The challenge of “complex” systems, with implications for self-organized critical (SOC) behavior

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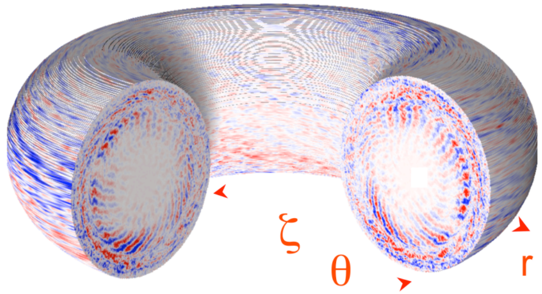
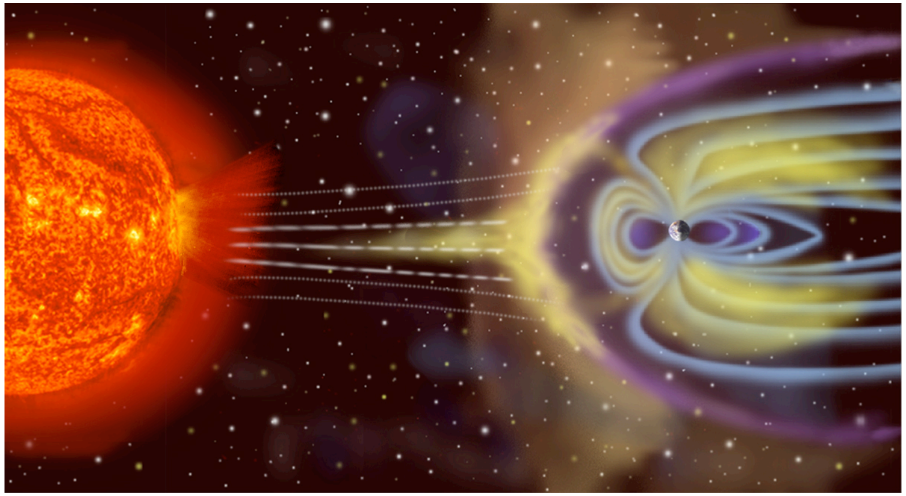
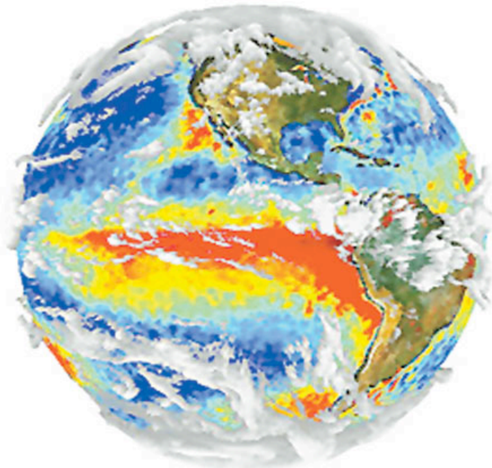
ISSI International Space Science Institute, Bern, Switzerland
15-10-2012, ore 14:30

The challenge of "complex" systems

- **By "complex" systems** we mean systems with "many" coupled degrees of freedom, interacting with external drive:
  - Burning plasma in a magnetic confinement device
  - Solar wind-magnetosphere-ionosphere interaction
  - Earth's global climate system







"So, even though there is apparently no science of complexity, there is much science to be learned from studying complex systems."

Leo P. Kadanoff, Phys. Today **54**, 39 (2001)

Is it only statistics or not?

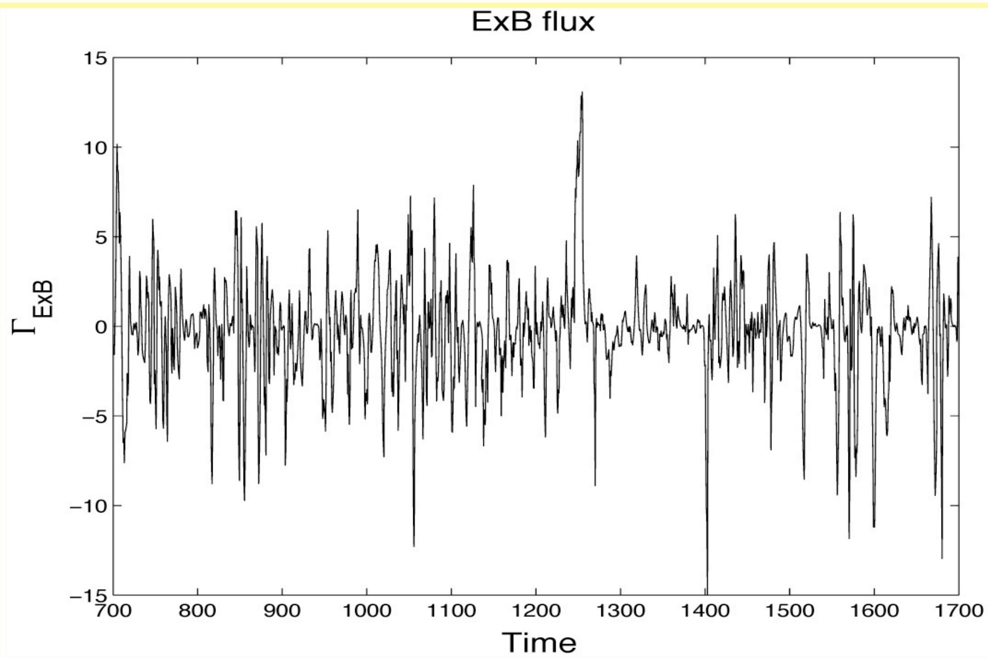
- "**Complexity science**: a new and fast-growing area of interdisciplinary science that seeks to understand those aspects of natural systems that are dominated by their collective interactions rather than their individual parts"

  Global Science in the Antarctic Context: British Antarctic Survey Core Programme 2005-2010, p.20

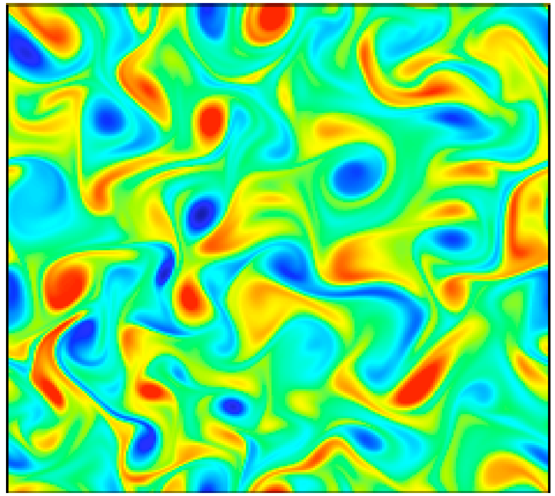
- **In plasmas** (both in space and laboratory plasma), behavior is often governed by multiple nonlinear physical processes, that interact with each other, and operate across a broad distribution of time and spatial scales (e.g., from millimeter to meter in laboratory)

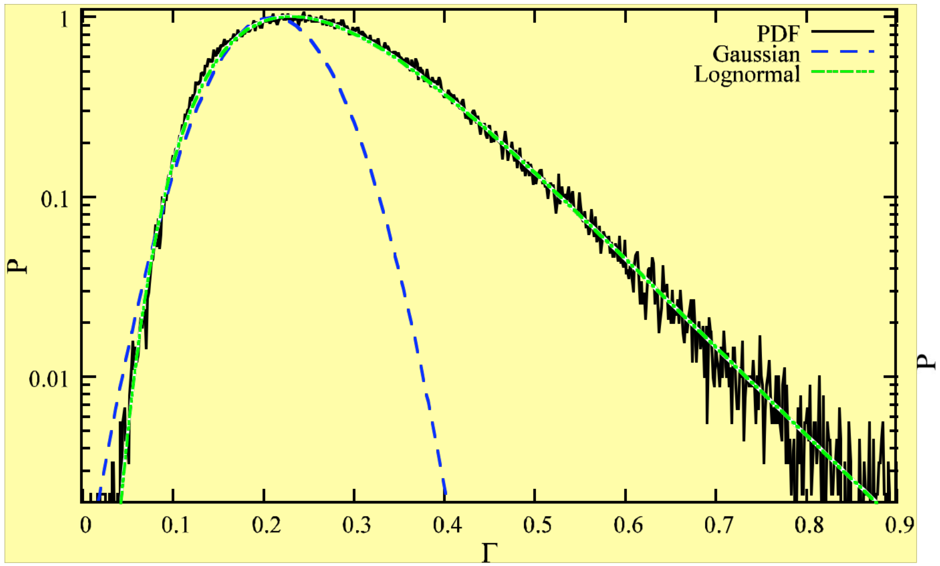
- The quest for **quantitative methods** that extract model-independent information from observed nonlinear signals from real plasmas: in particular, statistics. But what about first-principle models?

2D drift-wave turbulence

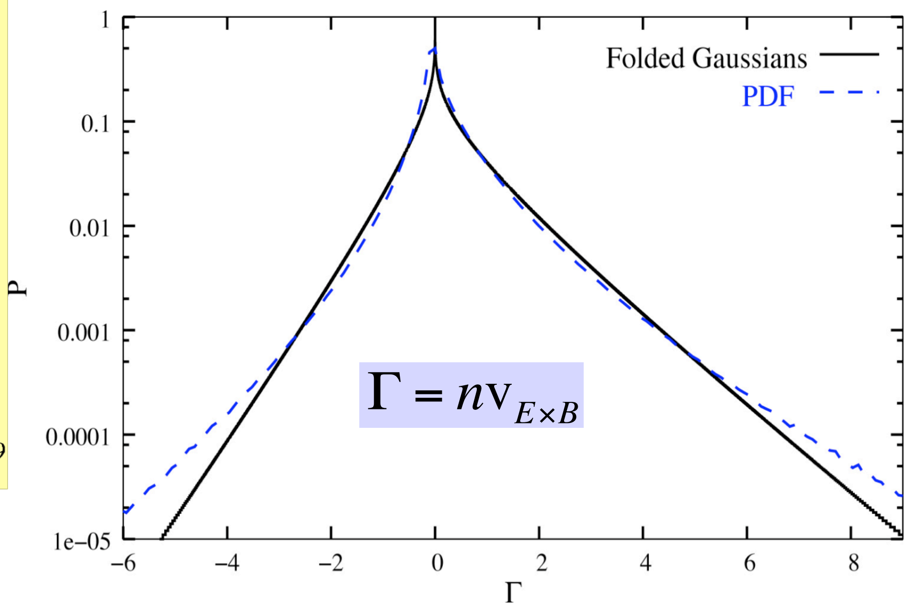


Bursting flux!



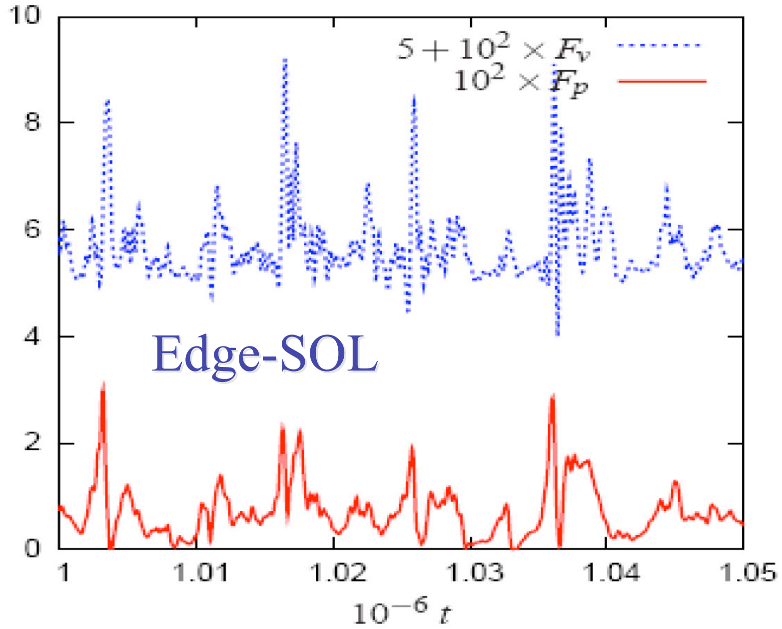


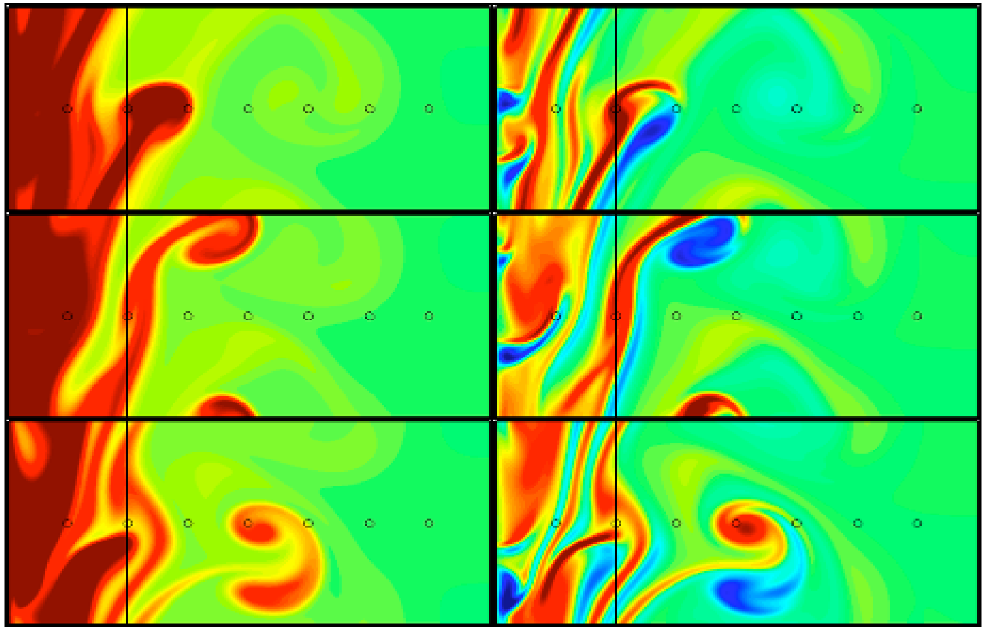
Flux PDF (point-wise)

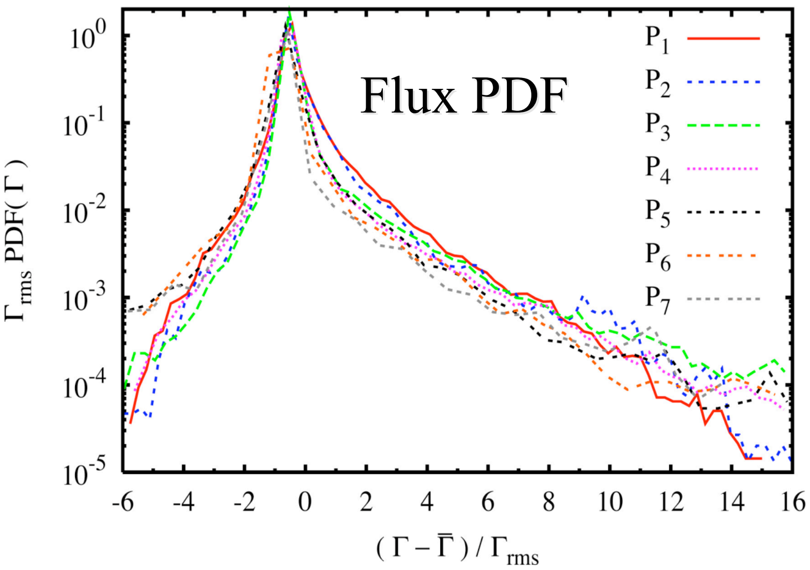


V. Naulin et al., Phys. Lett. A 321, 355 (2004)

Global model for interchange driven turbulence (Edge-SOL)

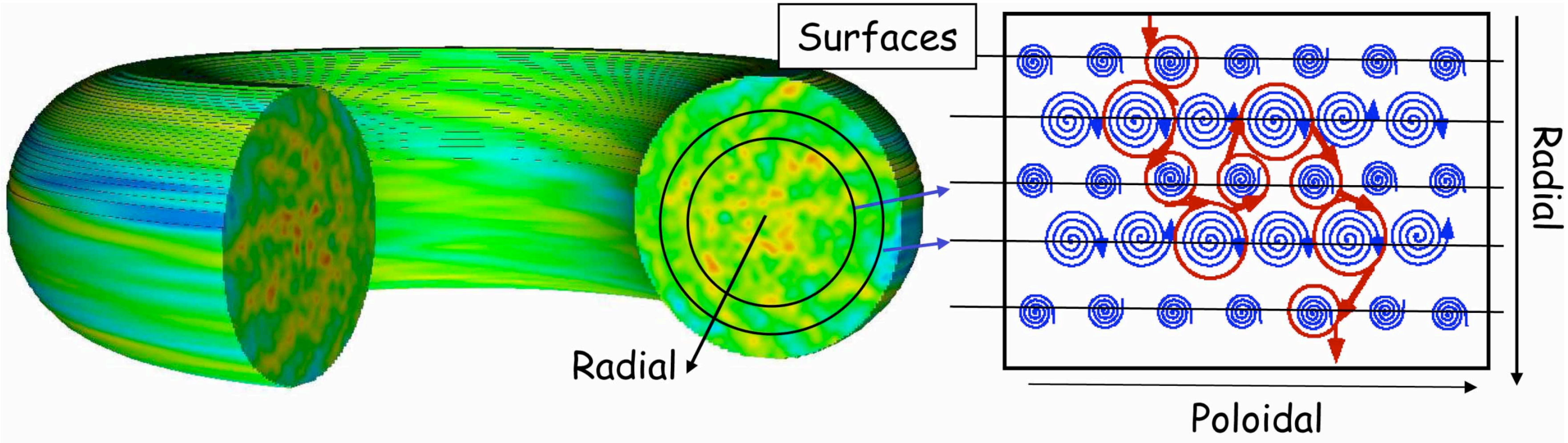






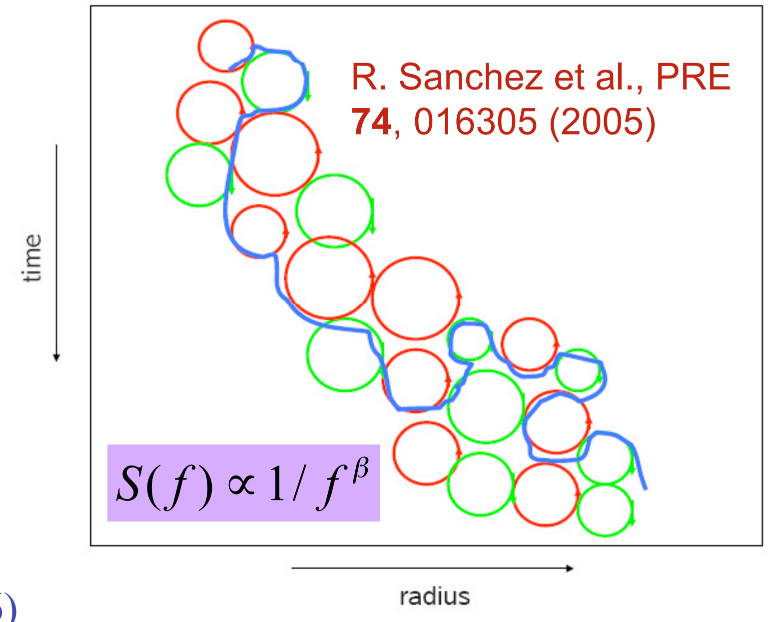
Particle density (left) and vorticity (right) during a burst. In the edge/scrape-off layer (SOL) region transport is strongly intermittent and characterized by large-amplitude, radially propagating blob-like structures of particles and heat, generated close to the last closed flux surface. Garcia et al., PPCF **48**, L1 (2006)

