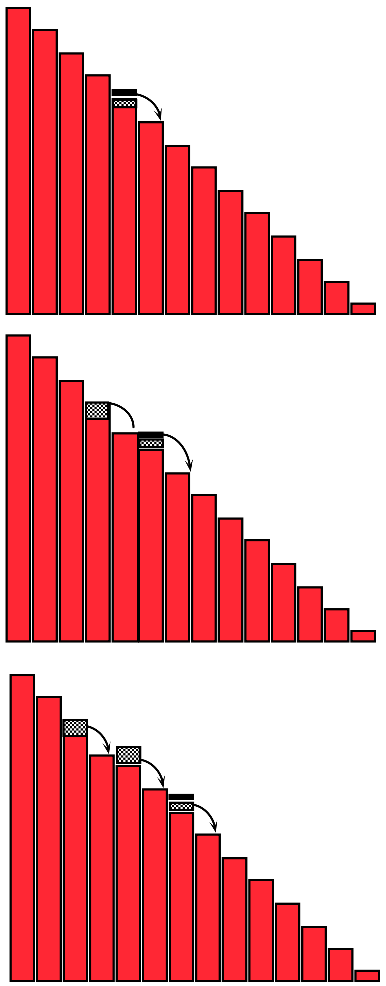
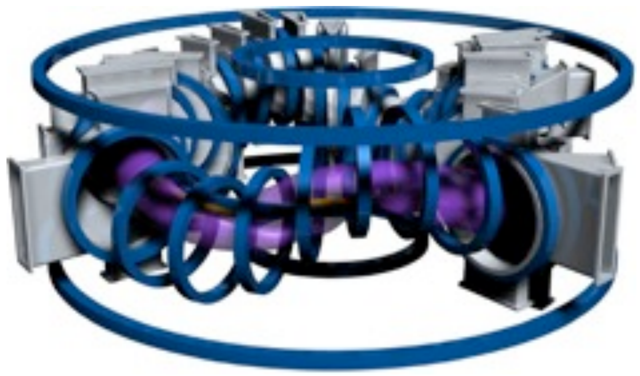
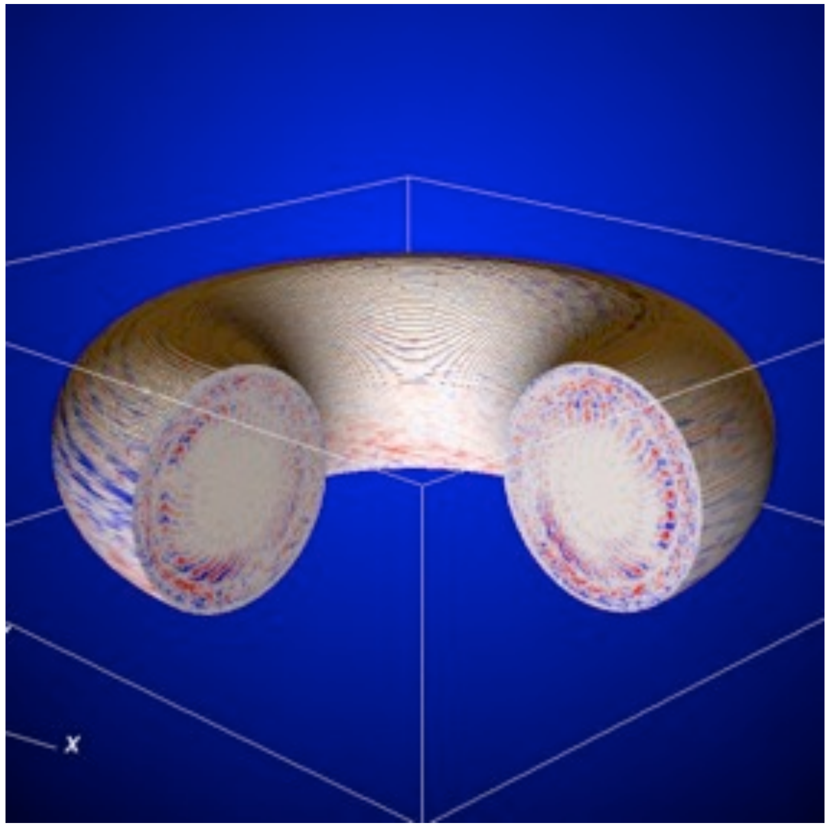
SELF-ORGANIZED CRITICALITY AND MAGNETICALLY CONFINED FUSION PLASMAS

Universidad Carlos III de Madrid







Raul Sanchez

Departamento de Física

Universidad Carlos III de Madrid, Leganés, SPAIN

ISSI Team Meeting - Self-Organized Criticality and Turbulence
October 14-19, 2012, Bern, Switzerland

A very large team effort.....

Universidad Carlos III de Madrid

David Newman, University of Alaska at Fairbanks, USA

Ben Carreras* and Vickie Lynch, Oak Ridge National Laboratory, USA

Luis García, Universidad Carlos III de Madrid

Viktor Decyk and Jean-Noel Leboeuf*, UCLA, USA

Boudewijn van Milligen and Ivan Calvo, CIEMAT, Madrid

Jose Angel Mier, Universidad de Cantabria (prev. @ UC3M)

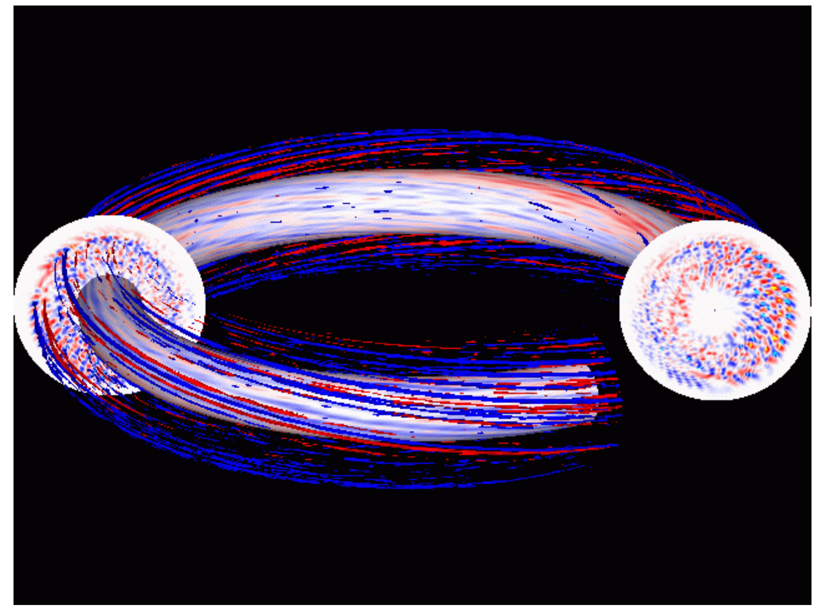
Ryan Woodard, ETZH, Switzerland (prev. @ UAF)

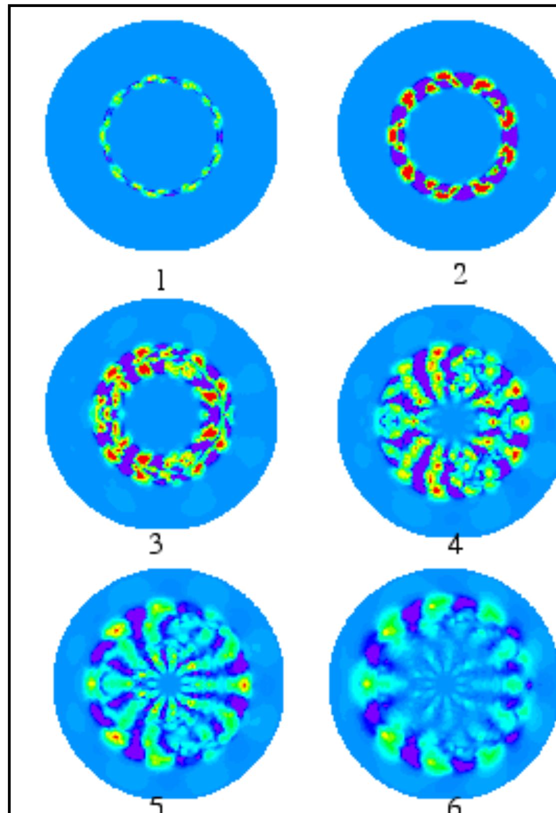
Debasmita Samaddar, ITER, Cadarache, France (prev. @ UAF)

(*)retired

Contents

Universidad Carlos III de Madrid





(V.E.Lynch, Oak Ridge National Lab, U.S.A.)

Self-Organized Criticality

The Directed Running Sandpile

SOC and magnetically-confined, toroidal fusion plasmas

Experimental evidence for SOC in fusion plasmas

Effective models of transport for turbulent fusion plasmas at a SOC state

SELF-ORGANIZED CRITICALITY

Universidad Carlos III de Madrid

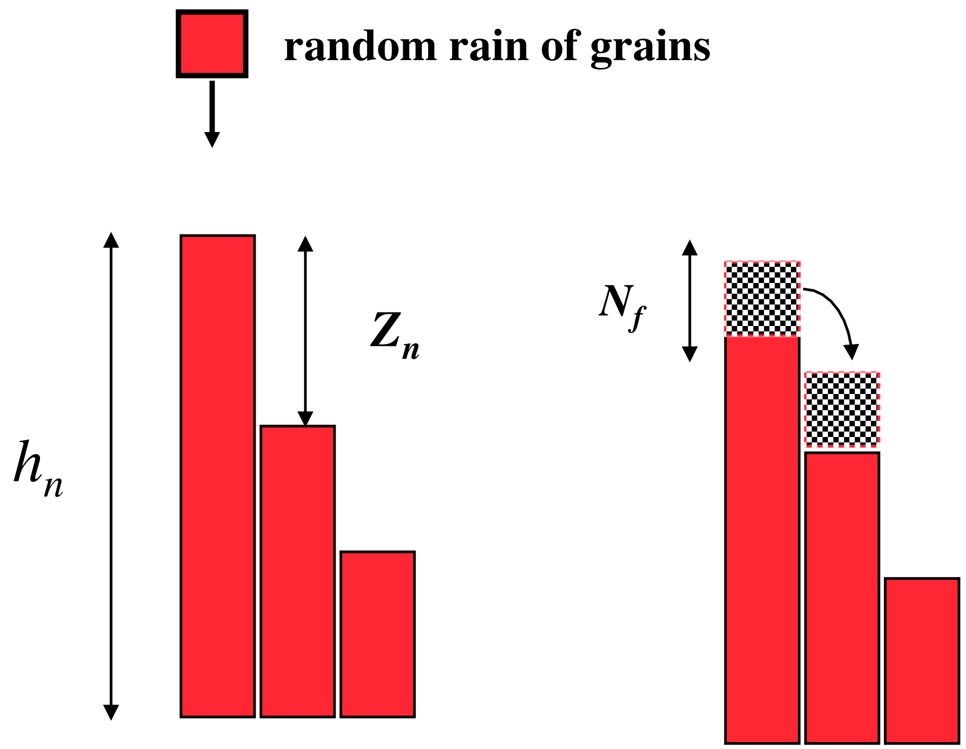
SELF-ORGANIZED CRITICALITY: concept introduced by P. Bak in 1987 to attempt to explain 1/f noise in many natural systems.

- Typically, requires the existence of a threshold that separates a quiescent phase from an active phase in which fast transport/redistribution takes place.
- It also requires a certain randomness/unpredictability in the system.

Bak's paradigmatic toy-model: the sandpile.

- Grains of sand dropped on a sandpile whose cells can go critical and turn when a certain local threshold condition is overcome.

Bak's original model was driven infinitely slow, with sand-addition being stopped as soon as an avalanche starts, and restarted when it stops.



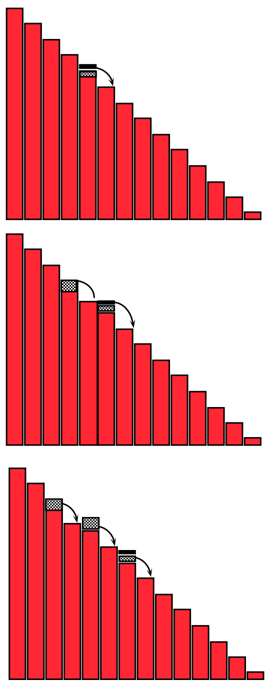
Steady-state (SOC state) exhibits self-similarity, long-term correlations and other properties typical of equilibrium critical states but without the need of external tuning.

DIRECTED RUNNING SANDPILE

Universidad Carlos III de Madrid

The directed running sandpile (DRS), using as the critical threshold the local slope, is much closer in spirit to the real situation that turbulent fusion plasmas experience. It does not stop the external drive while avalanches take place.

[See: T. Hwa and M. Kardar, Phys. Rev. A 45, 7002 (1992)]

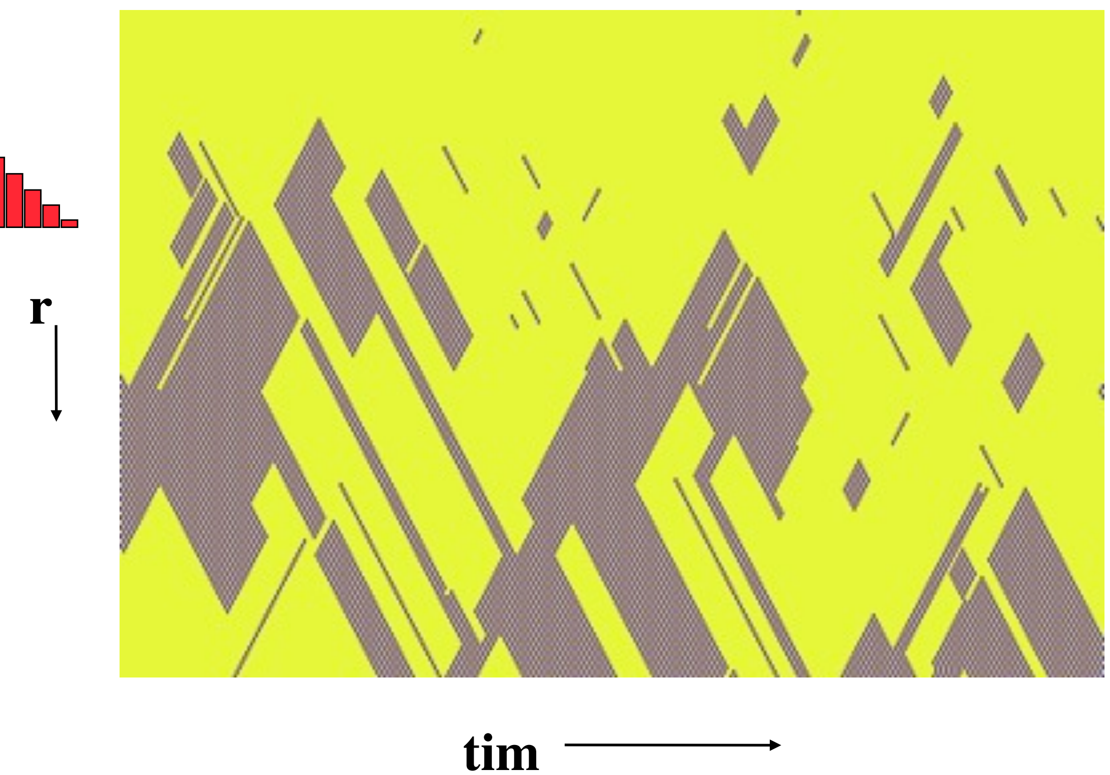


SOC-state exhibits self-similar, scale-free statistics of avalanche sizes and energies

It can maintain a net non-zero outflux in spite of being submarginal on average

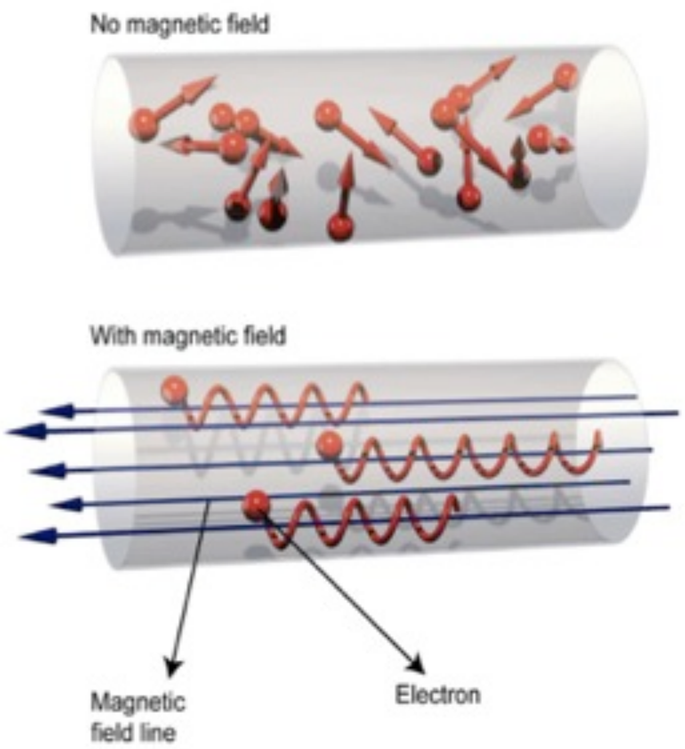
Intermittent transport is strongly correlated in time via shaping of the height profile

The shape of the height profile at the SOC-state is rather insensitive to the location of the source as long as the system is not overdriven

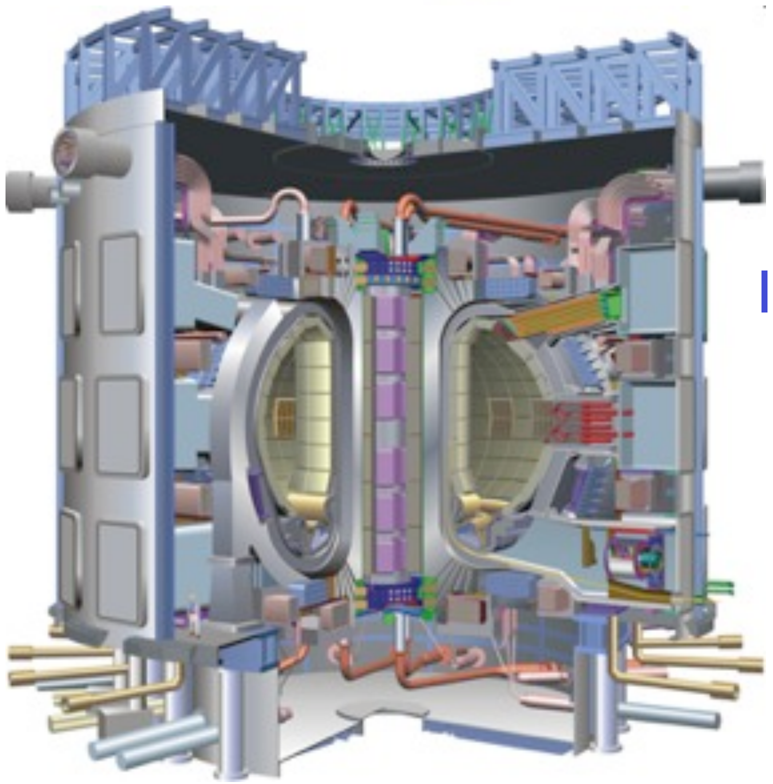


Avalanche overlapping is a real issue (invoked to explain the 1/f spectrum by HK)

