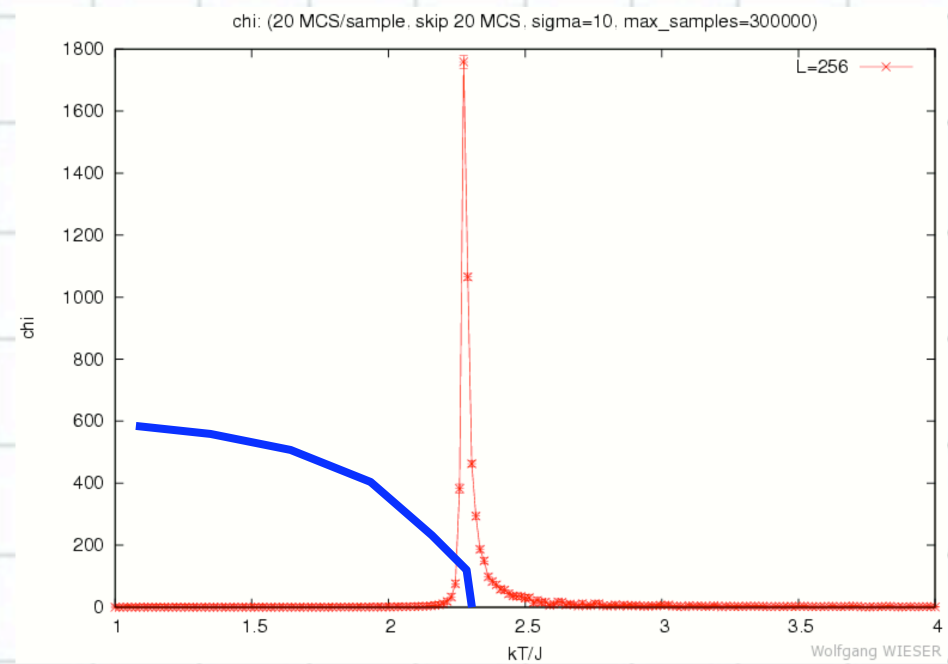
Dynamics near a critical state

Reminder - critical state



$T<T_C$ $\quad$ $T=T_C$ $\quad$ $T>T_C$



Ising model $H = -J \sum_{i,j} S_i S_j$

$S_i \in \{-1, 1\}$

http://www.triplespark.net/sim/isingmag/

What to probe?

- Focus on correlation functions

$$C(\mathbf{r},t) = \langle A(\mathbf{r}_0,t_0)A(\mathbf{r}_0+\mathbf{r},t_0+t)\rangle_{\mathbf{r_0},t_0} - \langle A(\mathbf{r}_0,t_0)\rangle^2$$

Event analysis

- Identify control parameter (humidity, background activity)
  Plot event sizes versus control parameter

Rain

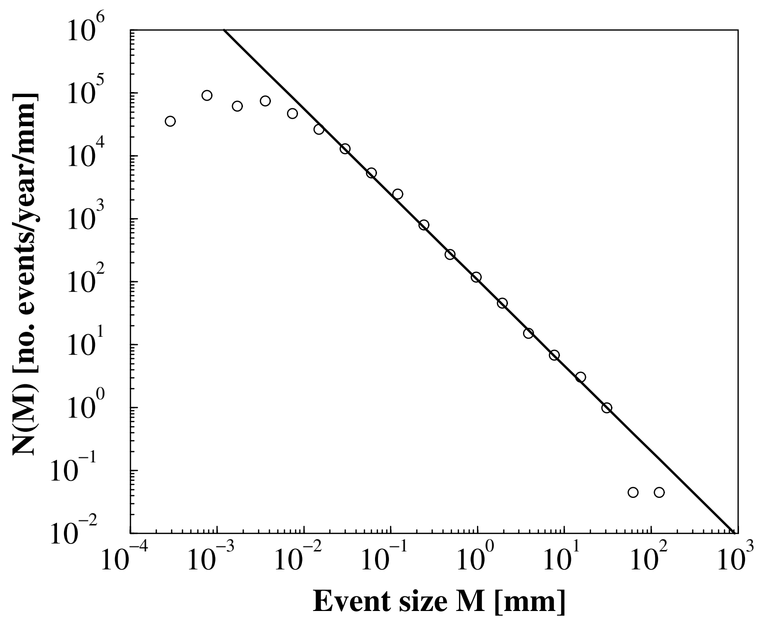


FIG. 1. The number density of rain events per year $N(M)$ versus event size M (open circles) on a double logarithmic scale. A rain event is defined as a sequence of consecutive nonzero-rain rates (averaged over 1 min). This implies that a rain event terminates when it stops raining for a period of at least 1 min. The size M of a rain event is the water column (volume per area) released. Over at least 3 decades, the data are consistent with a power law $N(M) \propto M^{-1.36}$, shown as a solid line.

