Black Hole Variability: Self-Organized Criticality (?)

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Team Meeting @Bern 16/10/2012

“shishi-odoshi” (or deer frightening)

Outline

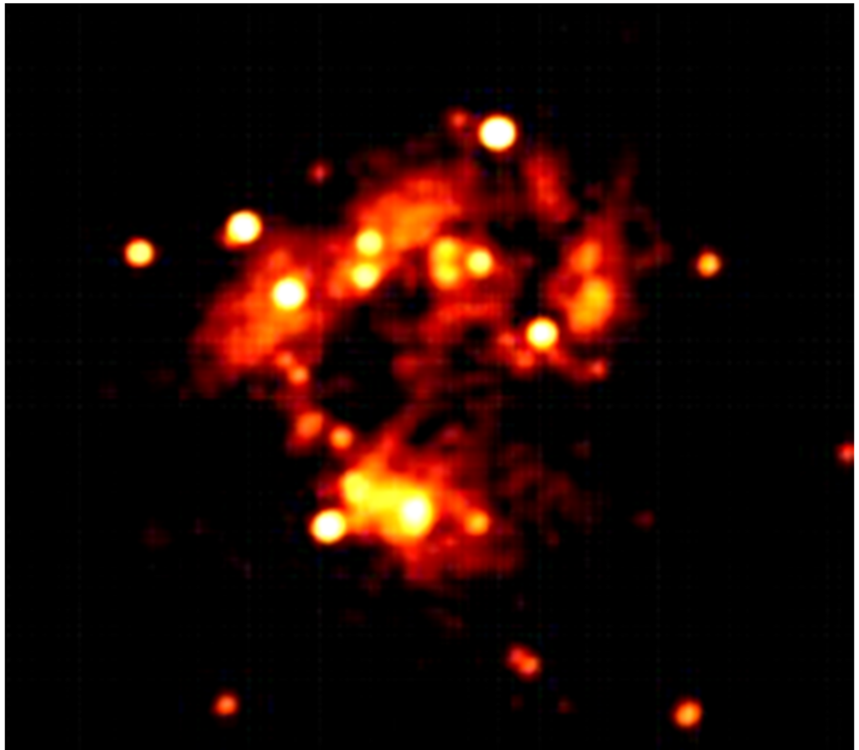
- Introduction
  - Where are black holes and how are they observed?
  - Fluctuations from black hole objects
  - PSDs and "shot" (flare) characteristics
- SOC model for black hole variability
  - Motivation and goal
  - CA rule and results
  - Issues: lognormal distribution, …
- Summary

How are black holes observed?

The vicinity of BHs = "extreme universe"

Very hot (over 1 million K) plasmas fill the space,

emitting X-rays



← X-ray image of a nearby galaxy

Luminous points are mostly black holes.

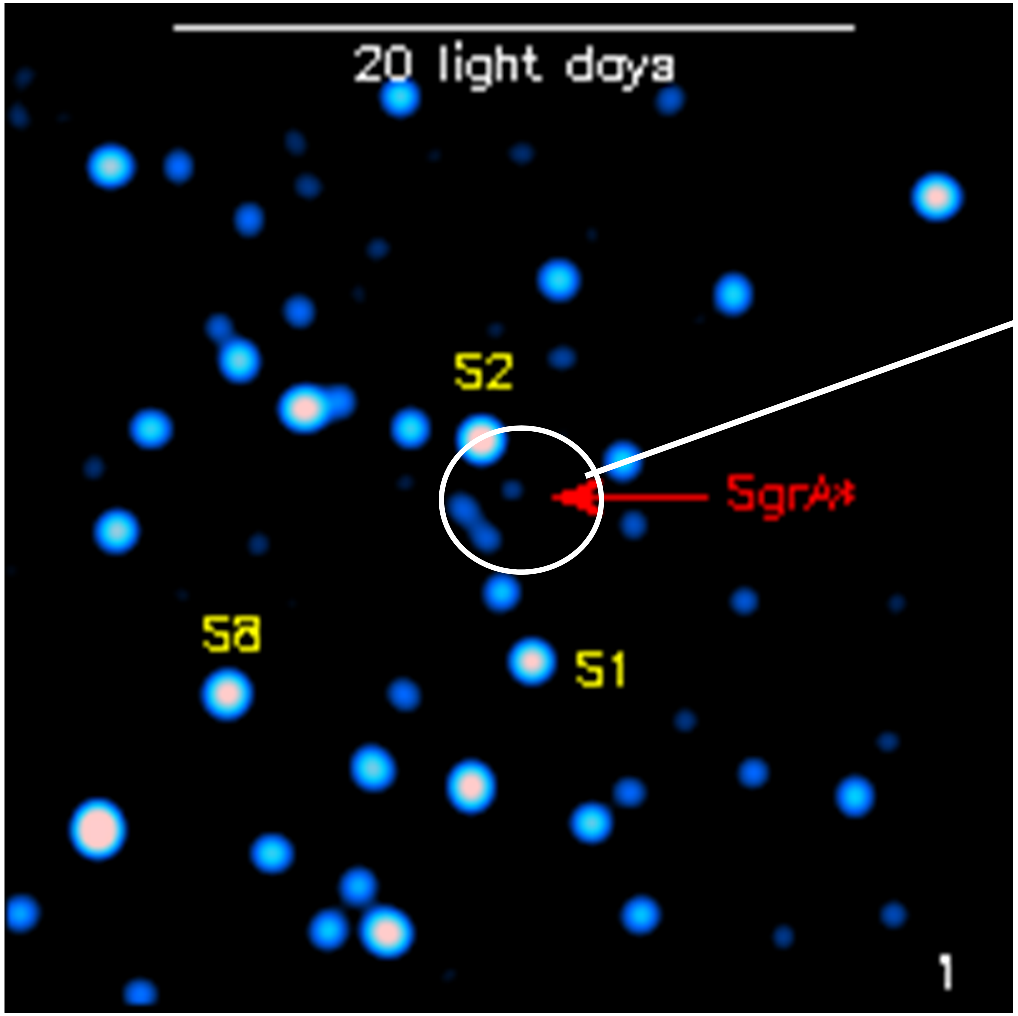
Typical luminosity $\sim 10^{5\text{-}6} \times L_{sun}$

(Fabbiano et a. 04)

(Note: The solar luminosity is $L_{sun} = 4 \times 10^{33}$ erg s^{-1})

Our Galactic Center

Supermassive black hole of 3 million solar masses



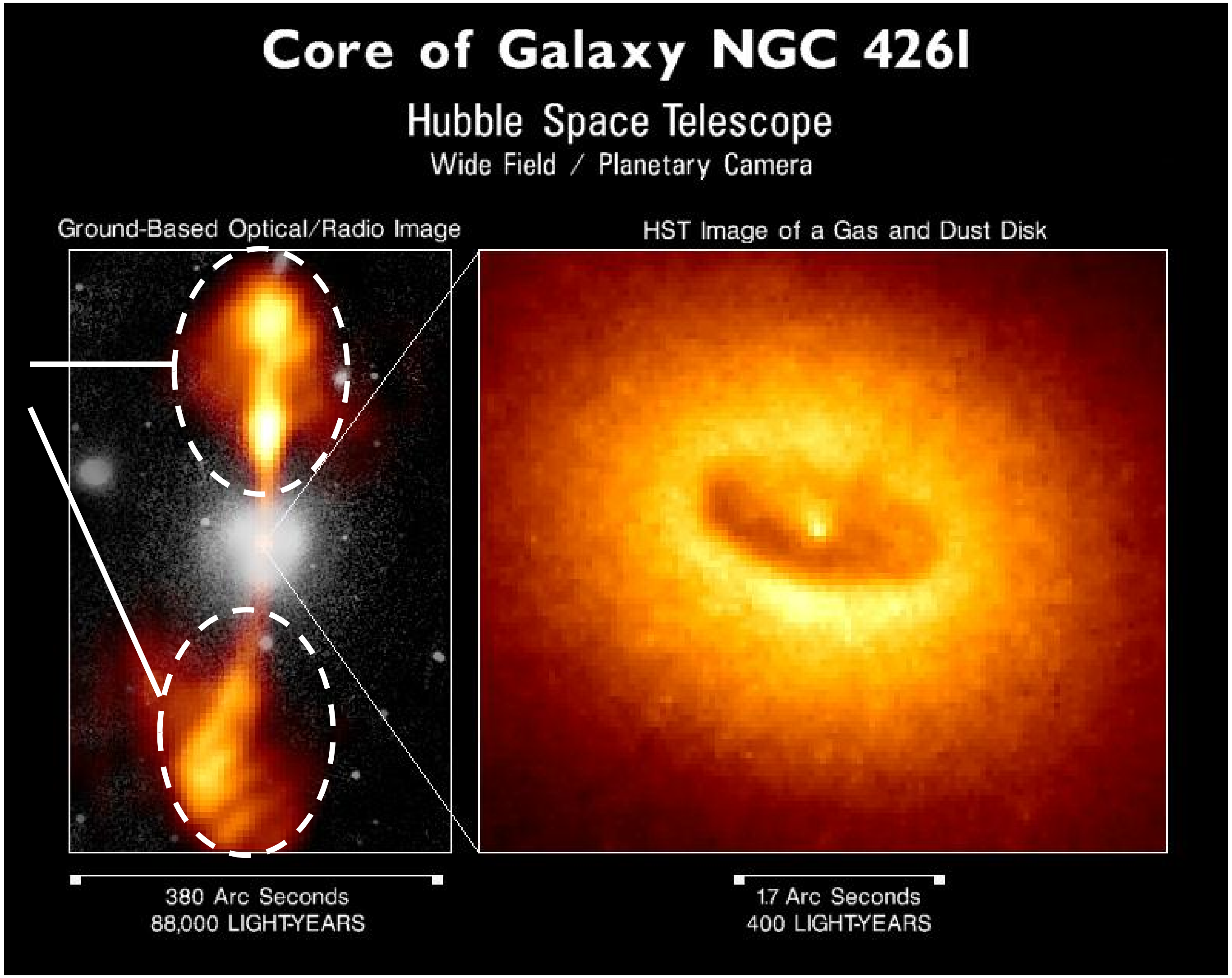
This variable source is a black hole!

Black holes cannot shine, but gas around them can shine bright!

http://www.mpe.mpg.de/ir/GC/index.php

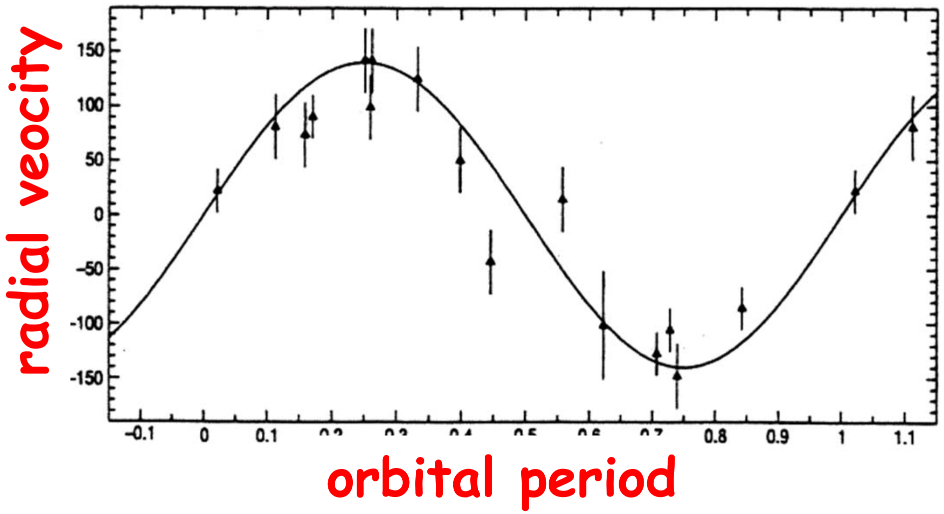
Supermassive black holes (10^{5-9} M_{sun}) in galactic nuclei

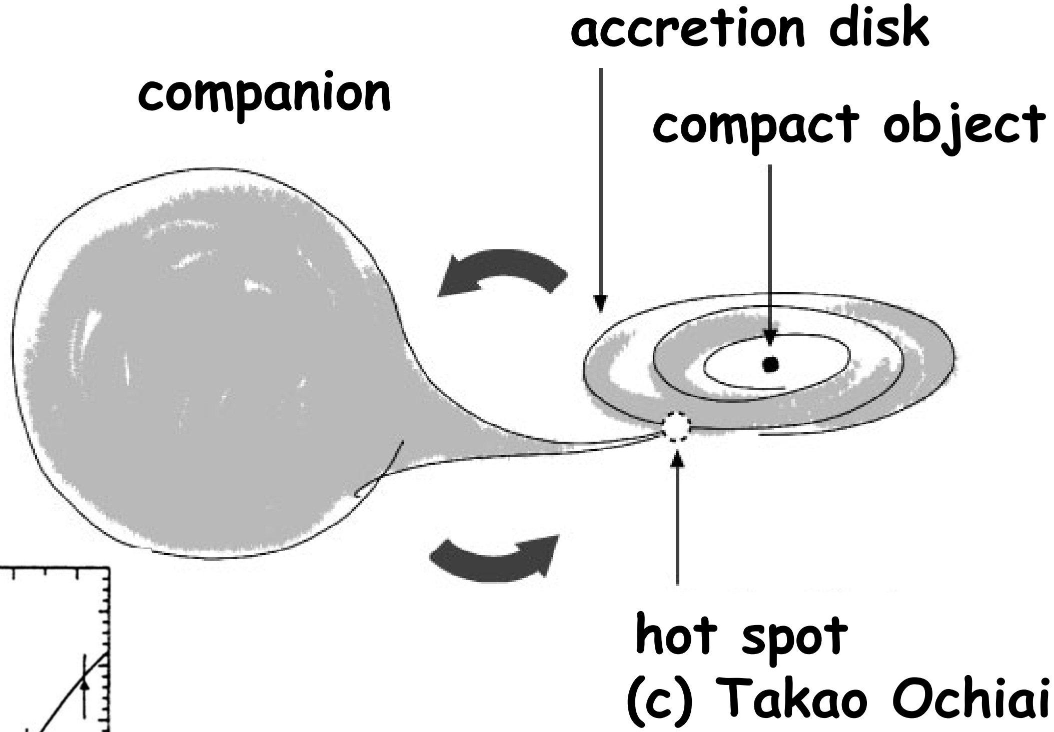
jets



Stellar-mass black holes (~10 M_{sun}) in close binaries

- Binaries containing a normal star and a stellar-mass BH
- ~10^{7-9} stellar-mass BHs in our Galaxy.
- Binary rotation