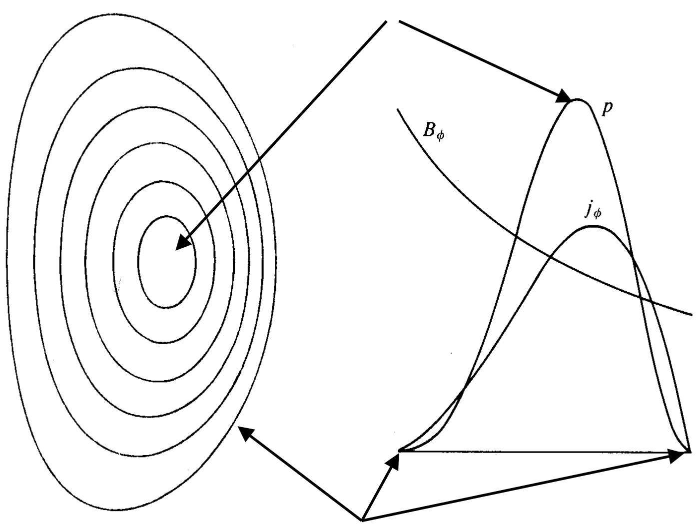
FUSION TOROIDAL PLASMAS



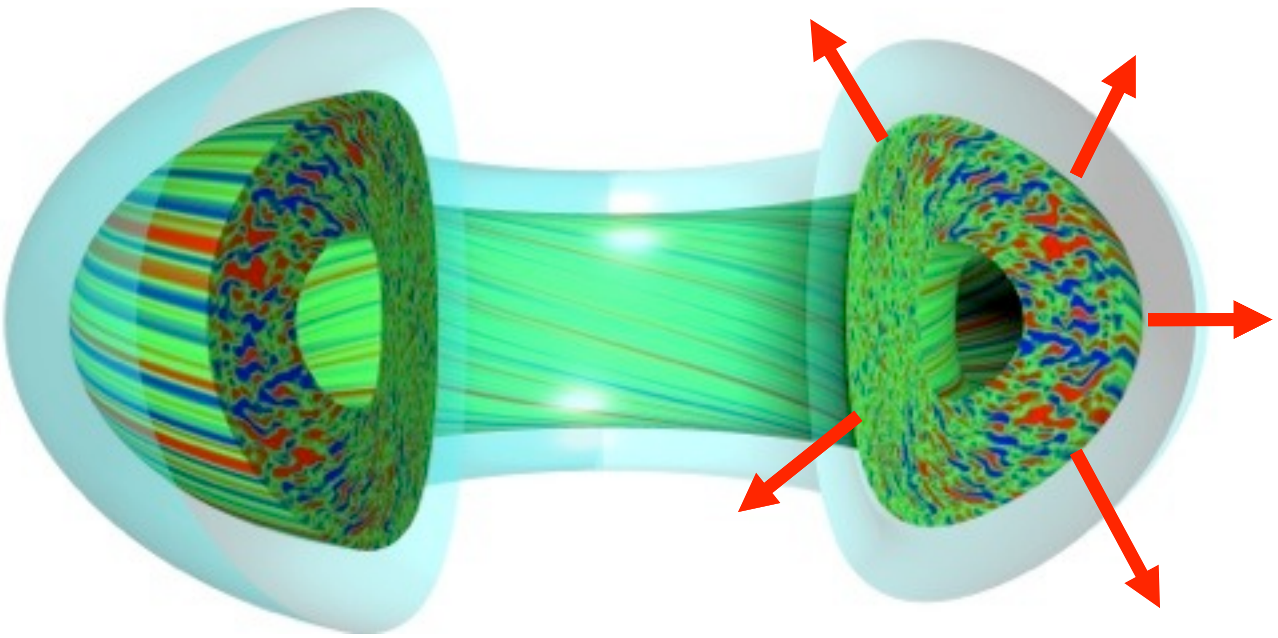
Helical magnetic fields can be used to confine hot plasmas long enough to produce energy



ITER



Turbulence dominates radial losses of energy and particles

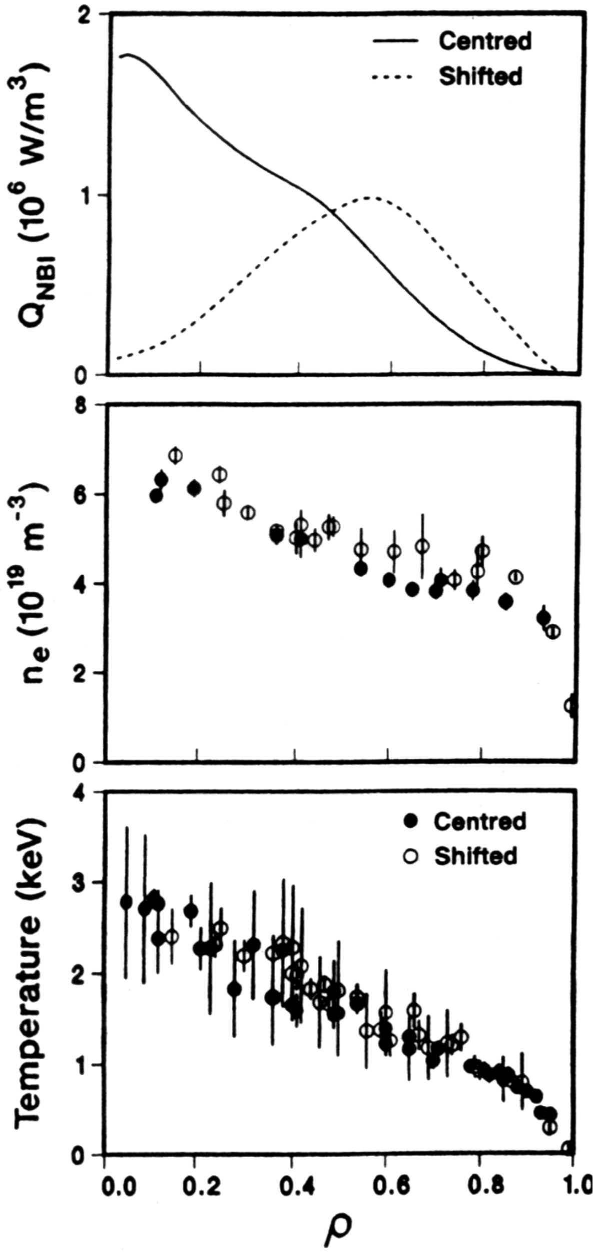


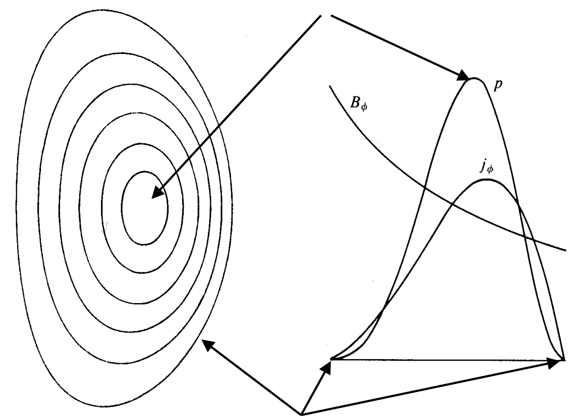
GYRO, General Atomics

L-MODE TOKAMAK PLASMAS

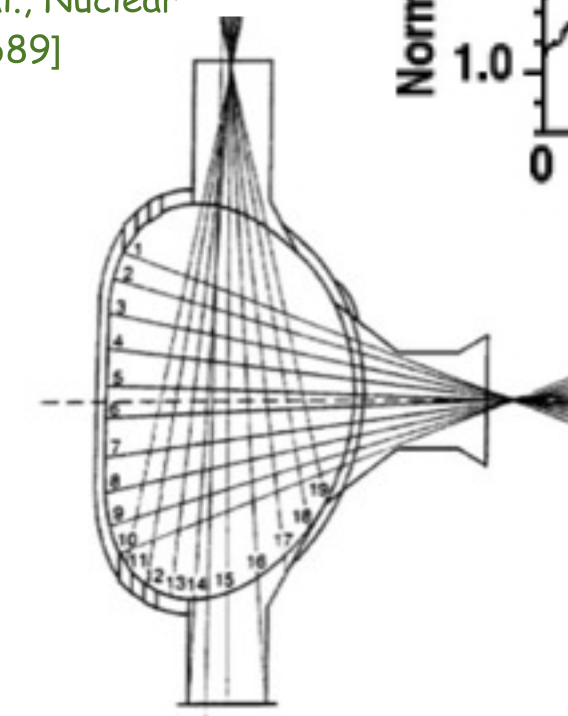
Universidad Carlos III de Madrid

Profiles insensitive to external drive location

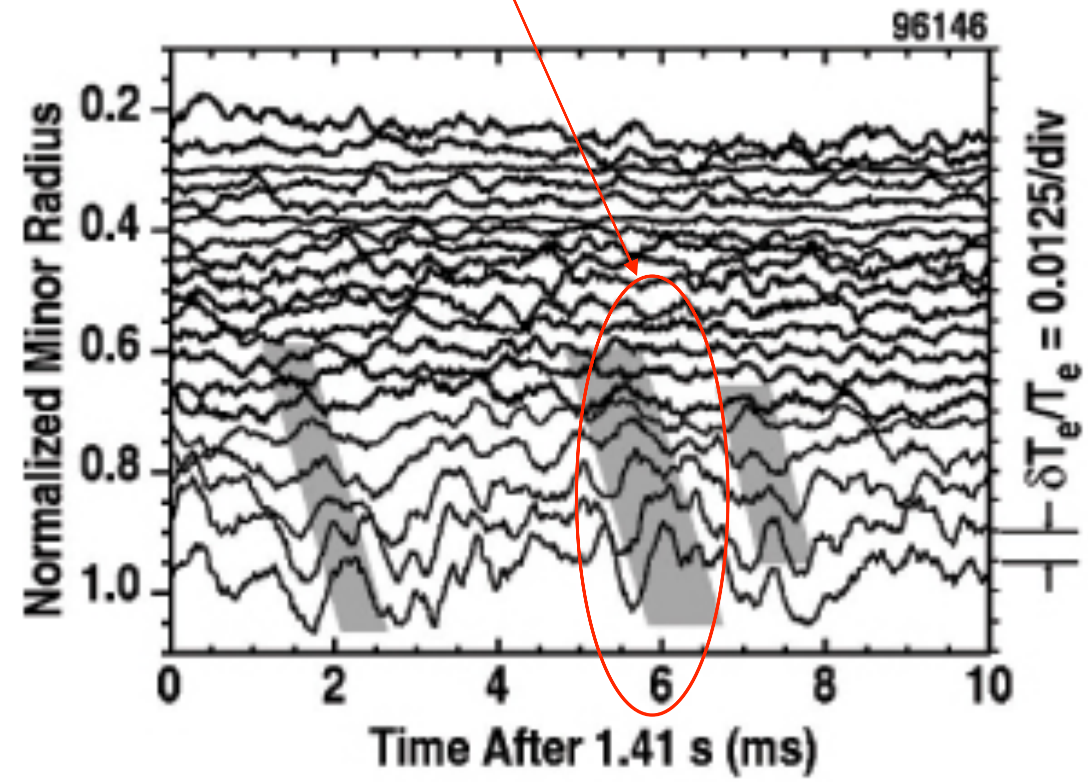




[See: Schissel et al., Nuclear Fusion **32** (1992) 689]



Radial avalanches seen in ECE diagnostic



[P.Politzer et al, Phys.Plasmas **9** (2002) 1962]

SOC hypothesis suggested as an explanation?

Lots of activity (not comprehensive...)

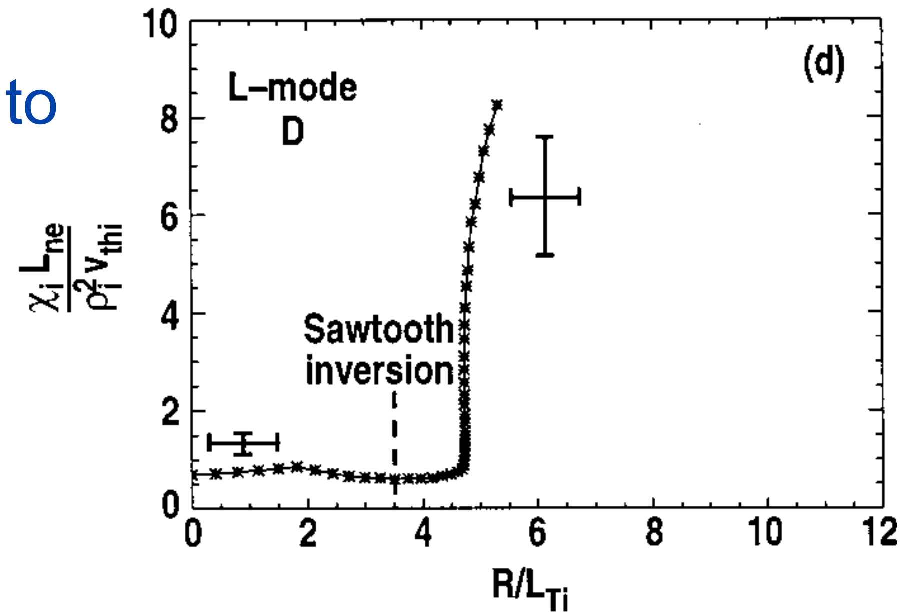
Universidad Carlos III de Madrid

P.H. Diamond et al, Phys. Plasmas 2, 3640 (1995) – Theory, PDE
D.E. Newman et al, Phys. Plasmas 3, 1858 (1996) – Sandpile, near-marginal dyns, shear flow effects
B.A. Carreras et al, Phys. Plasmas 3, 2904 (1996) – Simulation (Interch. fluid), near marginal, avalanches
R. Dendy et al, Pl. Phys. Contr. Fusion 39, 1947(1997) – Sandpile, near-marginal dyns
X. Garbet et al, Phys. Plasmas 5, 2836 (1998) – Simulation (gyrofluid, DTEM), near marginal, flux-driven
Y. Sarazin et al, Phys. Plasmas 5, 4214 (1998) – Simulation (gyrofluid, ITG), near marginal, flux-driven
T. Rhodes et al., Phys. Lett. A 253, 181 (1998) – Experimental, tokamak, fluctuation statistics
B.A. Carreras et al., Phys. Rev. Lett. 80, 4438 (1998) – Experimental, Hurst exponents, edge fluctuations
M.A. Pedrosa et al., Phys. Rev. Lett. 82, 3621 (1999) – Experimental, power spectra, edge fluctuations
B.A. Carreras et al., Phys. Rev. Lett. 83, 3653 (1999) – Experimental, self-similarity, edge fluctuations
P. Beyer et al, Pl. Phys. Contr. Fusion 41, A757 (1999) – Experimental, tokamak, edge fluctuations
B. A. Carreras et al, Phys. Rev. E 60, 4770 (1999) – Sandpile, Effective Tr. Model (CTRWs)
B. A. Carreras et al., Phys. Plasmas 8, 5096 (2001) – Simulation (Interch., fluid), Eff. Tr. Model (CTRWs)
R. Sanchez et al., Nucl. Fusion 41, 247 (2001) – Sandpile, diffusion effects, ELMs
S. Chapman et al, Phys. Rev. Lett. 86, 2814 (2001) – Sandpile, ELMs
E. Spada et al, Phys. Rev. Lett. 86, 3032 (2001) – Experimental (against) – W. times, No s-similarity
R. Sanchez et al, Phys. Rev. Lett. 88, 068302 (2002) – Sandpile, Wait. times
P.A. Politzer et al, Phys. Plasmas 9, 1962 (2002) – Experimental, avalanche viz., ECE, tokamak
L. García et al, Phys. Plasmas 9, 841 (2002) – Theory, PDE (continuum sandpile)
R. Sanchez et al, Phys. Rev. Lett. 90, 185005 (2003) – Experimental – W. times, Response to Spada's PRL
I Gruzinov et al, Phys. Plasmas 10, 569 (2003) – Sandpile, ELMs
V. Tangri et al, Phys. Rev. Lett. 91, 025011 (2003) – Theory, PDE (continuum sandpile)
Y Xu et al, Phys. Plasmas 11, 5413 (2004) – Experimental, Tokamak, edge fluctuations
B.Ph. van Milligen et al, Phys. Plasmas 11, 2272 (2004) – Theory, Effective Tr. Model (CTRWs/Master Eqs.)
D. del-Castillo-Negrete et al, Phys. Plasmas (2004) – Simulation (Interch., fluid), Eff. Tr. Model (Fract. Diff. Eqs)
B. Dudson et al, Pl. Phys. Contr. Fusion 47, 885 (2005) – Experimental, spherical tokamak
L. Marrelli et al, Phys. Plasmas 12, 030701 (2005) – Experimental, reverse field pinch
R. Sanchez et al, Phys. Rev. E 74, 016305 (2006) – Theory, Eff. Tr. Models (FBM/FLM -> FDEs)
J.A. Mier et al, Phys. Plasmas 13, 102308 (2006) – Simulation (DTEM, fluid), effect of diffusion
F. Sattin et al, Phys. Rev. Lett. 96, 105005 (2006) – Sandpile, include WTs
J. A. Mier et al, Phys. Rev. Lett. 101, 165001 (2008) – Simulation (DTEM, fluid), Eff. Tr. Model (Lagr. Diag.)
Y. Sarazin et al, Nucl. Fusion 50, 054004 (2010) – Simulation (ITG, gyrokinetics), near-marg., avalanches
H. Isliker et al, Phys. Plasmas 17, 082303 (2010) – Sandpile, ITG-like
J. Maggs et al, Phys. Rev. Lett. 107, 185003 (2011) – Experimental (against) – Lorenzian, random pulses
S. Tokunaga et al, Phys. Plasmas 19, 092303 (2012) – Experimental, tokamak, enhanced modes
B. van Milligen et al, Phys. Rev. Lett. 109, 105001 (2012) – Experimental – Response to Magg's PRL
GZ dos Santos et al, Phys. Lett. A 376, 753 (2012) – Experimental, tokamak