From: Peters, Herlein and Christensen
PRL 88, 018701 (2002)

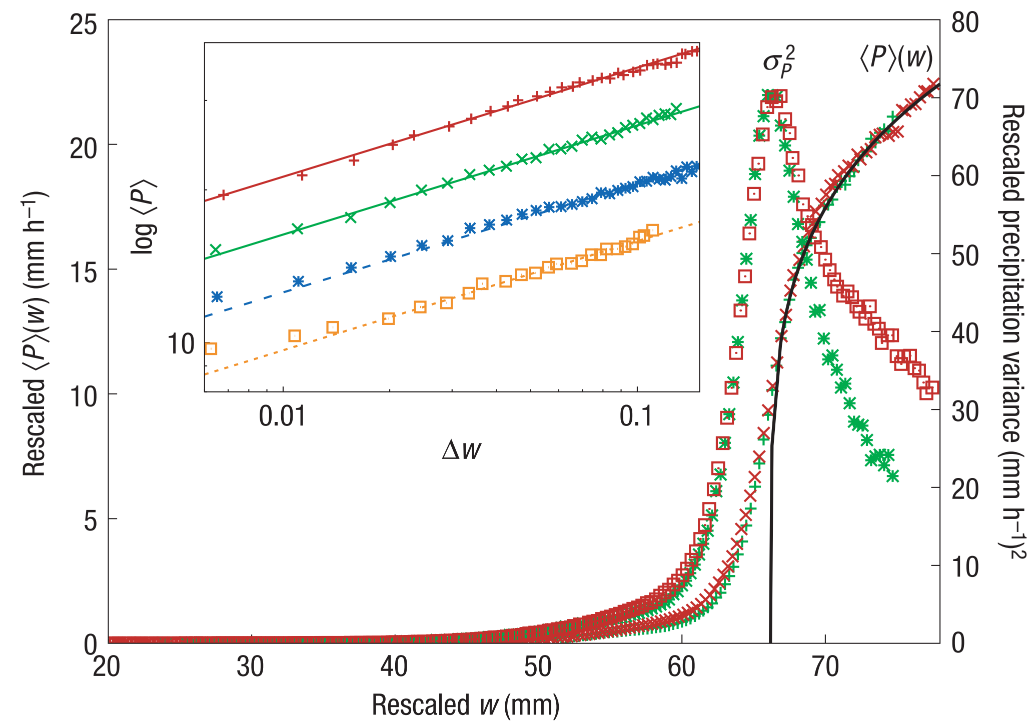
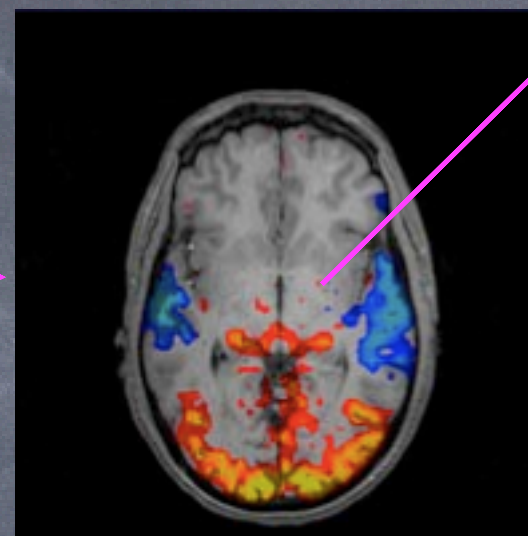
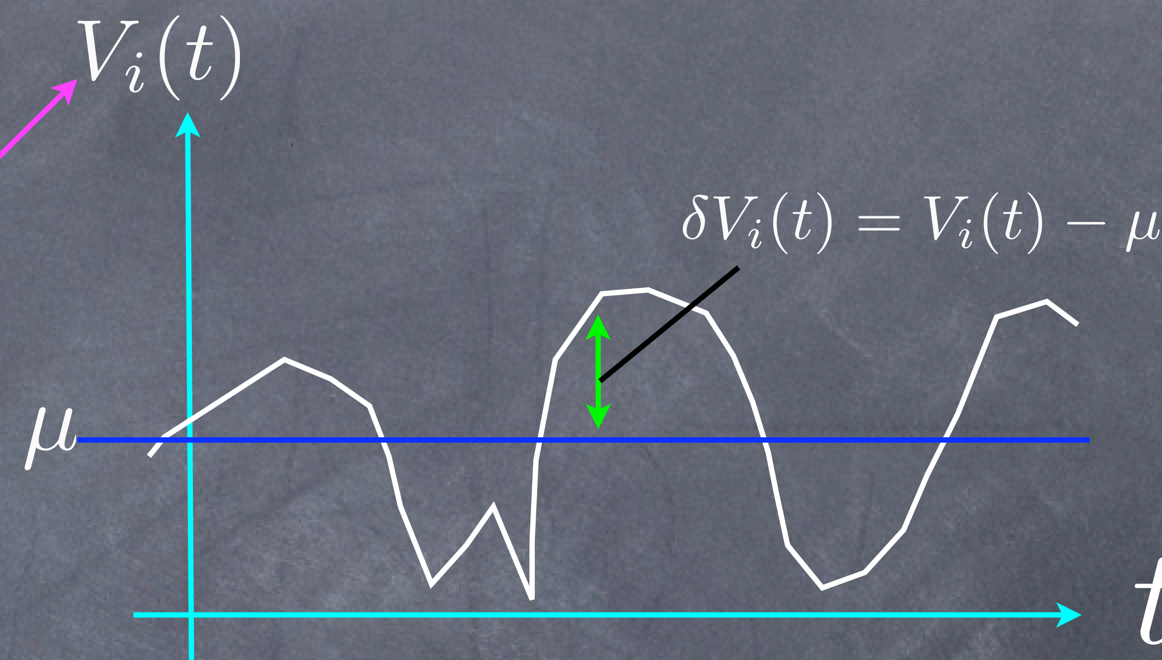