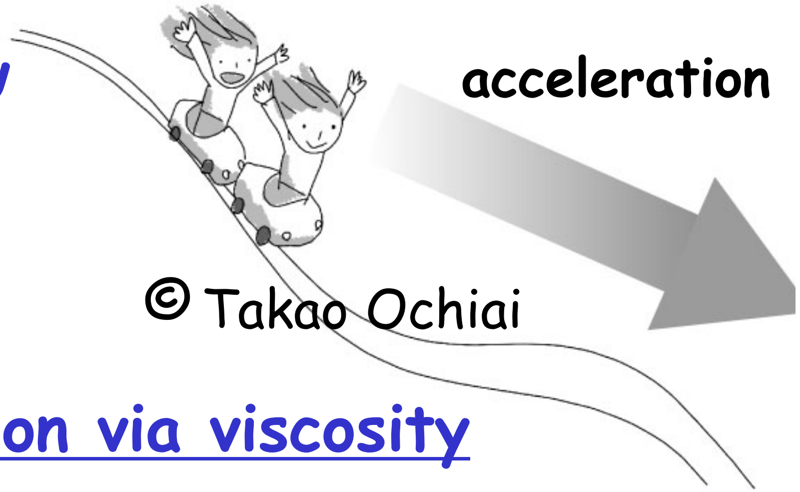




$$a \approx 3\times10^{9}\left(\frac{M_1}{M_{sun}}+\frac{M_2}{M_{sun}}\right)^{1/3}\left(\frac{P}{day}\right)^{2/3} \text{m}$$

Basics of accretion disk theory

- Spherical (radial) accretion flow cannot shine bright, since
  Grav. energy → kinetic energy



- Key: fast rotation + slow accretion via viscosity

  Two roles of viscosity (← magnetic origin)
  - Angular momentum transport → gas accretion
  - Energy dissipation → viscous heating → radiation

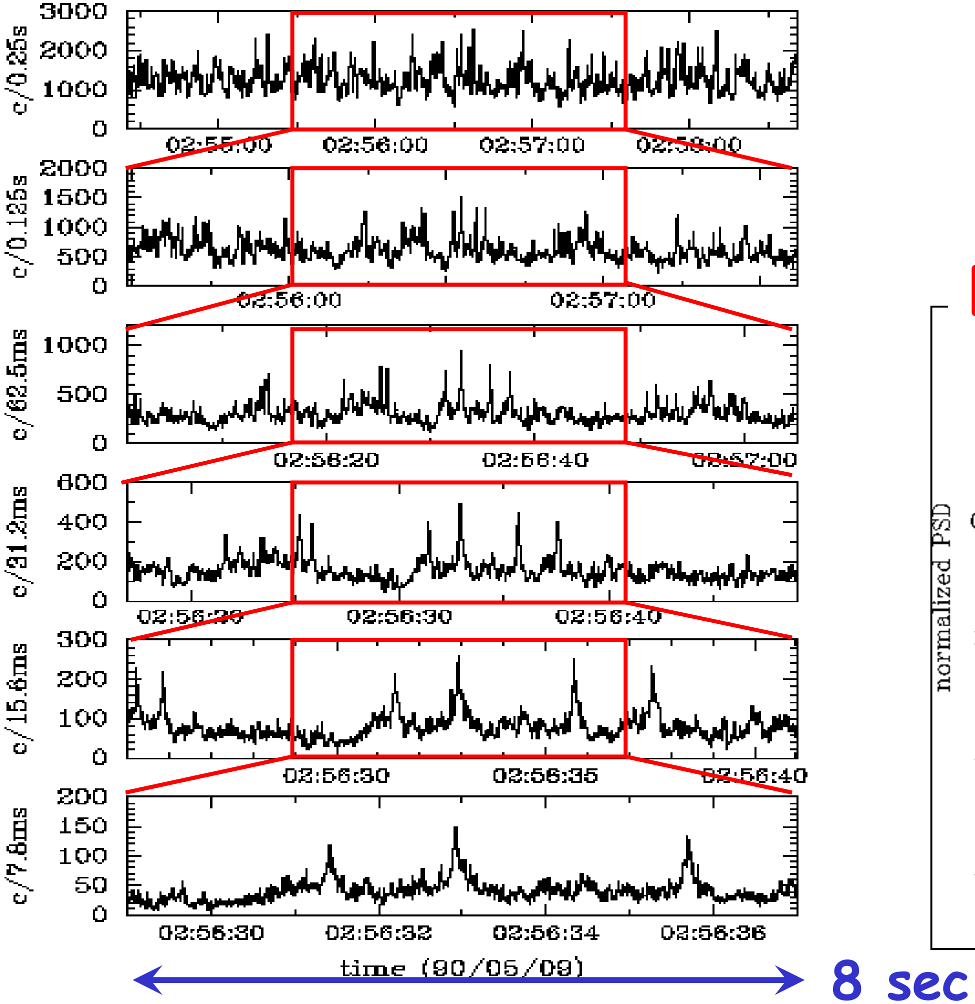
  ⇒ Grav. energy → rotation → thermal → radiation

- Time separation: $t_{accretion} \gg t_{thermal}, t_{rotation} \cdots$

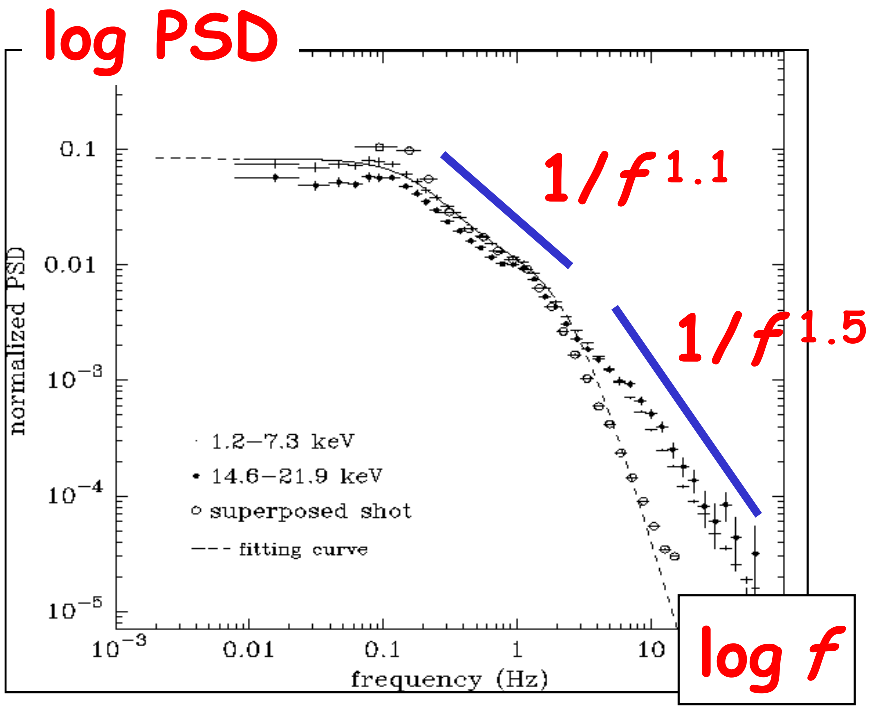
Stellar-mass BH variability

Negoro (1995, D-thesis)

X-ray intensity Cyg X-1



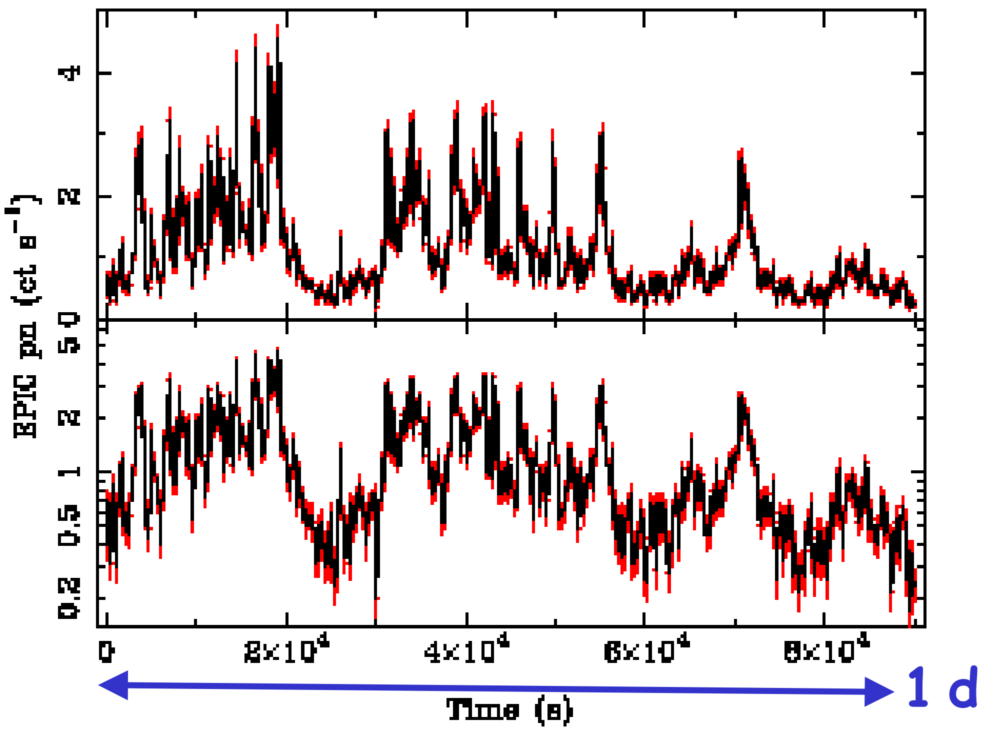
Power Spectral Density (PSD) ↓



Supermassive BH variability

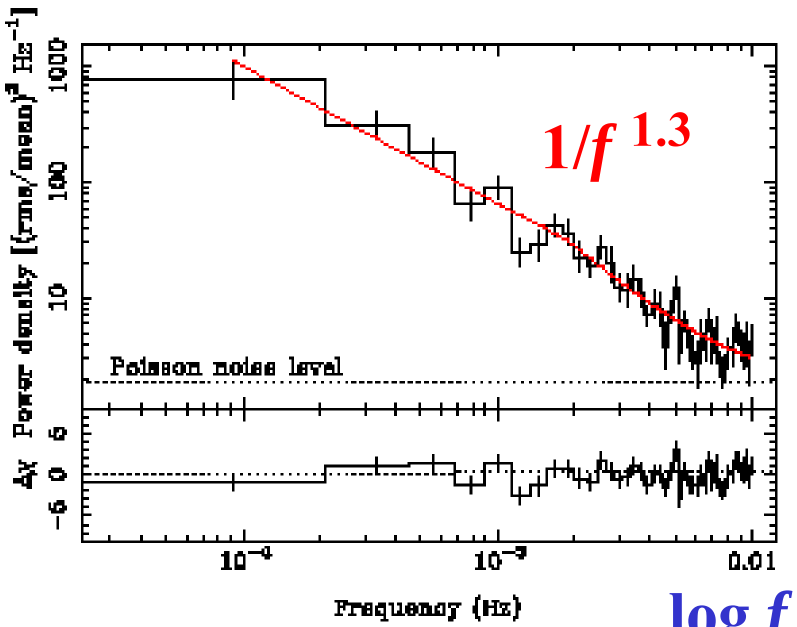
Vaughan et al. (2005)

NGC4395



← X-ray light curve and PSD ↓

log PSD

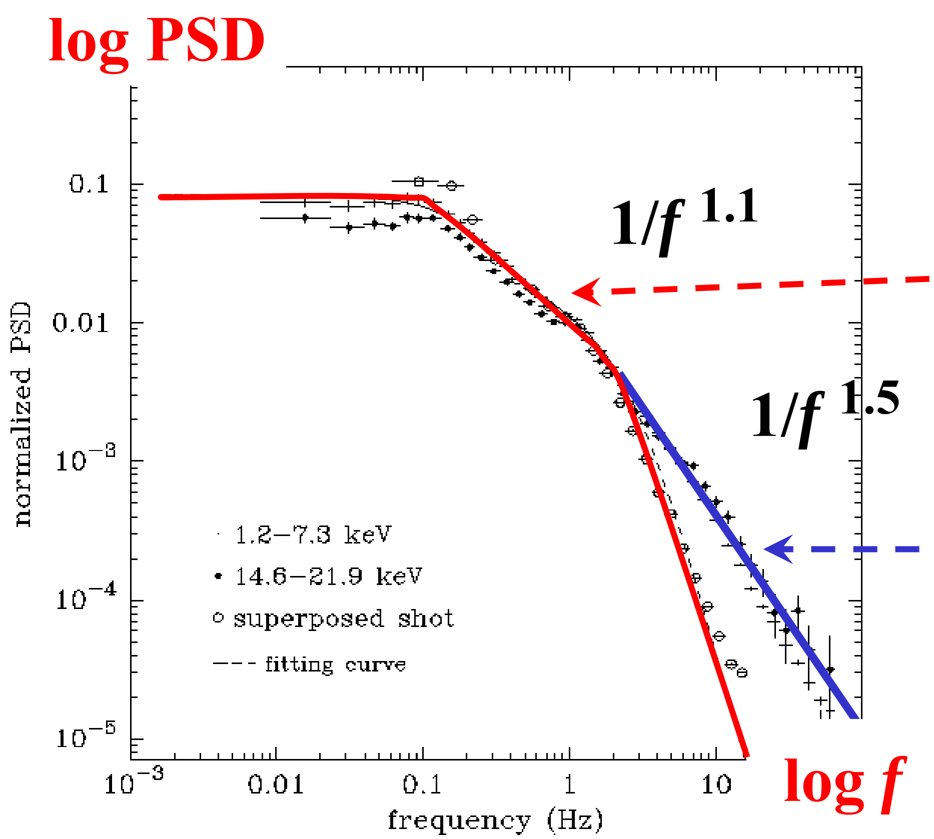


log f

Again, we find sequence of shots (flares)!!