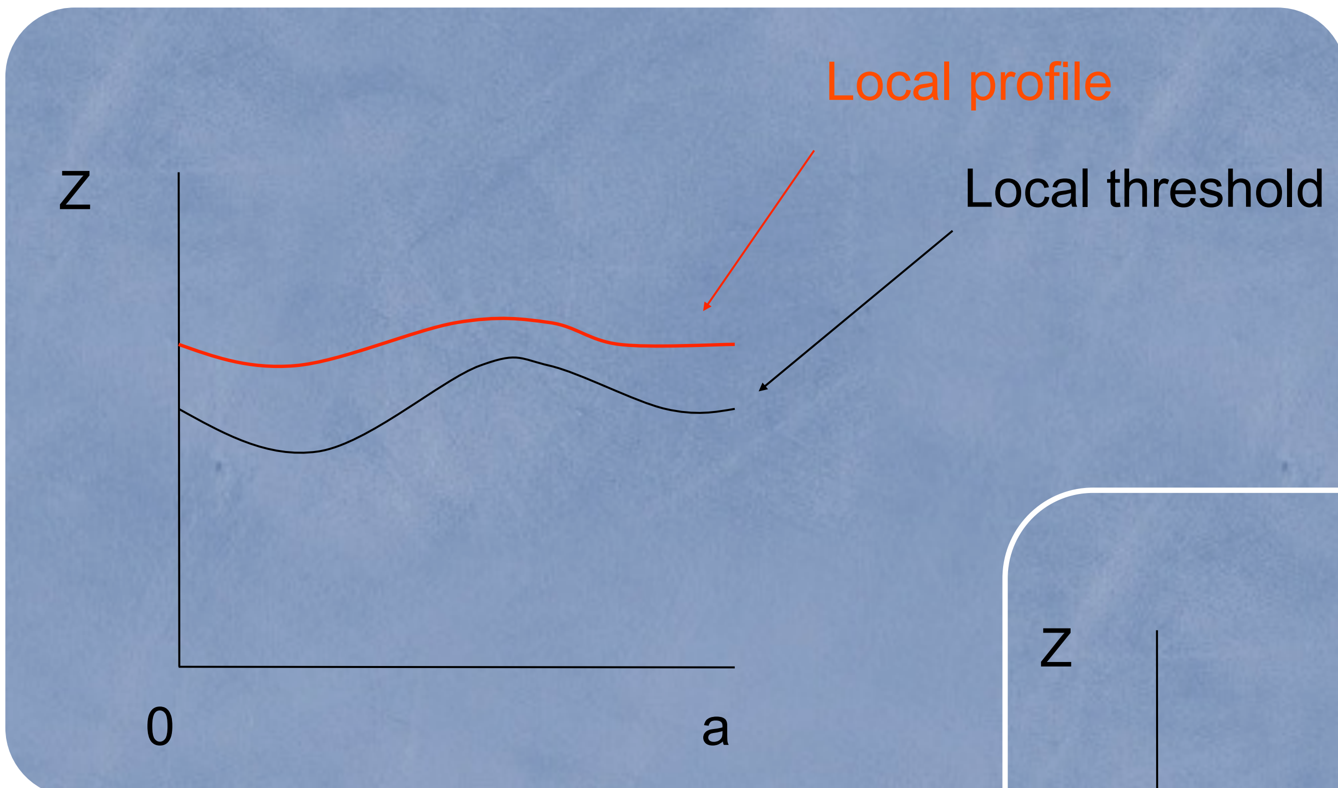
NEAR-MARGINAL TURBULENCE

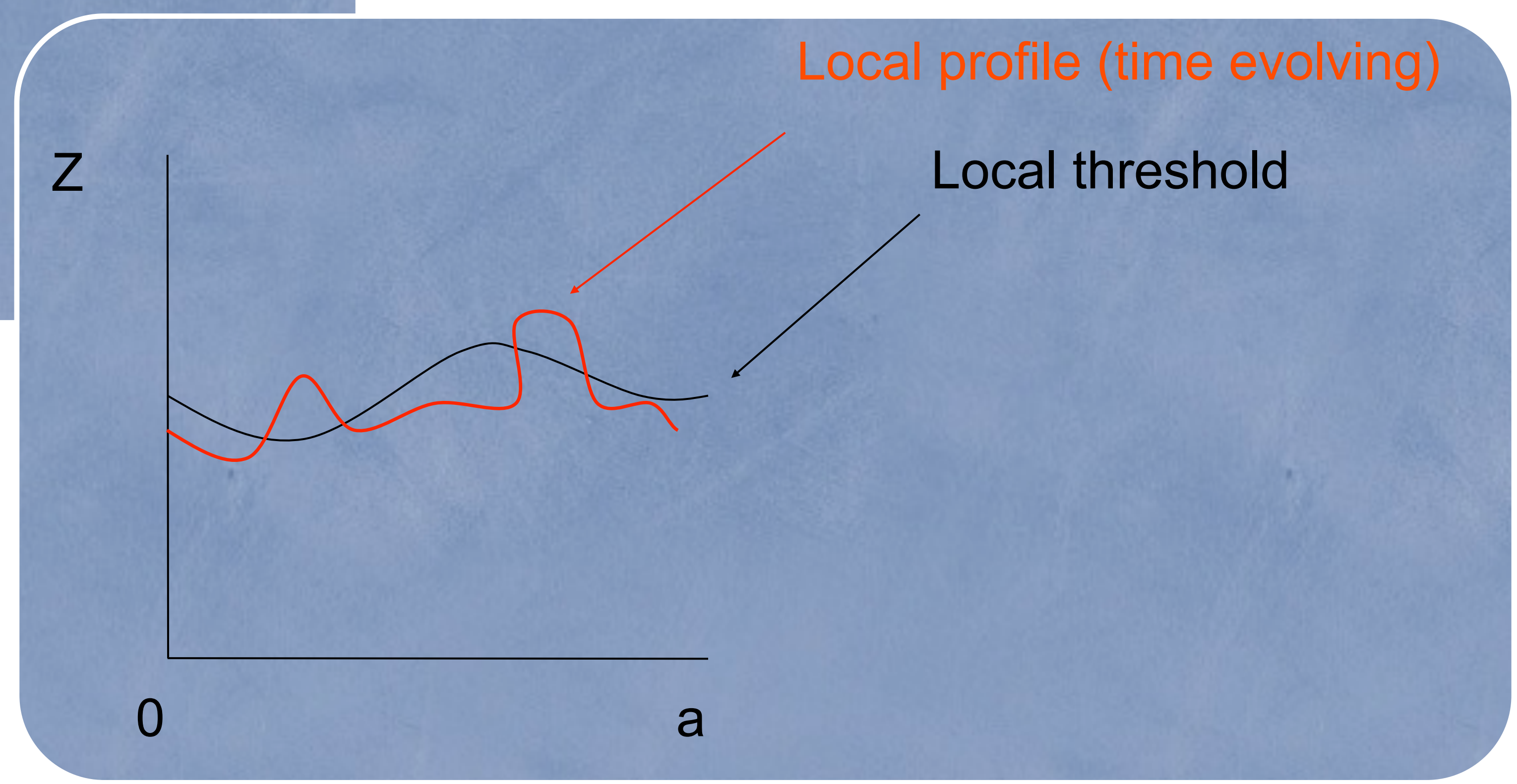
Universidad Carlos III de Madrid

Some of the aforementioned phenomenology could be understood if the confined plasma has its profiles close to marginal values for the onset of turbulence.

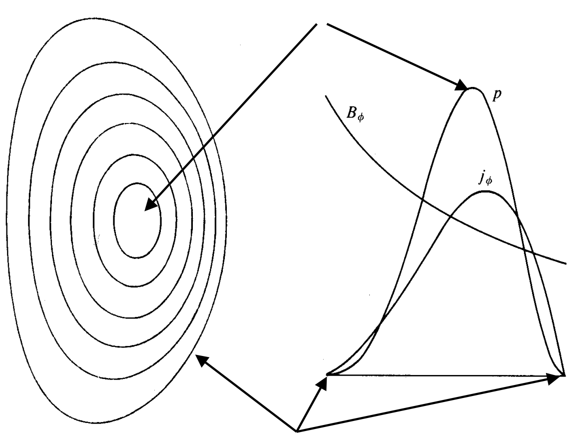


[See: D.R. Baker et al, Physics of Plasmas 8, 4138 (2001)]





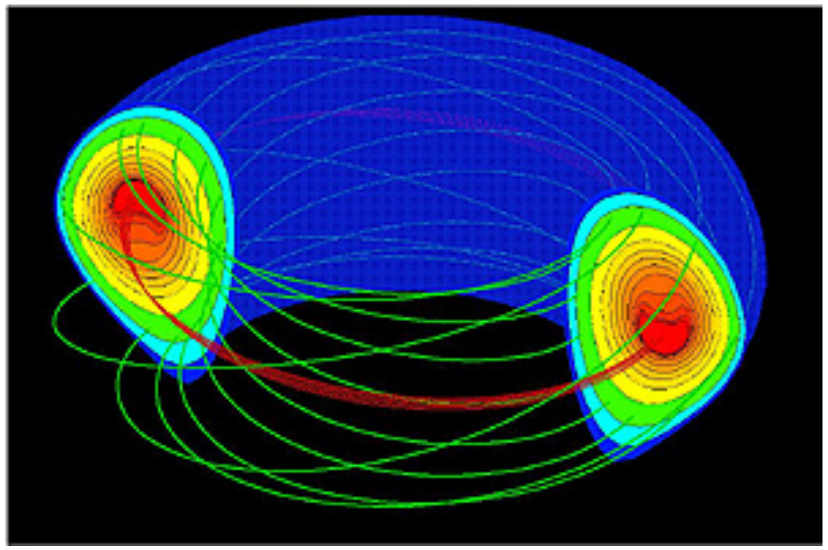
[See: P.H. Diamond and T.S. Hahm, Physics of Plasmas 3640, (1995)]

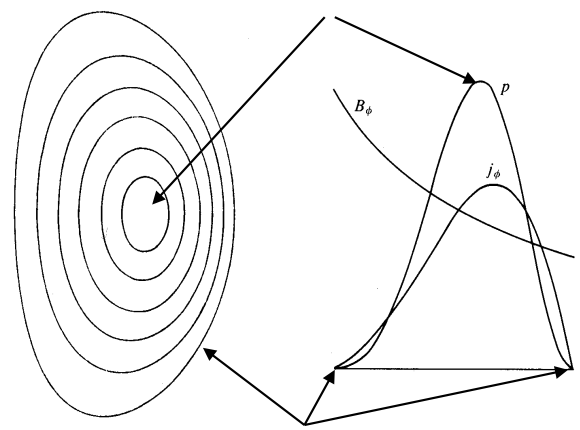


SOC AND TOKAMAK PLASMAS

Universidad Carlos III de Madrid

An analogy between HK's sandpile an a magnetically confined toroidal fusion plasma can be easily done if it is near-marginal conditions.





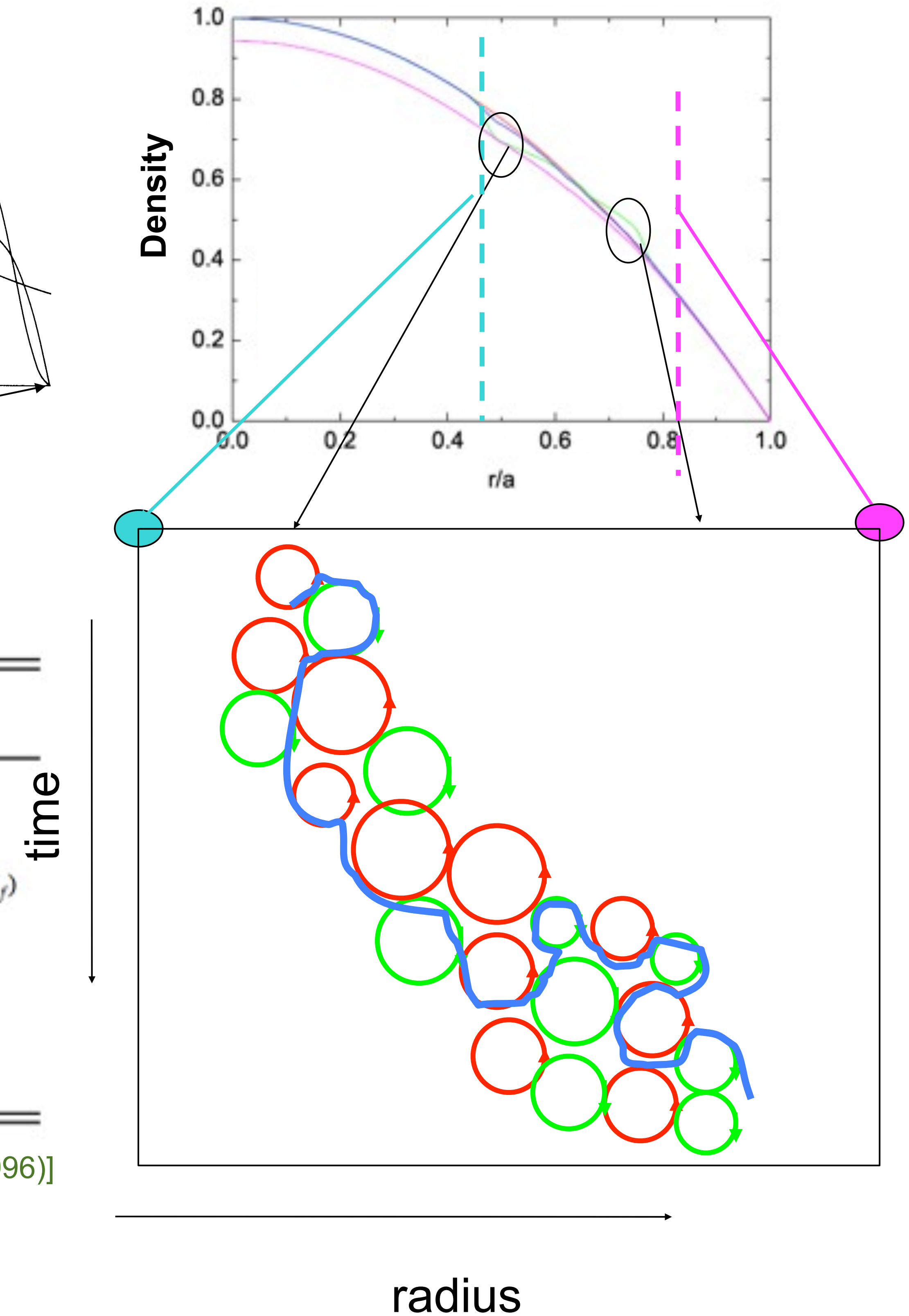


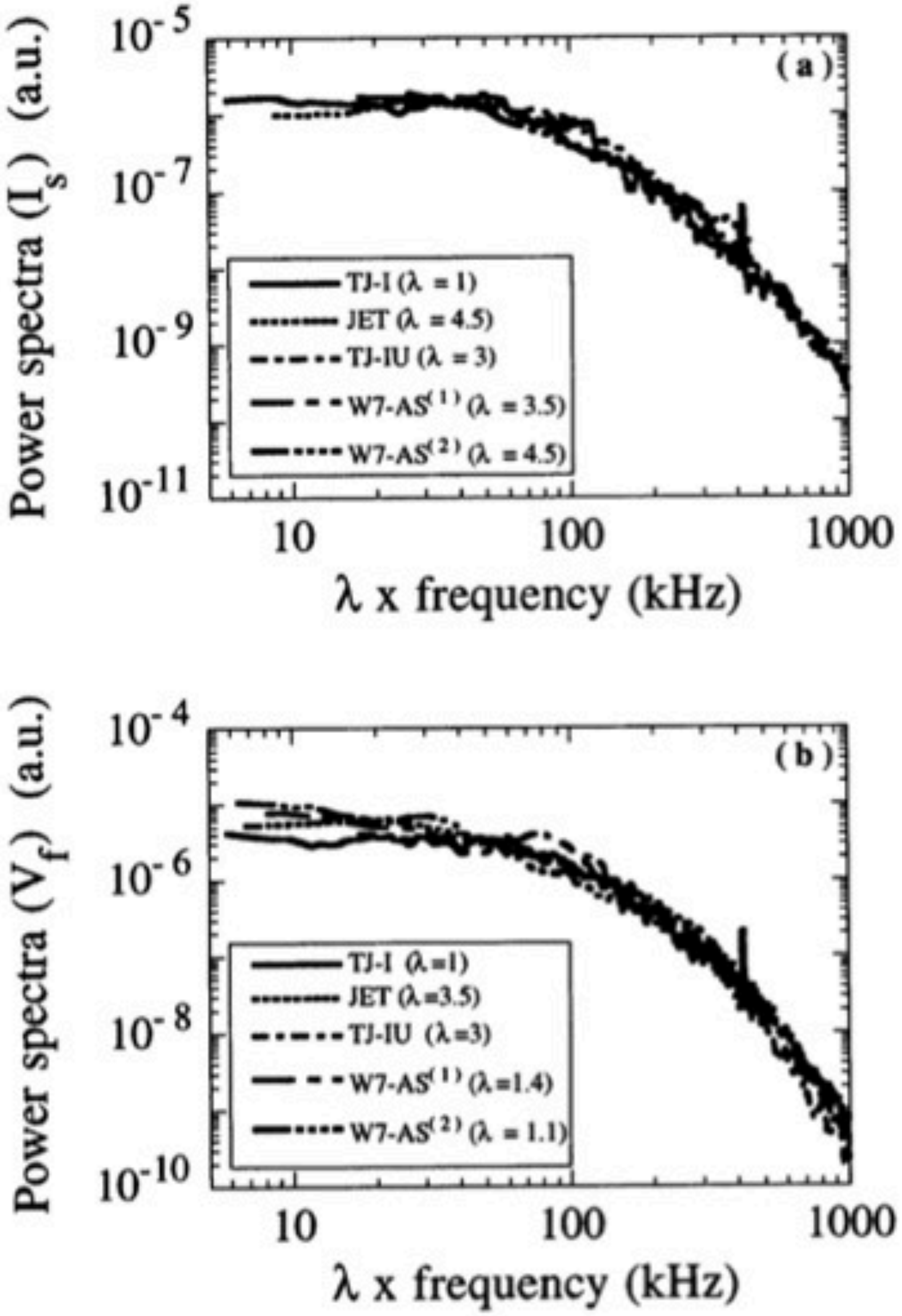
TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

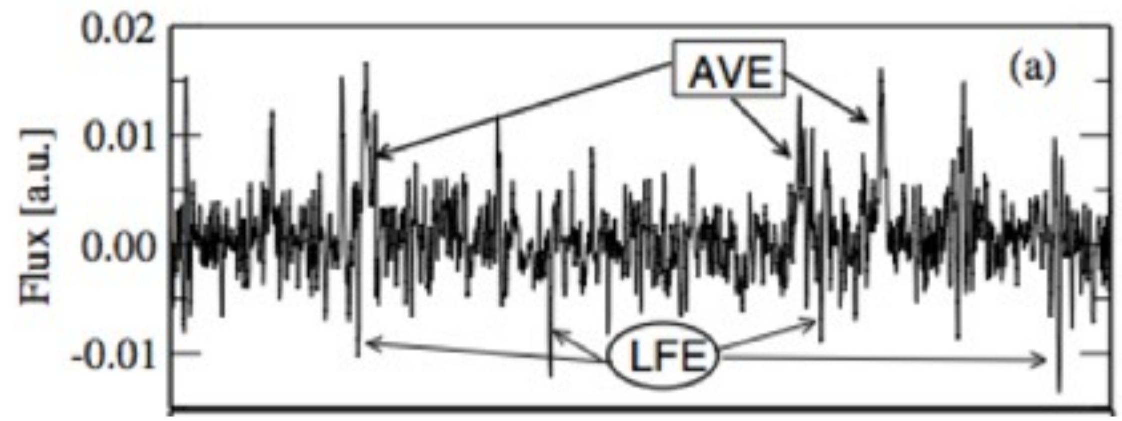
Turbulent transport in toroidal plasmas	Sandpile model
Localized fluctuation (eddy)	Grid site (cell)
Local turbulence mechanism:	*Automata rules*:
Critical gradient for local instability	Critical sandpile slope (Z_{crit})
Local eddy-induced transport	Number of grains moved if unstable (N_f)
Total energy/particle content	Total number of grains (total mass)
Heating noise/background fluctuations	Random rain of grains
Energy/particle flux	Sand flux
Mean temperature/density profiles	Average slope of sandpile
Transport event	Avalanche
Sheared electric field	Sheared flow (sheared wind)

[See: D.E. Newman et al, Phys. Plasmas 3, 1858 (1996)]

EVIDENCE FOR SOC: Power spectra

Universidad Carlos III de Madrid





1/f regions in power spectra such as those exhibited by DRS were considered a trademark of SOC then.