Avalanche-type transport at the plasma edge



PROPERTIES:

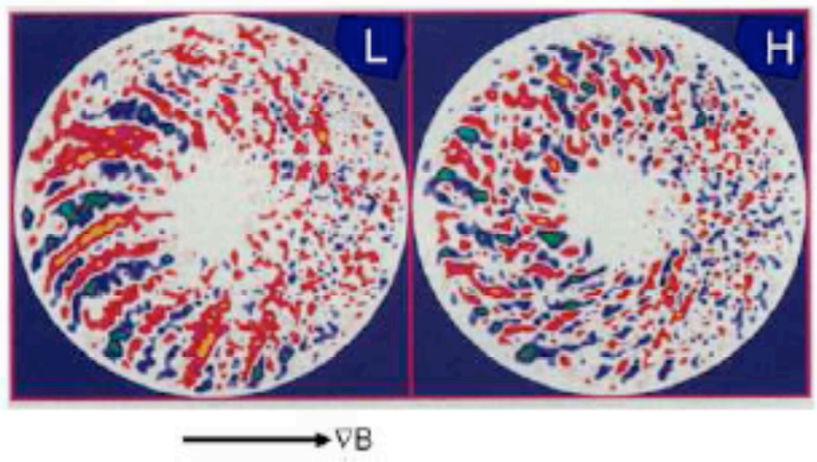
1. **PDF's** of radial displacements of tracer particles are non-Gaussian with algebraic tails
2. **Anomalous** scaling of the confinement time, τ, with the system size, L, $\tau = L^{\alpha}$, of low confinement mode plasma, with $1<\alpha<2$



D. del-Castillo-Negrete, Phys. Plasmas **13**, 082308 (2006)

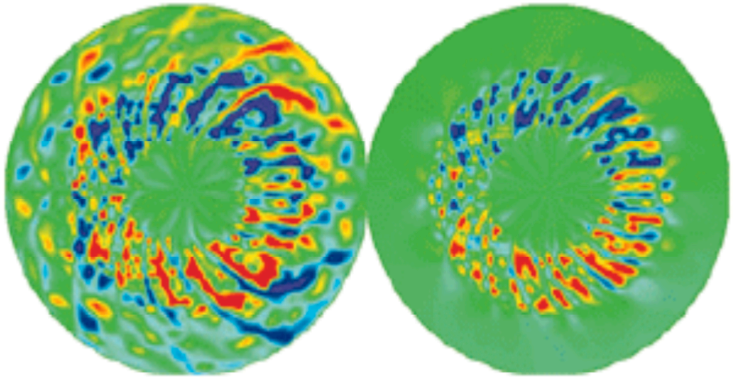
Coexistence of collective effects due to turbulence & SOC

Toroidal ITG gyro-kinetic simulations



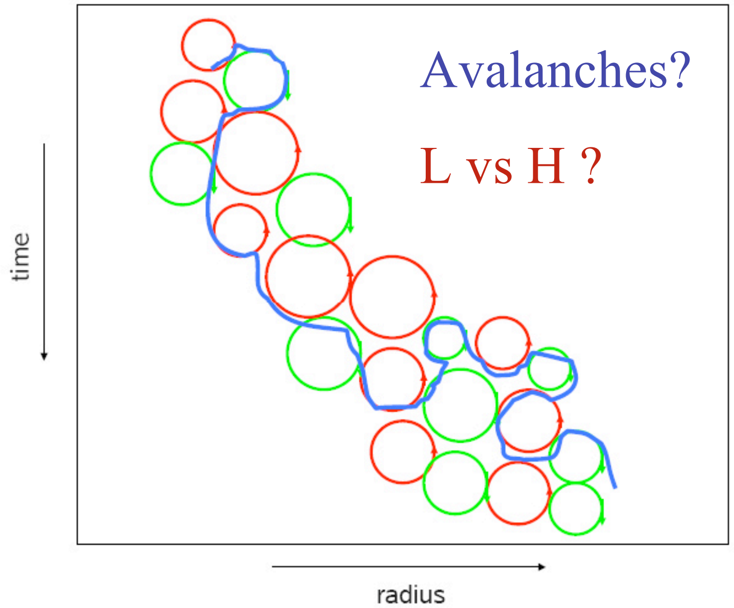
[See: Z. Lin et al, *Science* **281** (1998) 1835]

Fluid resistive interchange turbulence (cylinder)

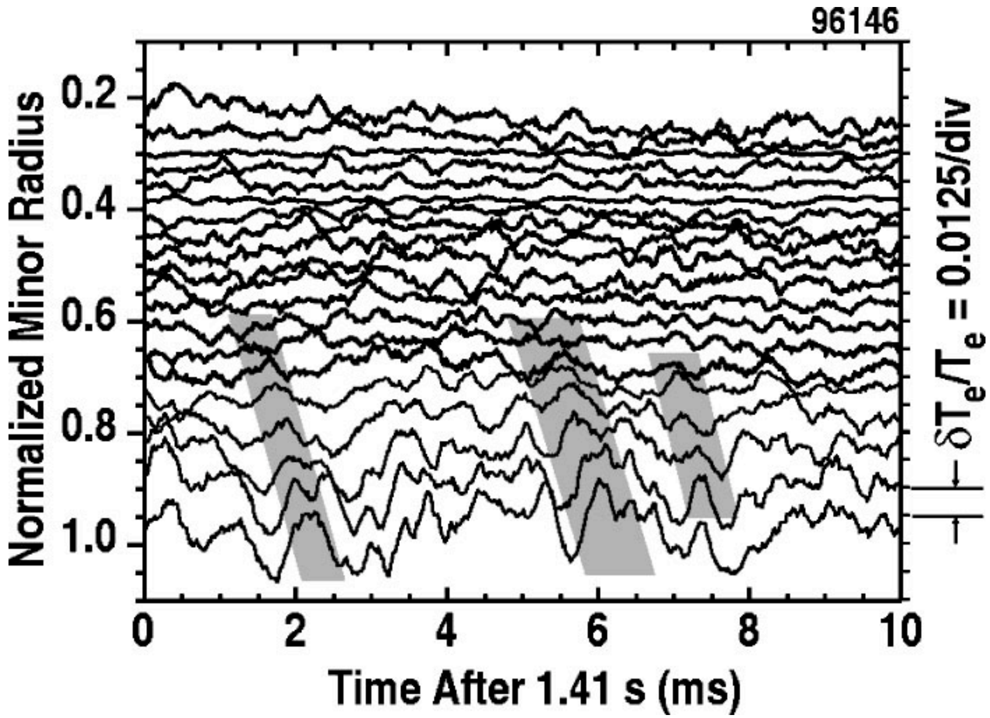


[See: B.A. Carreras et al, Phys. Fluids B 5 (1993) 1491]

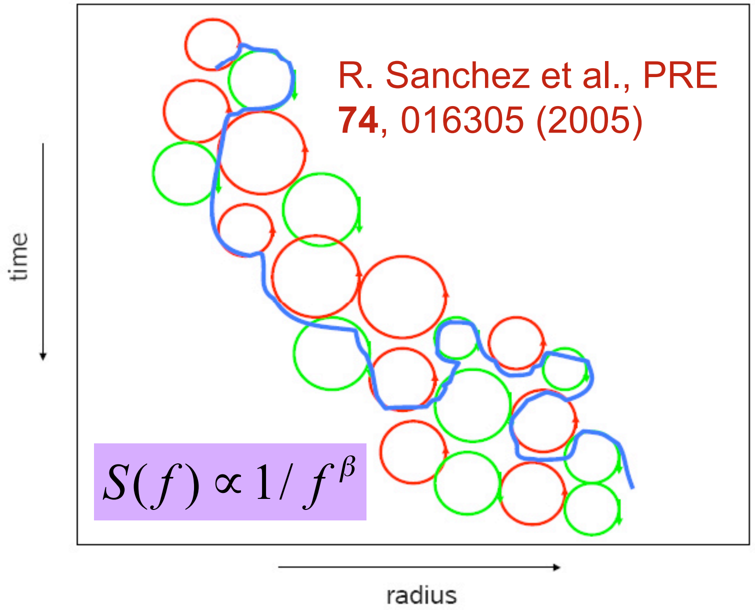
Classic measurements of avalanche-type transport include the analysis of **edge plasma turbulence**, yielding non-Gaussian probability distribution functions that are **long-tailed**. One unresolved problem here is the complex nature of **coexistence** of collective effects due to **turbulence, turbulent transport, and SOC.** In particular, which is the difference between the notions of turbulence and SOC?



L-mode tokamak plasmas



P. Politzer et al., Phys. Plasmas **9**, 1962 (2002)



SOC hypothesis suggested as an explanation?

The normalized electron temperature fluctuations ($\delta T/T$) for L-mode plasma discharges vs time in the DIII-D tokamak. ***The highlighted bands*** indicate examples of ***avalanche-like events,*** outwardly propagating disturbances, moving at 300 m/s.

Self-organized criticality?

Per Bak (1948-2002)

Self-Organized Criticality

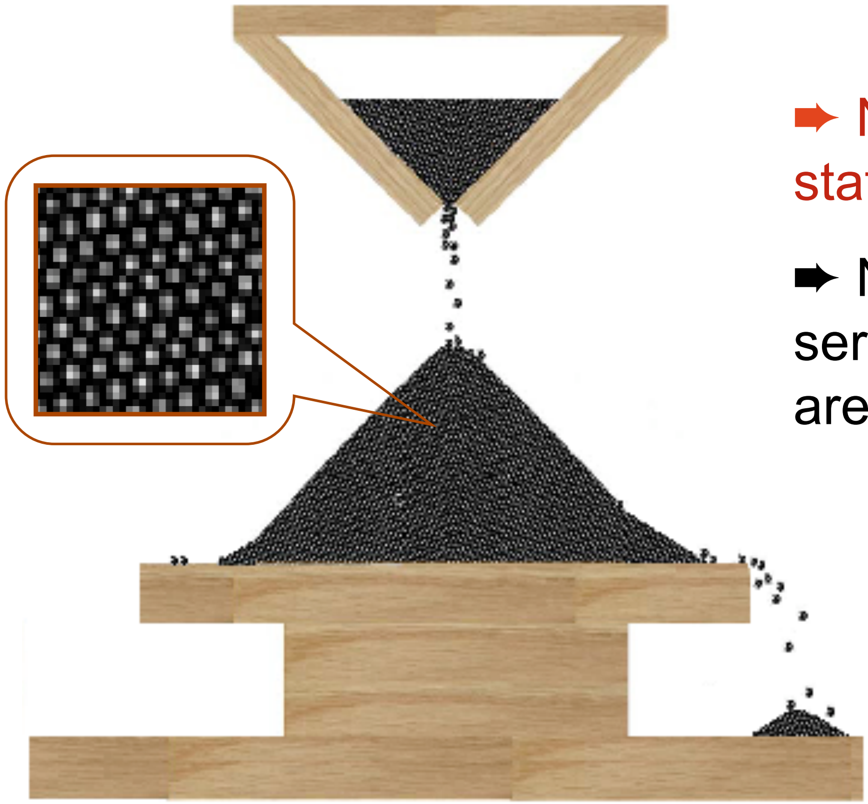
↓

A critical point as an attractor

"The system will become stable precisely at the point… where the noise signal cannot be communicated through infinite distances. At this point there will be no length scale in the problem so that one might expect the formation of a scale-invariant structure of minimally stable states."

Self-organized criticality (SOC) ➨BTW87

- A paradigm for complex dissipative systems that relax through bursts
- Self-organized state ➨ critical state (at the border of chaos) reached without fine tuning of any external or control parameters
- Critical state ➨ attractor, robust with respect to variations of parameters and with respect to randomness



Properties:

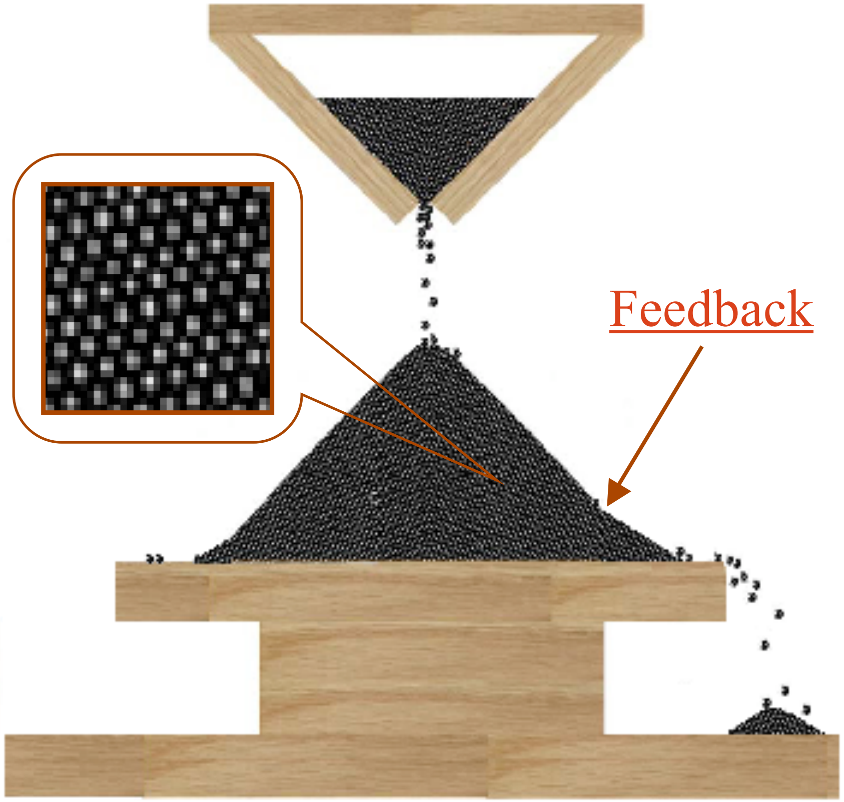
➨ Near equilibrium and far from the critical state, the system produces no avalanches

➨ Near criticality and far from equilibrium, series of relaxations of widely varying size are generated

But which are the dependencies on the driving rate?

And how to obtain the critical exponents from a general theory?

SOC: Cellular-automation (CA) models



If:

the local slope or pressure $z_{i,j}$ exceeds the critical value z_c

Then:

at the next time step (in two dimensions)

$$z_{i,j} \rightarrow z_{i,j} - 4, \qquad z_{i,j\pm1} \rightarrow z_{i,j\pm1} + 1,$$

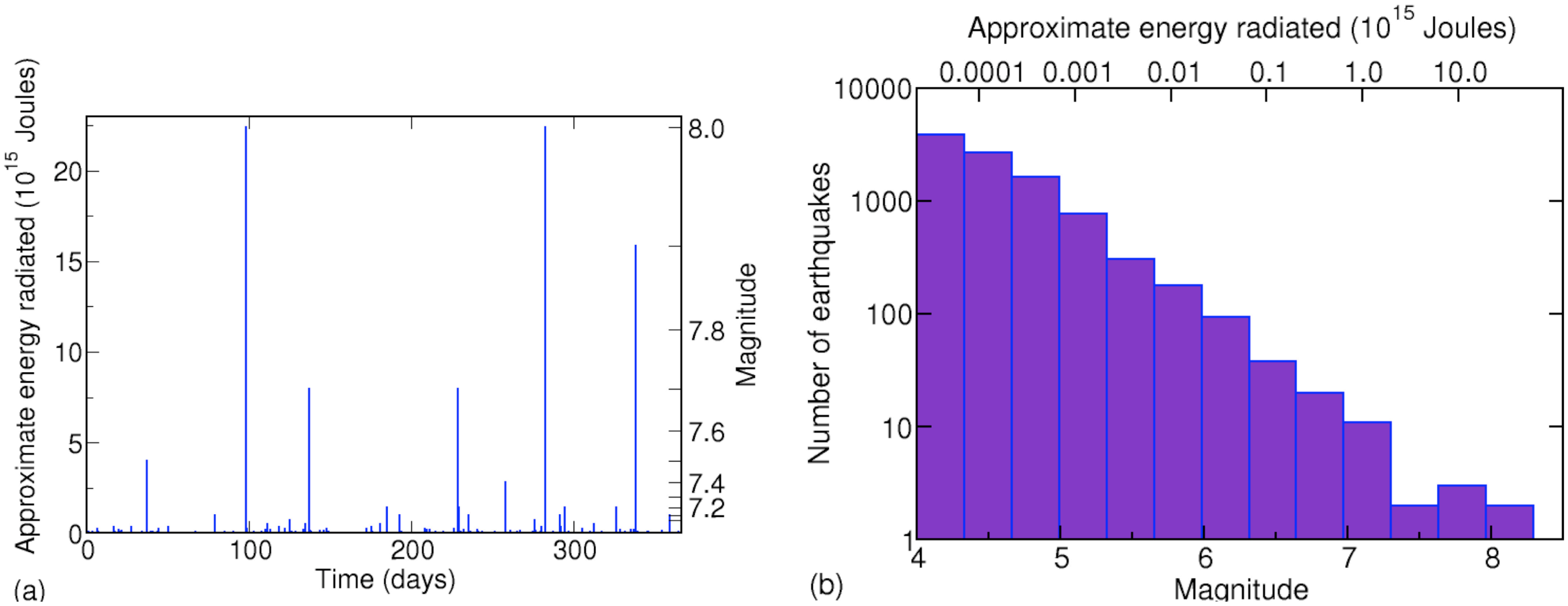
$$z_{i\pm1,j} \rightarrow z_{i\pm1,j} + 1$$