What determines the PSD shape of Cyg X-1?

Negoro (1995, D-thesis)



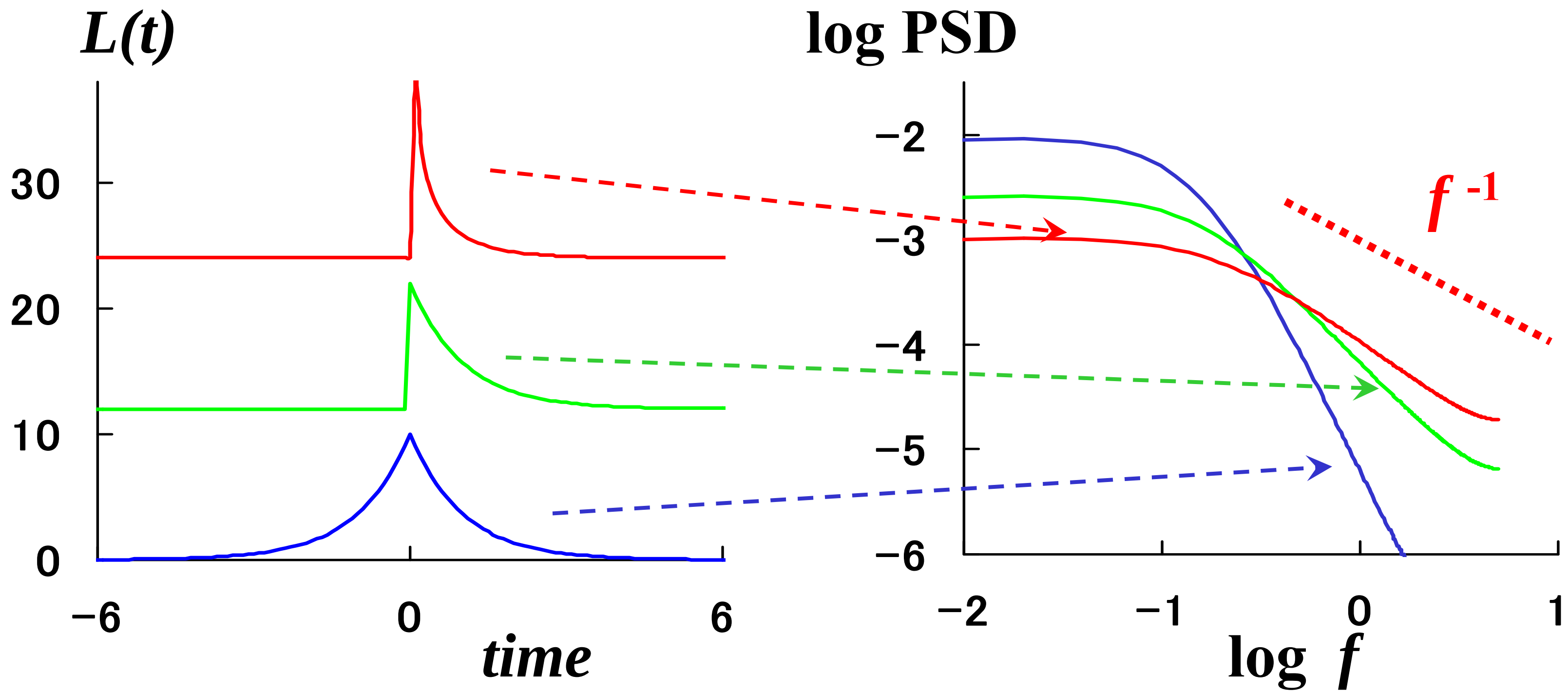
- *Shot profile determines the low-frequency part of PSDs.*
- *Shot distributions determines the high-frequency part of PSDs.*

Single shot profile

(a) power-law $\propto t^{-0.5} \exp(-t/t_0)$ $(t > 0)$

(b) single exp. $\propto \exp(-t/t_0)$ $(t > 0)$

(c) double exp. $\propto \exp(|t|/t_0)$



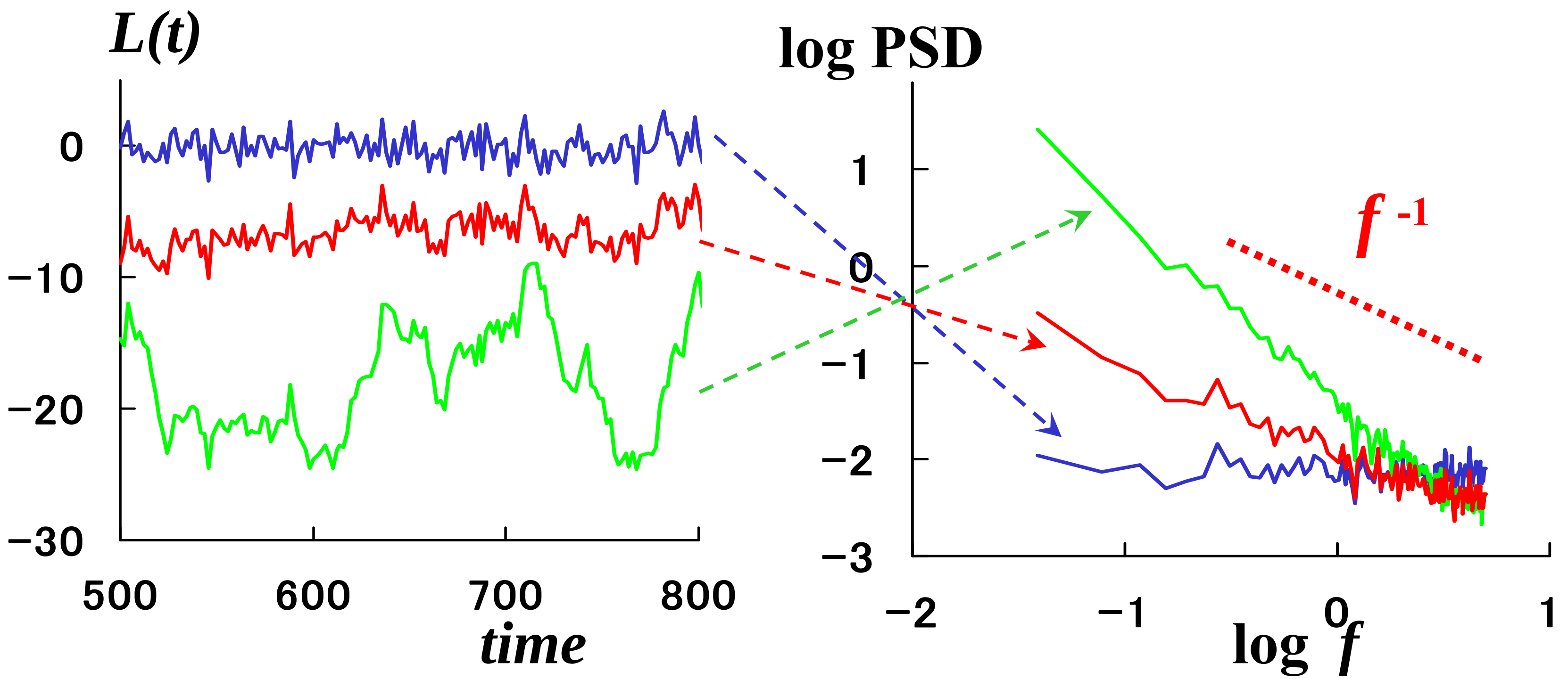
Random noise

(cf. Press 1978)

(a) white noise, $f_w(t)$

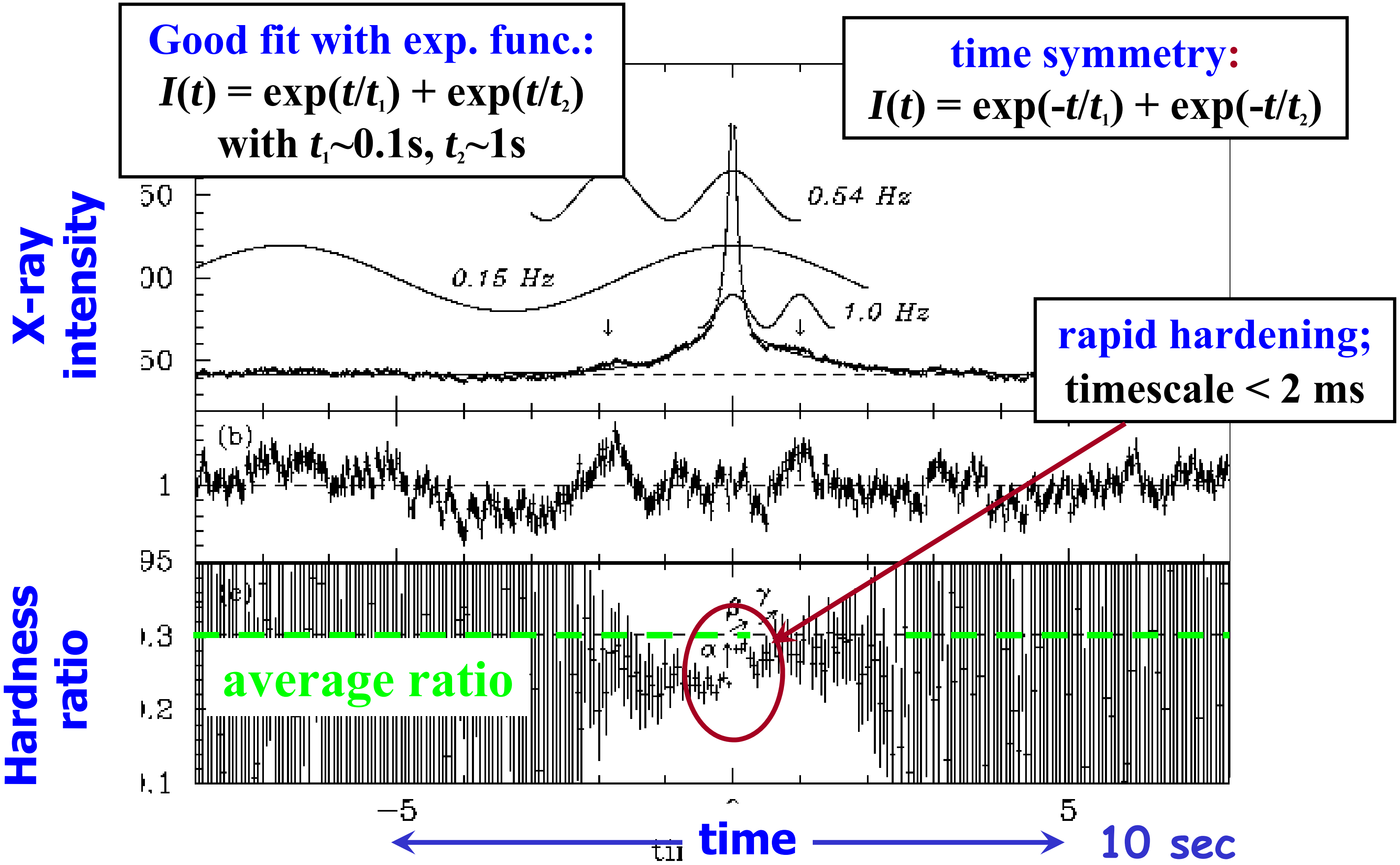
(b) half-integral of white noise, $\int^t f_w(t_0)\,(t-t_0)^{-1/2}\,dt_0$