1/f regions were sought for in fusion experiments, both tokamaks and stellarators.

Mainly, using edge fluctuation data measured with Langmuir probes. From these, time series of the turbulent fluctuations and turbulent fluxes can be obtained at a single radial location.

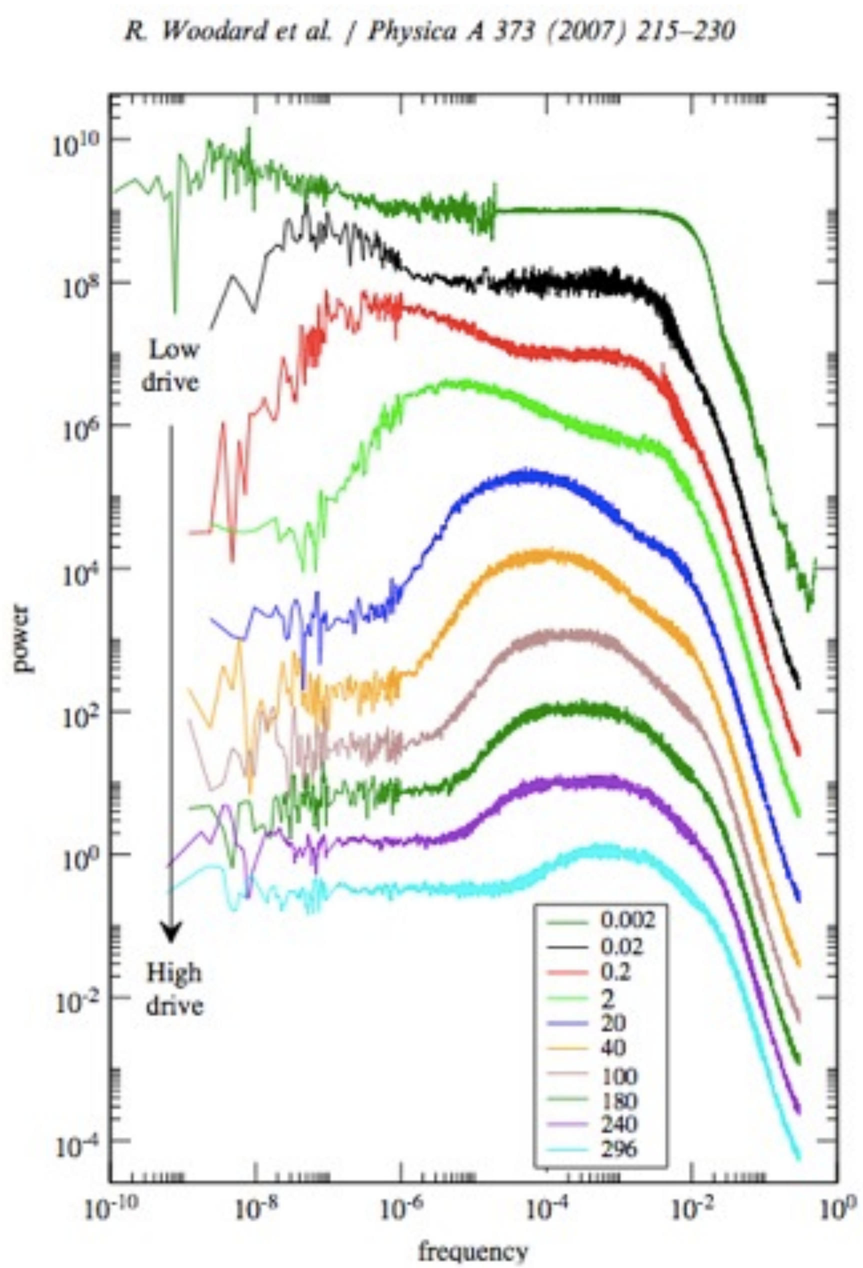
Self-similarity of power-spectra and power-laws close to 1/f were indeed reported.

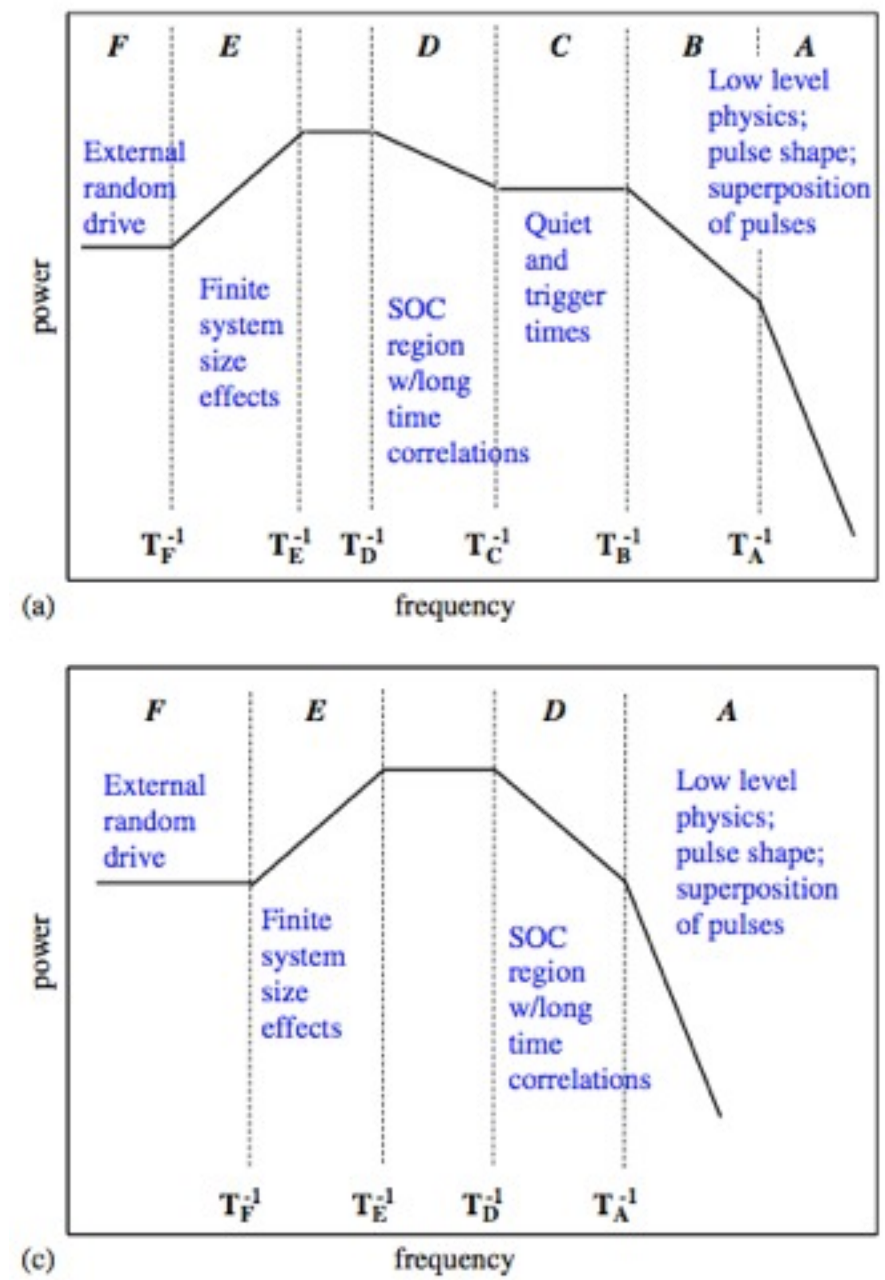
[See: M.A. Pedrosa et al, Phys. Rev. Lett. 82, 3621 (1999)]

EVIDENCE FOR SOC: Power Spectra?

Universidad Carlos III de Madrid

Warning: DRS spectra not always exhibit an 1/f, but an $1/f^s$ region, with $0<s<1$. The value of s is set by the competition between the different timescales in the problem (triggering, relaxation, etc.)





Hurst exponents: R/S analysis

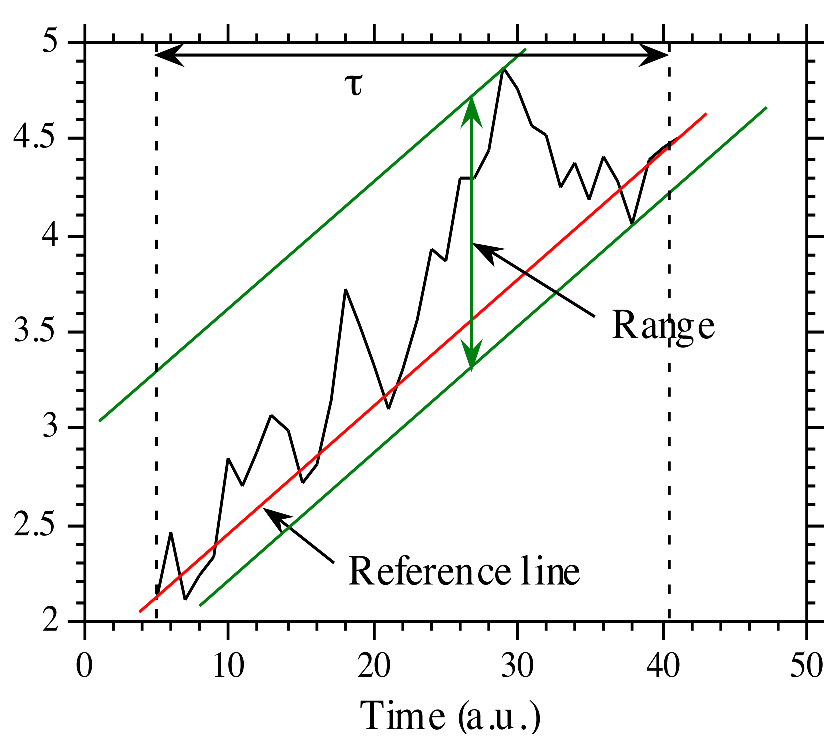
Universidad Carlos III de Madrid

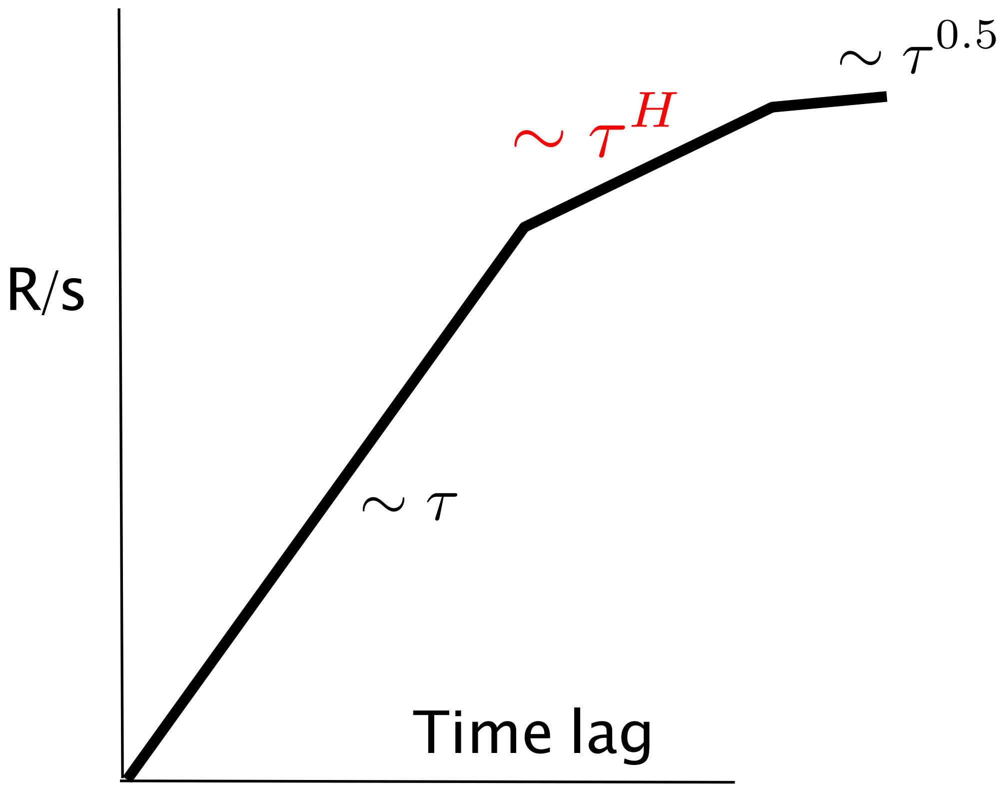
The Hurst exponent was introduced by Hurst (1952) to quantify correlation in time series.

He assumed that any stationary signal can be thought of as the ordered sequence of displacements of a particle. If the signal is random, the motion of such a particle will be that of a random walk, and the distance from its initial position will grow on average as $t^{1/2}$.

If the signal contains positive correlations between successive displacement, such distance will grow with a stronger exponent, t^H, $H > 1/2$. Similarly, if negative correlations exist, it will grow with $H < 1/2$.

H, the Hurst exponent, can be determined in many ways. One of the most popular ones is the R/S method, which is one of the more un-sensitive to noises.





EVIDENCE FOR SOC: Hurst exponents

Universidad Carlos III de Madrid

It is found that Hurst exponent values (for activity or local/global fluxes) are more robust than power-spectra exponents to capture DRS SOC dynamics.

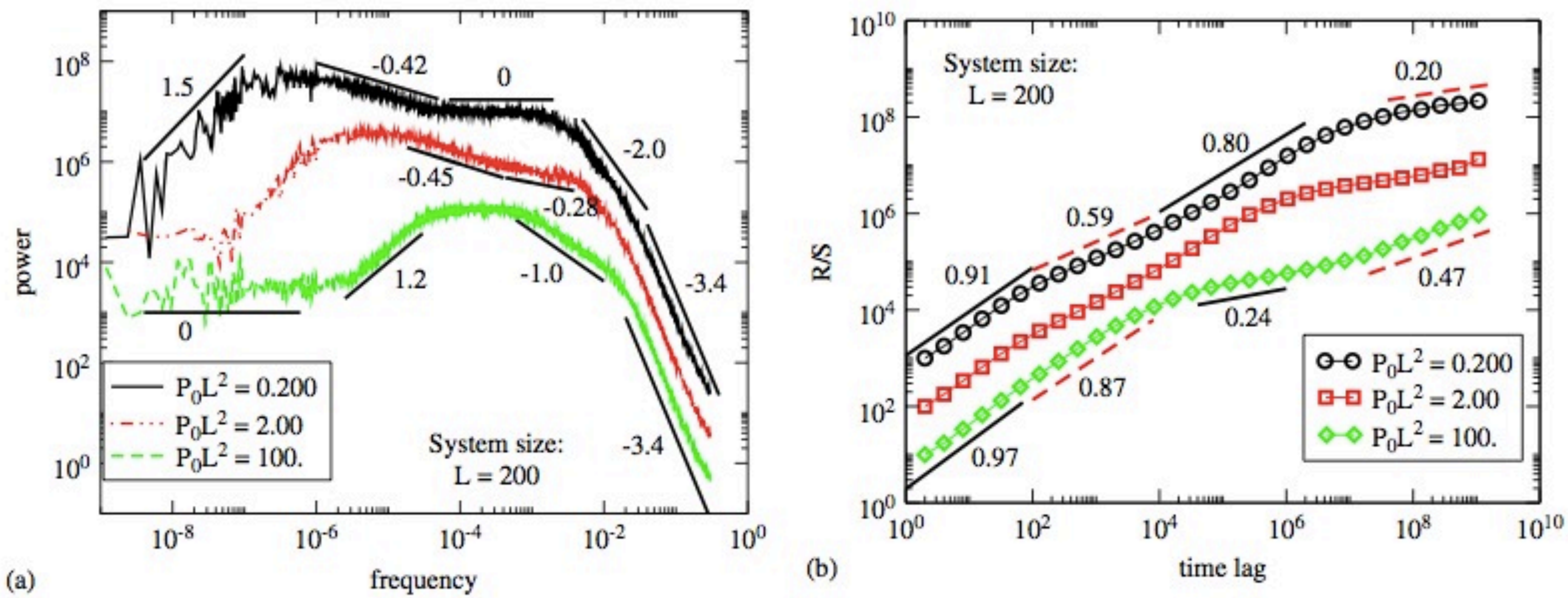


Fig. 3. (Color online) (a) Power spectra and (b) R/S analysis of activity for fixed system size and low, medium and high driving rates. The y values of both measures have been shifted for easier viewing. In the spectra (R/S), six (five) regions of low-drive and four regions of high-drive are shown by solid lines. Lines are power laws; numbers show values of β in spectra and H in R/S. Lowest frequency f^0 region of spectra and $H = 0.5$ of R/S for the low-drive case is not seen because of the finite size of the time series. Its existence is assumed based on the f^0 regions seen in the spectra of higher drive cases.