In model 2 the slope is increased by repeatedly letting

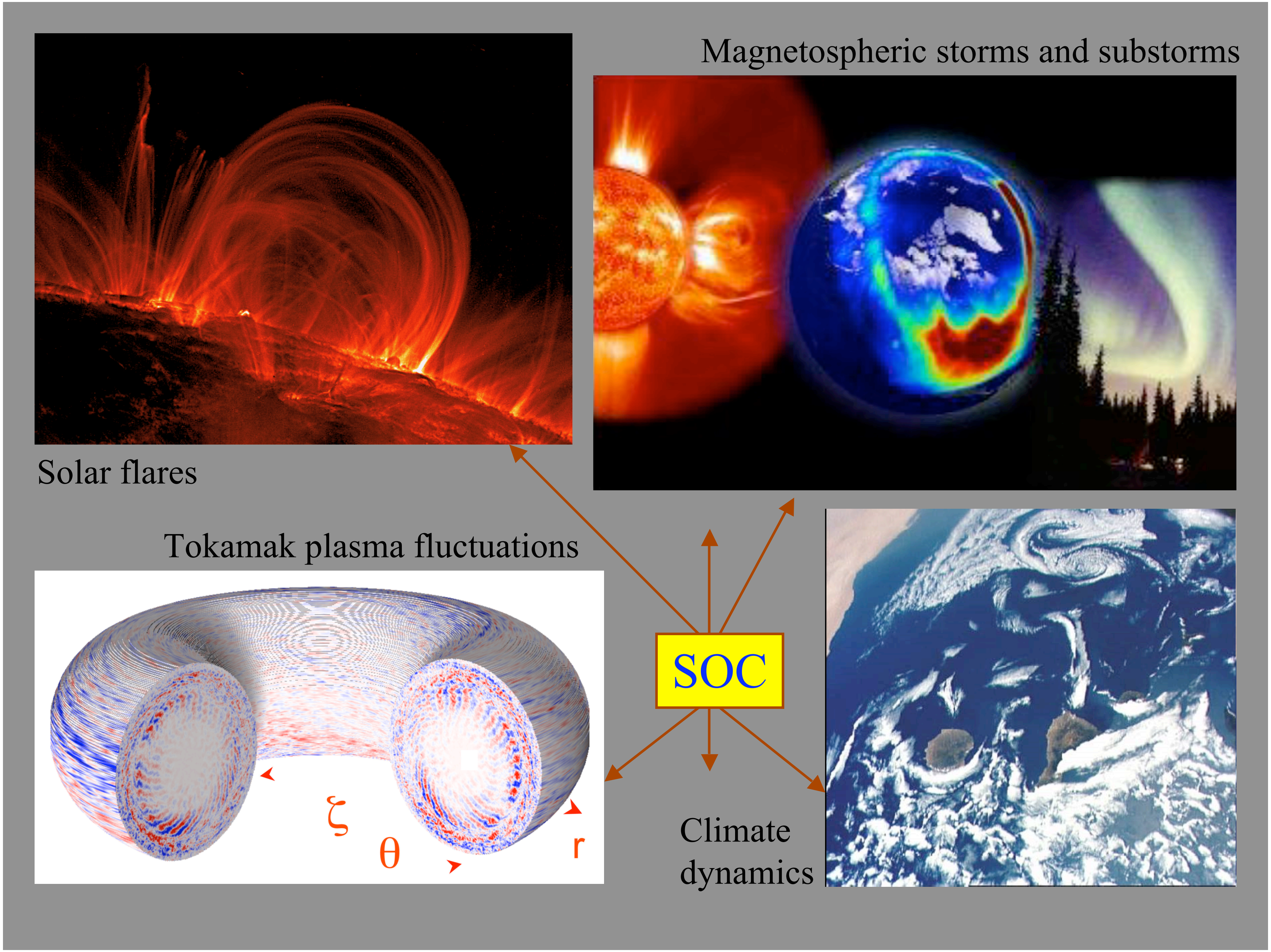
$$z_{i,j} \rightarrow z_{i,j} + 1$$

at random sites (i, j) and allowing the system to relax following the dynamical rule above. The boundary condition is $z = 0$.

Example: The Earth Crackles



The Earth responds to the slow strains imposed by continental drift through a series of earthquakes: impulsive events well separated in space and time. Earthquakes come in a wide range of sizes, from unnoticeable trembles to catastrophic events. The smaller earthquakes are much more common: the number of events of a given size forms a power law called the Gutenberg-Richter law. One would hope that such a simple law should have an elegant explanation. (Adapted from: Sethna et al., preprint)


Magnetospheric storms and substorms
Solar flares
Tokamak plasma fluctuations
SOC
ζ
θ
r
Climate
dynamics

Self-organized criticality: Good introductory reading

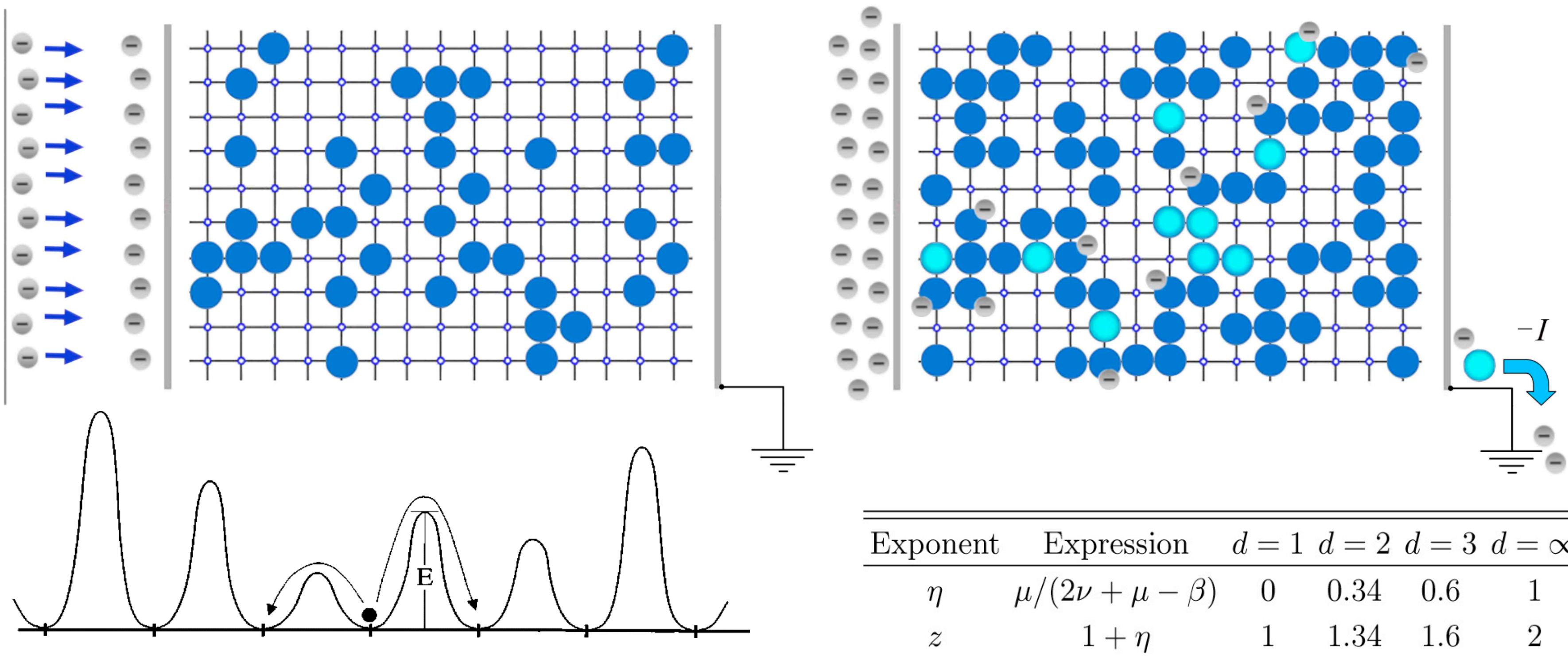
1. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
2. C. Tang and P. Bak, Phys. Rev. Lett. **60**, 2347 (1988).
3. Y.-C. Zhang, Phys. Rev. Lett. **63**, 470 (1989).
4. D. Sornette, J. Phys. I France **2**, 2065 (1992).
5. A. Vespignani and S. Zapperi, Phys. Rev. E **57**, 6345 (1998).

ISI Web of Science: > 3,220...

Books:

1. H.J. Jensen, *Self-Organized Criticality*, Cambridge Univ. Press, Cambridge, 1998.
2. M.J. Aschwanden, *Self-Organized Criticality in Astrophysics. The Statistics of Non-linear Processes in the Universe*, Springer, 2011.

Dynamic polarization random walk (DPRW) model

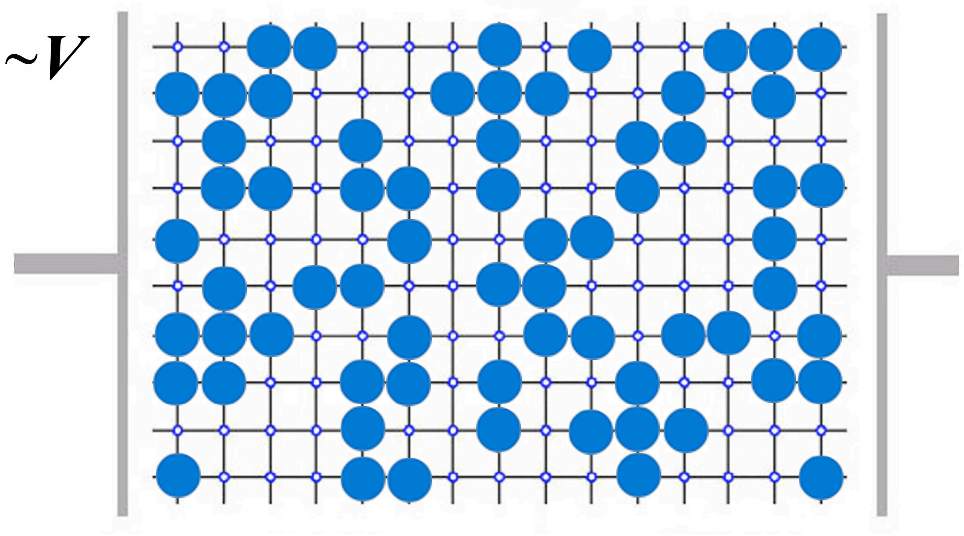


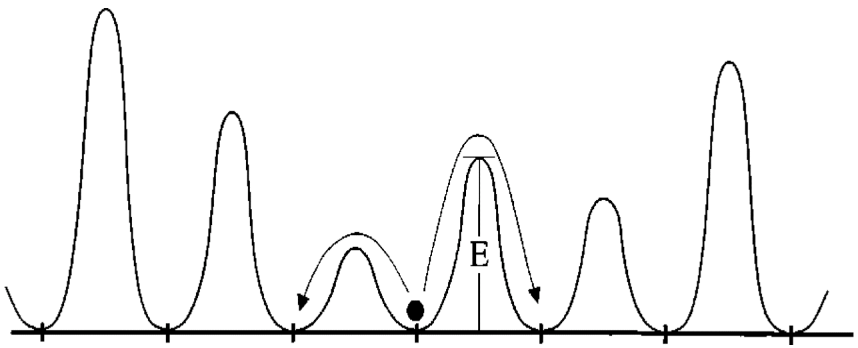
Exponent	Expression	$d=1$	$d=2$	$d=3$	$d=\infty$
η	$\mu/(2\nu+\mu-\beta)$	0	0.34	0.6	1
z	$1+\eta$	1	1.34	1.6	2
γ	$1-\eta$	1	0.66	0.4	0
α	$2-2\eta$	2	1.3	0.8	0
H	$1/z$	1	0.75	0.6	1/2
τ	$3-\alpha z/d_f$	1	2.1	2.5	3

➨ Random walk hopping on a self-adjusting percolation cluster, combined with the basic theory of a dynamic polarization response

➨ In the limit of vanishing external forcing, the model leads to a description in terms of a fractional extension of the diffusion equation associated with a Levy type distribution of durations of relaxation events, and a to a set of critical exponents consistent with the BTW sand-pile SOC model

Random walks on a percolation cluster





$$\sigma_{ac}(\omega) \propto \omega^{\eta}$$

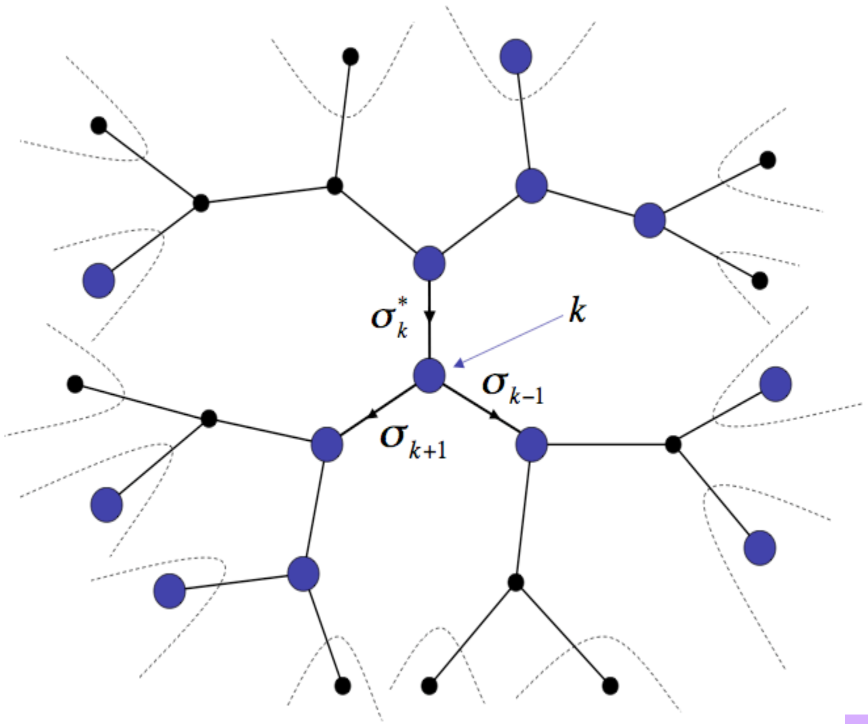
$$\eta = \mu/(2\nu + \mu - \beta)$$

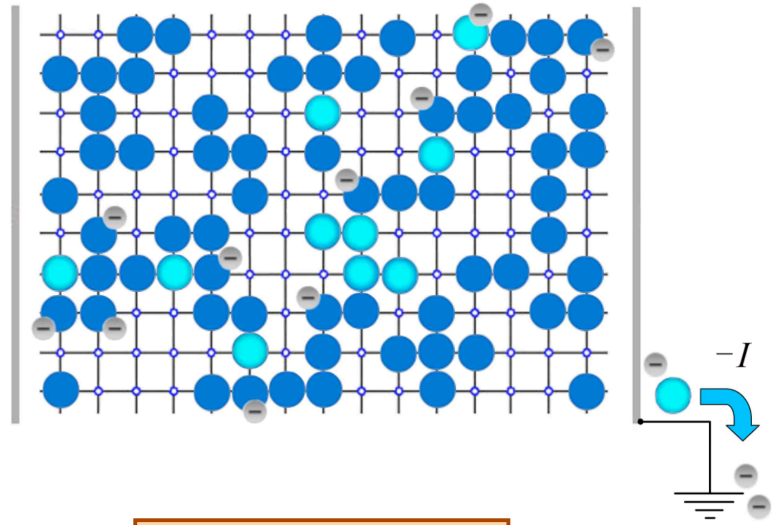
$$\sigma_{ac}(\omega) = \frac{ne^2}{k_B T} D(\omega)$$

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial^{\eta}}{\partial t^{\eta}} \frac{\partial^2}{\partial x^2} \left[Dn(x,t) \right]$$

1. Y. Gefen, A. Aharony, and S. Alexander, Phys. Rev. Lett. **50**, 77 (1983).
2. A. V. Milovanov, Phys. Rev. E **79**, 046403 (2009).

DANSE formalism for the hole wave function





$$\boxed{\left\langle \Delta n^2(t) \right\rangle \propto t^{1/3}}$$

$$\frac{\partial f(t,\Delta n)}{\partial t} = \frac{\partial^{2/3}}{\partial t^{2/3}} \frac{\partial^2}{\partial \Delta n^2} \left[f(t,\Delta n) \right]$$

$$\beta_{\max} = 1/\ln 2 \approx 1{,}4427$$

$$\boxed{i\hbar \frac{\partial \psi_n}{\partial t} = \widehat{H}_L \psi_n + \beta |\psi_n|^2 \psi_n} \longleftrightarrow \widehat{H}_L \psi_n = \varepsilon_n \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

Fractional calculus: as old as calculus itself…

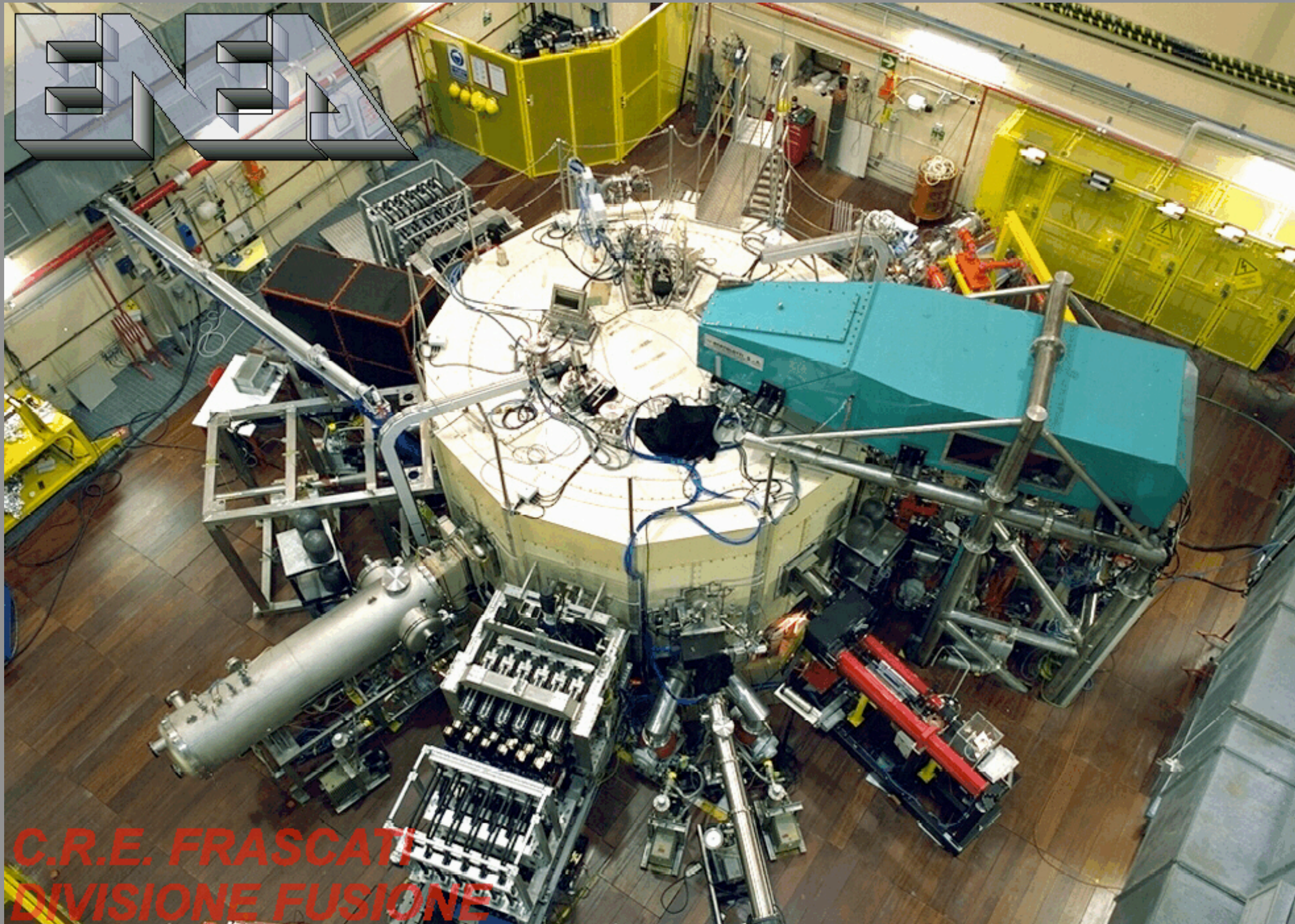
Gottfried Wilhelm Leibnitz.