(c) integral of white noise, $\int^t f_w(t_0)\,dt_0$

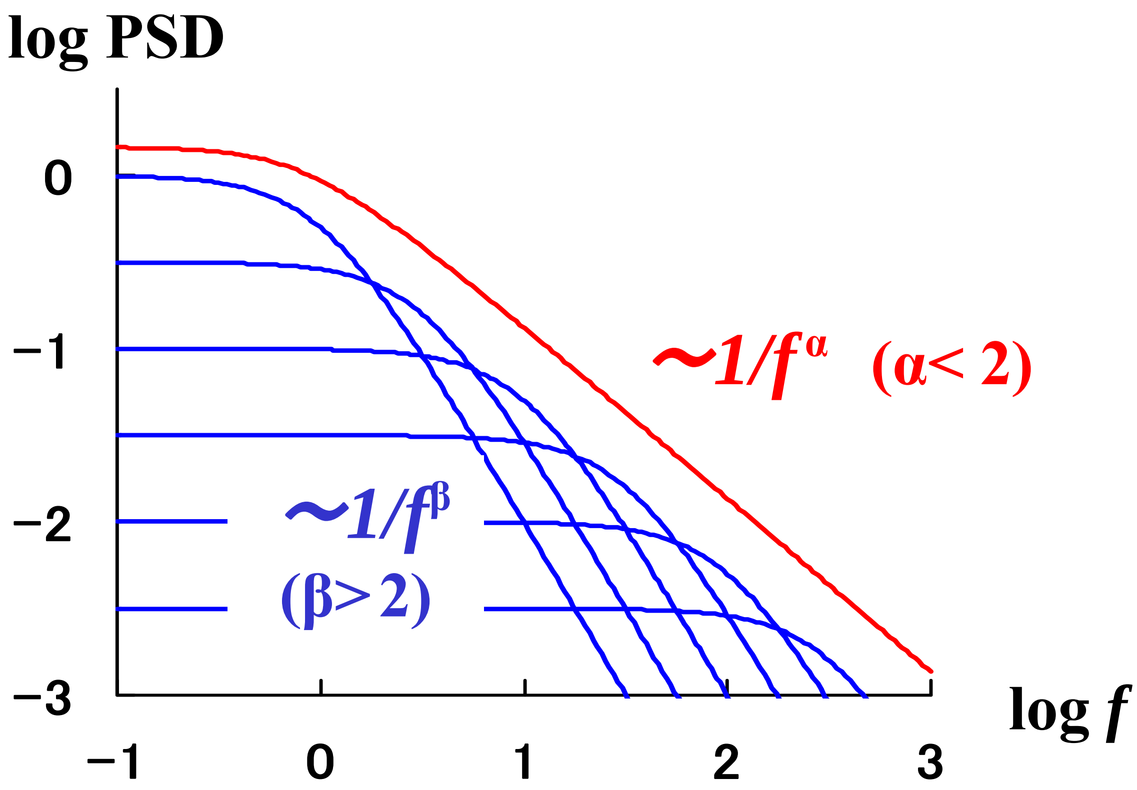


Superposed shot of Cyg X-1

Negoro et al. (1994); Negoro, Kitamoto, Mineshige (2001)



Smooth size distribution of shots (or flares)



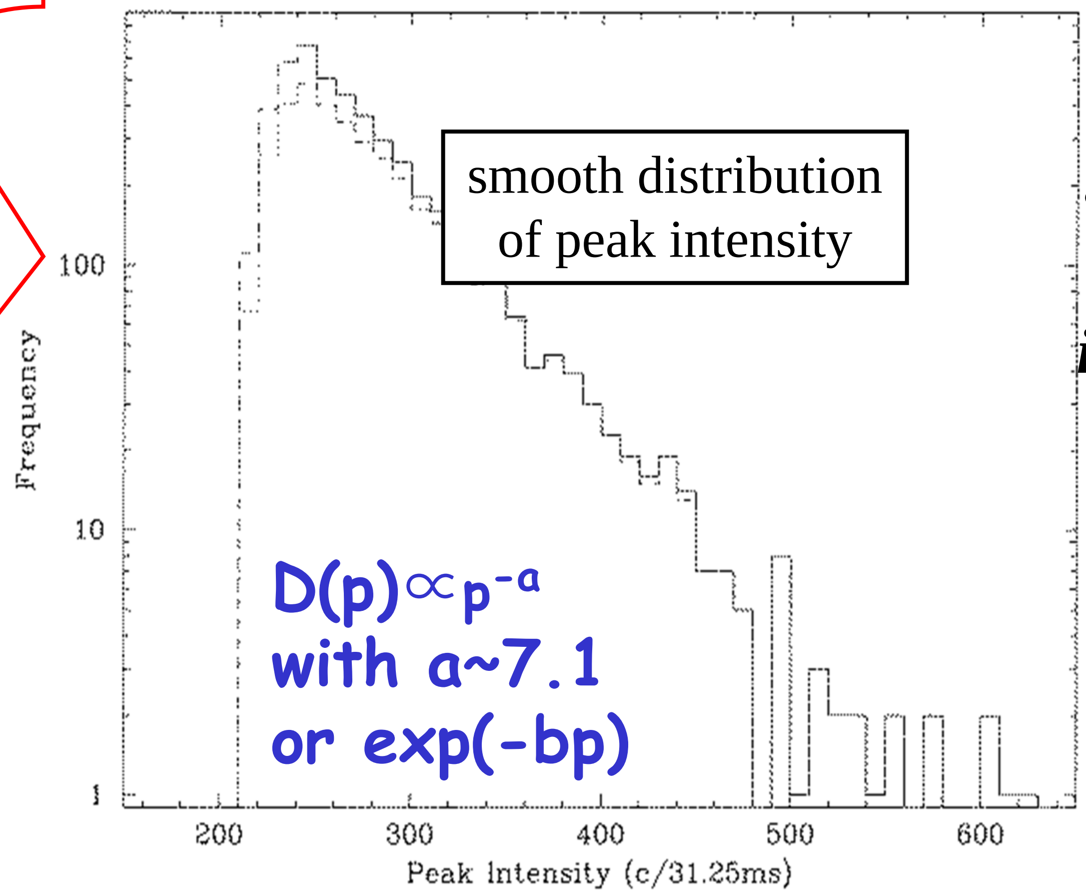
Power-law shot size (time) distribution determines $1/f^{\alpha}$ slope.

Individual shot profiles are not very important.

Statistical properties of shots (Cyg X-1)

Negoro et al. (1995); Negoro & Mineshige (2002)

Shot (flare) size is smoothly distributed.



- ***Power-law peak-intensity & peak-interval distributions***

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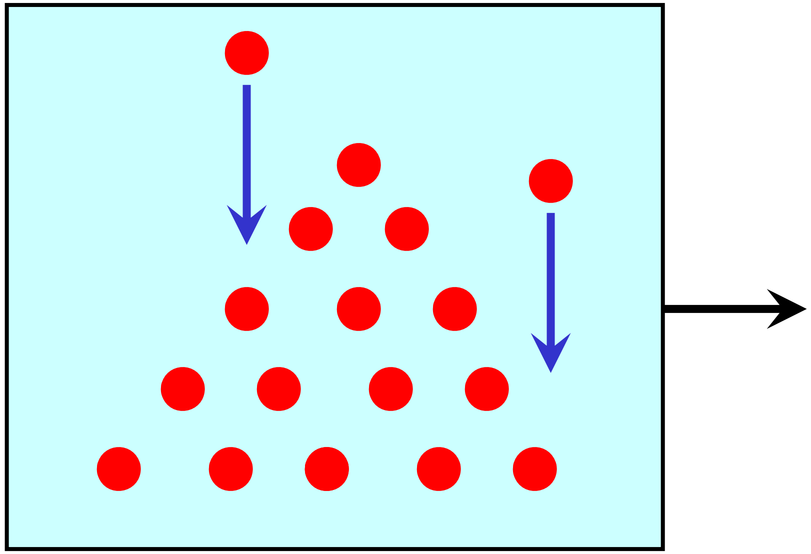
SOC modelling: Motivation & goal

- Make a simple model to reproduce the basic observational features
  - Aperiodic (random) fluctuation light curves
  - 1/f-type fluctuation power spectra
- Model both of spatial and temporal variations
  - The presence of various shots (flares) indicates critical behavior of accretion disks
- Goal: Obtain good insight for understanding physical mechanism producing variability

Self-Organized Criticality (SOC)

Bak et al. (1988, PRL)

- Basic notion to relate spatial fractals and temporal $1/f^{\beta}$ fluctuation
- Sand-pile model: when slope > $(\text{slope})_{crit}$ ⇒ avalanche
  ⇒ dissipation

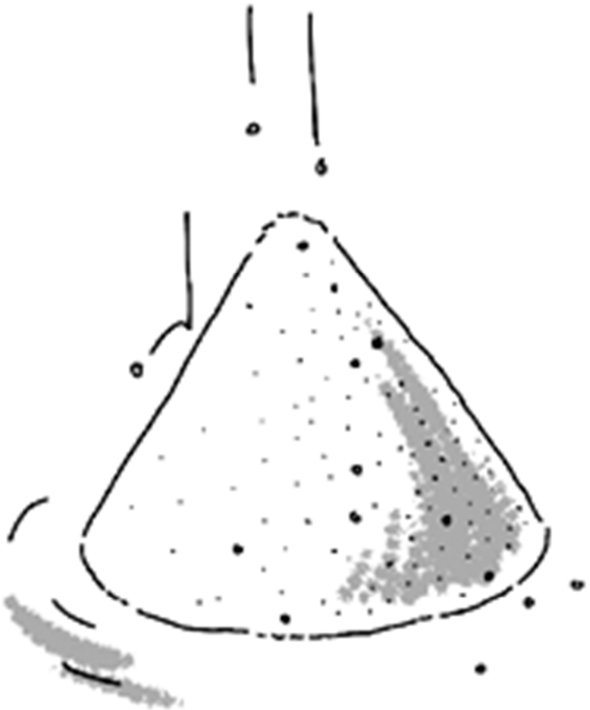


- power-law (fractal) size distribution
- $1/f^{\beta}$ type temporal fluctuations

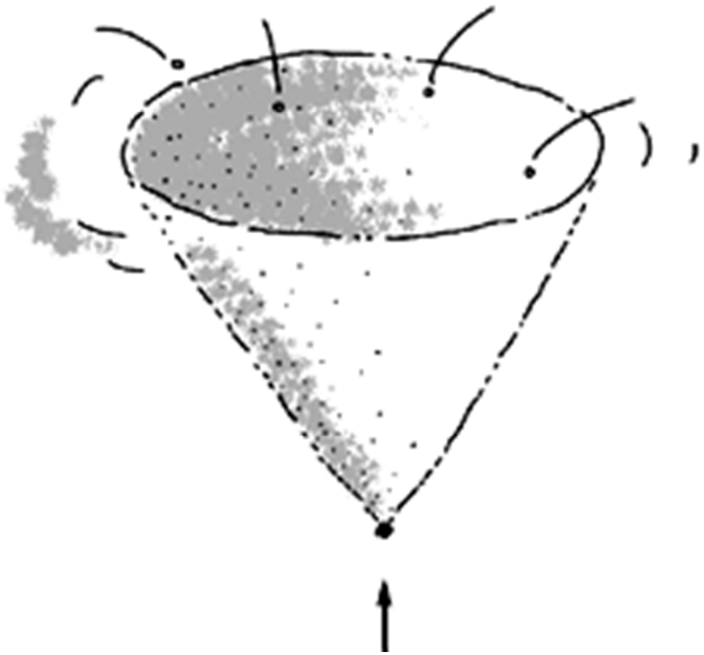
- How to model black hole accretion disks??

Sand pile vs. black hole accretion

Mineshige, Ouchi, & Nishimori (1994)



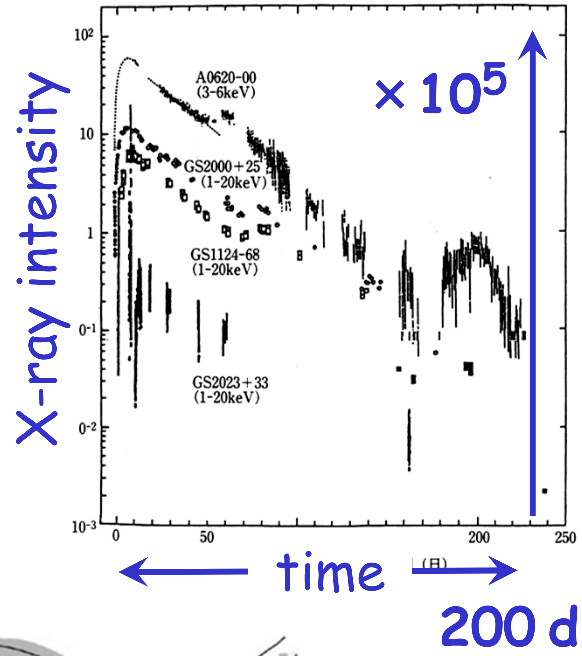
sand pile

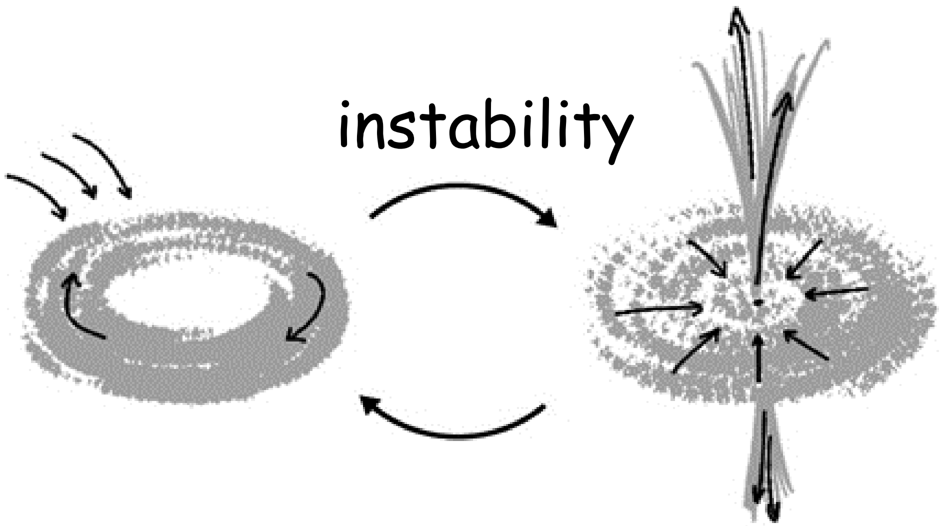


black hole

Disk instability model for dwarf novae/X-ray novae

Low temperature accretion disks alternate between two states: mass accumulation and mass release phases.