[See: R. Woodard et al, Physica A 373, 215 (2007)]

EVIDENCE FOR SOC: Hurst exponents

Universidad Carlos III de Madrid

A plethora of fusion experiments showed positively correlated Hurst exponents beyond the turbulent typical scales: the mesoscale.

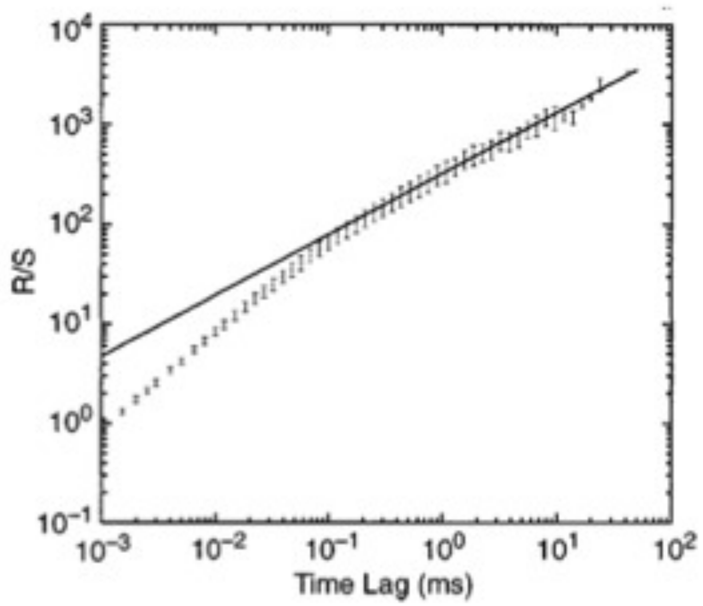
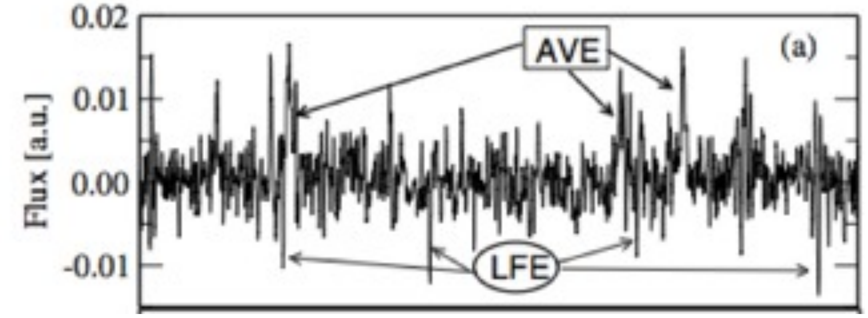


FIG. 9. A plot of the expected value of R/S as a function of the time lag for the same time record of W7-AS as in Fig. 1 (Ref. 33).



[See: B. Carreras et al, Phys. Plasmas 5, 3632 (1998)]

TABLE III. A summary of the analysis results.

Device	Number of time series	$\langle H \rangle_{in}$	$\langle H \rangle_{out}$	τ_D (μs)	Self-similarity range (ms)
TJ-I	9	0.64±0.03	0.70±0.04	3.0	0.02–1.0
JET limiter	4	⋯	0.52±0.04	29.0	0.1–2.0
JET divertor	4	⋯	0.63±0.03	19.0	0.1–2.0
TJ-IU	21	0.64±0.03	0.67±0.01	6.0	0.1–2.0
W7-AS ι_a=0.243	24	0.62±0.01	0.60±0.04	20.0	1–20
W7-AS ι_a=0.355	29	0.72±0.07	0.66±0.06	19.0	1–20
ATF	20	0.71±0.03	0.92±0.07	34.0	1–12
RFX	29	0.69±0.04	⋯	3	0.03–3.0
Thorello	10	0.55±0.04	⋯	6	0.05–5.0

EVIDENCE FOR SOC: Waiting times

In the late-90s, it was suggested that the statistics of waiting-times between avalanches could be used as a test for SOC.

In the original Bak sandpile and in DRS, waiting-times followed Poisson (exponential) statistics, due to the randomness of the drive.

This statement must be made more precise, though. When sufficiently large avalanches are considered, correlations are indeed apparent, and power-laws appear.

Furthermore, for non-random drives, power-laws may appear with no thresholding. These, however, are a reflection of drive correlations, not dynamical ones. Via thresholding, the SAME power-law can be made to appear as in the random drive case!

[See: R. Sanchez et al, Phys. Rev. Lett.88, 068302 (2002)]

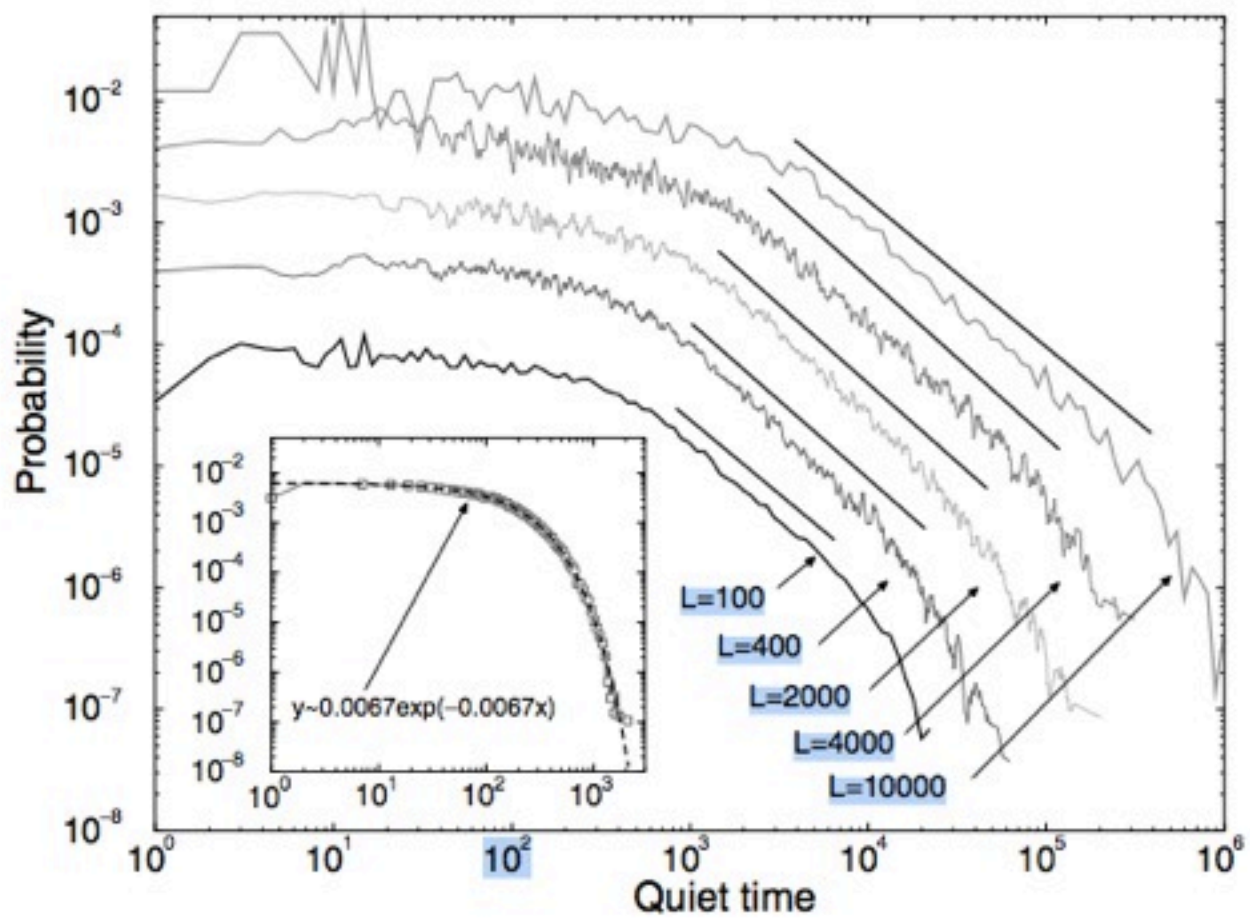


FIG. 2. Quiet times PDFs, for sandpiles with $L = 100$, 400, 2000, 4000, and 10 000, respectively thresholded using $d_t = 30$, 75, 200, 400, and 800. The two upper and two lower PDFs have been, respectively, shifted up and down by a half and a full decade to allow for easier comparison. In the inset, the PDF is shown without thresholding.

EVIDENCE FOR SOC: Waiting times

Universidad Carlos III de Madrid

The waiting-time statistics of edge fluctuation data from tokamaks and stellarators are consistent with SOC waiting-time statistics.

[See: R. Sanchez et al, Phys. Rev. Lett. 90, 185005 (2003)]

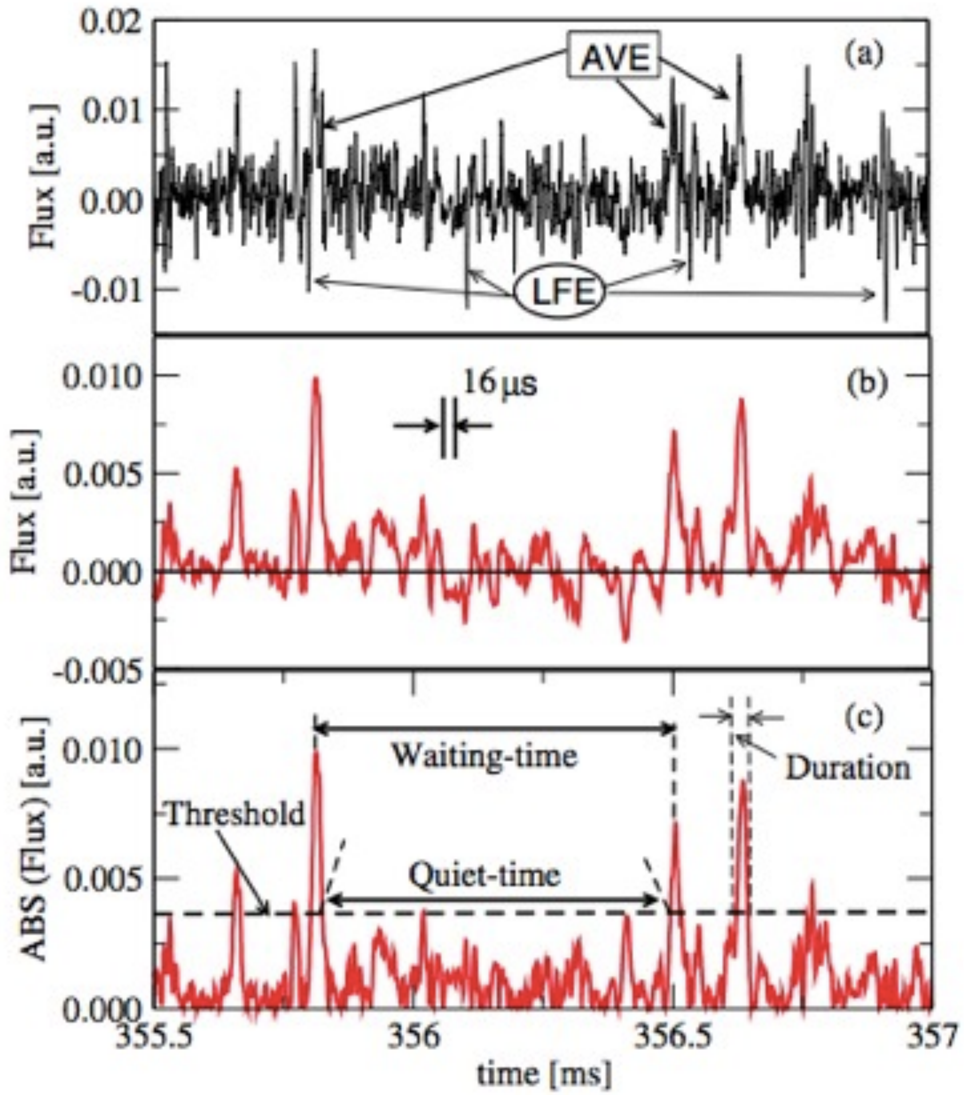


FIG. 1 (color online). (a) Detail of raw W7-AS flux signal; (b) same signal averaged with $m = 32$; (c) absolute value of averaged signal together with a sketch of relevant definitions.

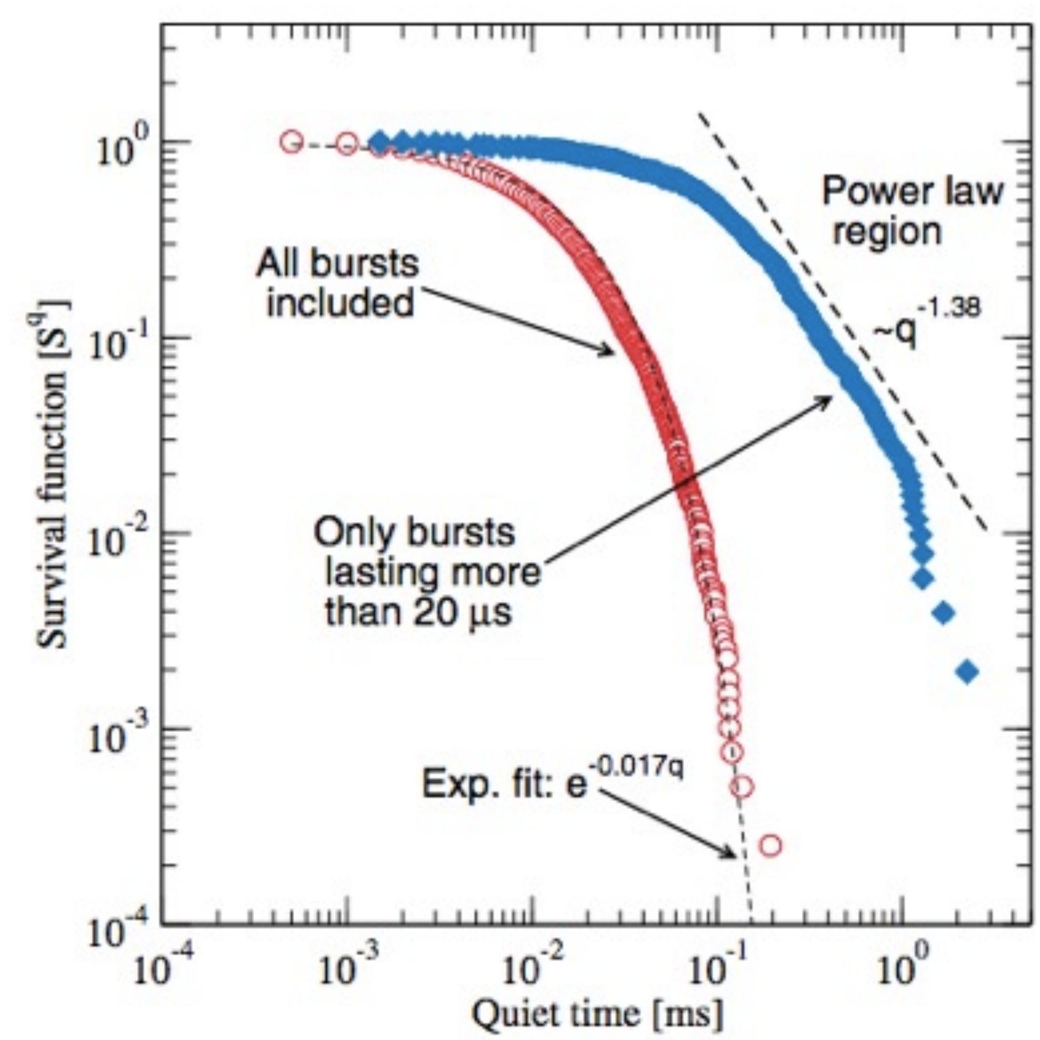
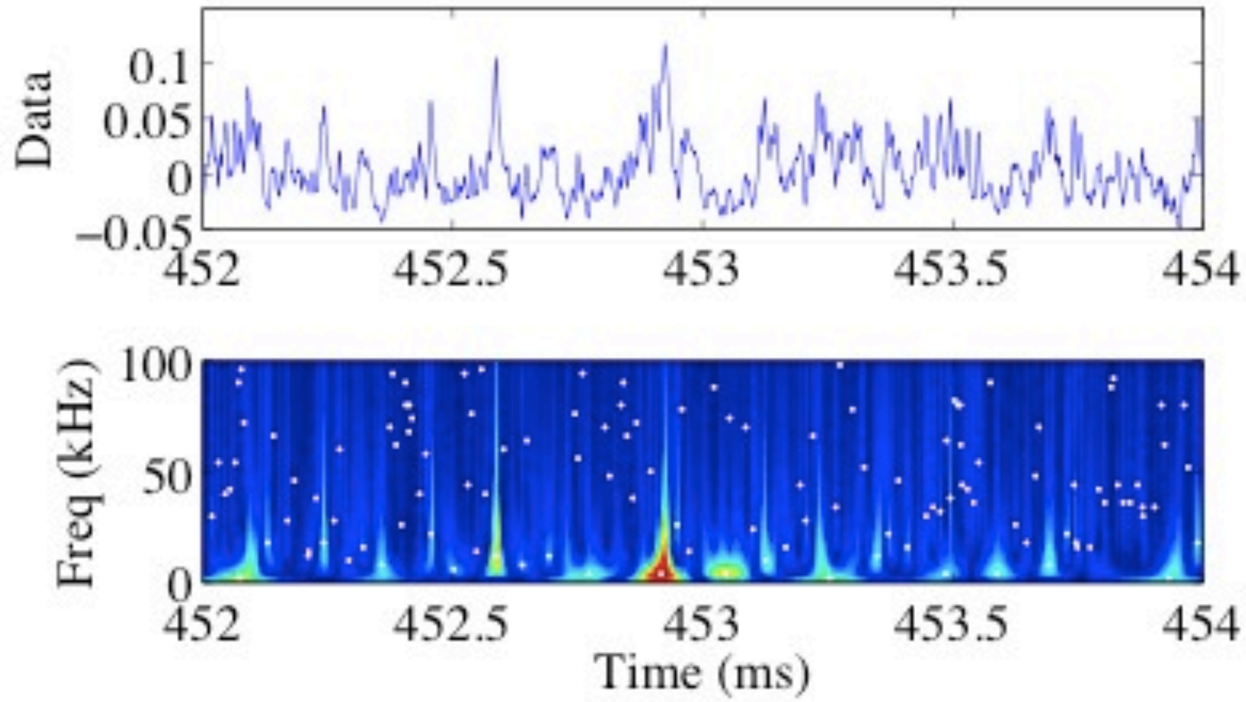


FIG. 2 (color online). Examples of quiet-time survival functions for W7-AS shot No. 35427.