"Thus it follows that $d^{1/2}x$ *will be equal to* $x\sqrt{dx:x}$*, ... from which one day useful consequences will be drawn."*

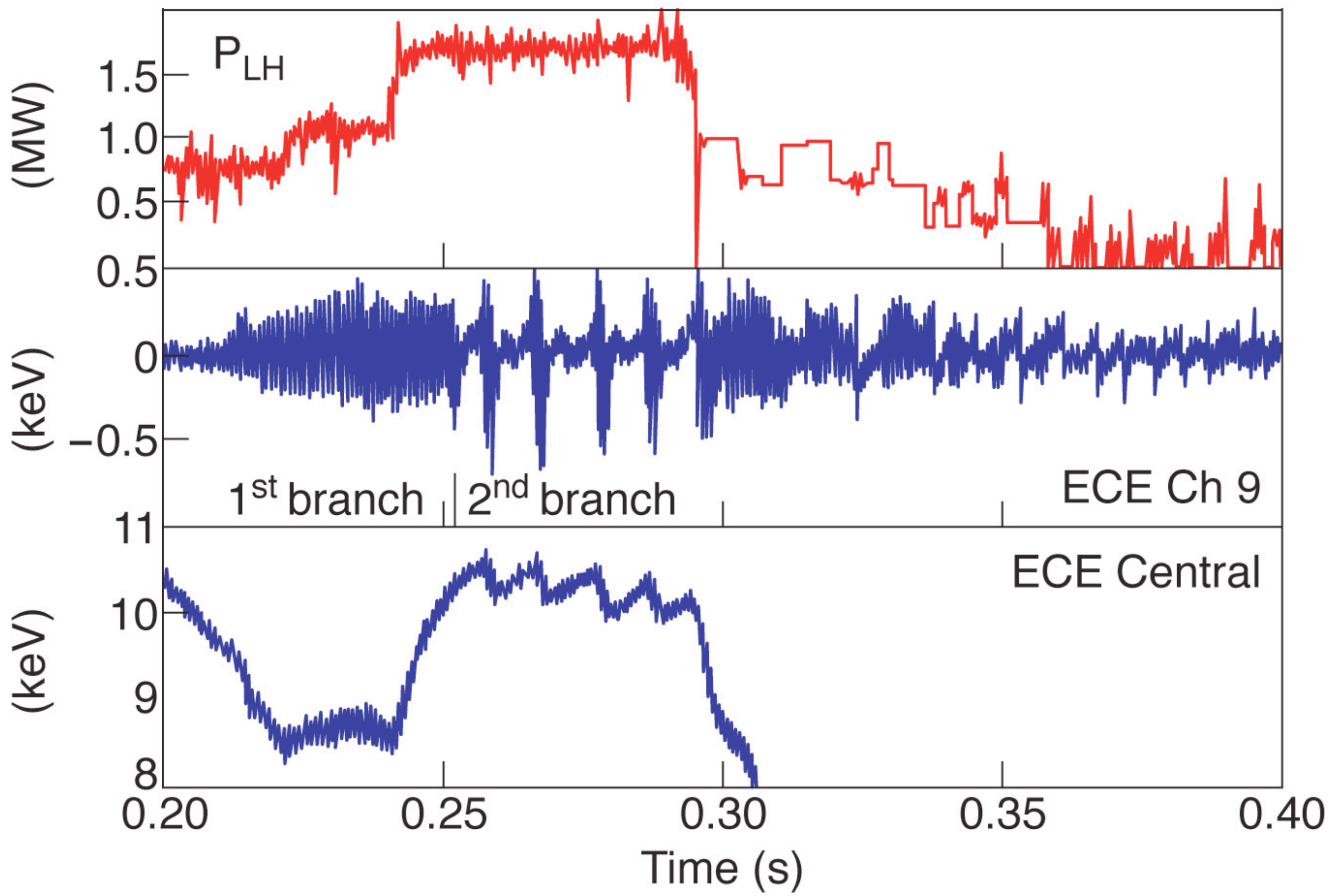
Gottfried von Leibnitz,
in a reply to Guillaume de l'Hôpital
(1695)

$$\frac{d^{\alpha}}{dx^{\alpha}}x^{\mu}=\frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)}x^{\mu-\alpha}$$

Frascati Tokamak Upgrade



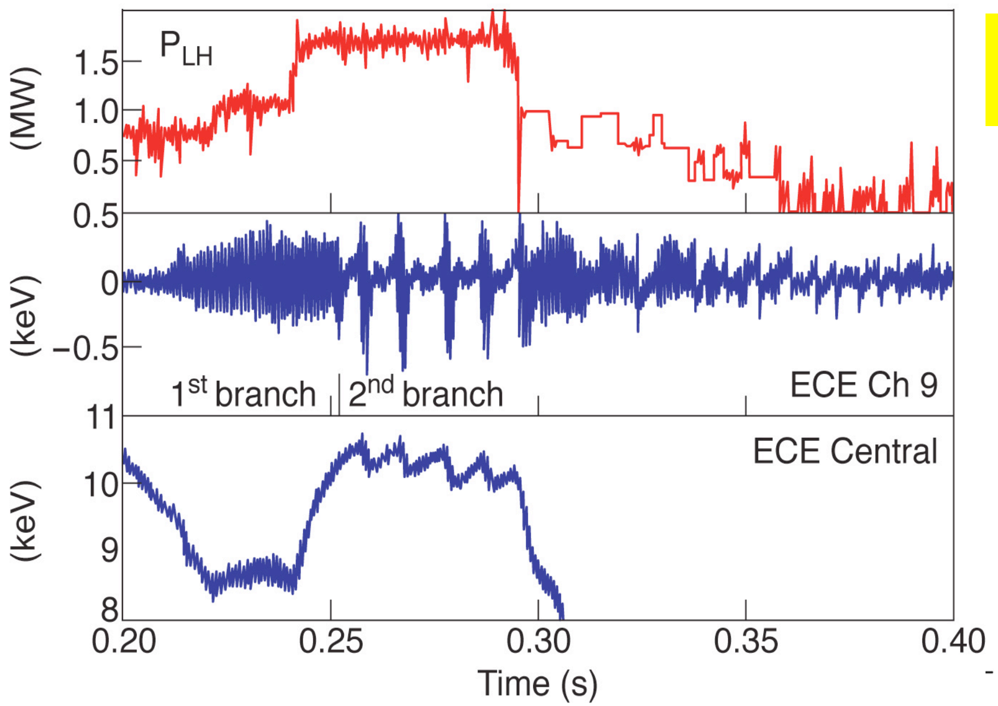
Electron fishbones in FTU (limiter tokamak)

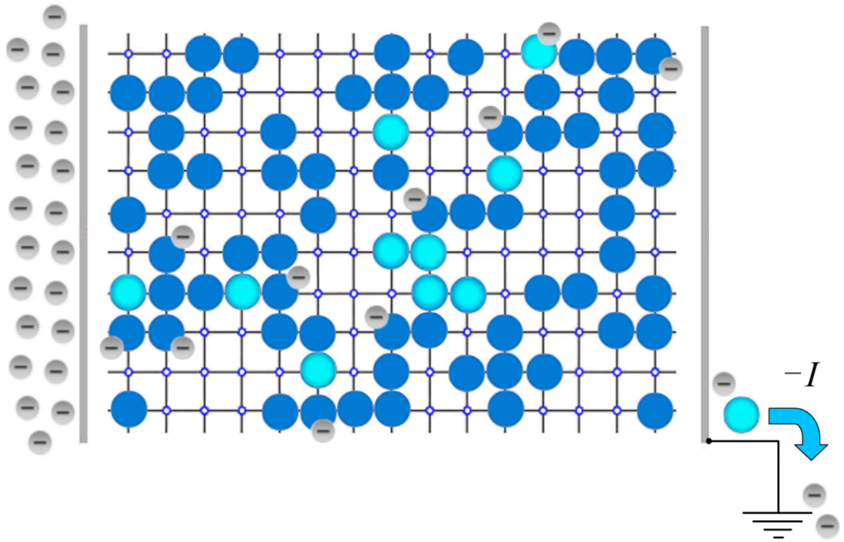


LH absorbed power, fast electron temperature fluctuations, and central radiation temperature in FTU shot #20865. Source: F. Zonca et al., NF **47**, 1588 (2007)

- During high power LH injection, an evident transition in the electron fishbone signature takes place from almost steady state NL oscillations (fixed point) to regular bursting behavior (limit cycle).

Fishbone instability of SOC





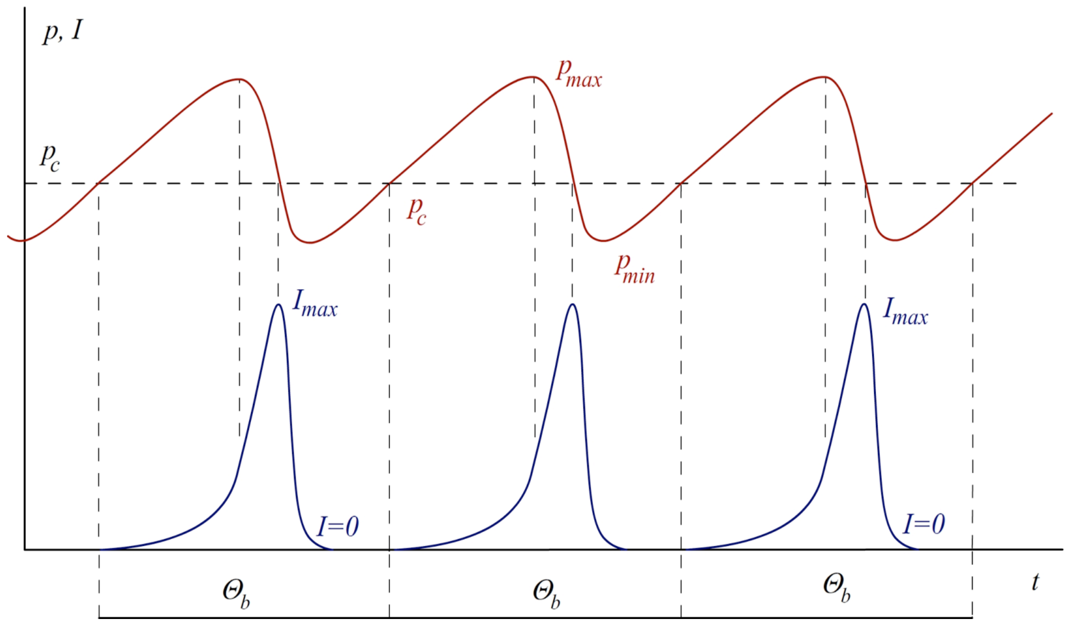
$$dp/dt = Z_+ - I\theta(p - p_{\min})$$

$$dI/dt = WI\left|p - p_c\right|^{z\nu}\,\mathrm{sign}(p - p_c)$$

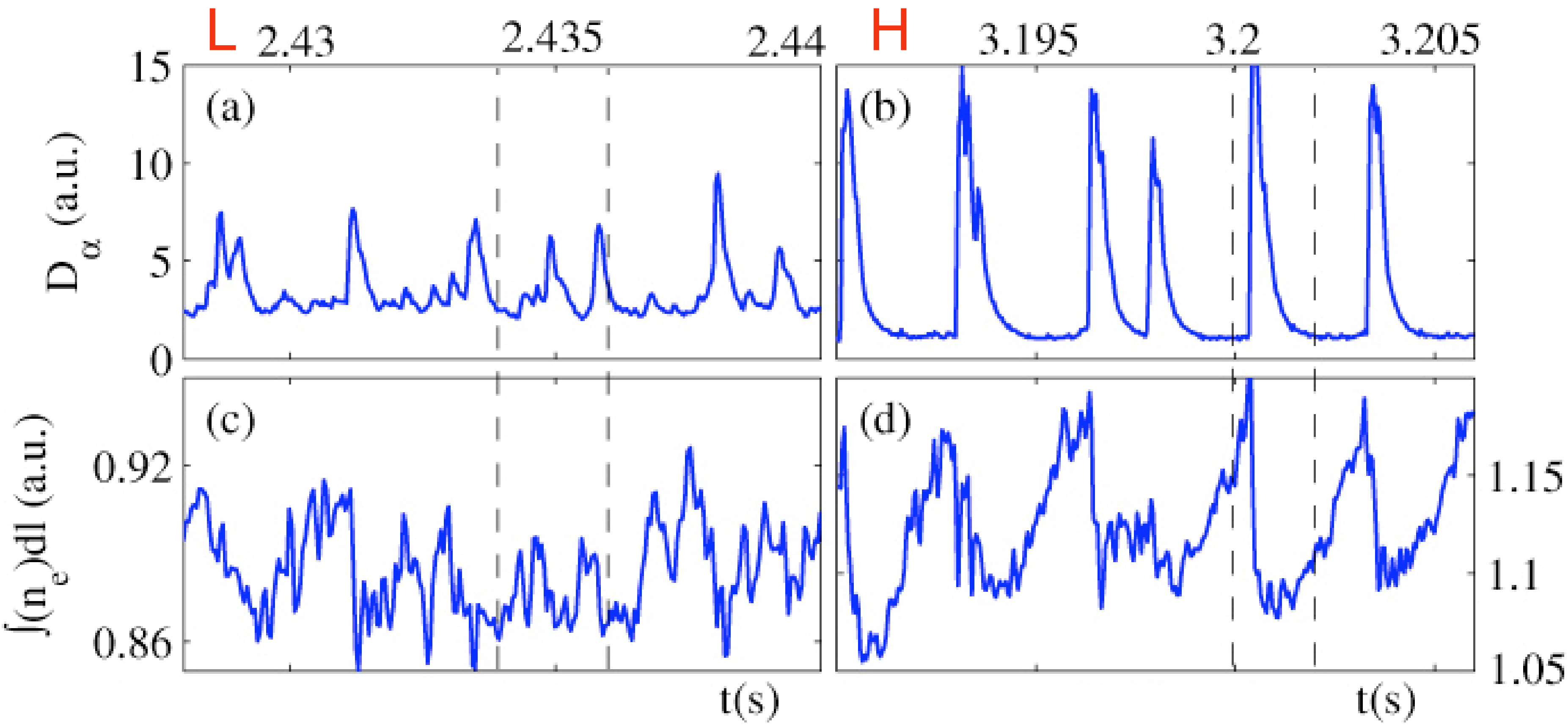
$$I_{\max} \approx W(p_{\max} - p_c)^{z\nu+1}$$

$$\Delta \approx (1/W)(p_{\max} - p_c)^{-z\nu}$$

$$\Theta_b \approx (p_c - p_{\min})/Z_+$$



Fishbone triggering ELMs in TEXTOR (limiter tokamak)

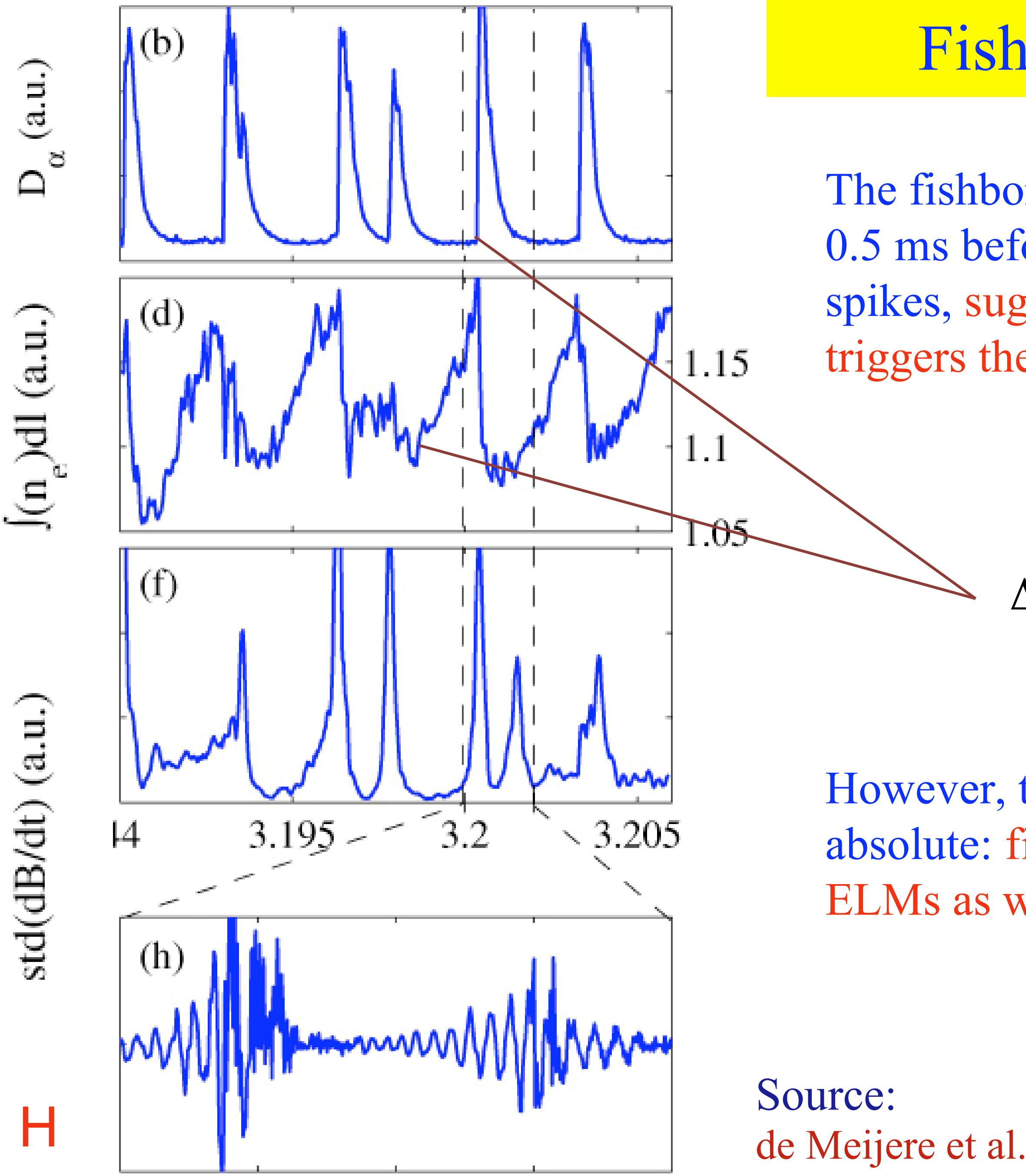


Top: Signal from filtered photodiode looking at limiter (ELMs)