Figure 1 Order parameter and susceptibility. The collapsed (see text) precipitation rates $\langle P \rangle(w)$ and their variances $\sigma_P^2(w)$ for the tropical eastern (red) and western (green) Pacific as well as a power-law fit above the critical point (solid line). The inset shows on double-logarithmic scales the precipitation rate as a function of reduced water vapour (see text) for western Pacific (green, 120E to 170W), eastern Pacific (red, 170W to 70W), Atlantic (blue, 70W to 20E), and Indian Ocean (pink, 30E to 120E). The data are shifted by a small arbitrary factor for visual ease. The straight lines are to guide the eye. They all have a slope of 0.215, fitting the data from all regions well.

From: Peters and Neelin,
Nature Physics 2, 393 (2006)

Rest state fMRI: activity fluctuations



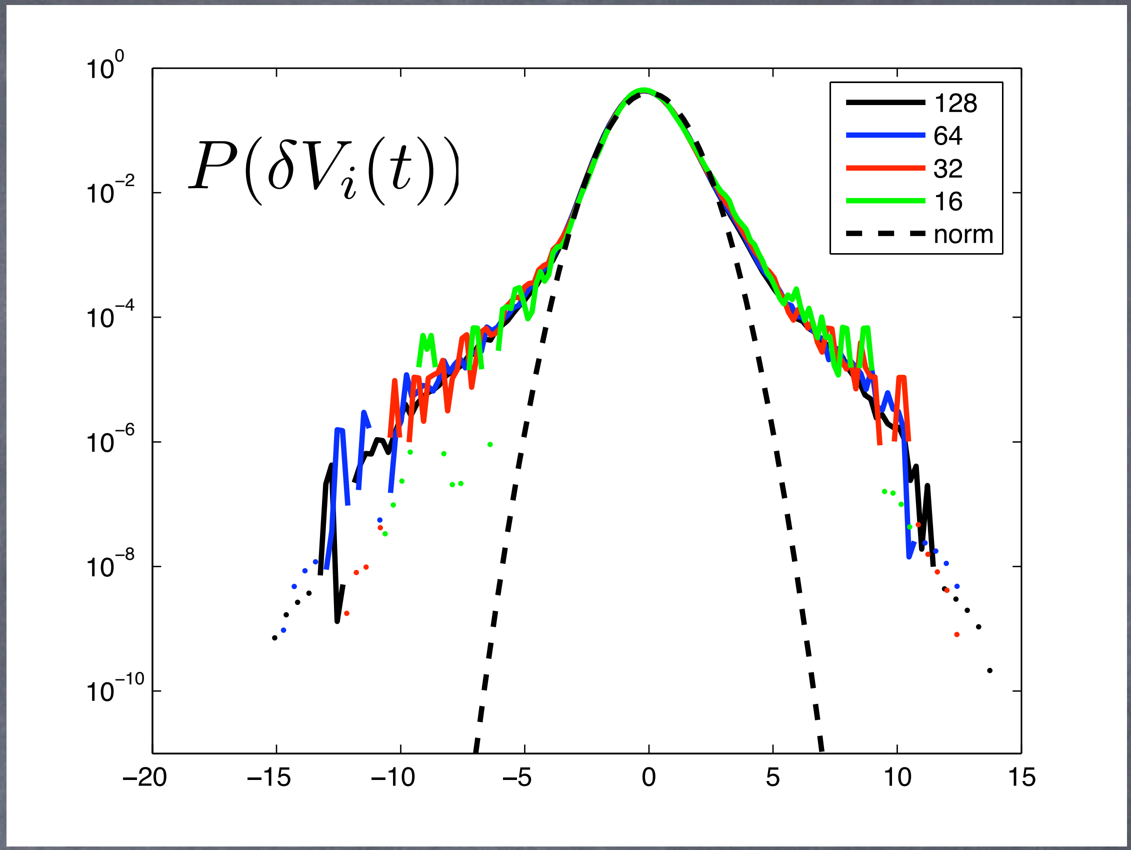


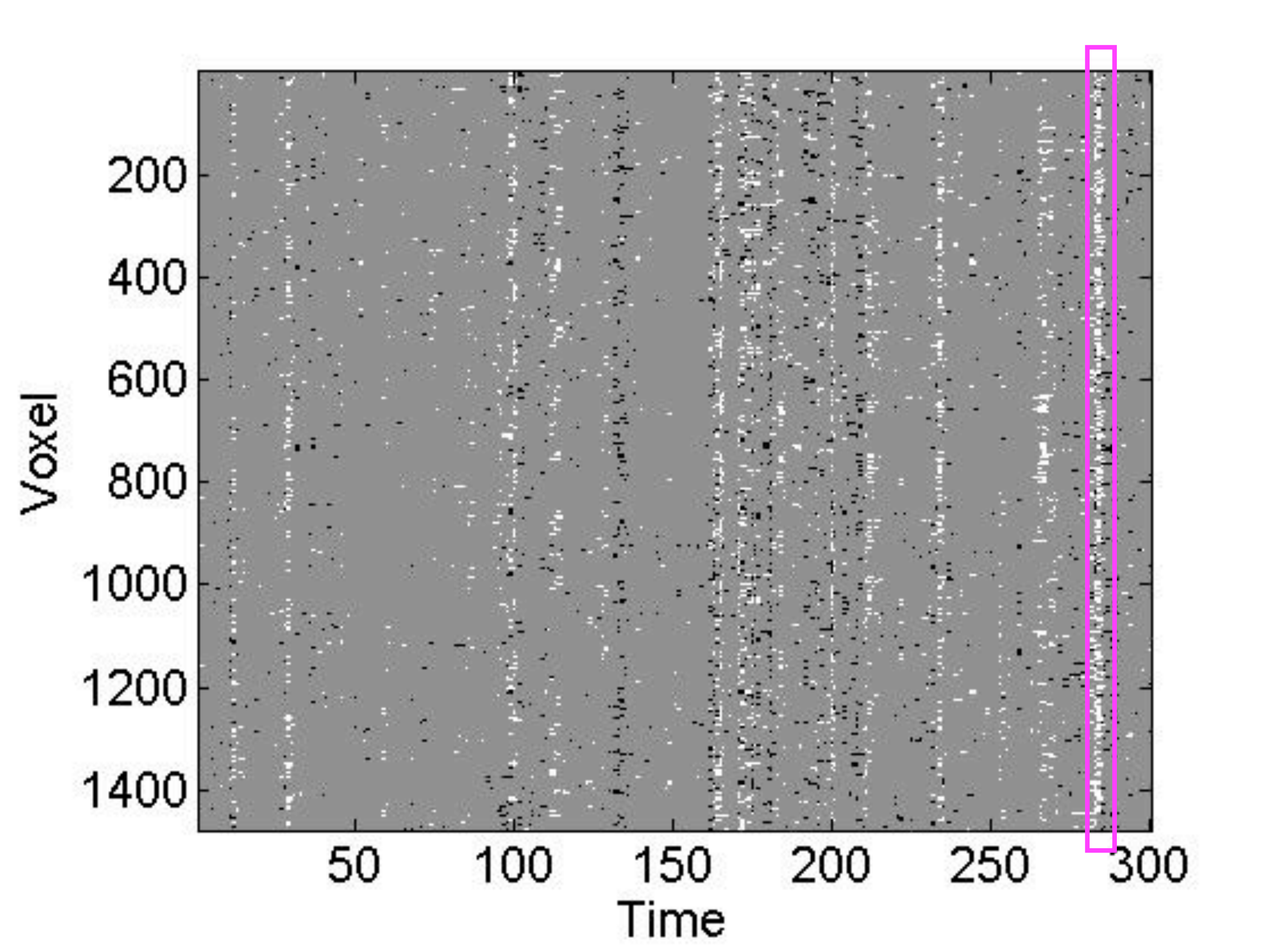
- $P(\delta V_i(t))$ probability density function
- Spatio-temporal structure of fluctuations above a certain size