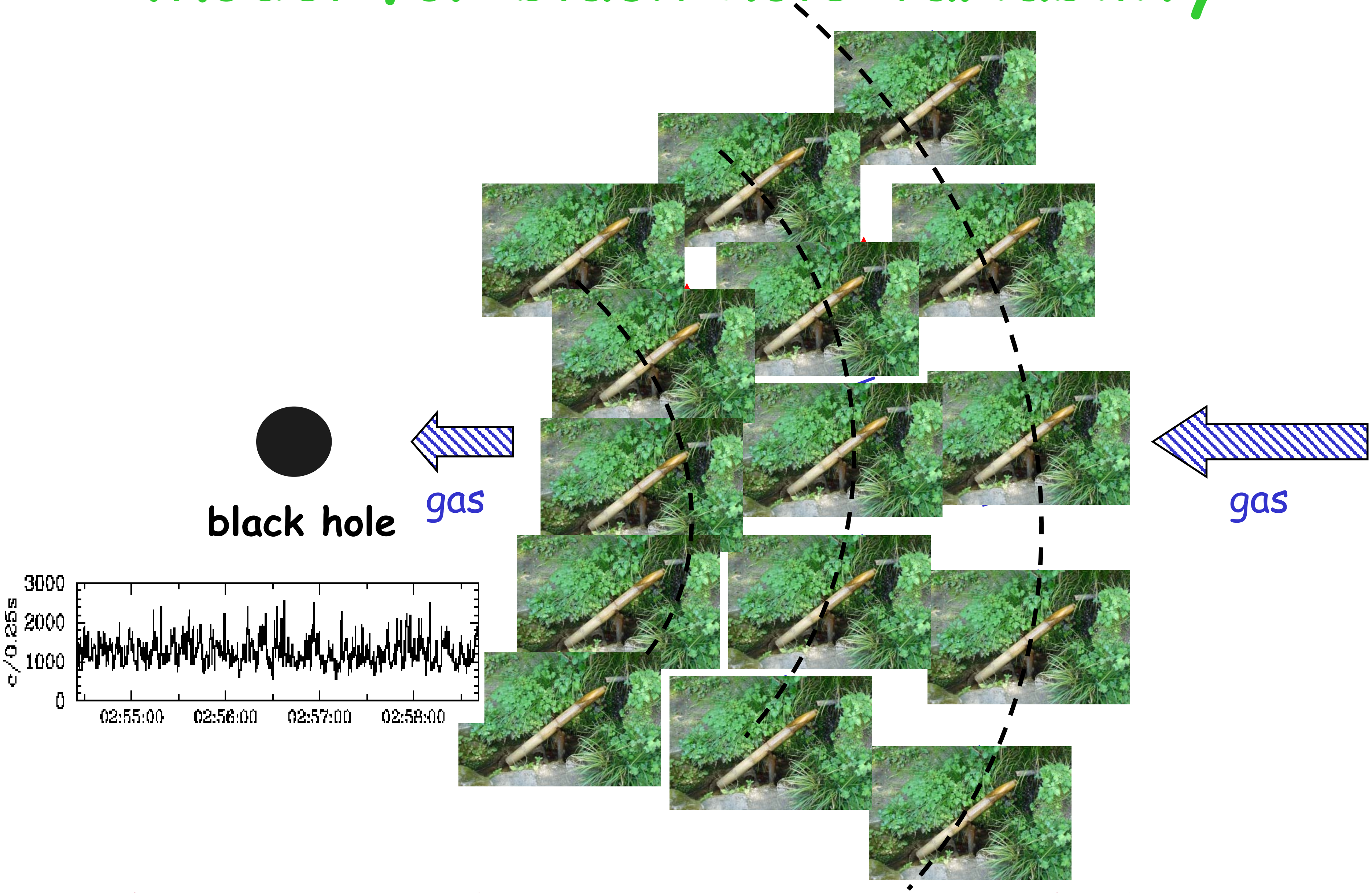




accumulation phase
(quiescence)

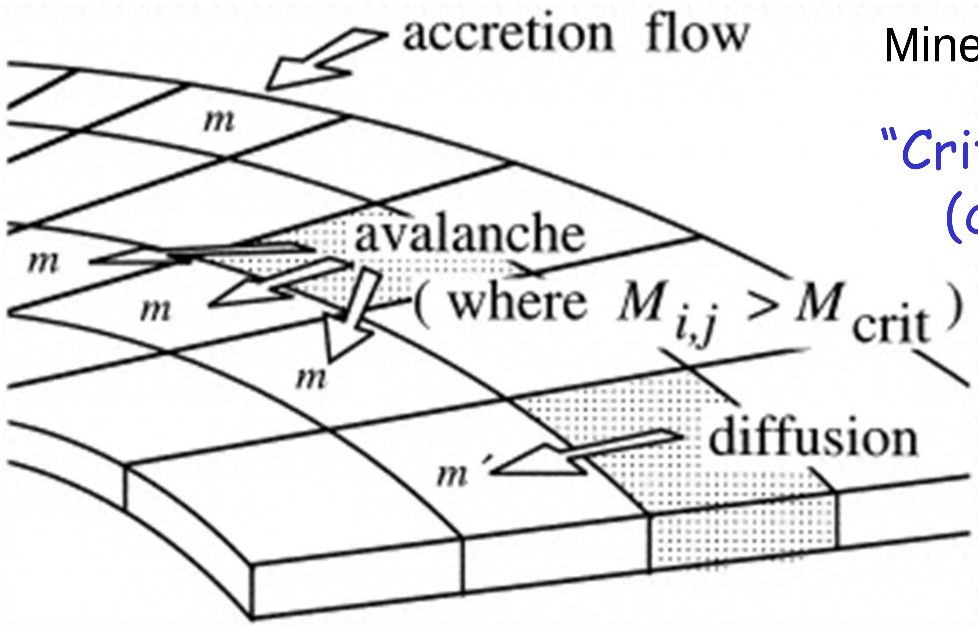
release phase
(outburst)

Model for black hole variability



2D/3D network of "shishi-odoshi" produces 1/f fluctuation.

Cellular automaton rule (original)



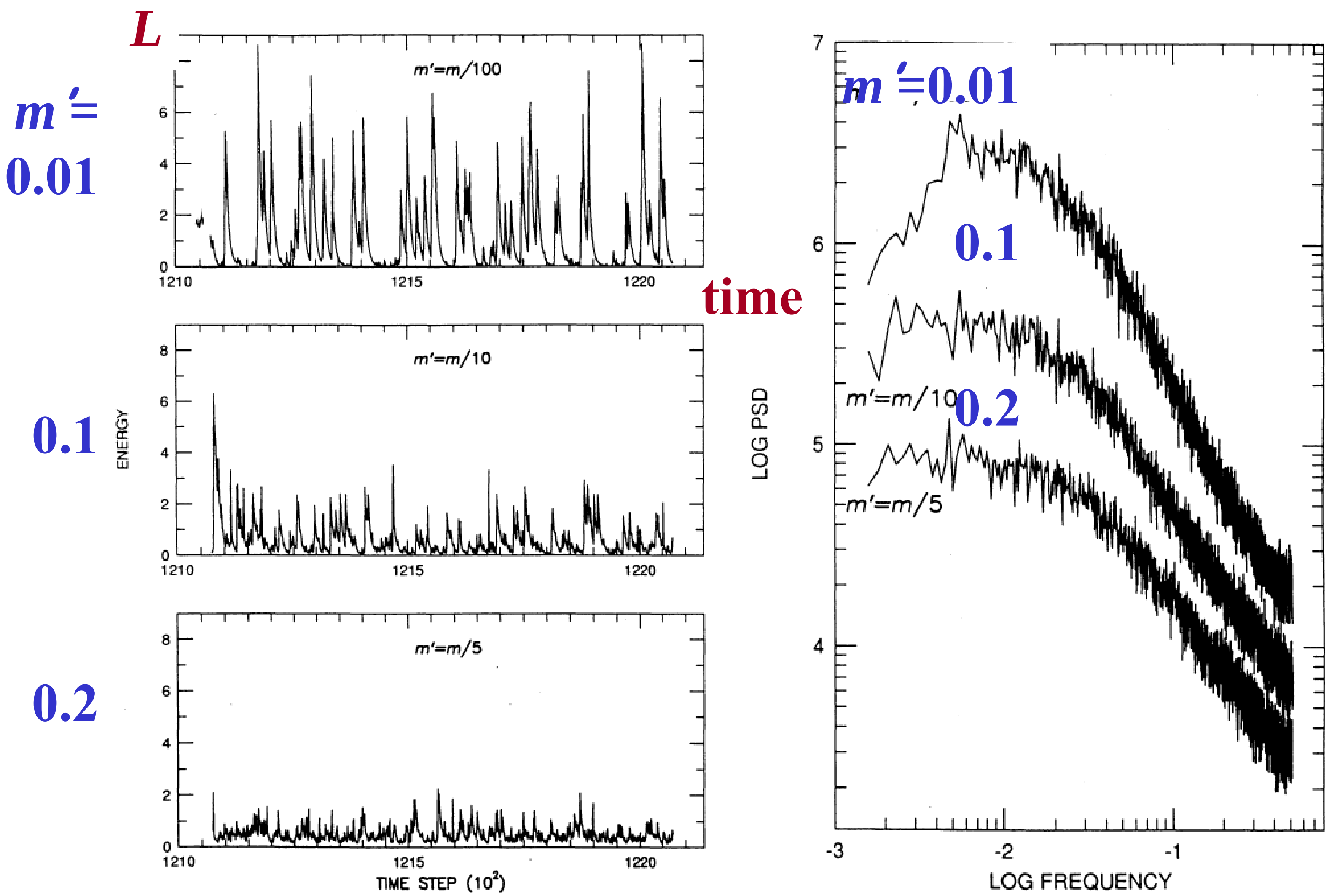
Mineshige et al. (1994)

"Critical height model"
(cf. Manna Model)

1. **Add one particle with mass *m* in the outermost ring.**
2. **If the mass of the sell (M_{ij}) exceeds the critical value, let an avalanche and energy dissipation occur.**
3. **Small mass with $m' << m$ gradually diffuses inward.**
4. **Calculate luminosity, according to $L \propto \Delta m/r$ (Δm is accreted mass).**
5. **Repeat 1.-4. for over 10^5 times.**

Results 1. light curve & PSD

Takeuchi et al. (1995)