Bottom: Line-integrated electron density along a vertical chord (fishbone)

1.6 MW of co-current NBI (56 keV D; tangential at plasma center)

Source: de Meijere et al., PPCF **54**, 105024 (2012)



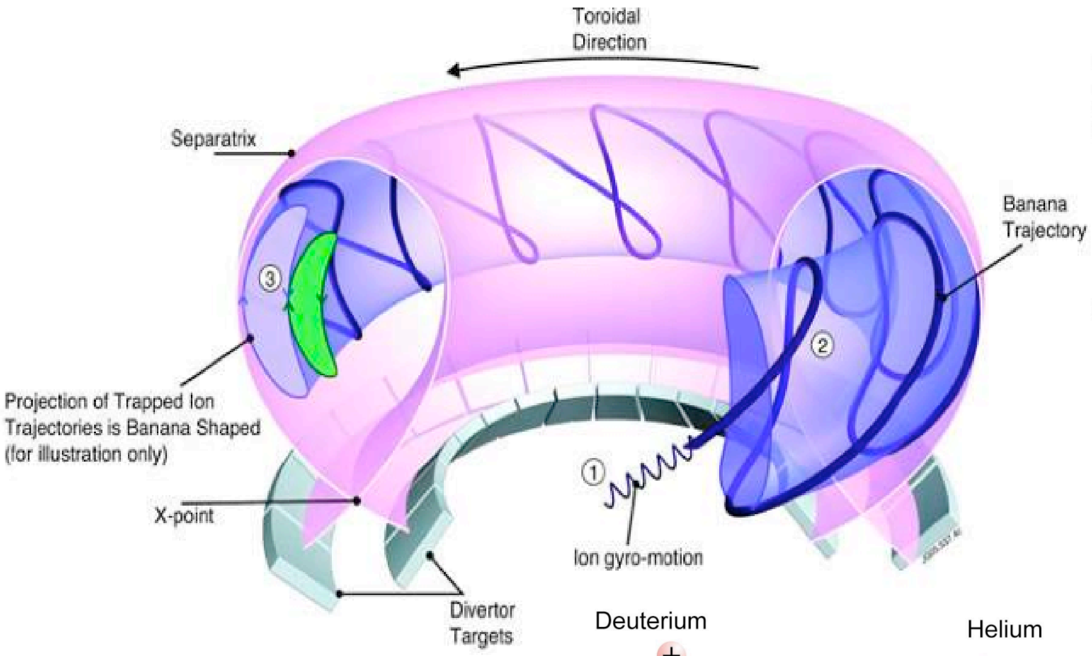
Fishbone vs ELMs

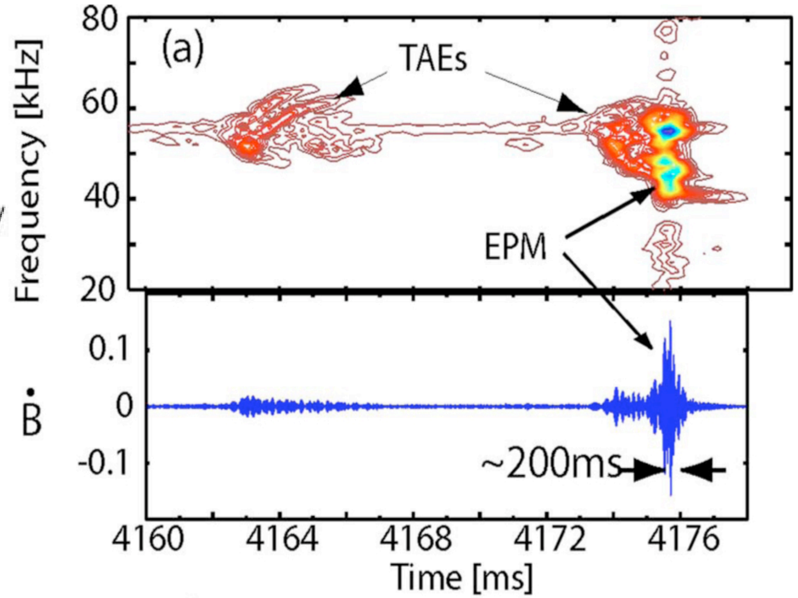
The fishbone typically starts to grow 0.5 ms before the photodiode signal spikes, suggesting that the fishbone triggers the ELM

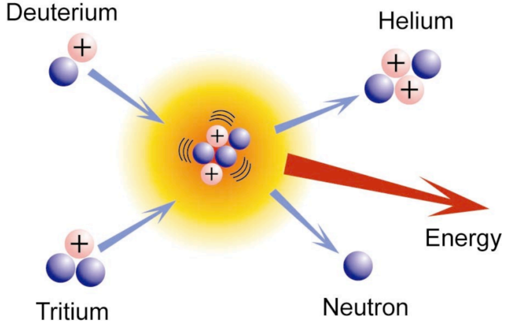
However, this correlation is not absolute: fishbones can occur between ELMs as well..

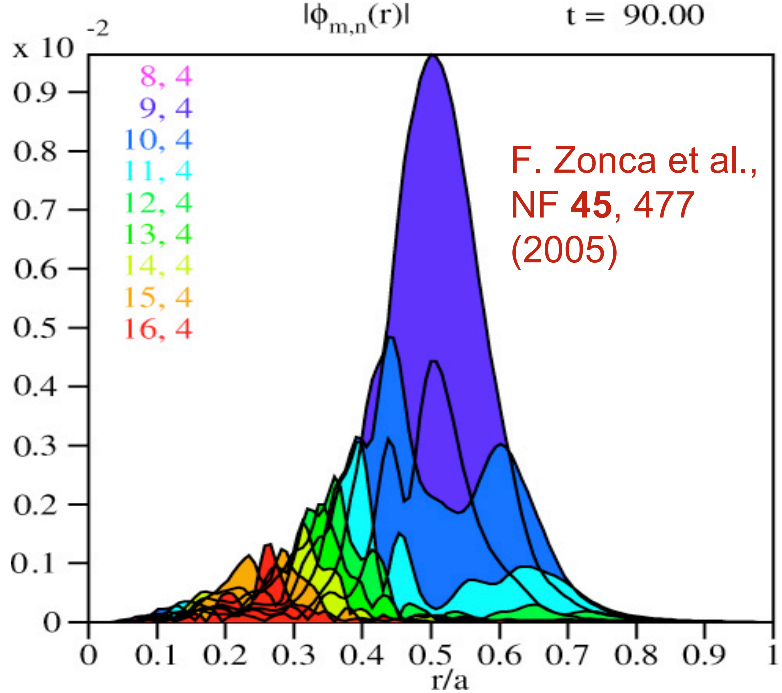
Source:
de Meijere et al., PPCF **54**, 105024 (2012)

Energetic-particle modes (EPM)

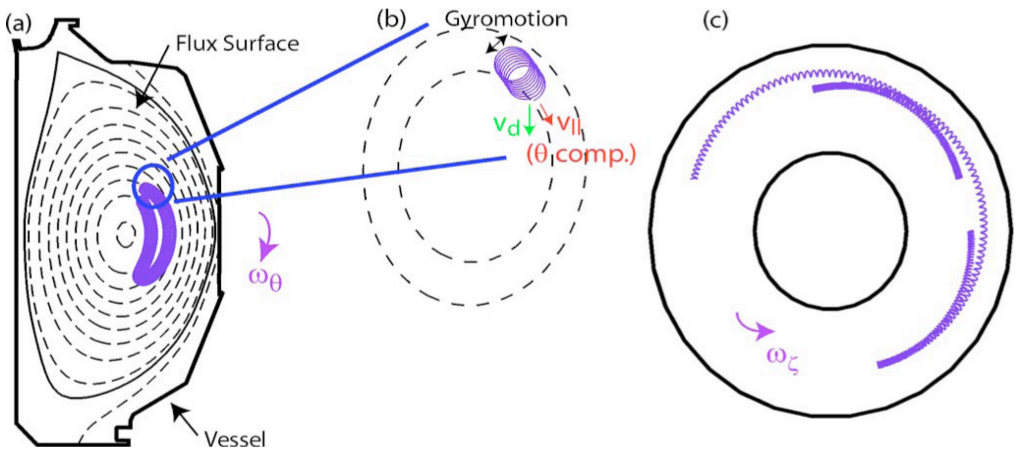




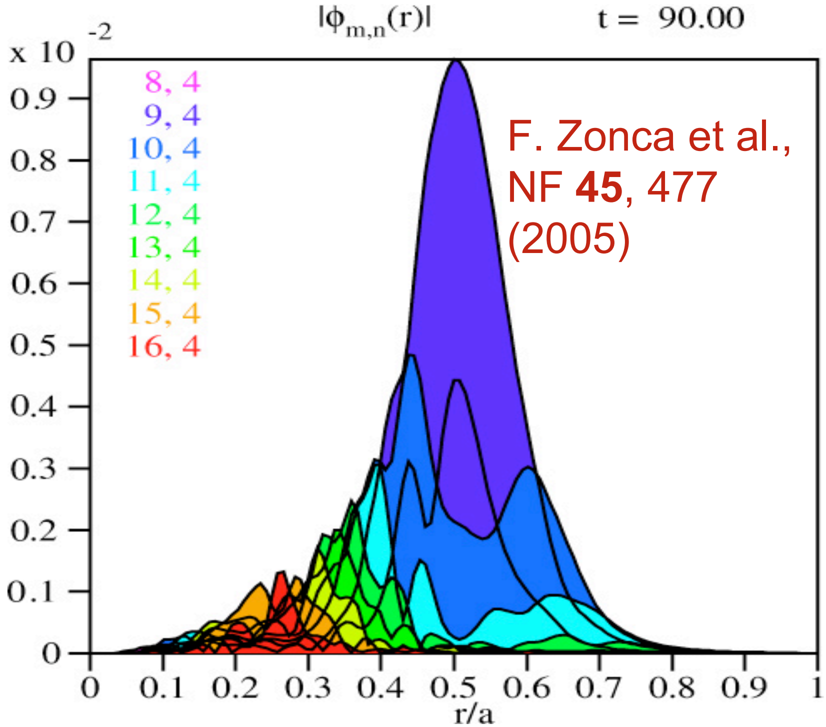




F. Zonca et al., NF **45**, 477 (2005)



EPM: A case for fractional parabolic (NLS) equation



F. Zonca et al., NF **45**, 477 (2005)

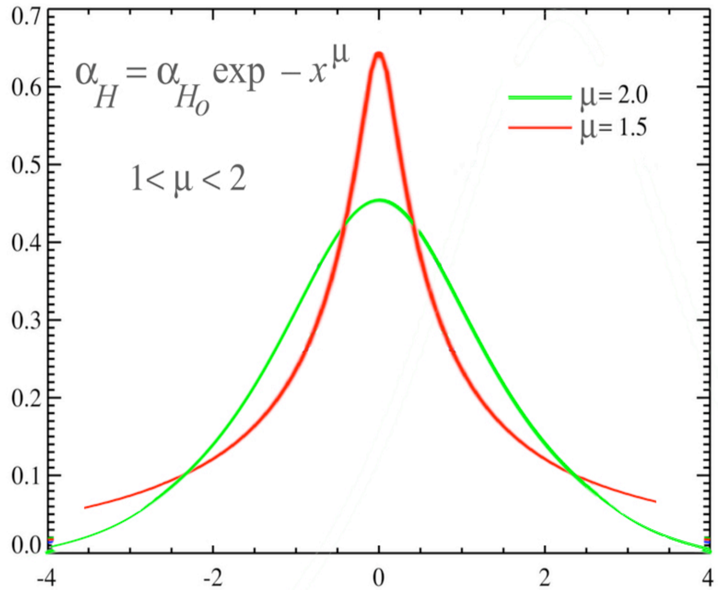
➢ NL physics of EPM wave-packets:

$$(D_R + iD_I)\partial_t A_0 = \alpha_H (\Lambda_R + i\Lambda_I)\partial_t A_0 + +i\eta A_0 \partial_t^{-1} \nabla_r^2 (\alpha_H |A_0|^2)$$

$$\alpha_H = -R_0 q^2 (d\beta_H / dr)$$

$$A_0 = A_0(r - v_{gr} t)$$

NL saturation regime:



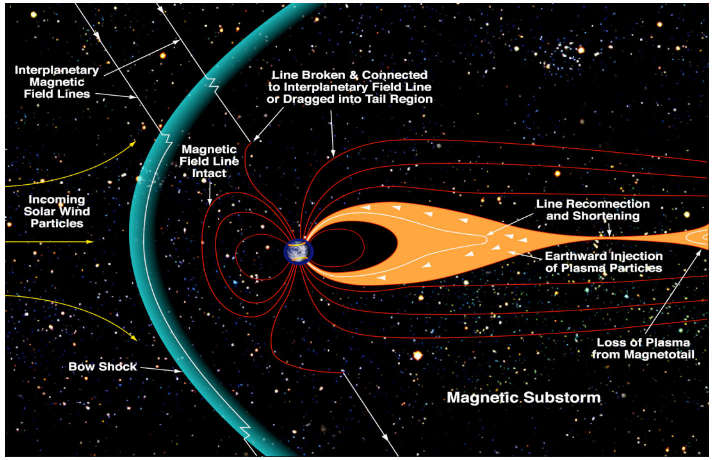
➢ NL Fractional envelope eq:

$$\nabla_\xi^2 A_0 = \kappa^2 A_0 - \sigma^2 A_0 \nabla_\xi^{2-\mu} |A_0|^2$$

$$\nabla_\xi^{2-\mu} \Psi = \frac{1}{\Gamma(\mu - 1)} \frac{\partial}{\partial \xi} \int_{-\infty}^{\xi} \frac{\Psi(\xi')}{(\xi - \xi')^{2-\mu}} d\xi'$$

Spontaneously occurring turbulence & FGL

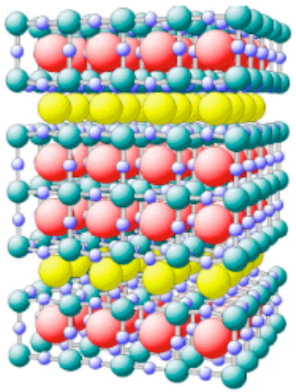
- The key word is "spontaneously": the transition to turbulence is thermodynamically favourable below a critical B_n value
- The turbulent state is stable and is characterized by a self-similar geometry at and below the transition point



The formalism of FGL equation:

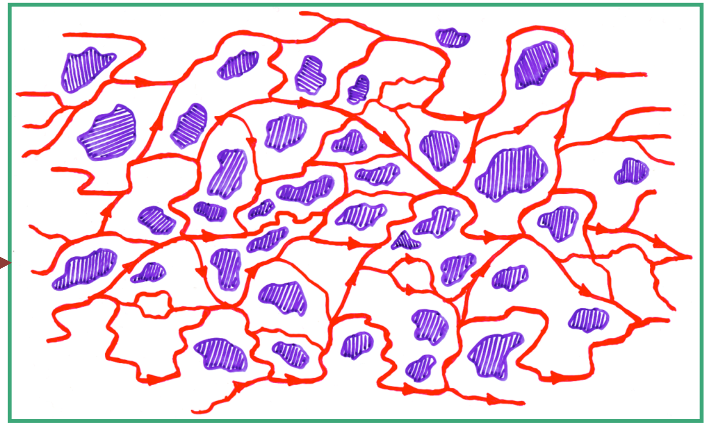
$$Q_\mu \nabla^\mu_{-x}\nabla^\mu_{x}\Psi + \alpha_\mu (T - T_c)\Psi + b_\mu |\Psi|^2 \Psi = 0$$

$$F = F_n + \int_{-\infty}^{+\infty} dx \left[A_\mu |\nabla^\mu_x \Psi|^2 + a_\mu |\Psi|^2 + \frac{1}{2} b_\mu |\Psi|^4 \right]$$



$$a_\mu = \alpha_\mu (T - T_c)$$

$$T = \frac{B_n^2}{8\pi}, \quad T_{crit} = \frac{B_{n,crit}^2}{8\pi}$$



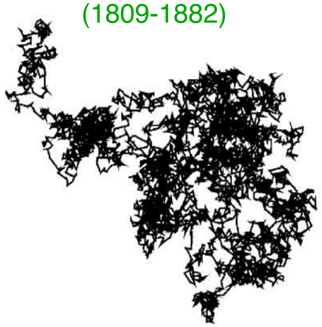
Milovanov and Rasmussen, Phys. Lett. A **337**, 75 (2005)