fMRI burst activity

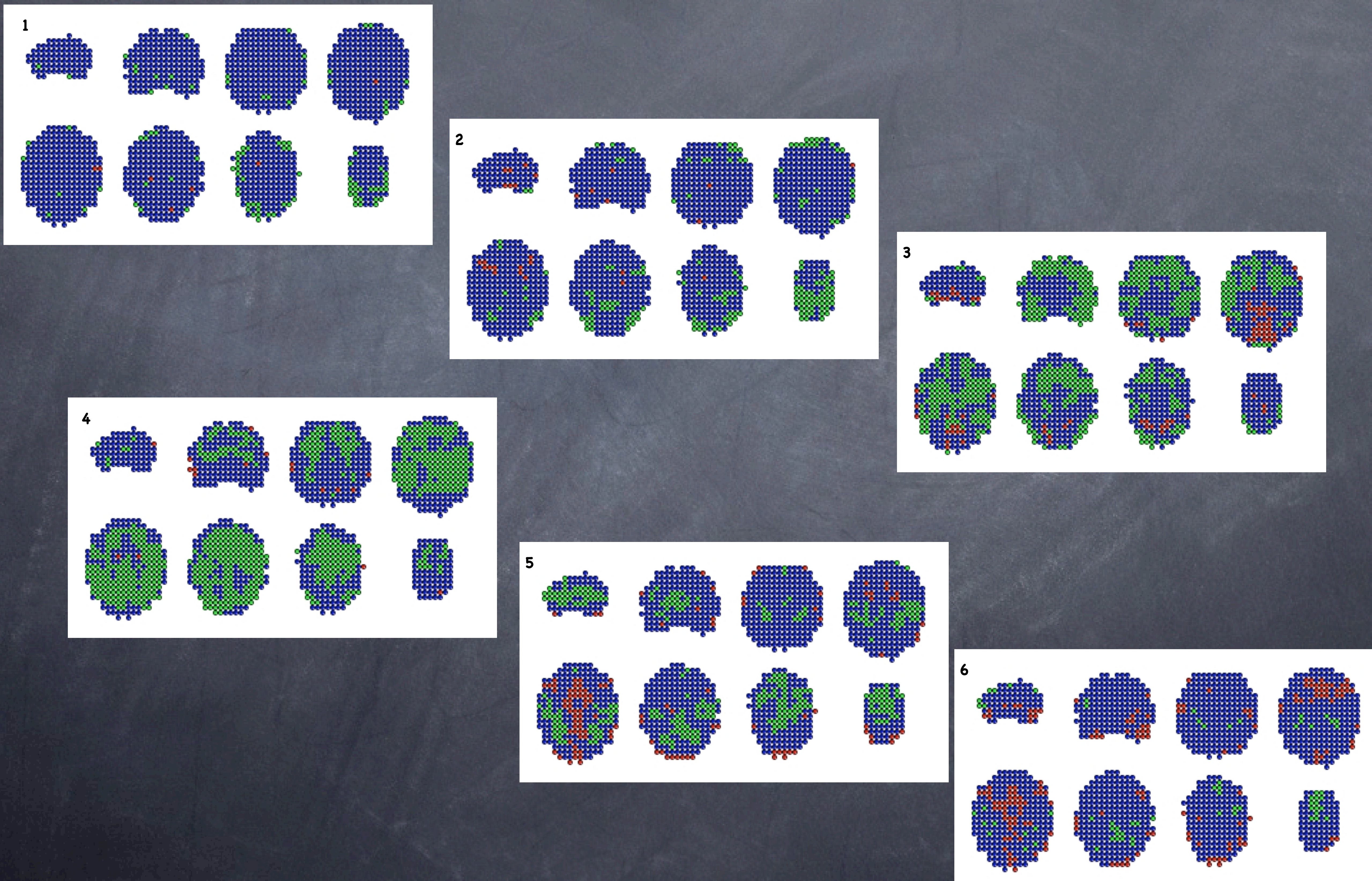
Two coarse graining steps
32x32x8

White > 2σ
Black < 2σ



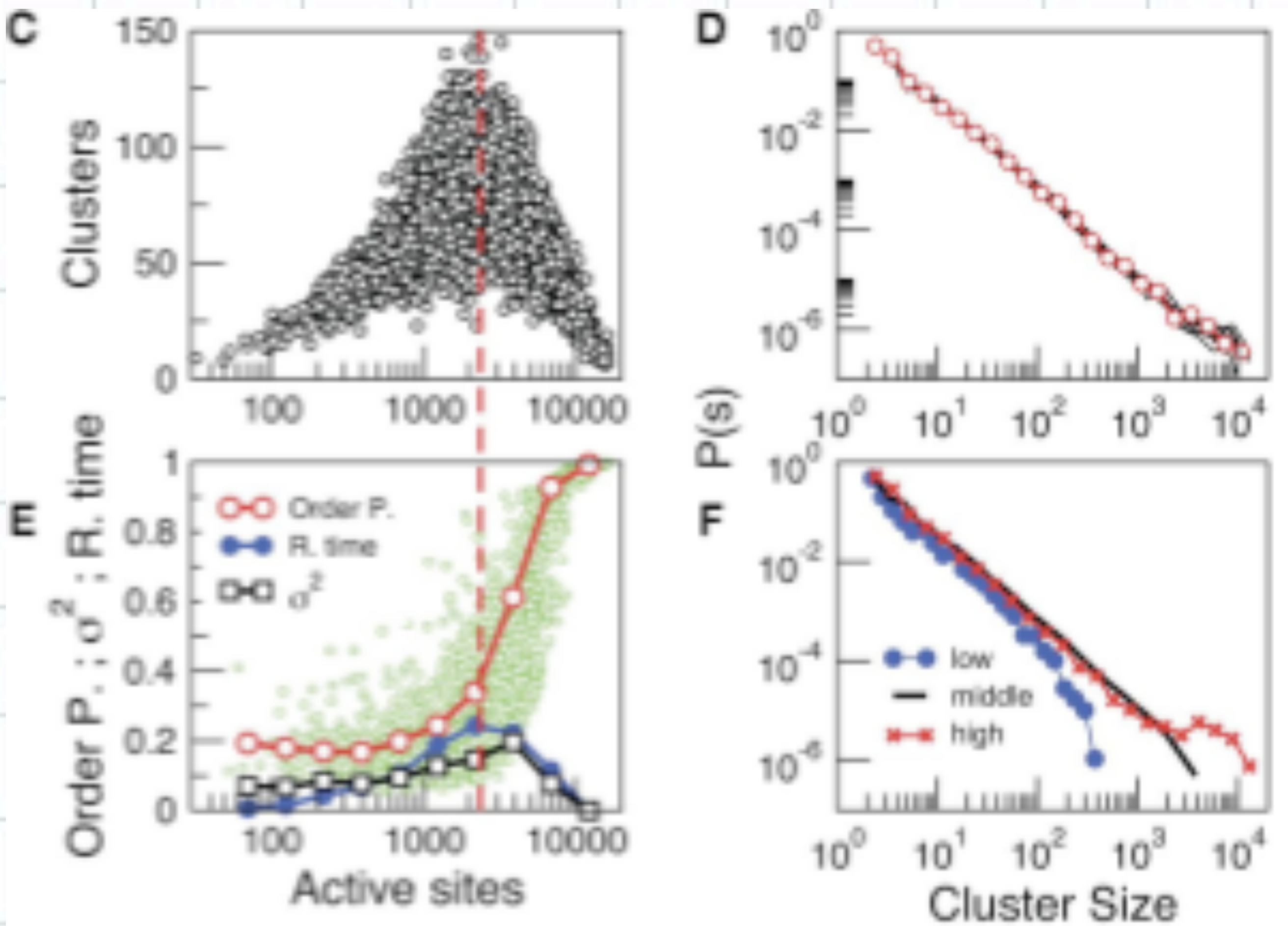


Temporal evolution of fMRI burst activity



Complexity & Networks

Brain



From: Tagliazucchi, et al.
frontiers in Physiology
Feb 2012

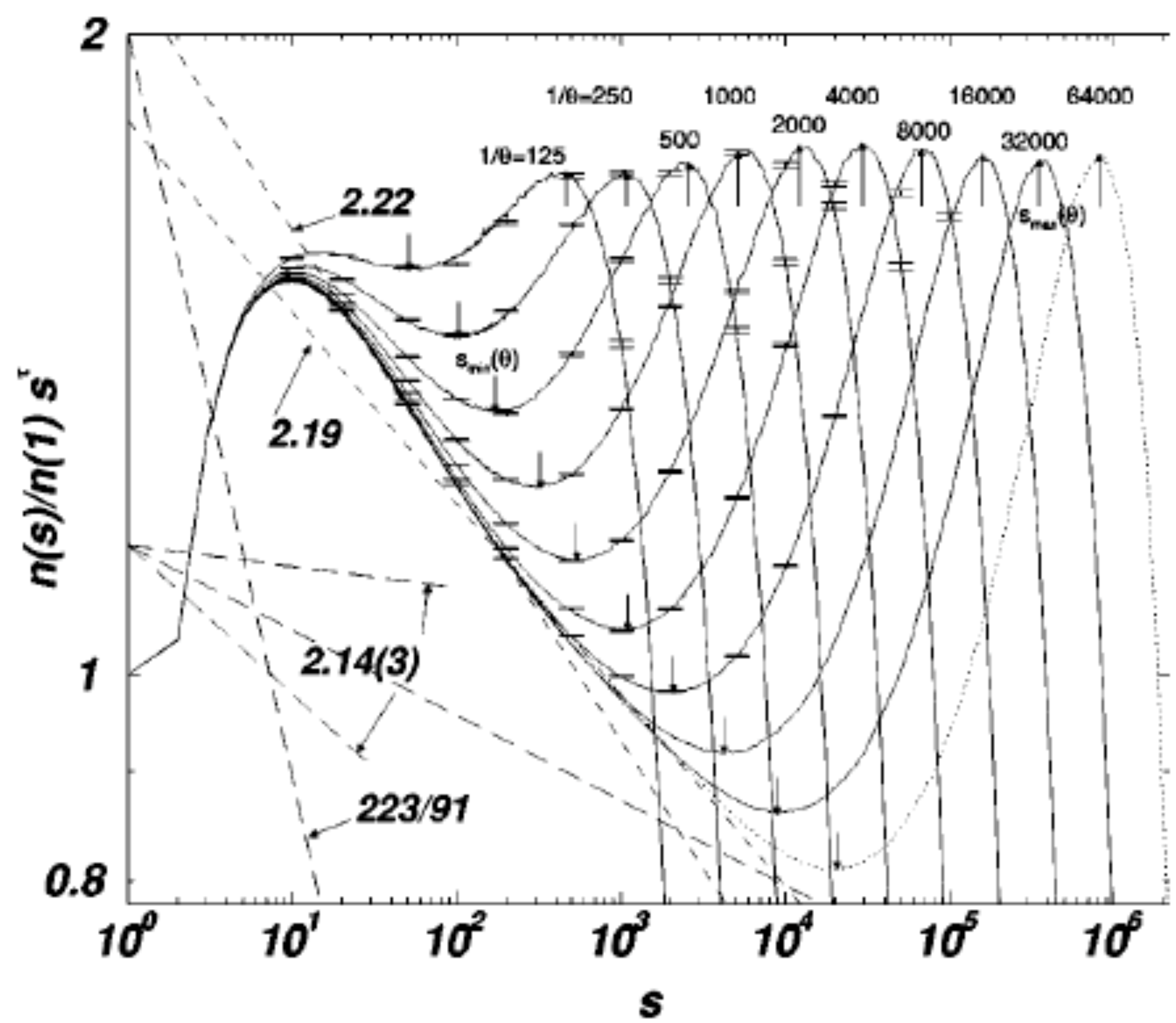
of voxels) in one individual. **(C)** Instantaneous relation between the number of clusters vs. the number of active sites (i.e., voxels above the threshold) showing a positive/negative correlation depending whether activity is below/above a critical value [~2500 voxels, indicated by the dashed line here and in **(B)**]. **(D)** The cluster size distribution follows a power law spanning four orders of magnitude. Individual statistics for each of the ten subjects are plotted with lines and the average with symbols. **(E)** The order parameter, defined here as the (normalized) size of the largest cluster is plotted as a function of the number of active sites (isolated data points denoted by dots, averages plotted with circles joined by lines). The calculation of the residence time density distribution (R. time, filled circles) indicates that the brain spends relatively more time near the transition point (which corresponds to about 0.4 of the largest giant cluster observed). Notice that the peak of the R. Time in this panel coincides with the peak of the number of clusters in **(C)**. Note also that the variance of the order parameter (squares) increases as expected for a phase transition. **(F)** The computation of the cluster size distribution calculated for three ranges of activity (low: 0–800; middle: 800–5000; and high >5000) reveals the same scale invariance plotted in **(D)** for relatively small clusters, but shows changes in the cut-off for large clusters.