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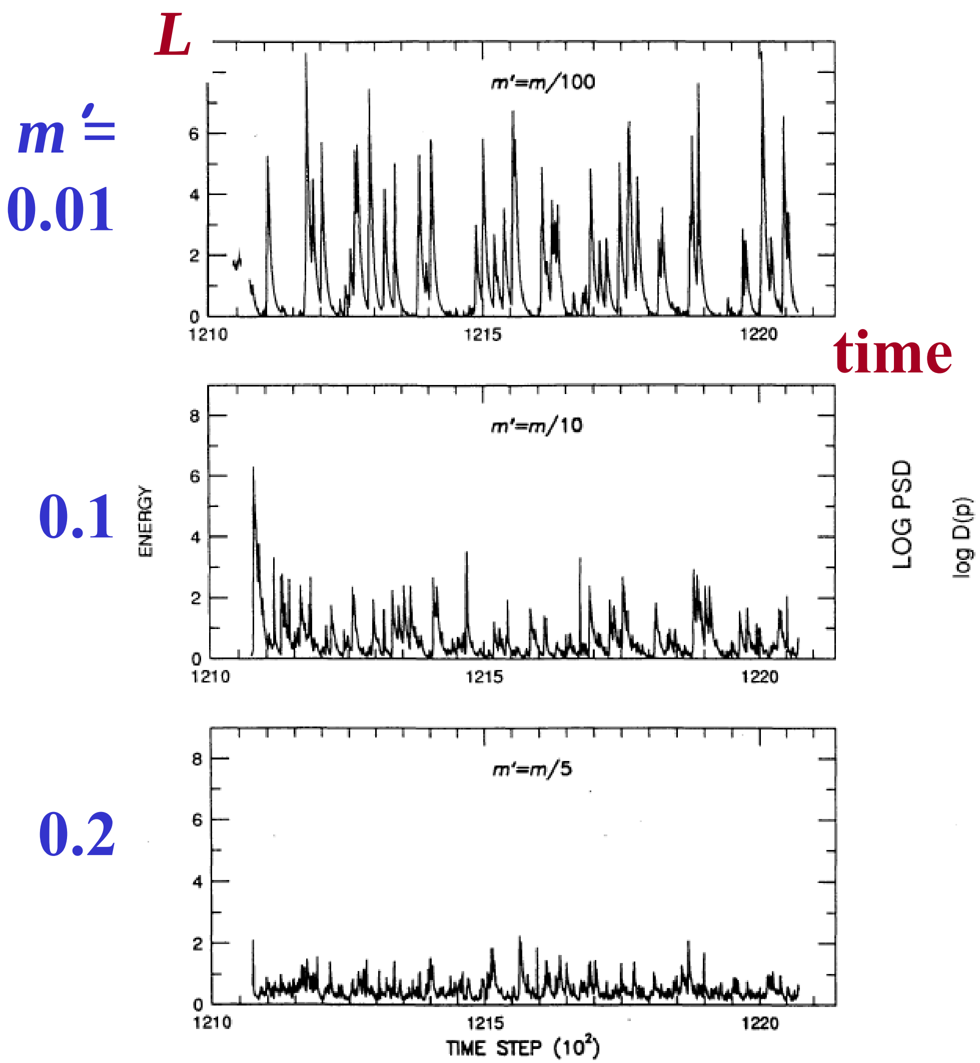
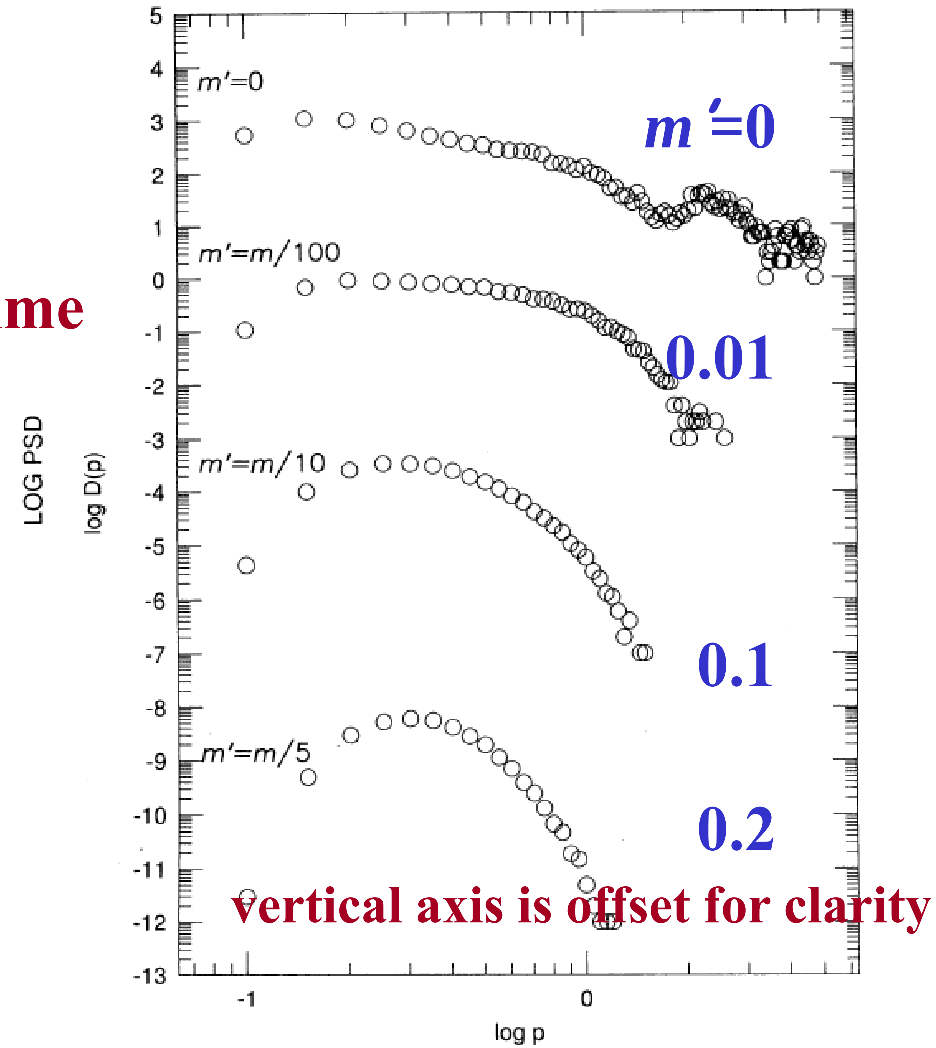


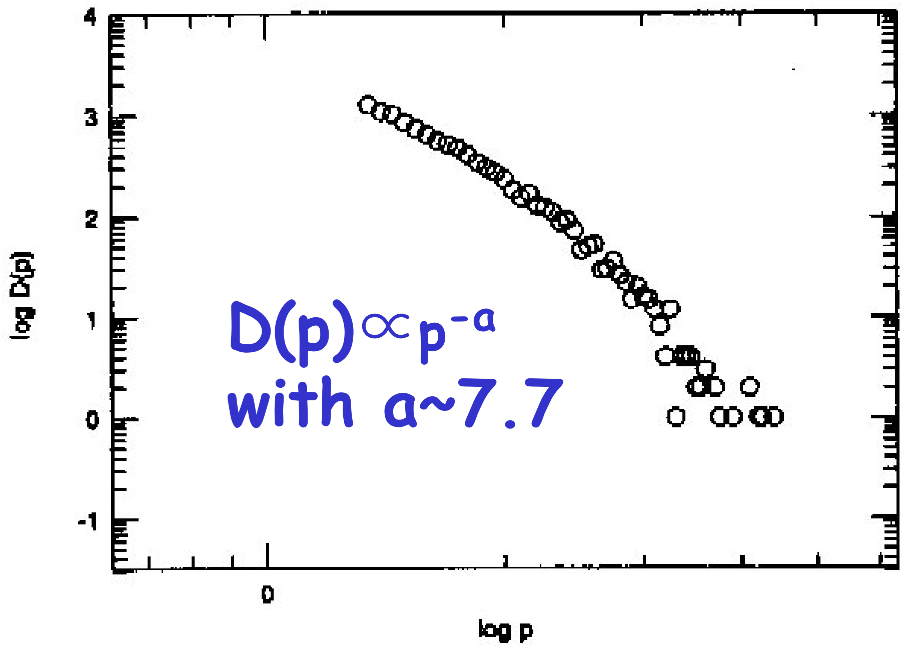
peak intensity distribution



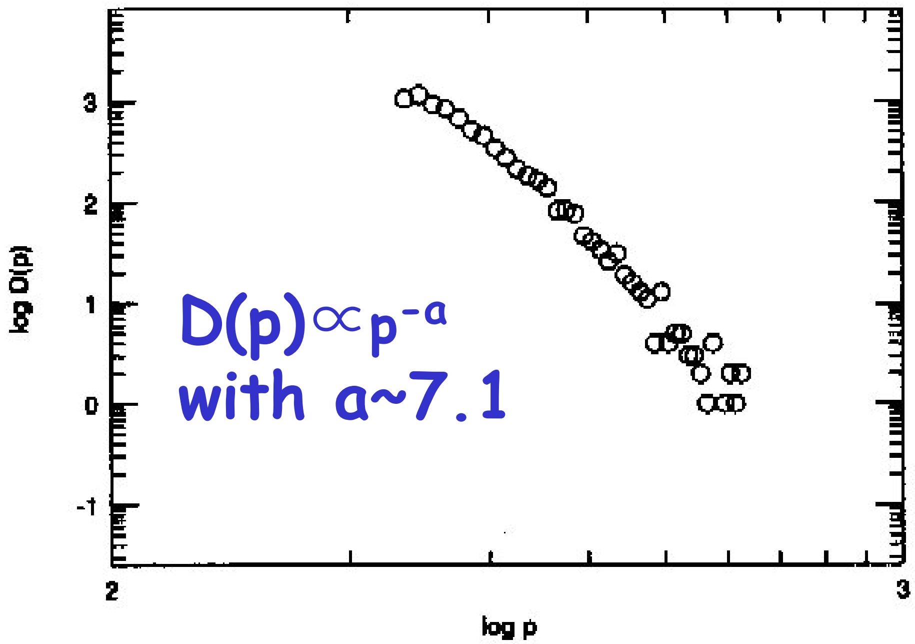
Results 2. Peak intensity distribution

Negoro et al. (1995)

Roughly power-law → supporting the SOC model



SOC model

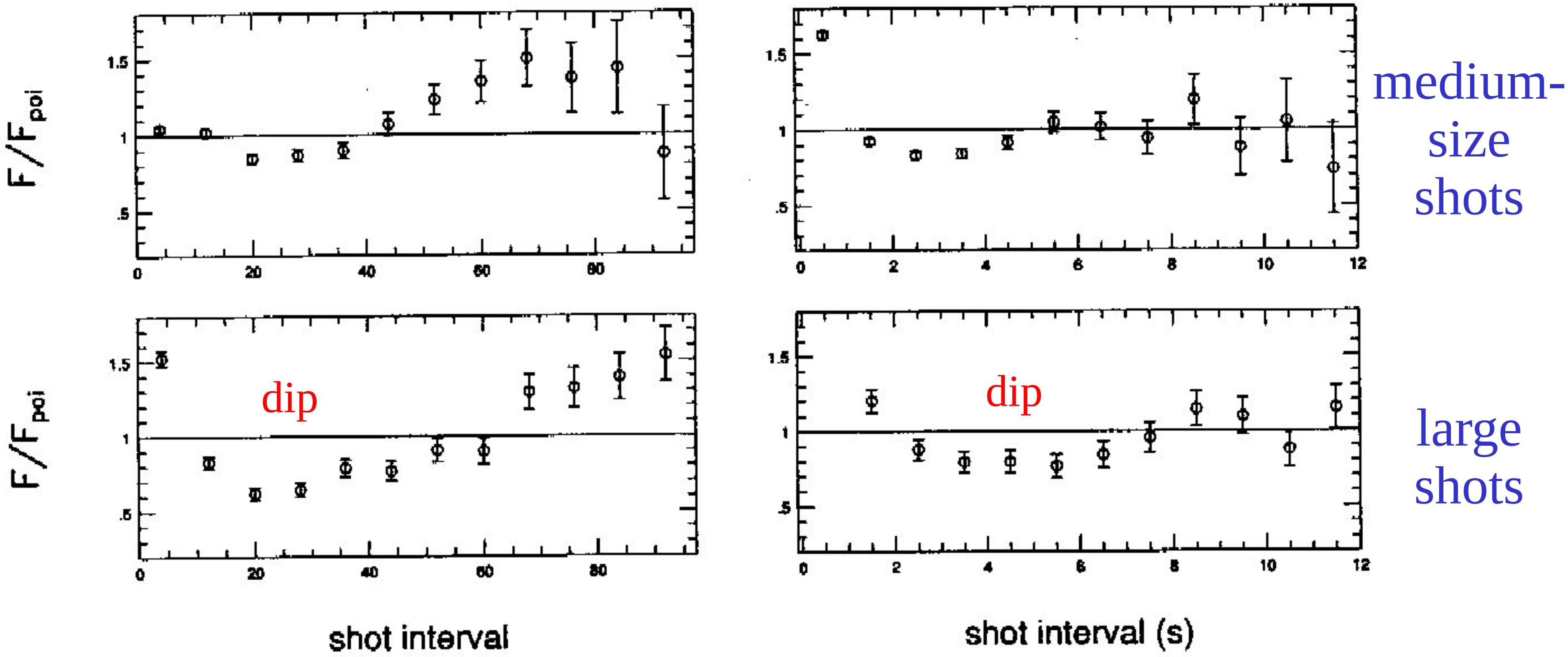


observation (Cyg X-1)

Results 3. Peak interval distribution

Negoro et al. (1995)

Dip in the peak interval distribution → "reservoir"



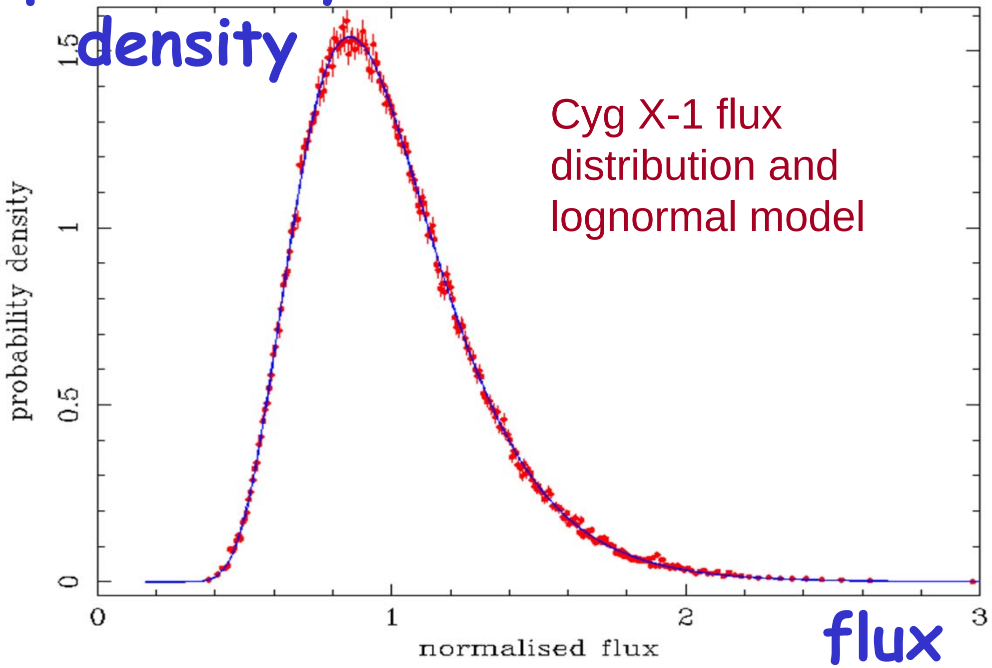
SOC model **observation (Cyg X-1)**

Issue: lognormal flux distribution

(Uttley, McHardy & Vaughan 2005)

Lognormal flux distribution is found in both of stellar-mass and supermassive black holes.