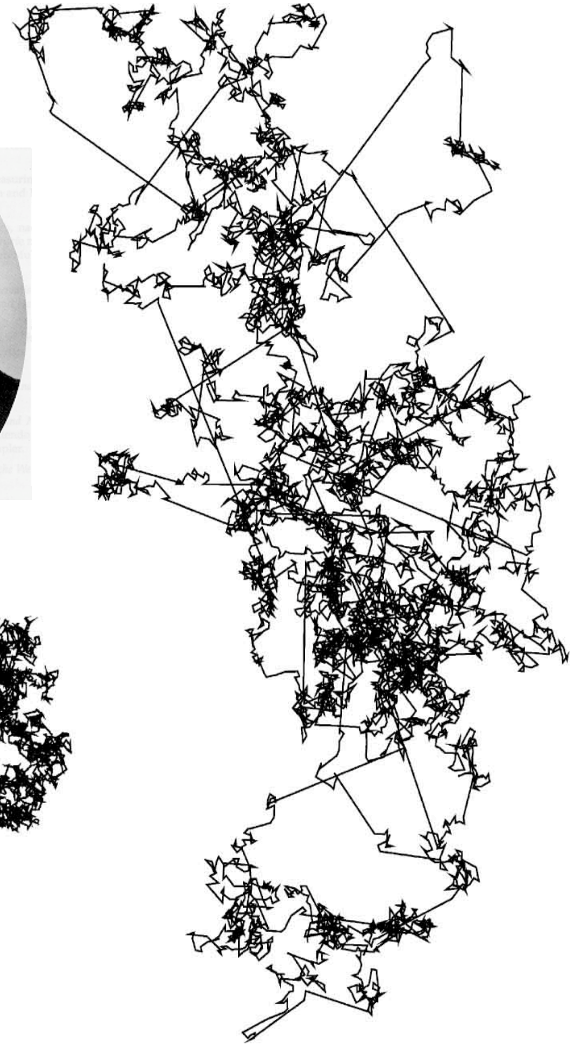
$$\mu = 1 + D - d$$

Bi-fractional diffusion equation

$$\boxed{\frac{\partial^{\beta}}{\partial t^{\beta}} n(x,t) = \frac{\partial^{q}}{\partial |x|^{q}} \left[D_{q,\beta} n(x,t) \right]}$$

J. Liouville
(1809-1882)





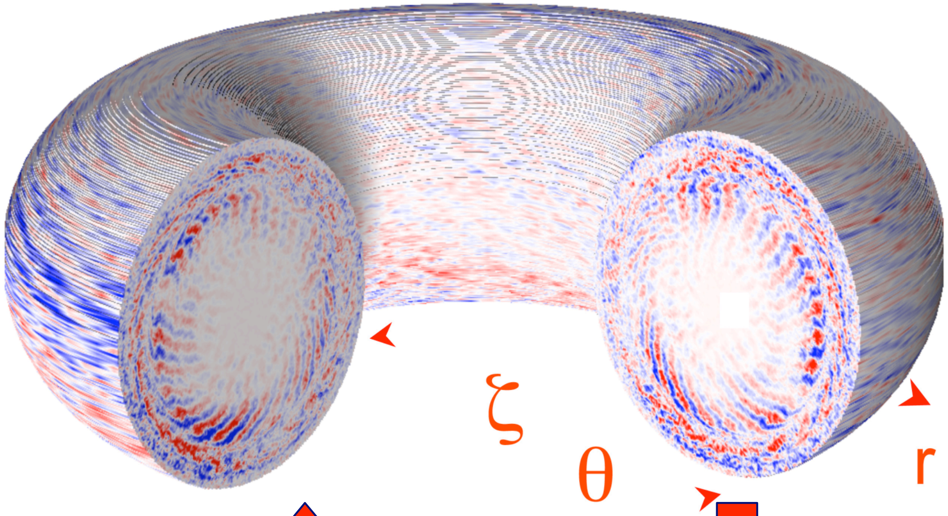
$$\frac{\partial^{\beta}}{\partial t^{\beta}} n(x,t) = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial t} \int_{0}^{t} dt' \frac{n(x,t')}{(t-t')^{\beta}}$$

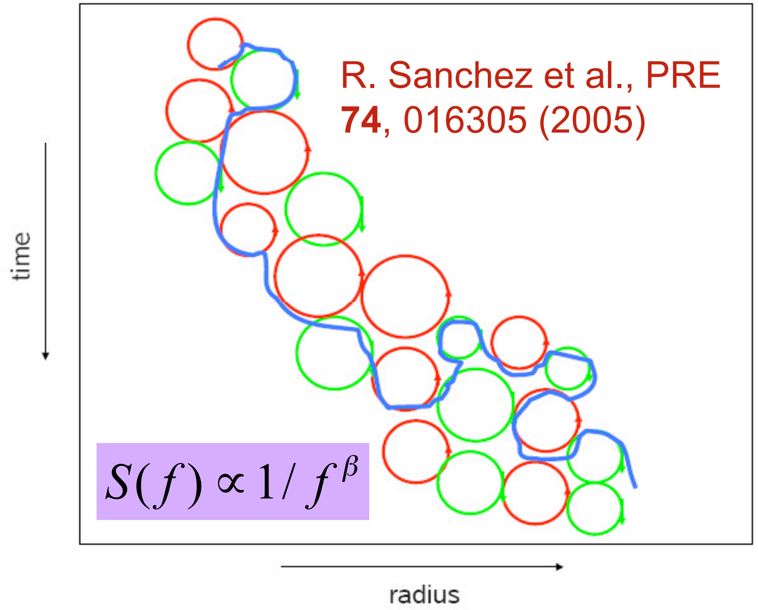
$$0 < \beta \leq 1$$

Frigyes Riesz
(1880-1956)

$$\frac{\partial^{q}}{\partial |x|^{q}} n(x,t) = \frac{\cos^{-1}(\pi q/2)}{2\Gamma(p-q)} \times$$

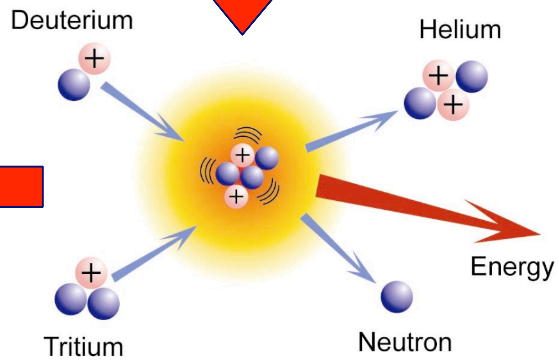
$$\times \left[\frac{\partial^{p}}{\partial x^{p}} \int_{-\infty}^{x} dx' \frac{n(x',t)}{(x-x')^{q-p+1}} - \frac{\partial^{p}}{\partial(-x)^{p}} \int_{x}^{+\infty} dx' \frac{n(x',t)}{(x'-x)^{q-p+1}} \right]$$

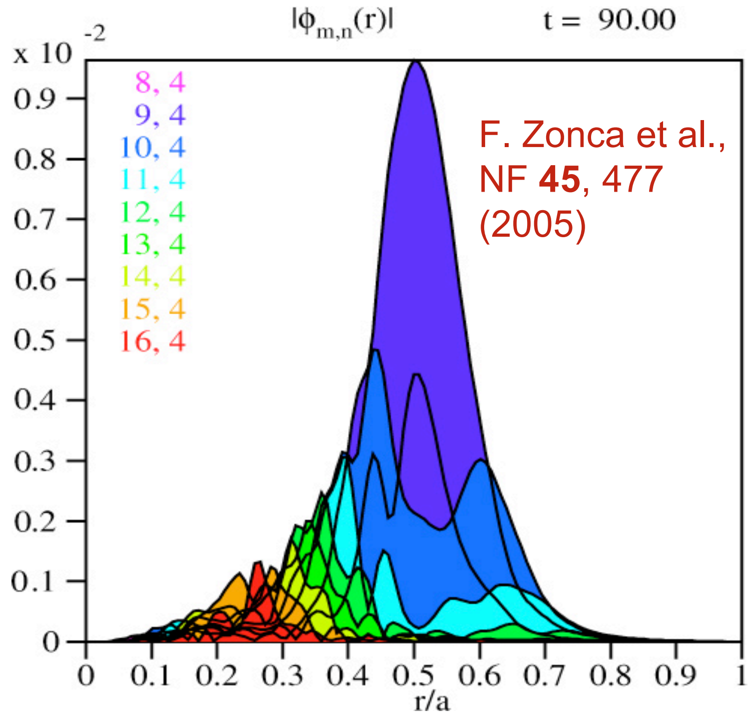




R. Sanchez et al., PRE **74**, 016305 (2005)

Fusion reactivity coupled via collective processes with the **background turbulence**



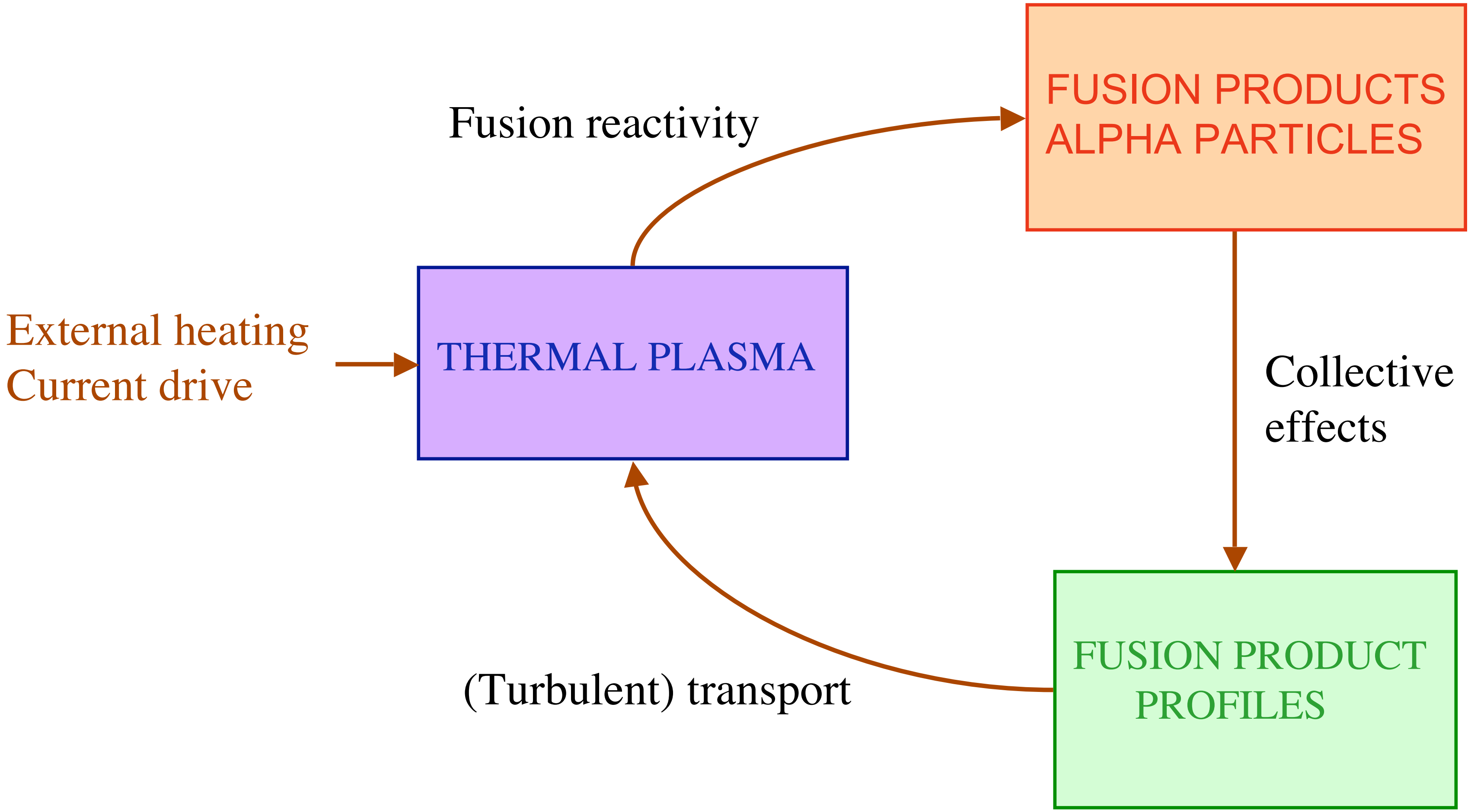


F. Zonca et al., NF **45**, 477 (2005)

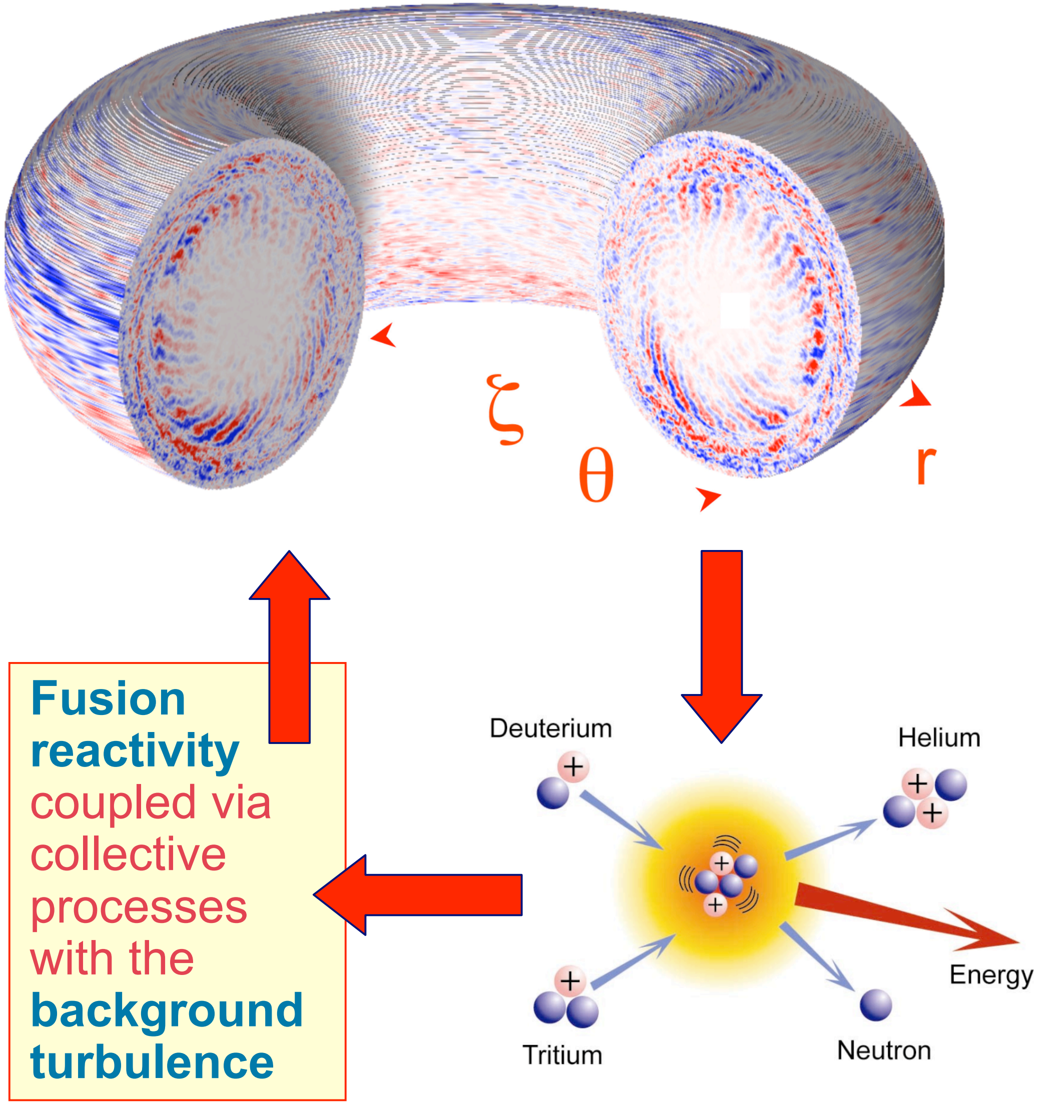
Co-existence of turbulence and the energetic particle physics

Burning plasma as complex system

A burning plasma is a complex self-organized system where among the crucial processes to understand are (turbulent) transport and fast ion/fusion product induced collective effects



Burning plasma as complex system (ctd)