Complexity & Networks

Drossel-Schwabl Forest fire model

Cluster size distribution does NOT behave as

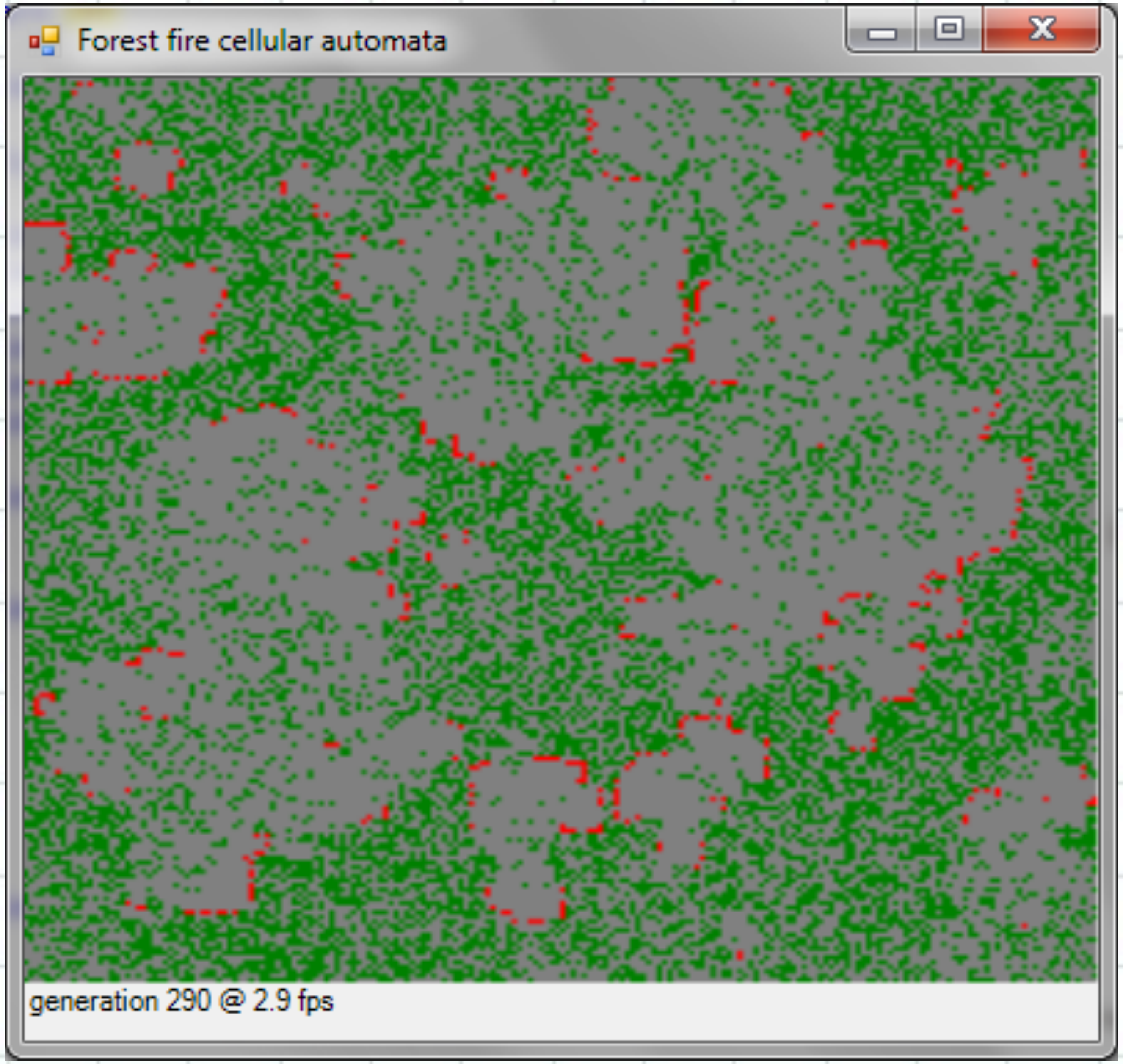
$$n(s) \propto t^{-\tau}$$



From G. Pruessner and H.J Jensen, *Broken scaling in the forest-fire model* Phys. Rev. E **65** 056707 (2002).

Complexity & Networks

Drossel-Schwabl Forest fire model



http://rosettacode.org/mw/images/5/5b/ForestFire-FSharp.png

Complexity & Networks

What to probe?

- Identify control parameter (humidity, background activity)
  Plot event sizes versus control parameter

- Focus on correlation functions

$$C(\mathbf{r},t) = \langle A(\mathbf{r}_0,t_0)A(\mathbf{r}_0+\mathbf{r},t_0+t)\rangle_{\mathbf{r_0},t_0} - \langle A(\mathbf{r}_0,t_0)\rangle^2$$