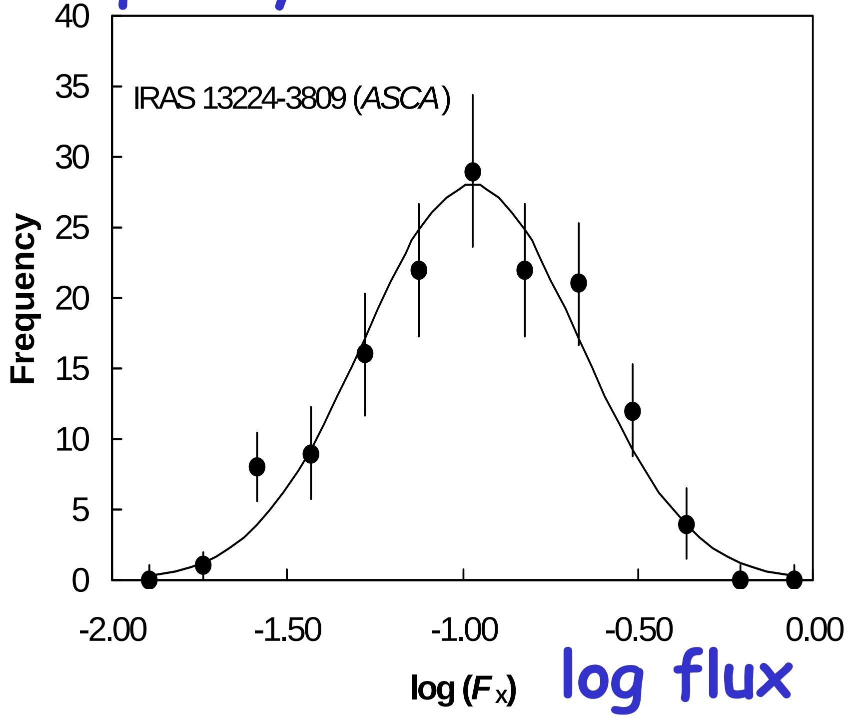
probability density



flux

(Uttley, McHardy & Vaughan 2004)

frequency



log flux

(Gaskell 2004)

What does this mean?

(Uttley et al. 2005)

- A big issue: Log-normal flux distribution:

  ⇒ Variability process is multiplicative, not additive.

  If $f(t)=\Sigma f_i \rightarrow f(t)$ obeys Gaussian

  If $f(t)=\Pi f_i(t) \rightarrow \log f(t)$ obeys Gaussian

  ⇒ Rules out shot-noise, SOC, and multiple independently varying regions

- Another issue: the CA rule is phenomenological (and not so physical)
- Let us consider a revised model.

From SOC model for solar flares

Lu & Hamilton (1991), Isliker et al. (1998)

Induction equation describes how magnetic field (B) evolves with time.

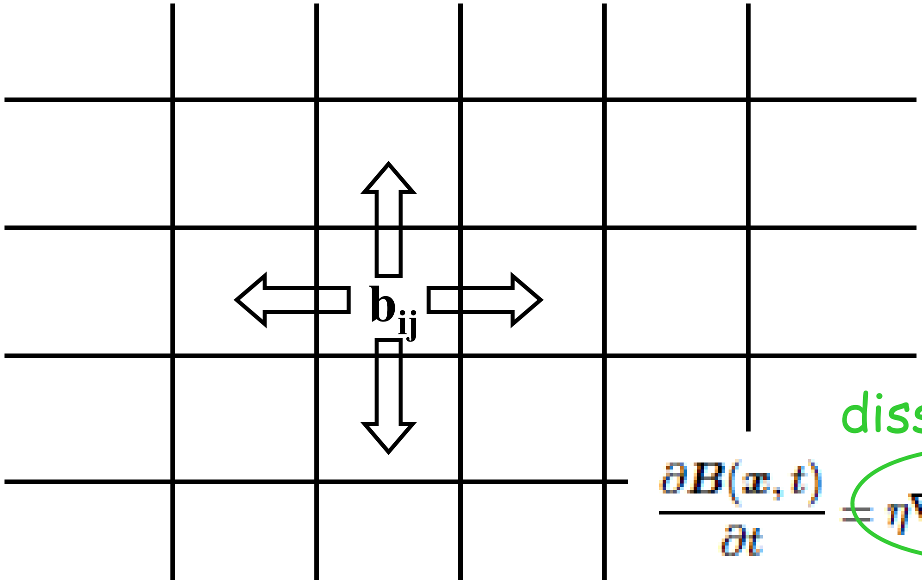
$$\frac{\partial B(x,t)}{\partial t} = \underline{\eta \nabla^2 B(x,t)} + \underline{\nabla \wedge (v \wedge B)}$$

⇧ dissipation due to magnetic reconnection

⇧ fluctuations due to gas motion (v)

$\Rightarrow$ **We simulate evolution of magnetic fields.**

Cellular automaton rule (revised)



dissipation fluctuations

$$\frac{\partial B(x,t)}{\partial t} = \eta \nabla^2 B(x,t) + \nabla \wedge (v \wedge B)$$

1. **Add random fluctuations on a quantity b_{ij} at each site.**
2. **Calculate $\nabla^2 b_{ij}$. If it exceeds a critical value, let an avalanche (energy dissipation) occur.**
3. **All the values of b_{ij} flow inward; $b_{ij} \rightarrow b_{ij+1}$**
4. **Calculate luminosity as $L_1 \propto |\nabla^2 b_{ij}|$ and $L_2 \propto \Sigma(b_{ij})^2$.**
5. **Repeat processes 1.-4. for at least 10^5 times.**

Parameters

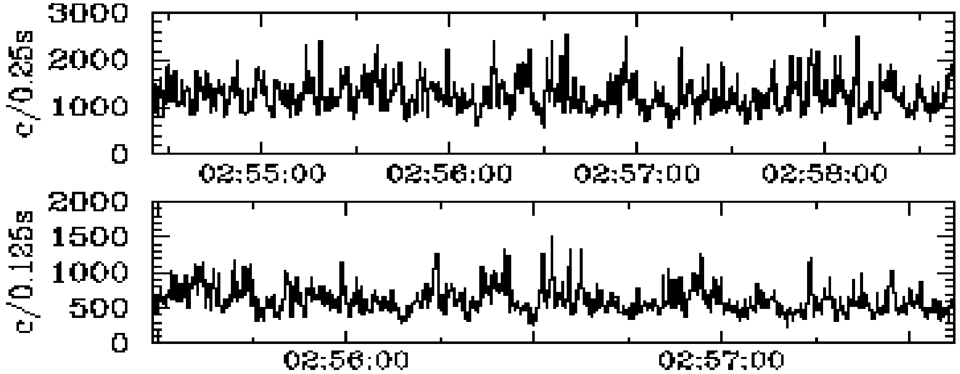
- η: L (luminosity) = L_1 (flare) + η L_2 (synchron)
- $\langle\delta\rangle$ = average fluctuation amplitude, $b_{ij} \rightarrow b_{ij}(1+\delta)$
- d_{crit} = critical $|\nabla^2 b_{ij}|/b_{ij}$, over which flare occurs

symbol	value(s)	meaning
$r_{\rm in}$	0.1	radius of the inner edge of the disk
$r_{\rm out}$	1.0	radius of the outer edge of the disk
N_r	512	number of the radial mesh points
N_φ	256	number of the azimuthal mesh points
$b^0_{i,j}$	1.0	initial magnetic field strength
η	0 – 0.1	luminosity ratio [see Eq. (14)]
$\langle\delta\rangle$	0.4 – 1.0	average fluctuation amplitude [see Eq. (4)]
$d_{\rm crit}$	5 – 15	critical value for db/b ($\propto \nabla^2 b$)

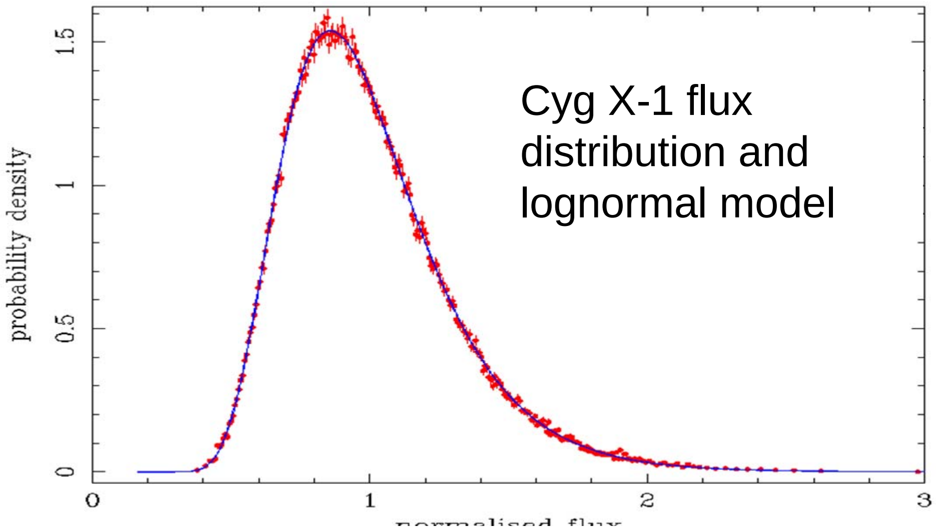
Observations of Cygnus X-1

Negoro (1995, D-thesis)

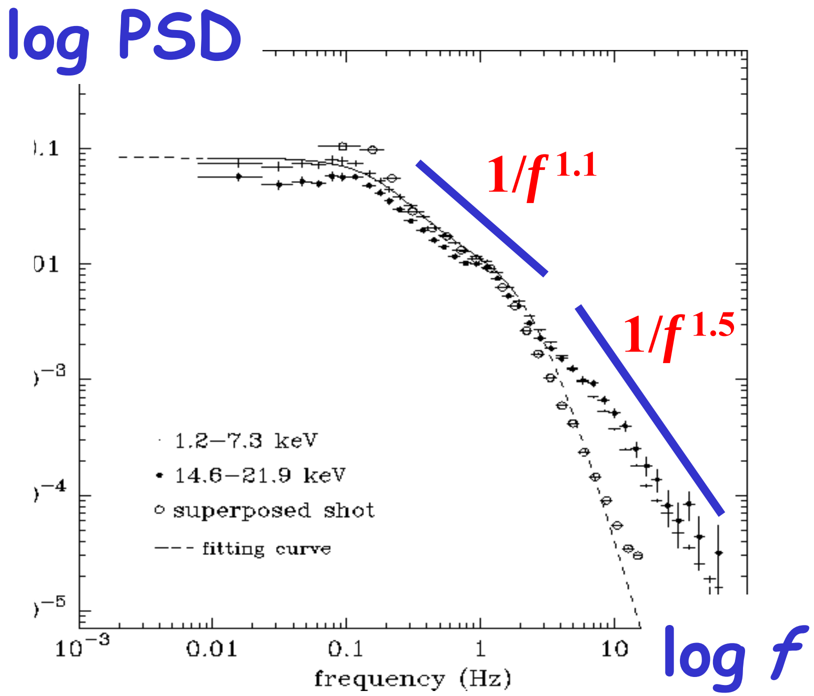
- X-ray intensity



light curve (left)
PDF (lower-left)
PSD (lower-right)

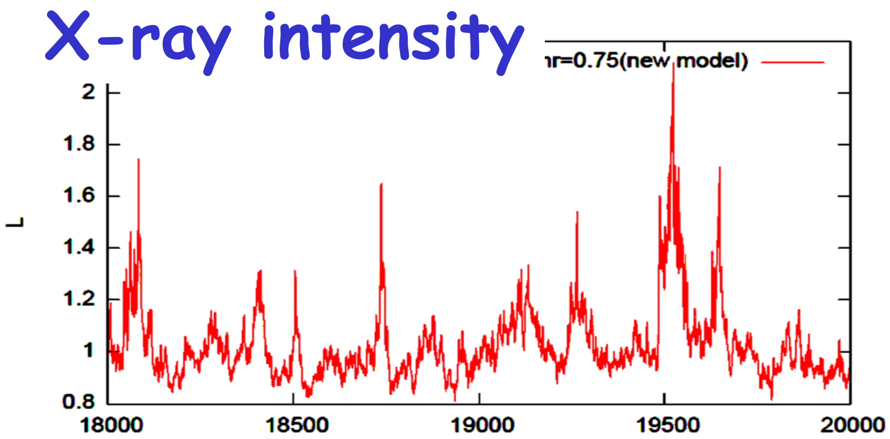


(Uttley, McHardy & Vaughan 2004)



Simulation results: overview

Miaeda et al. (2012?)