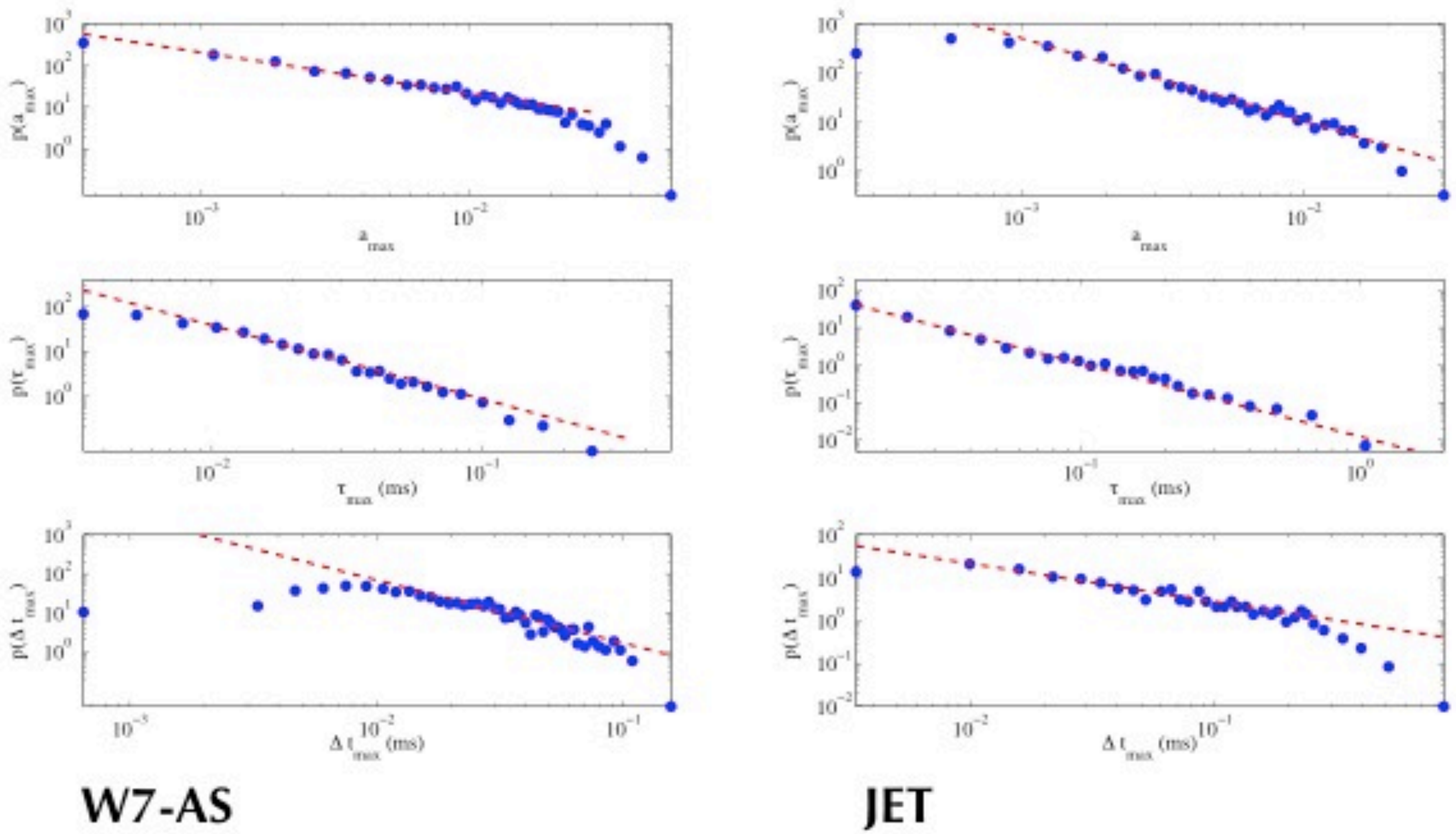
EVIDENCE FOR SOC: Statistics

Universidad Carlos III de Madrid



Finally, using wavelets, one can identify individual bursts in the fluctuation signals and obtain size, duration and waiting time statistics.

[See: B. Ph. van Milligen et al, Phys. Rev. Lett. 109, 105001 (2012)]



SOC hypothesis is not mainstream

Universidad Carlos III de Madrid

The SOC idea is not yet considered mainstream among fusion scientists.

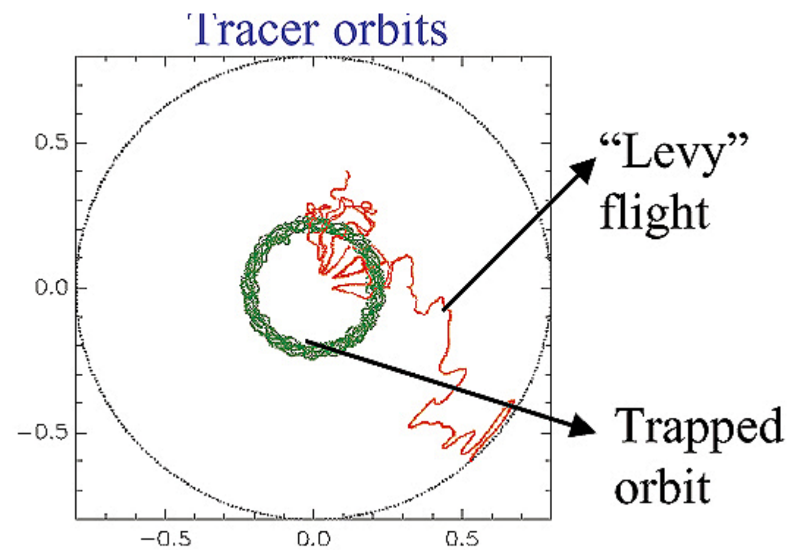
It has however changed the way simulations are done: DO self-consistent profile evolution!

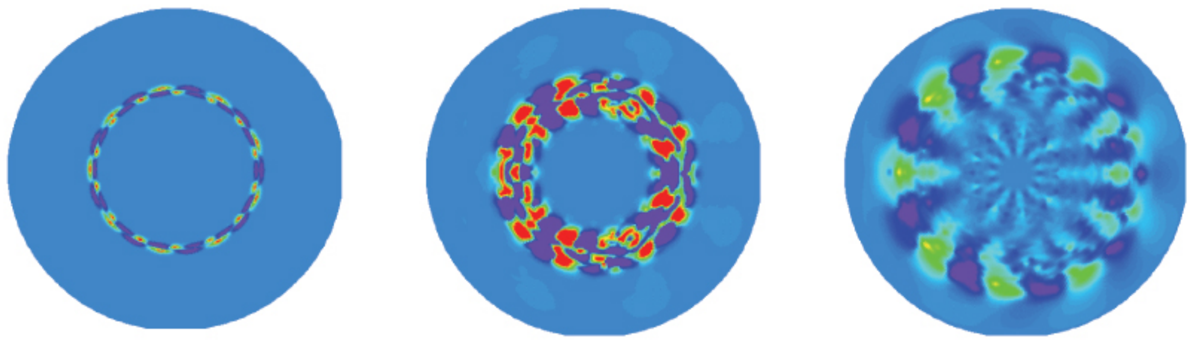
$$\left(\partial_t + \tilde{V}\cdot\nabla\right)\nabla^2_{per}\tilde{\Phi} = \frac{B_0}{m_i n_0 r_c}\frac{1}{r}\frac{\partial\tilde{p}}{\partial\theta} - \frac{1}{\eta m_i n_0 R_0}\nabla^2_{\parallel}\tilde{\Phi} + \mu\nabla^4_{\parallel}\tilde{\Phi}$$

$$\left(\partial_t + \tilde{V}\cdot\nabla\right)\tilde{p} = \frac{\partial\langle p\rangle}{\partial r}\frac{1}{r}\frac{\partial\tilde{\Phi}}{\partial\theta} + \chi_{per}\nabla^2_{per}\tilde{p} + \chi_{\parallel}\nabla^2_{\parallel}\tilde{p}$$

$$\frac{\partial\langle p\rangle}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}r\left\langle\tilde{V}_r\tilde{p}\right\rangle = S_0 + D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\langle p\rangle}{\partial r}\right)$$

[See: B.A. Carreras et al, Phys. Plasmas. **3**, 2906 (1996)]





Pure SOC needs to be combined with other transport processes that may alter the dynamics: poloidal sheared flows (enhanced regimes, ELMs), parallel transport (open field lines in linear devices and/or SOL), etc.

SOC or non-SOC distinction matters!

Universidad Carlos III de Madrid

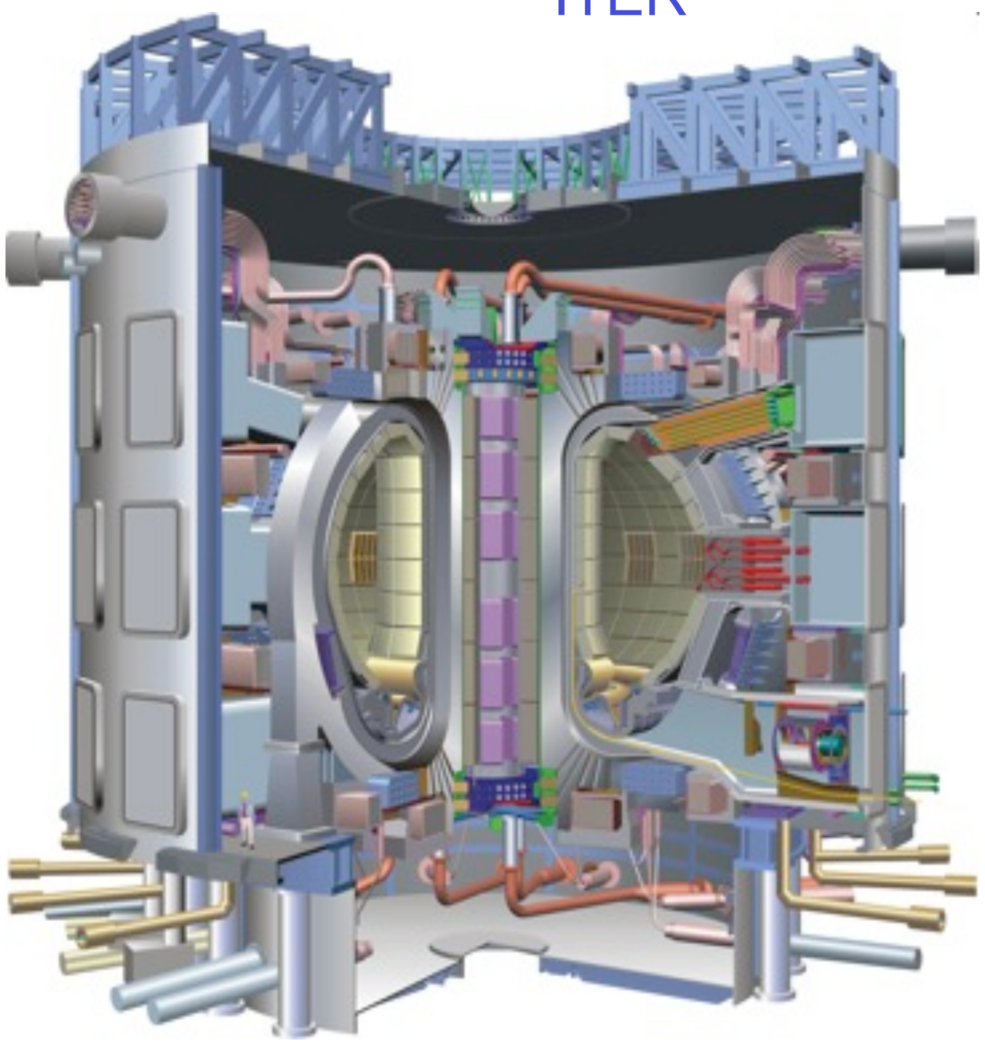
But SOC may have very important implications for the future of fusion:

Intermittent transport events exiting the device with a divergent power-law distribution yield very different peak-energy loads (that must be absorbed by first walls/divertors) than in the case of diffusive transport. This issue is central to the design and cost of these elements of a reactor!

Scaling of energy confinement with the system size is also very different in a SOC state compared with the more traditional diffusive-like scalings. Huge implications for size needed for fusion and its cost!

Whether ITER will be in a SOC state or not is unknown. But the ability of turbulence to locally get rid of excess free energy increases with a power of temperature. And ITER will be very hot!

ITER



Universidad Carlos III de Madrid

Modeling transport in fusion plasmas:

How does one effectively model transport and estimate confinement in a SOC state?

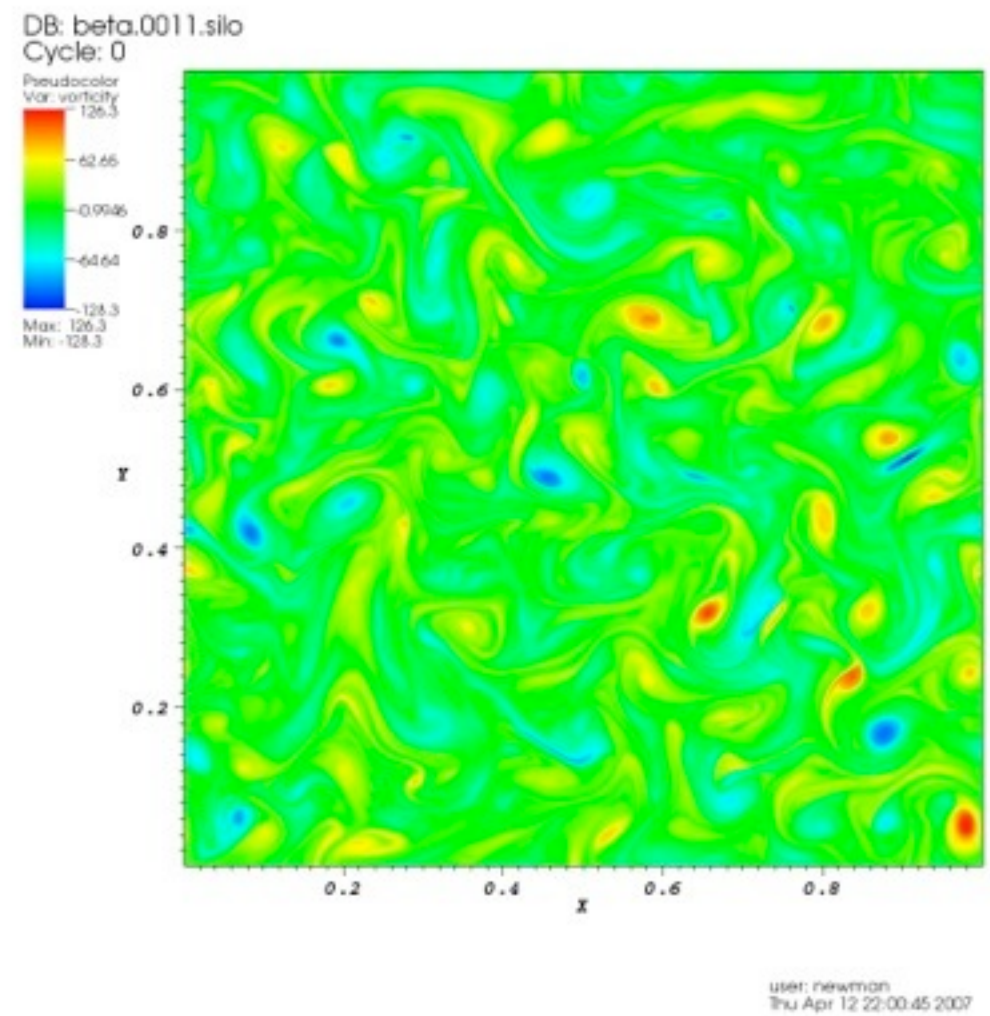
What are the proper “renormalized” fluxes that will do a good enough job?