Mutual interactions between

collective modes and **energetic ion dynamics**

with

drift wave turbulence and **turbulent transport**

can generate

long time-scale nonlinear behaviors

typical of self-organized complex systems

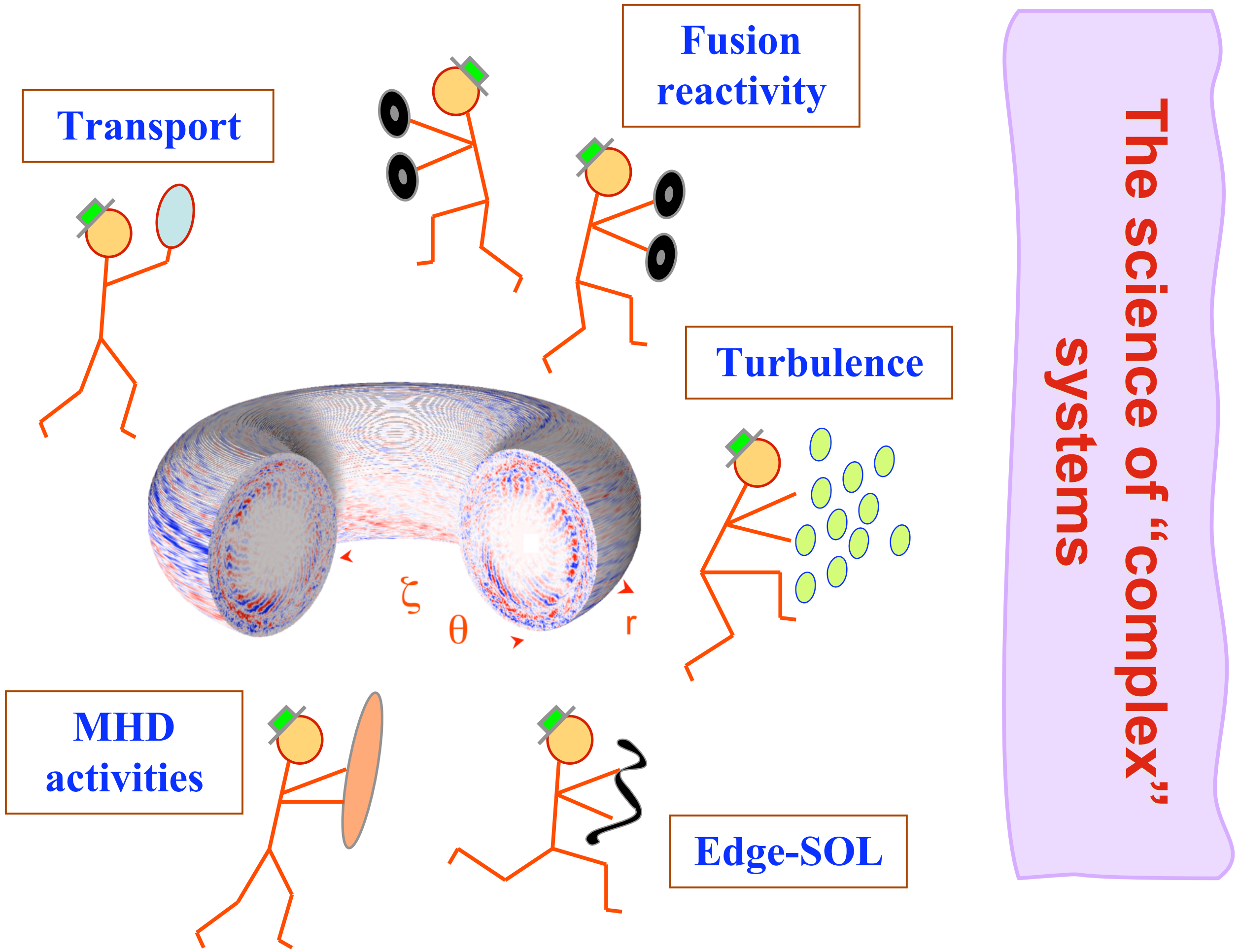
The juncture

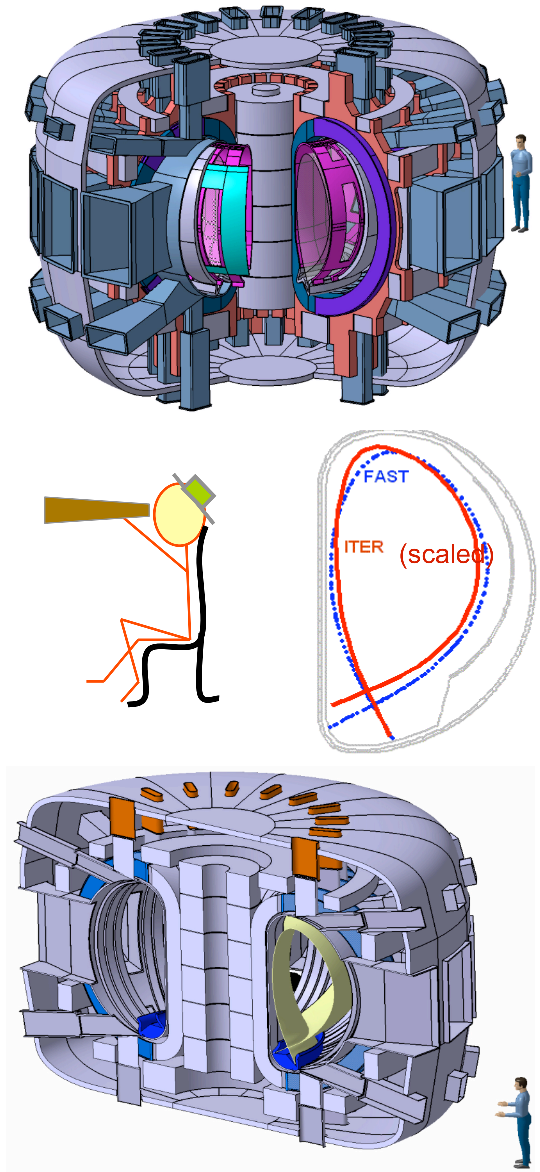
- Research activities in fusion plasma are now arriving at a crucial juncture that necessitates the understanding of "complexity" in the accessible and relevant operation regimes of burning plasma

- It is becoming clear that important questions that will be receiving attention in the coming years, particularly with the development of ITER and DEMO scenarios, are addressed toward the comprehension of burning plasma state as being self-organized, thresholded, nonlinear dynamical system with many interacting degrees of freedom

- Questions of non-locality, cross-scale coupling, and long-range correlation in a plasma system in which the energetic particles (MeV energies) and charged fusion products constitute a significant fraction of the total energy density.

Burning plasma as complex system





FAST: an Italian proposal for an EU satellite, aimed at supporting ITER operation regimes, relevant to DEMO physics and technology issues, complementing JT60-SA

A satellite: Rationale

To properly contribute to the development of fusion energy, a satellite should primarily be capable of:

- working in a dimensionless parameter range as close as possible to that of ITER
- exploiting, for the largest possible extent, the key ITER operational issues: ELMs, Plasma Wall interaction, Wall Load, Heating coupling
- exploring innovative solutions for the first wall/divertor relevant for ITER and DEMO, such as full-tungsten plasma facing components and advanced liquid metal divertor target
- ✓ investigating non linear dynamics that are relevant for the understanding of alpha particle behaviours in burning plasmas, by using fast ions accelerated by heating and current drive systems
- providing a suitable framework for model and numerical code benchmarks, as well as verification and validation, in ITER and DEMO relevant plasma conditions
- being capable of integrated and substantially higher capabilities than those available in existing experiments

FAST: Flexibility

FAST	H-mode reference	H-mode extreme	Hybrid	AT	AT2	Full NICD
I_p (MA)	**6.5**	8.0	5	3	**3**	2
q_{95}	3	2.6	4	5	3	5
B_T (T)	**7.5**	8.5	7.5	6	**3.5**	3.5
H_{98}	1	1	1.3	1.5	1.5	1.5
$<n_{20}>$ (m^{-3})	2	5	3	1.2	1.1	1
P_{th_H} (MW)	14 ÷ 18	22 ÷ 35	18 ÷ 23	8.5 ÷ 12	5 ÷ 7	5 ÷ 7
β_N	1.3	1.7	2.0	1.9	3.2	3.4
τ_E (s)	0.4	0.65	0.5	0.25	0.18	0.13
τ_{res} (s)	5.5	5	3	3	5 ÷ 6	2 ÷ 5
T_0 (keV)	13.0	9.0	8.5	13	13	7.5
Q	0.65	2.5	0.9	0.19	0.14	0.06
$t_{discharge}$ (s)	20	13	20	70	170	170
$t_{flat\text{-}top}$ (s)	13	2	15	60	160	160
I_{NI}/I_p (%)	15	15	30	60	80	>100
P_{ADD}(MW)	30	40	30	30	40	40

Ip=8.5MA and BT=8T scenario possible with additional 10MW NNBI

Port compatible with 10 MW (45° inclined) NNBI

The heating power coupled to the plasma, producing a P/R=22 MW/m, is:

- ICRH 30 MW at 60-80 MHz - fast particle generation
- LH 6 MW at 3.6 or 5 GHz - Current Drive and profile control
- ECRH 4 MW at 170 GHz - MHD control

FAST vs ITER (Base operation)

FAST

Plasma Current (MA)	6.5
B_T (T)	7.5
Major Radius (m)	1.82
Minor Radius (m)	0.64
Elongation k_{95}	1.7
Triangularity δ_{95}	0.4
Safety Factor q_{95}	3
$\langle n \rangle (m^{-3})$	2×10^{20}
Flat-top (s)	13
ICRH power (MW)	30
ECRH power (MW)	4
LH power (MW)	6

Ip = 8.5 MA and B_T = 8 T scenario possible with additional 10MW NNBI

ITER

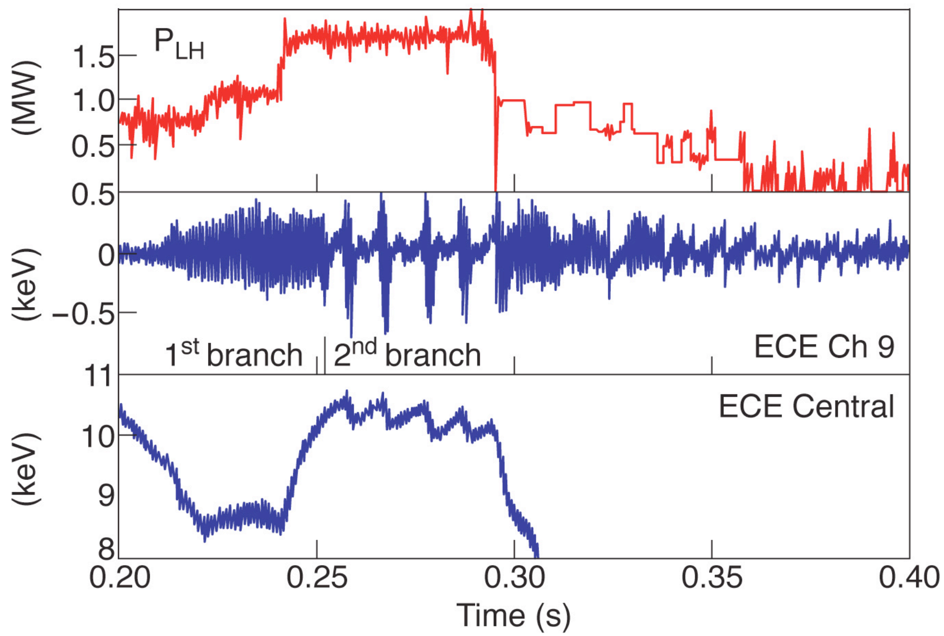
Plasma Current (MA)	15
B_T (T)	5.3
Major Radius (m)	6.2
Minor Radius (m)	2.0
Elongation k_{95}	1.7
Triangularity δ_{95}	0.4
Safety Factor q_{95}	3
$\langle n \rangle (m^{-3})$	0.9×10^{20}
Flat-top (s)	400
ICRH power (MW)	20
ECRH power (MW)	20
LH power (MW)*	40
NNBI power (MW)	33

* not for base operation
20 MW / 5 GHz LHCD power system under consideration for delivery

FAST: Accommodating new topics: electron fishbones

Alpha particles in reactor relevant conditions are characterized by small dimensionless orbits, similarly to the energetic electrons in present day plasma experiments

Fusion alphas in burning plasma predominantly transfer their energy to the electrons while slowing down and it is important to assess the peculiar effect of the energetic electron population on MHD stability



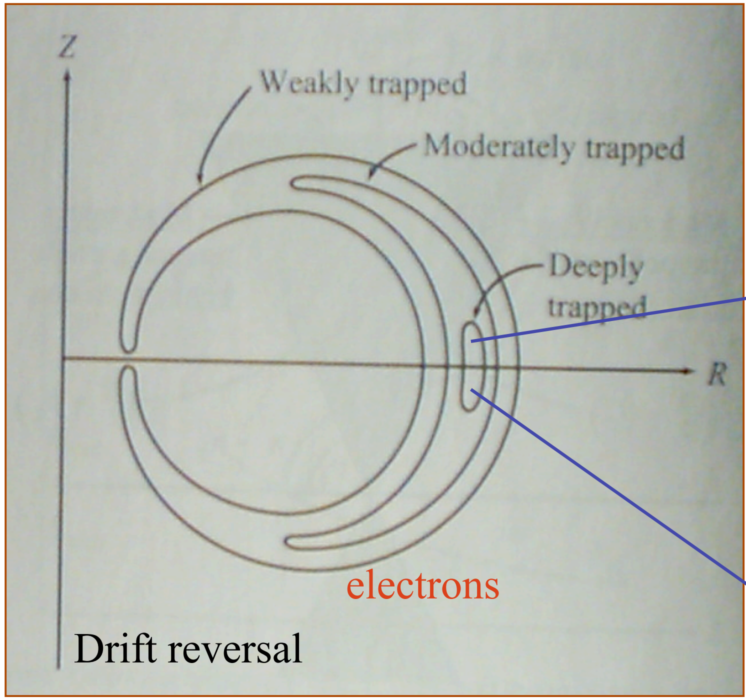
LH absorbed power, fast electron temperature fluctuations, and central radiation temperature in FTU shot #20865

FTU experimental facility:

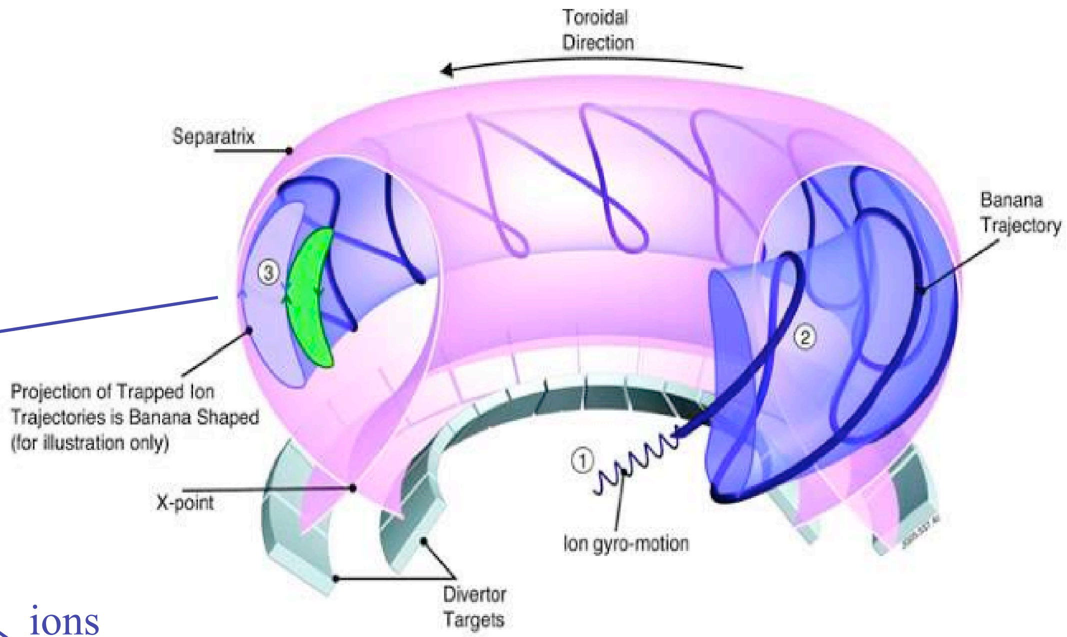
- Implementation of optimization techniques to maximize the RF power coupling versus the plasma conditions (Centioli et al., Fusion Engineering & Design **74**, 543, 2005)
- Absorbed LH power: up to 1.7 MW vs 1.1 MW ECRH power on DIII-D (Wong et al., PRL **85**, 996, 2000)

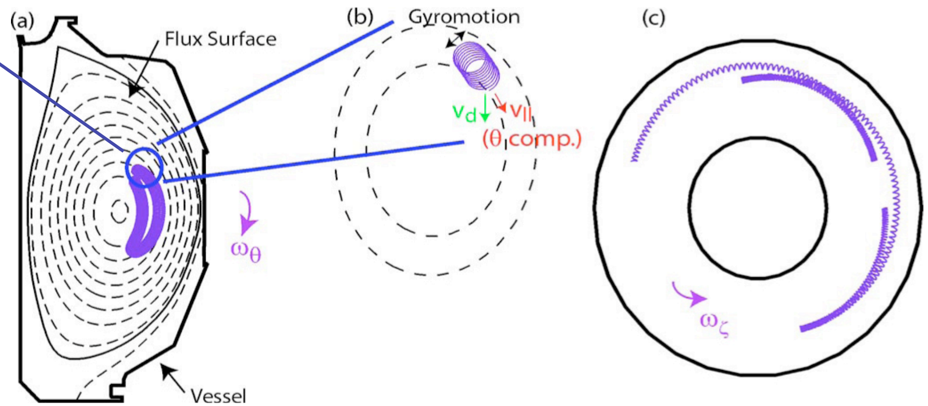
FTU: First evidence of NL bursting behavior excited by LH only

The physics: Drift reversal effect



ions

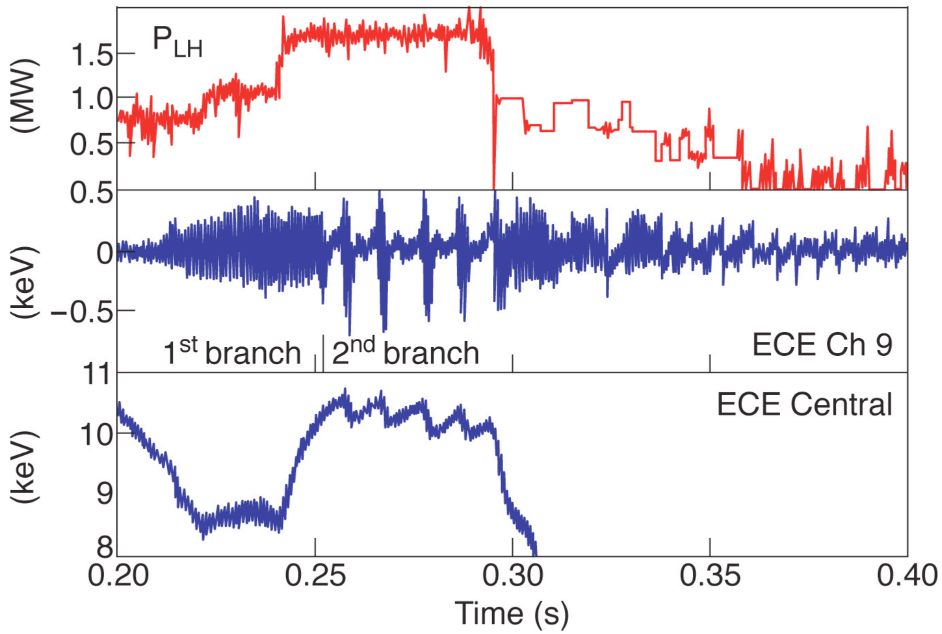




For barely trapped electrons, the precessional direction averaged over the entire orbit is parallel to that of the deeply trapped ions because these electrons spend more time on the high field side so that this part of the orbit is weighted more heavily.

Because of this drift reversal effect, the barely trapped suprathermal electrons in the same energy range as the fast ions from NBI can resonate with fishbone modes.

LH power injection: A route to complexity



LH absorbed power, fast electron temperature fluctuations, and central radiation temperature in FTU shot #20865

F. Zonca et al., Nuclear Fusion **47**, 1588 (2007)

Conjectures:

- Bursting phase: a continuum resonant mode
- Particle NL's dominate the fishbone cycle

Qualitative model:

$x \Leftrightarrow A_0$

$y \Leftrightarrow \beta_{h,res}$

$$\dot{x} = -\mu x + kxy$$

$$\dot{y} = \nu y - (\eta_1 - \eta_2) y^2 - pxy$$

Multiplication

Generic NL

Reconstruction: (Ext., Coulomb)

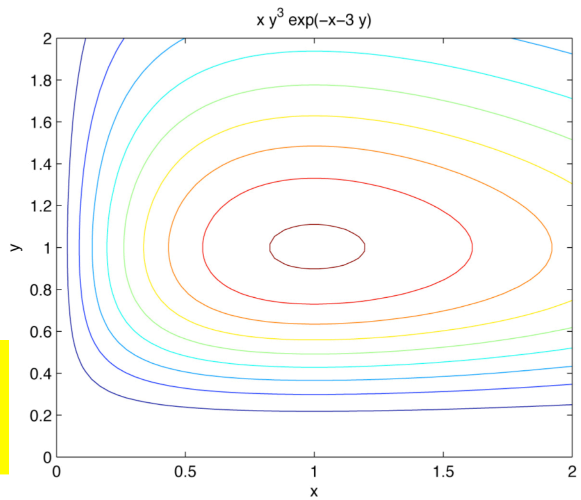
P-LH → Barely circulating → Mode drive

P-LH → Well circulating → Modify the current profile → Broaden the fraction of barely trapped particles characterized by drift reversal → Mode drive

The peculiarity of LH!