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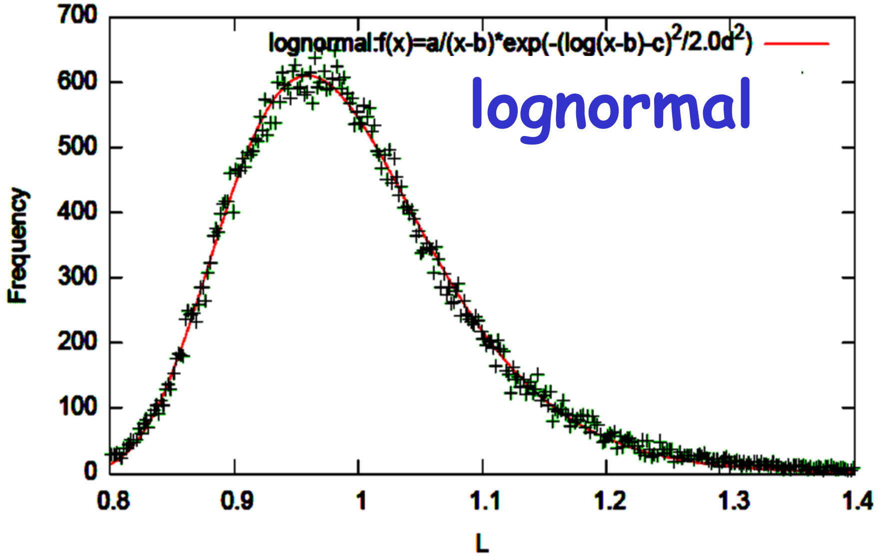
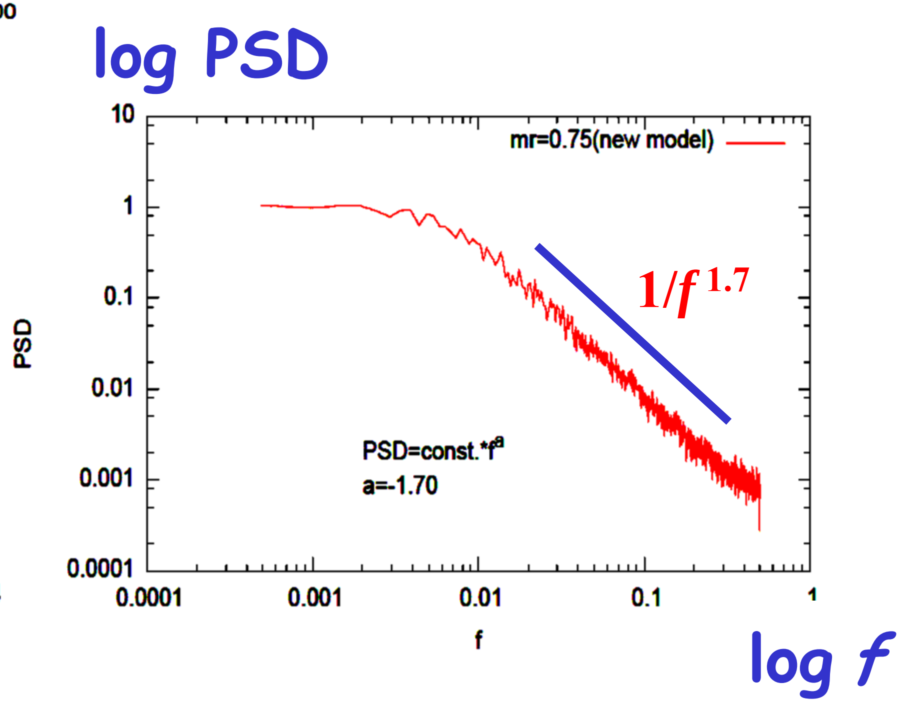
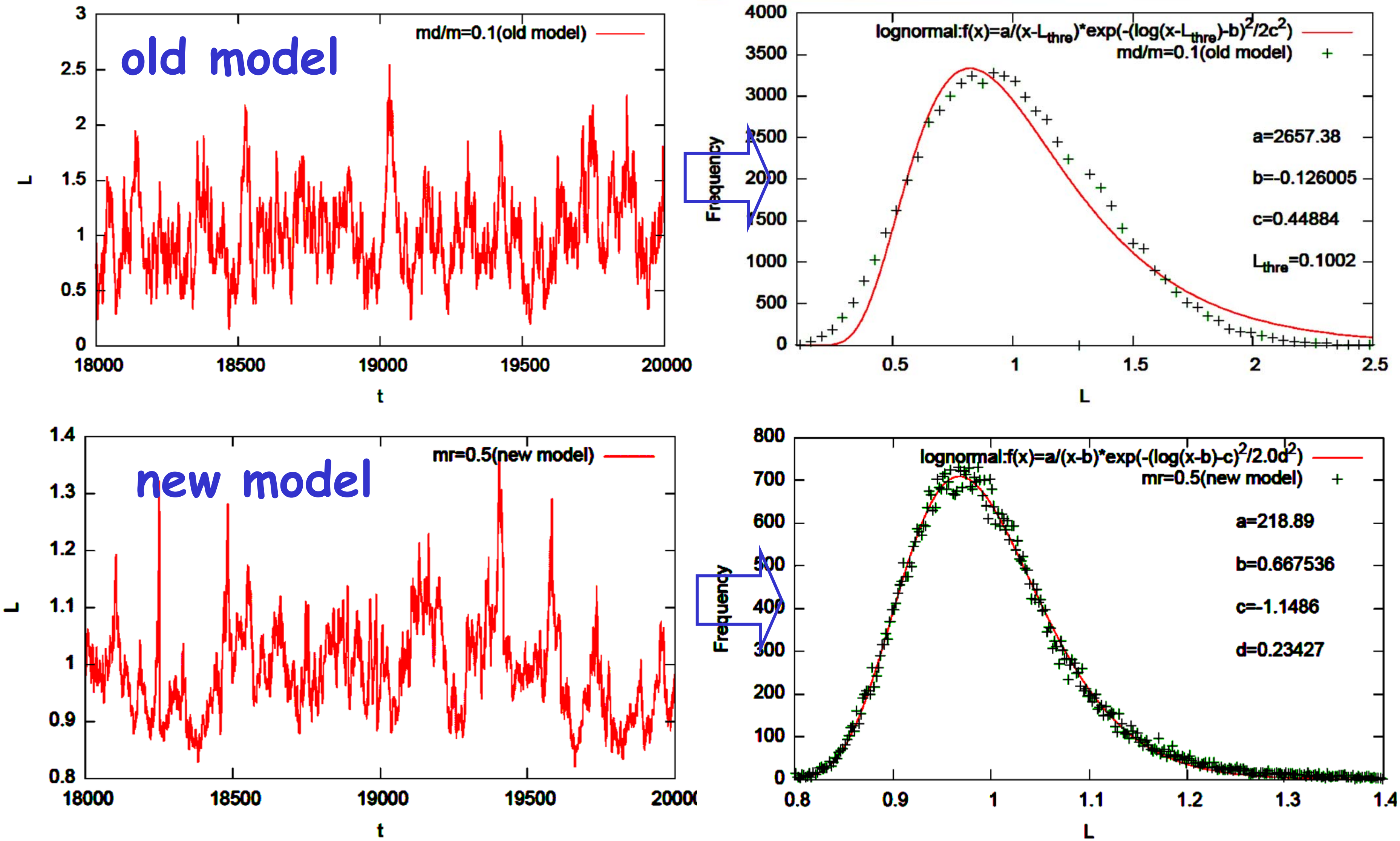
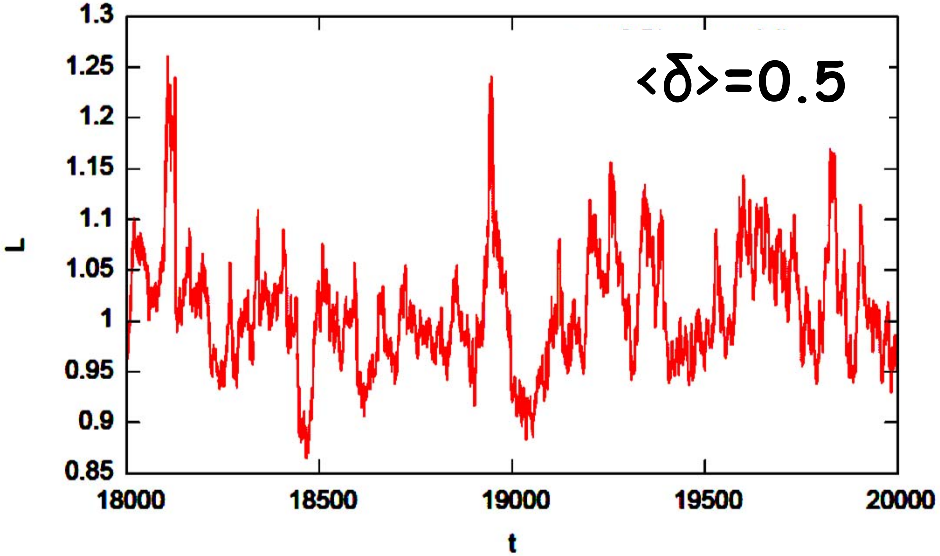




Old model vs. new model



Parameter dependence:<δ>

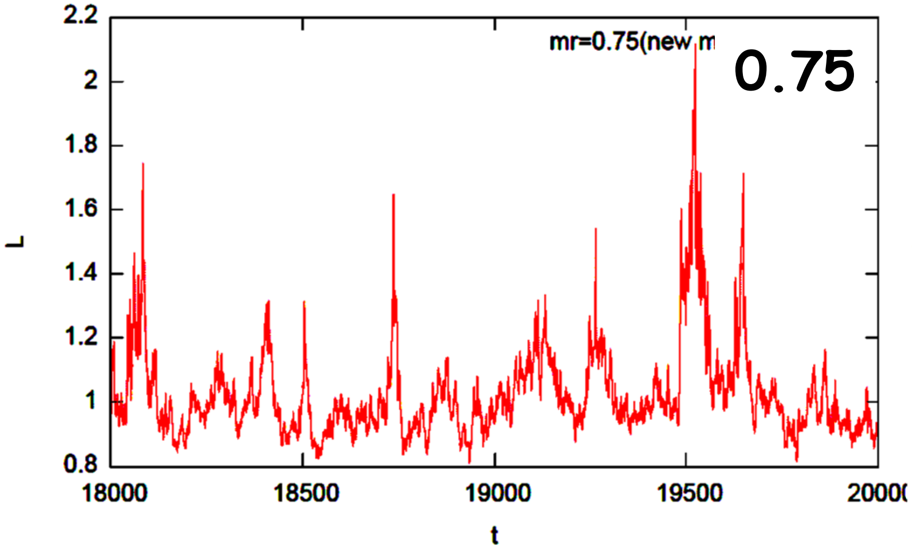


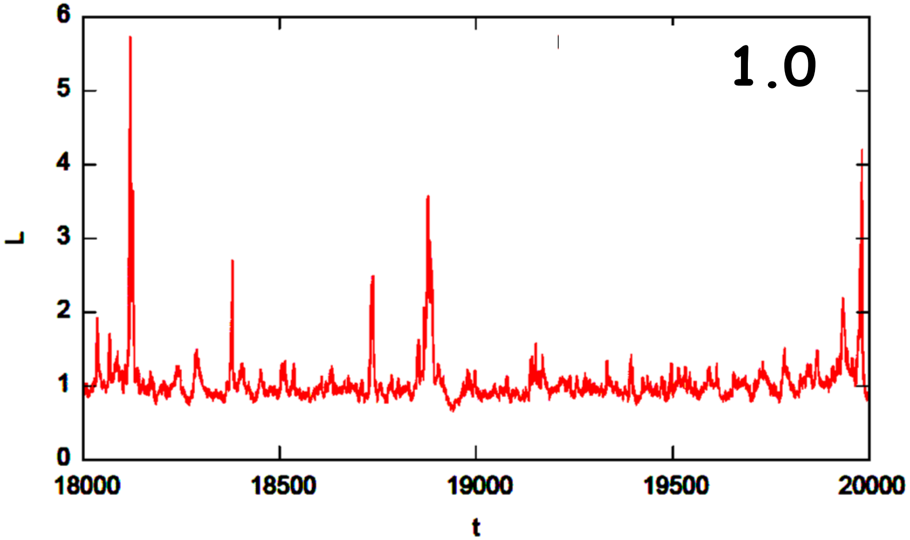
Parameter <δ> denotes mean fluctuation amp.

<δ>=0.5 (left)

0.75 (lower-left)

1.0 (lower-right)





Summary & future issues

Observations show rather complex time variation.

Basic observational features of black hole variability can be (roughly) reproduced by simple cellular automaton model(s).
The success of our model may indicate magnetic field fluctuations being the origin of black hole variability.

Fluctuation + dissipation + (local) interaction
→ 1/f type PSD, power-law size distribution,
lognormal flux distr..

Future issues:

Direct comparison with numerical (MHD) simulations.

Connection to the solar flares./ Global effects (?)

Application to other compact objects (gamma-ray bursts,…)