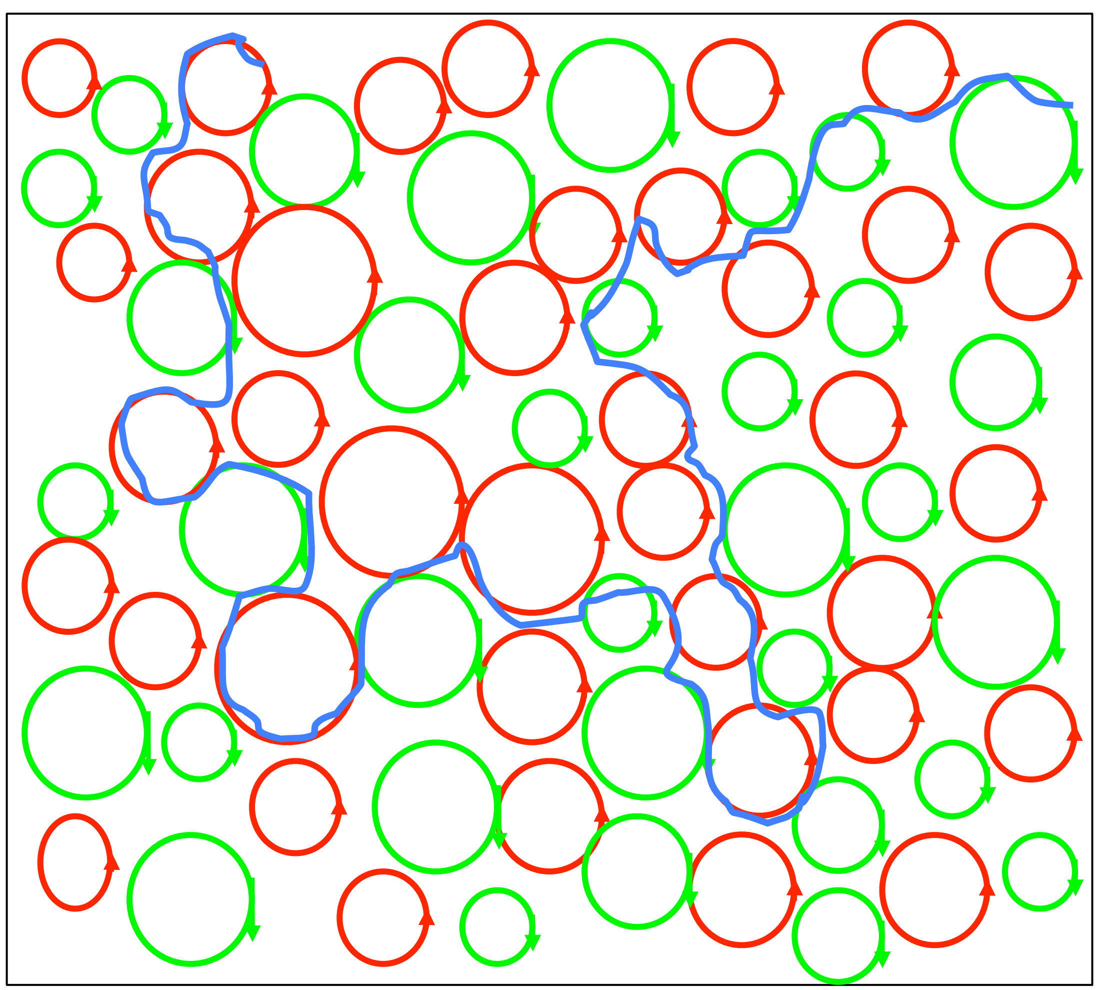
Modeling transport in fusion plasmas: EFFECTIVE TURBULENT DIFFUSION

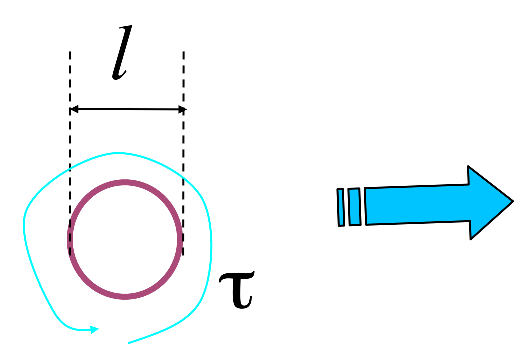
Universidad Carlos III de Madrid

Assumes:

- Characteristic scales
- Lack of memory







$$D \sim \frac{l^2}{\tau}$$

EFFECTIVE TURBULENT DIFFUSION: mathematical assumptions

Universidad Carlos III de Madrid

[See: R. Balescu, "Aspects of Plasma Turbulent Transport", IOP, Bristol (2005)]

$$\frac{\partial n}{\partial t} + (\mathbf{V} \cdot \nabla) n = 0$$

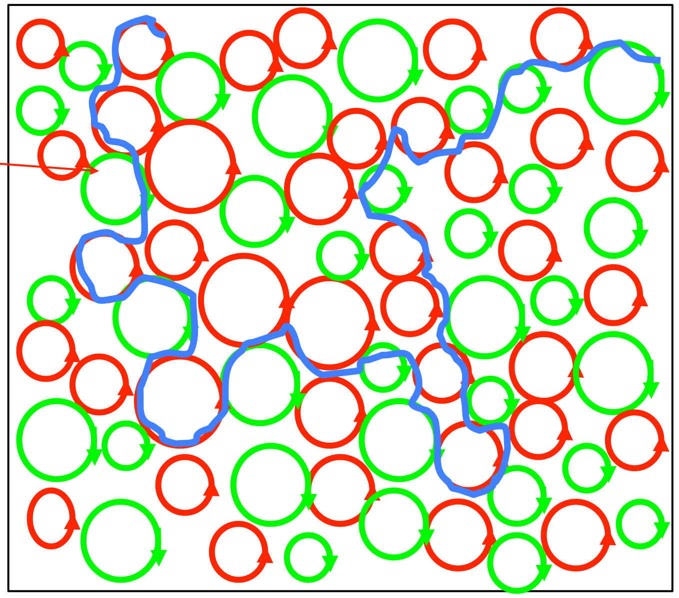
$$\frac{\partial n_0}{\partial t} = -\left\langle \tilde{\mathbf{V}} \cdot \nabla \tilde{n} \right\rangle$$

$$\frac{\partial \tilde{n}}{\partial t} + \tilde{\mathbf{V}} \cdot \nabla \tilde{n} = -\tilde{\mathbf{V}} \cdot \nabla n_0 + \left\langle \tilde{\mathbf{V}} \cdot \nabla \tilde{n} \right\rangle$$

$$\frac{\partial n_0}{\partial t} = \nabla \cdot \int_0^t \left\langle \tilde{\mathbf{V}}(\mathbf{r},t) \tilde{\mathbf{V}}(\mathbf{R}(t'|\mathbf{r},t),t) \nabla n_0(\mathbf{R}(t'|\mathbf{r},t),t) \right\rangle$$

Lagrangian trajectories

$$\frac{d\mathrm{R}}{d\tau} = \tilde{\mathrm{V}}(\mathrm{R},\tau), \quad \mathrm{R}(t) = \mathrm{r}$$



REDUCTION TO DIFFUSIVE DESCRIPTION: Lagrangian view

Universidad Carlos III de Madrid

$$\frac{\partial n_0}{\partial t} = \nabla \cdot \int_0^t \left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}(\mathbf{R}(t'|\mathbf{r},t),t)\nabla n_0(\mathbf{R}(t'|\mathbf{r},t),t)\right\rangle$$

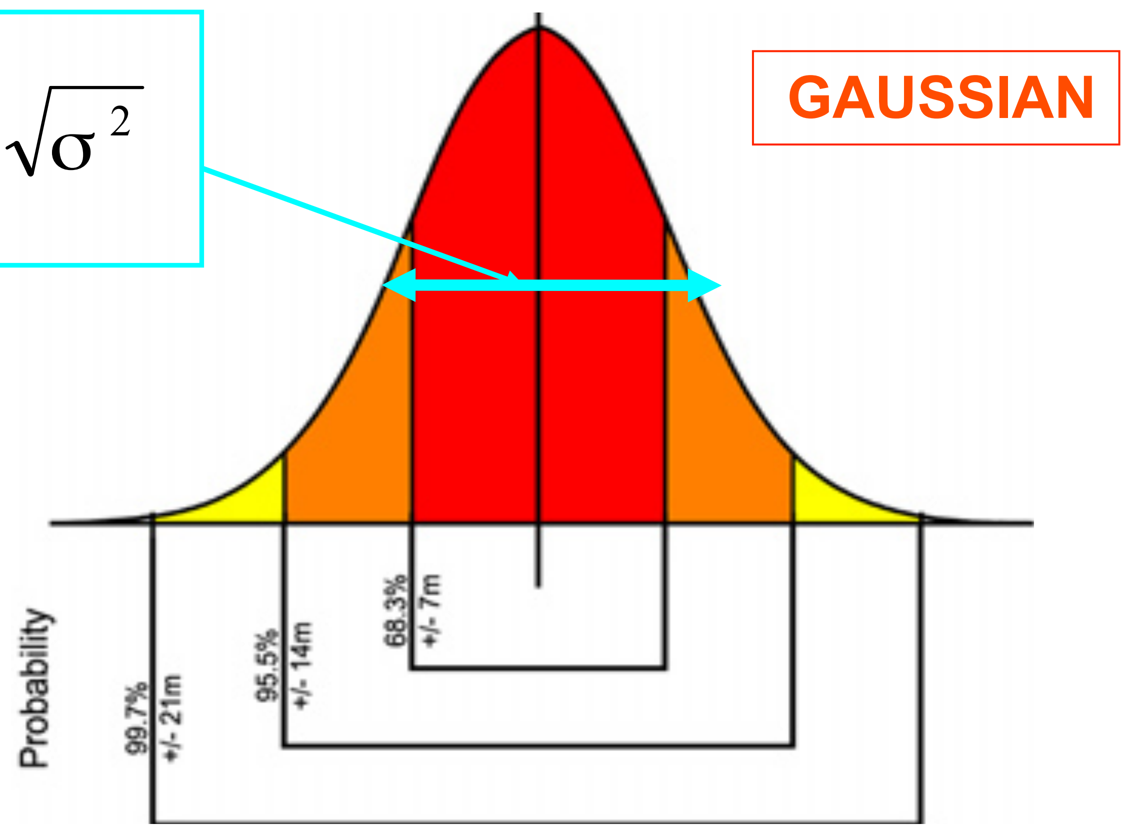
Characteristic Scales

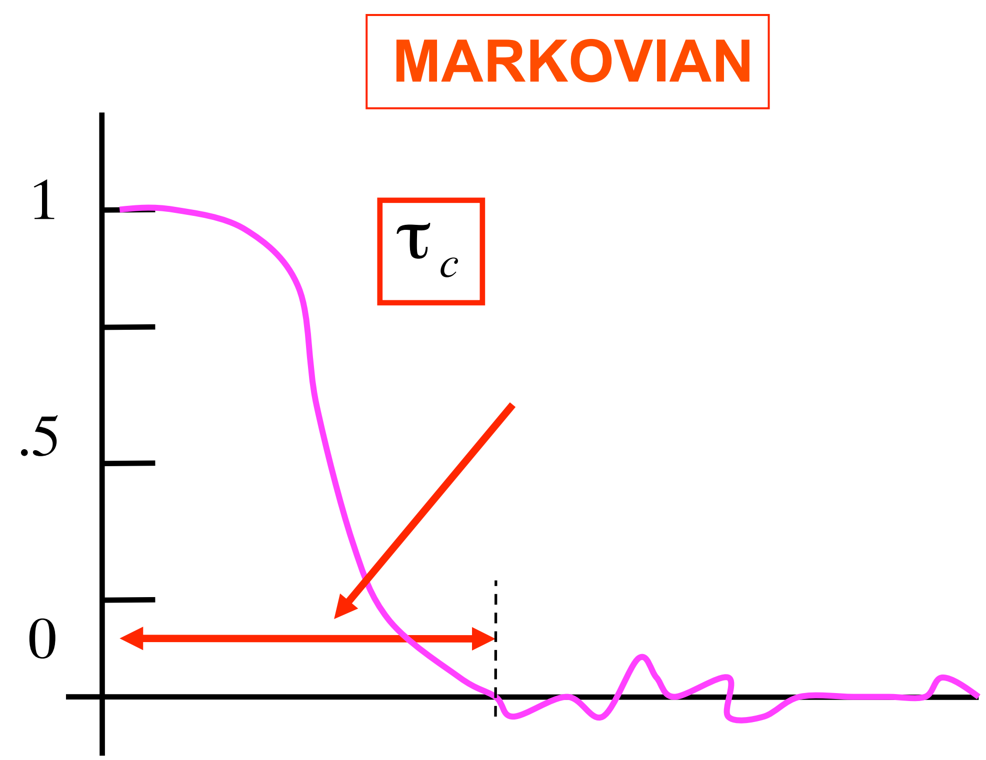
Lack of Memory

$$\frac{\partial n}{\partial t} \simeq D\frac{\partial n}{\partial x^2}$$

$$D \simeq \tilde{V}_c^2 \tau_c$$

$$V_c \sim \sqrt{\sigma^2}$$





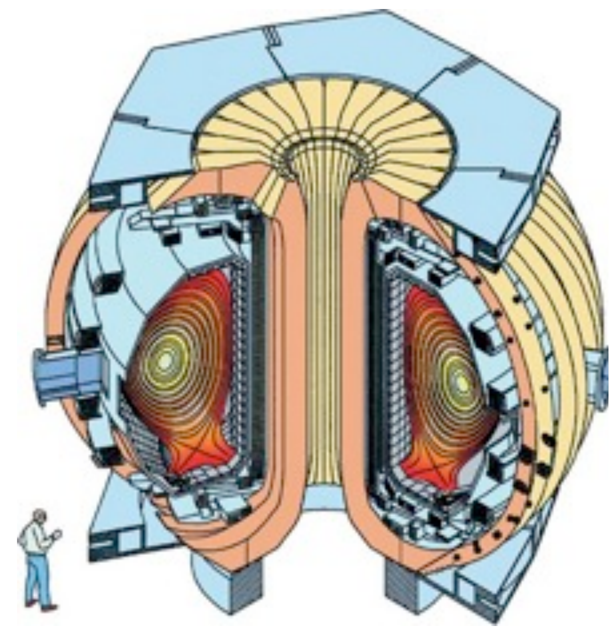
Does turbulent transport in tokamaks have characteristic scales?

Universidad Carlos III de Madrid

[See: G.R. McKee et al, IEEE Tran. Plasma Sci. (2002)]

SOC state

- Lack of characteristic scales
- Memory



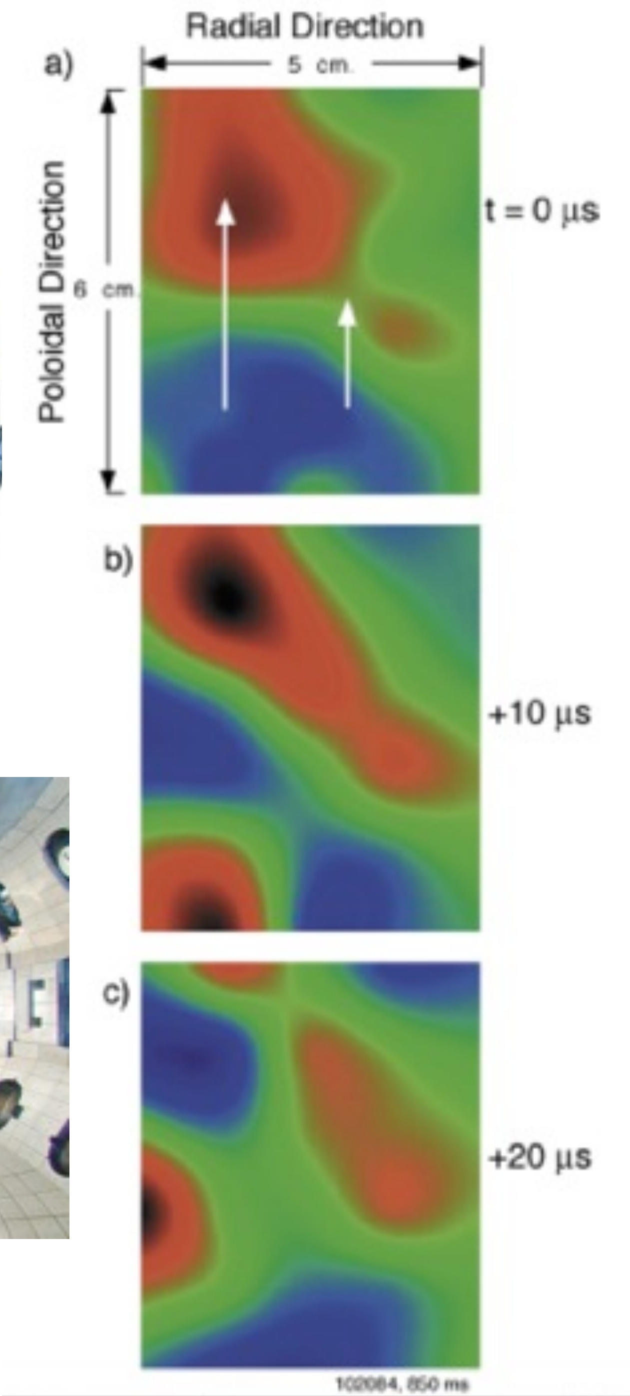


Fig. 1. Images of density fluctuations at 10-μs intervals, obtained over a 5-cm (radial) $\times$ 6-cm (poloidal) region at the outer plasma midplane, $\rho = 0.9$–1.05, in a low-confinement (L-mode) discharge. White arrows in (a) qualitatively indicate average flow direction and magnitude.

But from point (Eulerian) probe data, both a finite decorrelation length (i.e., typical length scale) and a finite decorrelation time (i.e., typical time scale and thus no long-term memory) are obtained for turbulence.

So how is it even possible that SOC play a role at all in this context?

NEAR-MARGINAL TURBULENCE

Universidad Carlos III de Madrid

Turbulent transport can develop SOC features in near-marginal regimes.

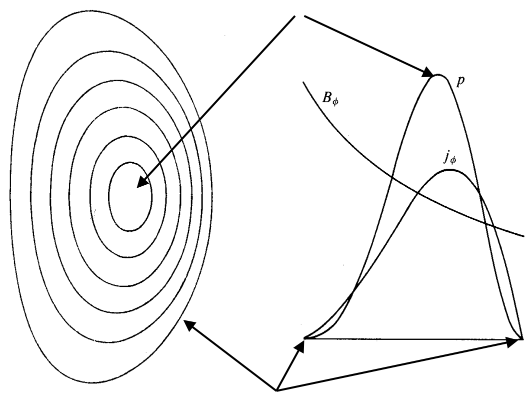
Why? Turbulence may switch on and off as free energy becomes or ceases to be available locally.