$\eta_1 < \eta_2$

Transition to stable limit cycle behavior (via center - a marginal periodic regime):

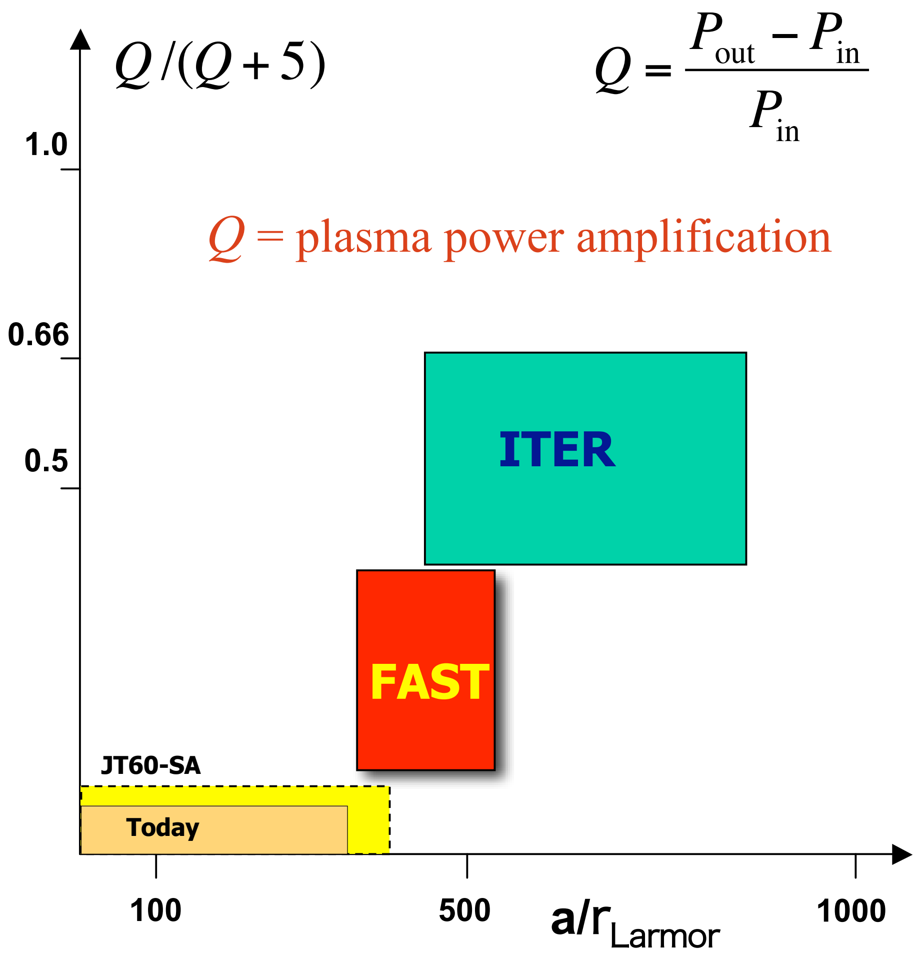


FAST: filling the gaps

FAST addresses ITER & DEMO operating condition **routinely**

Present-day machines can address **only separately** these conditions

FAST addresses the unexplored physics on the way to ITER: **Complexity**



In the line with **state-of-the-art** in plasma physics/NL physics

In summary, **FAST…**

- FAST is an intrinsically "multi-dimensional" → ideal for "non-linear" physics with many time scales and threshold phenomena

- Is flexible and integrates many physics issues, so far only addressed separately

✓ Can provide a unique opportunity to explore unexpected physics issues thus helping in avoiding long and costly scenario development in ITER

- Is an ideal test bed to exploit advanced divertor materials and configurations (W. Liquid Lithium) suitable for DEMO

- Can be a key facility for international collaboration, training of young scientist in a ITER-scale device and development of new diagnostics

- FAST and JT60-SA, being complementary satellites, can serve as strong and thorough basis and support for ITER

FAST: Synergetic effort required

The involvement of the largest possible number of Associations is mandatory to realize **FAST:**

The expertise of the EU Labs. is crucial for its design and fruitful operations (e.g.: H&CD, Diagnostics)

FAST is being designed and it has been proposed to be constructed and operated in the framework of a collaboration with other **Associations.**

Approfondendo gli aspetti scientifici - e´cruciale che la comunità di ricerca italiana, con la sua tradizione e le sue potenzialità, si faccia comunicatrice di una forte proposta per **esperimento satellite europeo di ITER**, a partire dalla **proposta FAST**, da costruirsi in **Italia**, con un programma di ricerca nazionale pienamente inserito nel contesto europeo

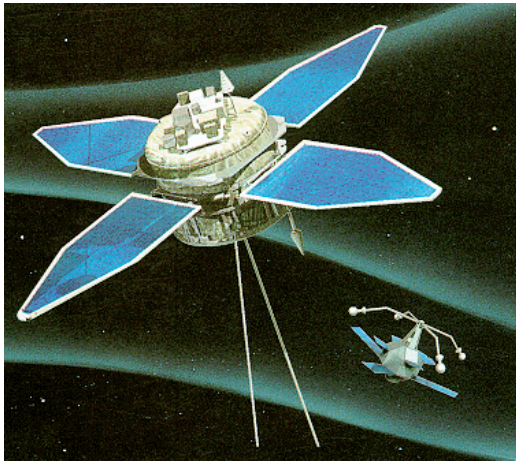
L'Istituto per le Ricerche Spaziali dell'Accademia delle Scienze Russa

Cosmic plasma and space exploration to the benefit of the fundamental science:

R. Z. Sagdeev
Dir 1973-1988

A. A. Galeev
Dir 1988-2002

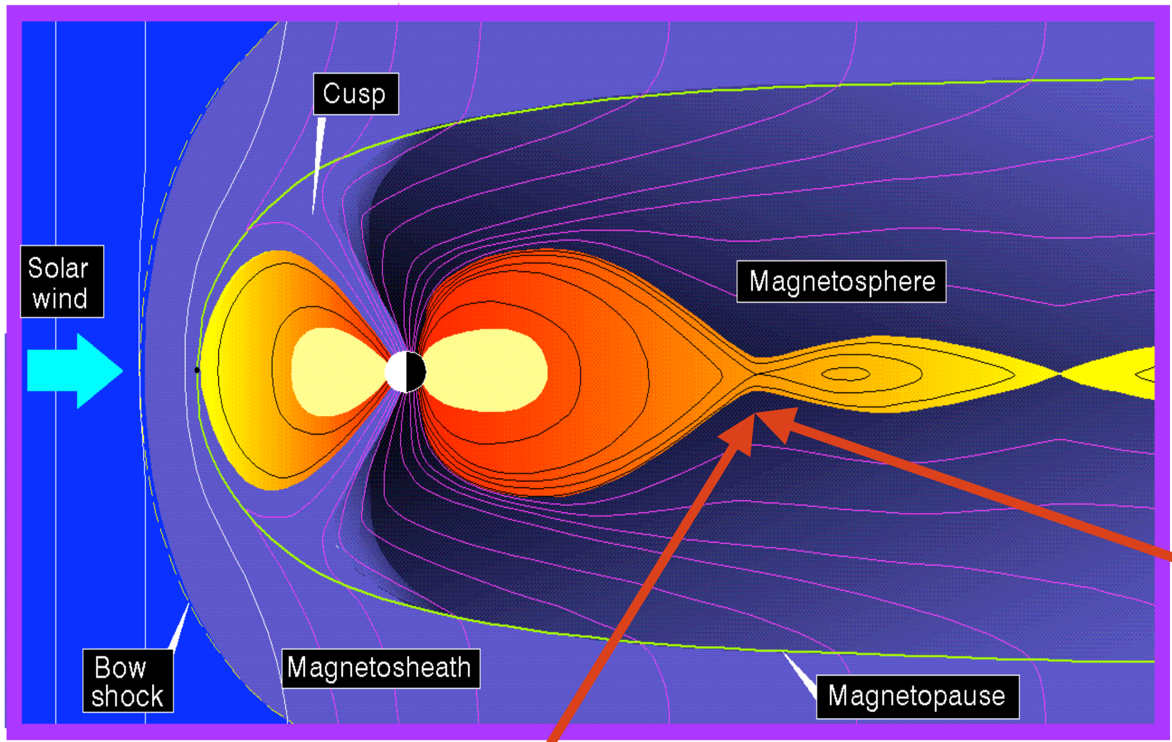
Neo-classical transport theory (1968)

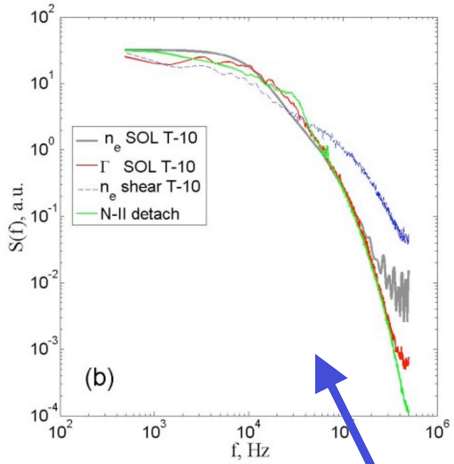


Cross-fertilization between fusion and space plasma research

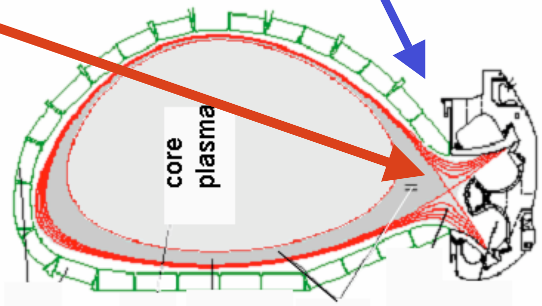
R. Z. Sagdeev
Dir 1973-1988

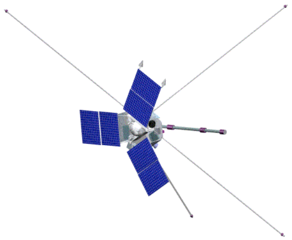
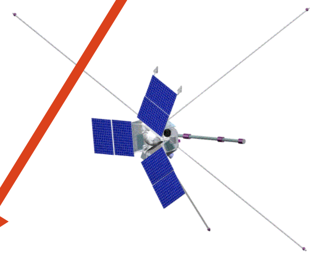
L. M. Zelenyi
Dir 2002-



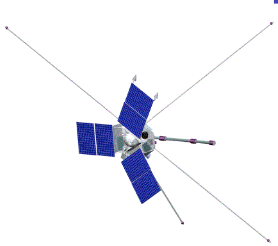
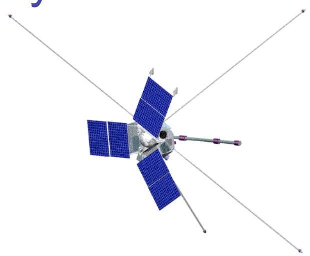


multi-satellite



ROY space system

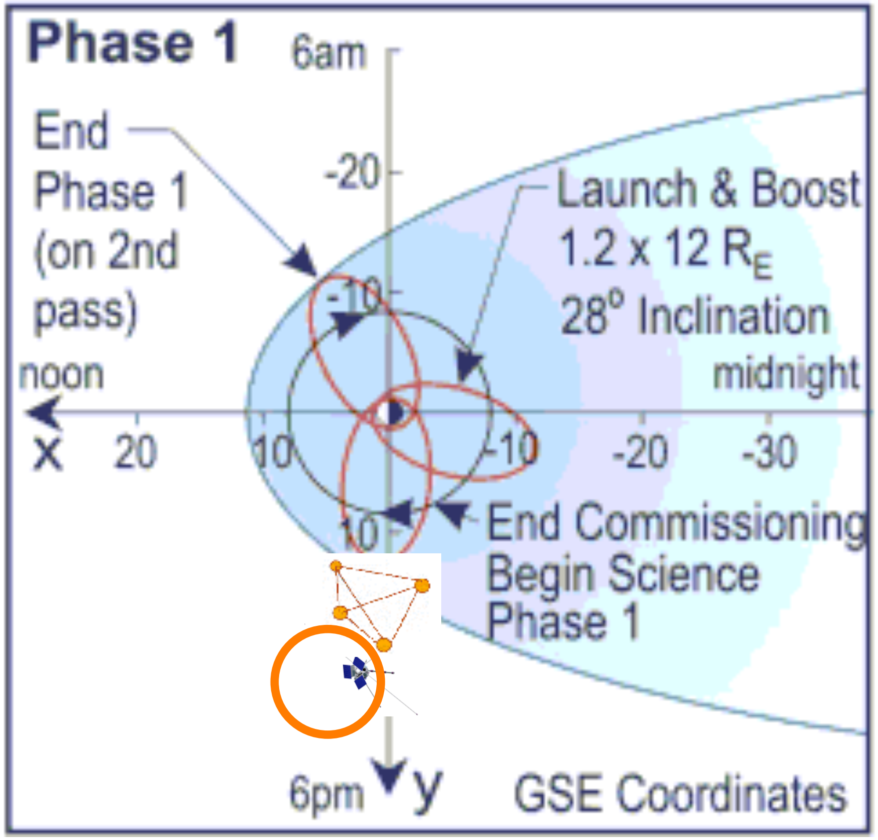
The IKI-led ROY ("Swarm") project in space research:

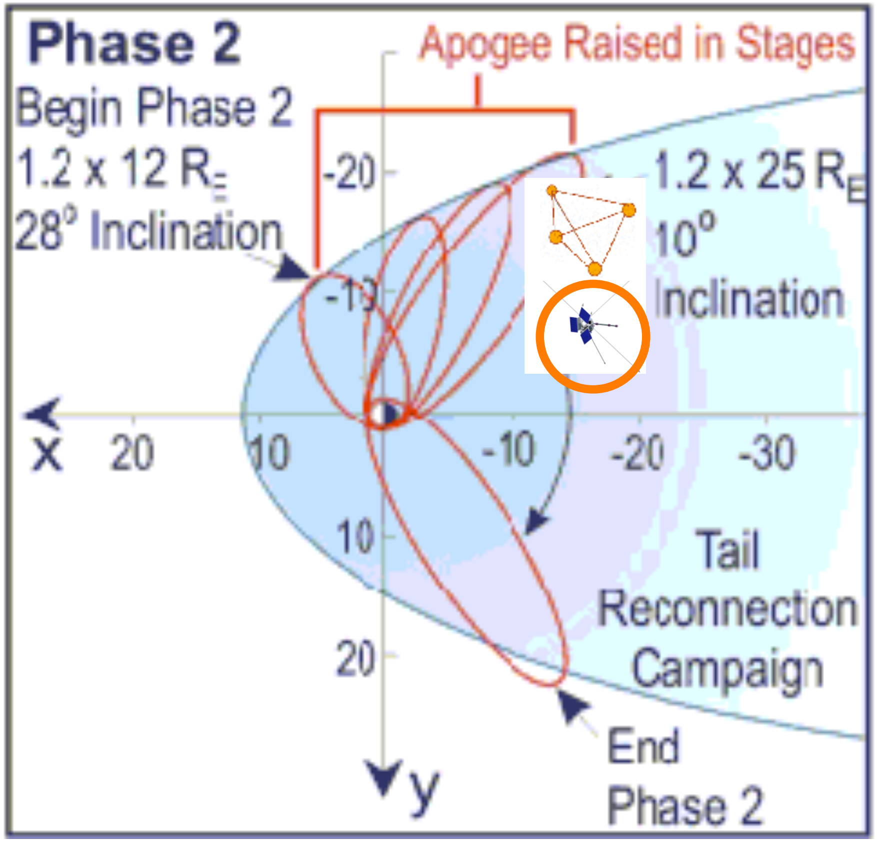
➡ A geo-space approach to divertor physics in tokamaks?

MAGNETOSHERE MULTI SCALE (MMS)

(NASA, launch October 14, 2014)

and STRANNIK (Странник)





Summary (though not a final summary yet..)

- **A family of unexplored physics issues** - crucial to comprehend controlled fusion burn - involving issues of complexity, self-organization, and cross-scale interaction in plasmas with a significant population of super-thermal energetic particles, could be addressed as part of the scientific rationale of the proposed European satellite for ITER, the Fusion Advanced Studies Torus (**FAST**) project.

- **Synergy is crucial -** not only does it involve collaborative effort from the various Associations, a contribution from university based science, and apparently diverse fields of research, will be most welcomed

- **FAST,** as an experiment to study nonlinear science in burning plasma as complex system, addresses the fundamental physics issues currently under consideration in geo-space research for the next-generation multi-satellite experiments such as **ROY**

The final: Lessons from "complex" systems:

To be ready to expect a physically rich behavior, with a bit of self-organization, long-range dependence, and cross-scale interaction, and to watch out for unexplored patterns of behavior, with multi-scale properties twisted with non-linear and coherent features

- "**Complex**" not a synonymous with "**complicated**" - we have found that the observed patterns of behavior could be cast in the framework of generalized SOC models, and kinetic equations with fractional time and space derivatives. Those equations are mathematically **well defined**, dating back to the foundations of the calculus itself

Recent publications:

A. V. Milovanov, Phys. Rev. E **79**, 046403 (2009);
A. V. Milovanov, Europhys. Lett. **89**, 60004 (2010);
A. V. Milovanov, New J. Phys. **13**, 043034 (2011);
A. V. Milovanov and A. Iomin, Europhys. Lett. **100**, 10006 (2012).

”Is it SOC or Not”?

- “What then is SOC good for? Let us consider some important questions. (1) Can we identify SOC as a well-defined distinct phenomenon different from any other category of behavior? (2) Can we identify a certain construction that can be called “a theory” of self-organized critical systems? (3) Has SOC taught us anything about the world that we did not know prior to the BTW’s seminal paper? (4) Is there any predictive power in SOC - that is, can we state the necessary conditions a system must fulfil in order to exhibit SOC? And if we are able to establish that a system belongs to the category of SOC systems, does that actually help us to understand the behavior of the system? With some caution I think it is meaningful to answer these questions in the affirmative.”

Self-Organized Criticality: H.J.Jensen (1998)