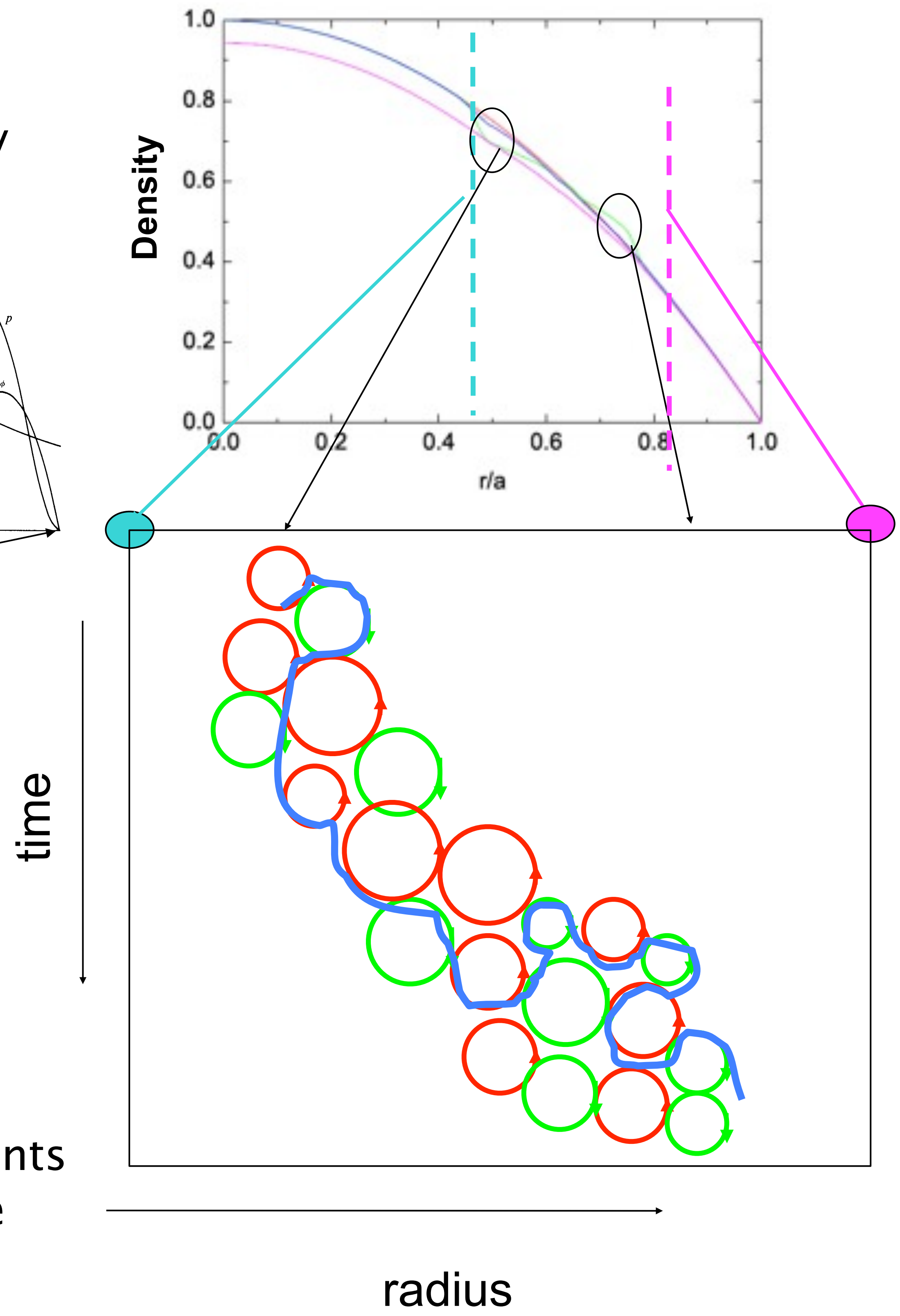
Lack of typical scales

Transport via avalanches

Spatio-temporal correlations

Memory stored in profile





This can happen despite the fact that local measurements still give finite decorrelation lengths and times that are set by the dominant instability!!

Stable distributions: Levy pdfs

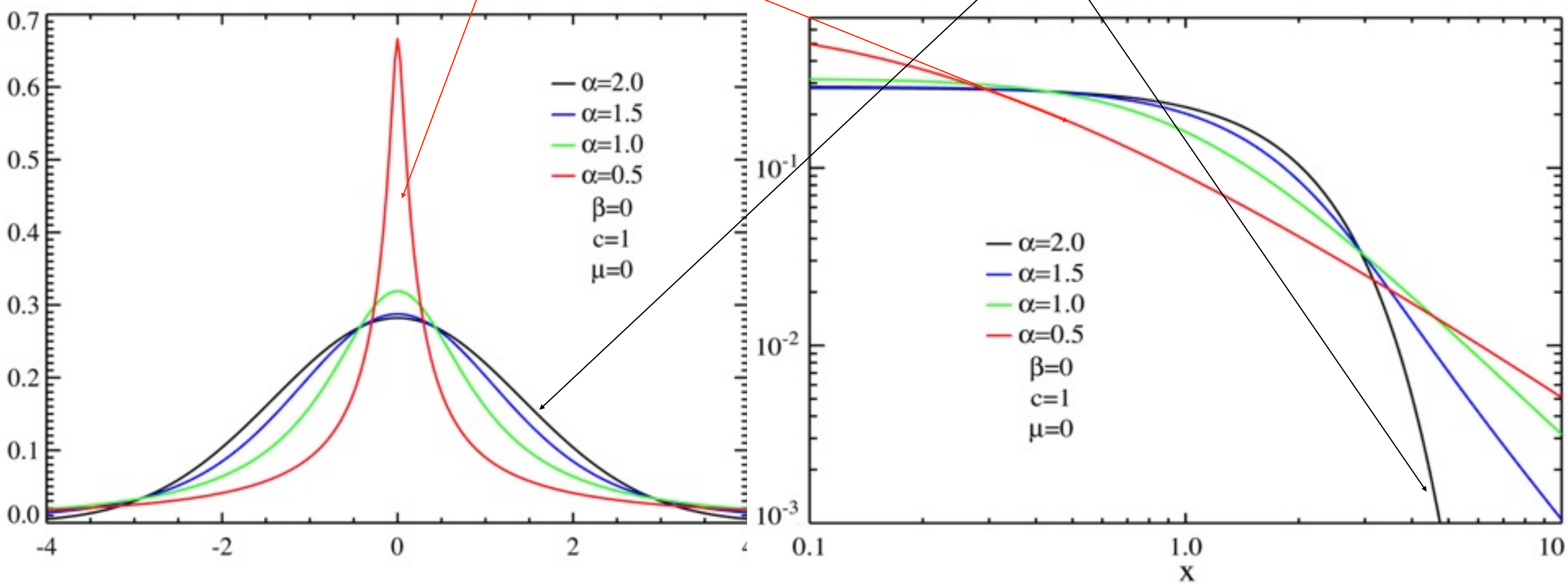
$$\frac{\partial n_0}{\partial t} = \nabla \cdot \int_0^t \left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}(\mathbf{R}(t'|\mathbf{r},t),t)\nabla n_0(\mathbf{R}(t'|\mathbf{r},t),t)\right\rangle$$

$$P_{\{\alpha,0,\sigma\}} \sim |x|^{-(1+\alpha)}(k) = \exp(-\sigma^\alpha |k|^\alpha), \quad \alpha \leq 2$$

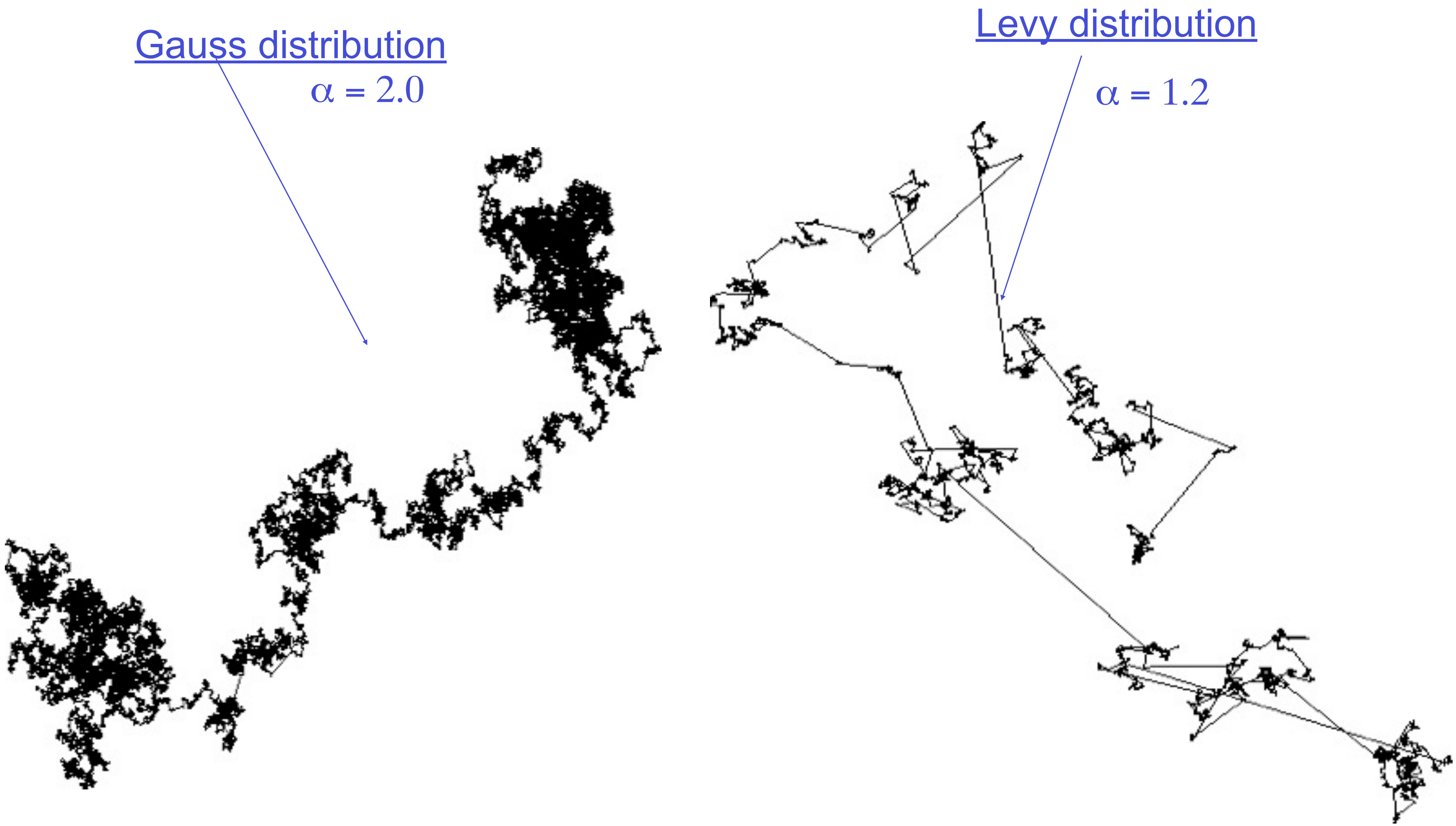
[See: G. Samorodnitsky and M. Taqqu, "Stable non-Gaussian distributions", Chapman and Hall, New York (1994)]

$$P_{\{\alpha,0,\sigma\}} \sim |x|^{-(1+\alpha)}$$

Gaussian: α = 2



Stable distributions: Levy pdfs



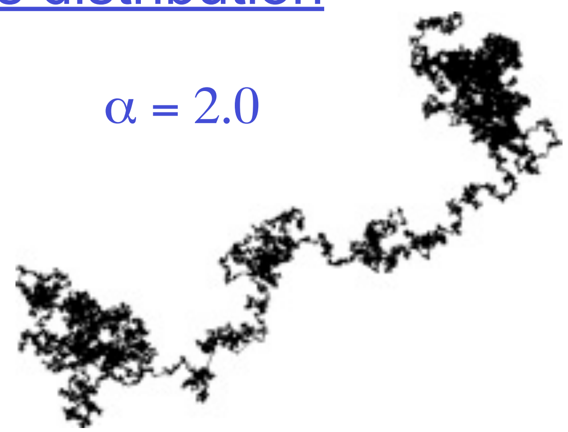
LAGRANGIAN MEMORY

Universidad Carlos III de Madrid

$$\frac{\partial n_0}{\partial t} = \nabla \cdot \int_0^t \left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}(\mathbf{R}(t'|\mathbf{r},t),t)\nabla n_0(\mathbf{R}(t'|\mathbf{r},t),t)\right\rangle$$

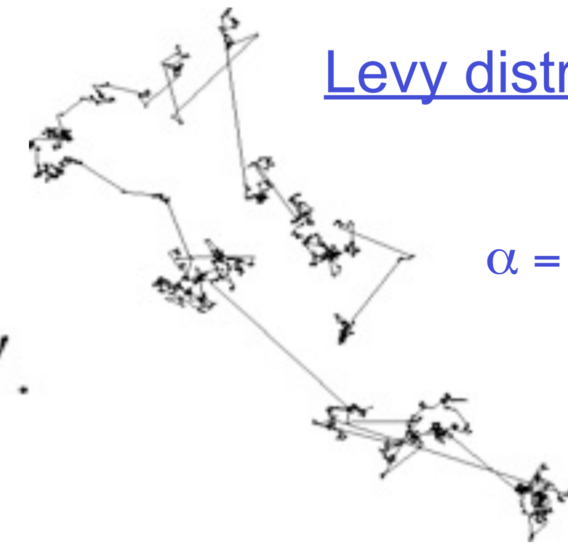
Gauss distribution

$\alpha = 2.0$



$$x(t) = x_0 + \int_0^t \xi_2(t')dt'.$$

Levy distribution



$\alpha = 1.2$

[See: B.B. Mandelbrot and J.W. van Ness, SIAM Review 10, 422 (1968); I. Calvo and R. Sanchez, J. Phys. A 32, 055003 (2009)]

H, Hurst exponent

$$x(t) = x_0 + \frac{1}{\Gamma\left(H - \frac{1}{\alpha} + 1\right)} \int_0^t dt'(t-t')^{H-1/\alpha}\xi_\alpha(t')$$

$H=1/\alpha$, random; $H > 1/\alpha$, correlated positively; $H < 1/\alpha$, correlated negatively

[See: H.E. Hurst, Trans. Am. Soc. Civ. Eng. 110, 770 (1951)]]

"Complex" EFFECTIVE TRANSPORT

Universidad Carlos III de Madrid

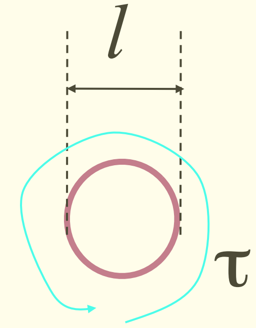
Typical scales

Lack of memory

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

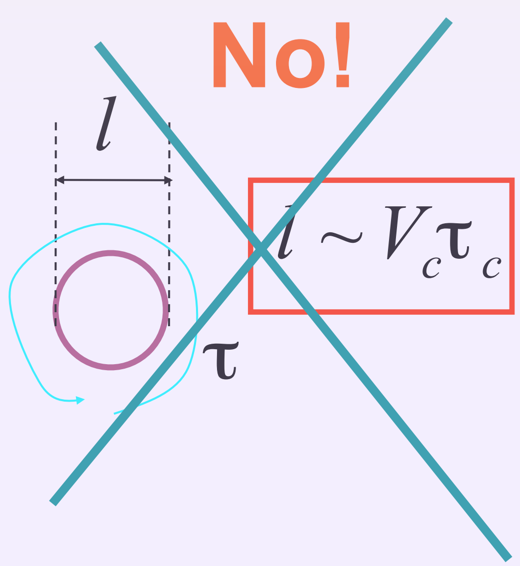
Standard Transport equations

Yes!



$l \sim V_c \tau_c$

No!



$$\frac{\partial n}{\partial t} = {}_0D_t^{1-\alpha H}\left[D\frac{\partial^\alpha n}{\partial |x|^\alpha}\right]$$

Fractional Transport equations

Lack of typical scales

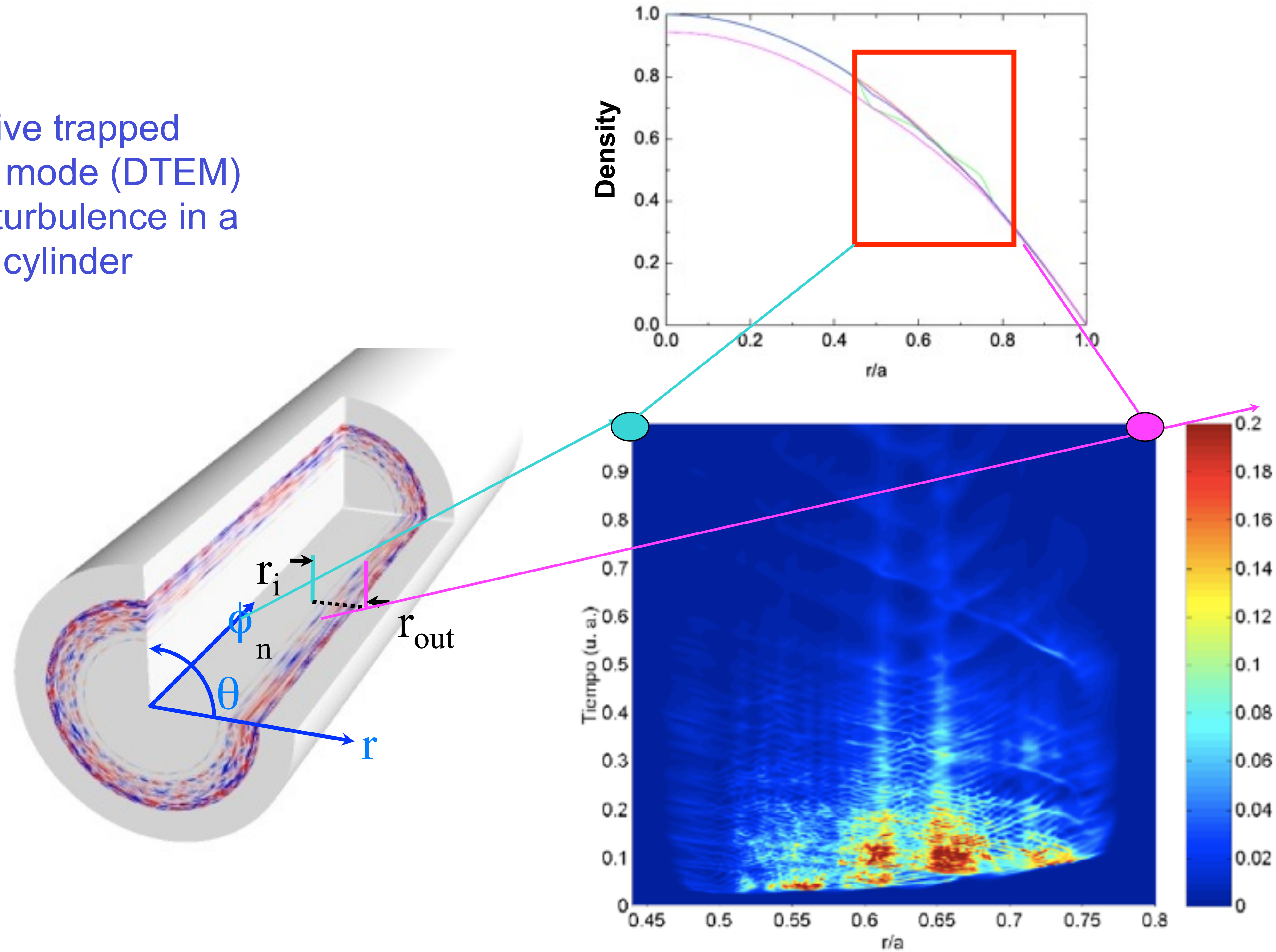
Spatio-temporal correlations

[See: R. Sanchez, B.ph. van Milligen, B.A. Carreras and D.E. Newman, Physical Review E 74, 016305 (2006)]

NEAR-MARGINAL SIMULATION:
An example of SOC transport

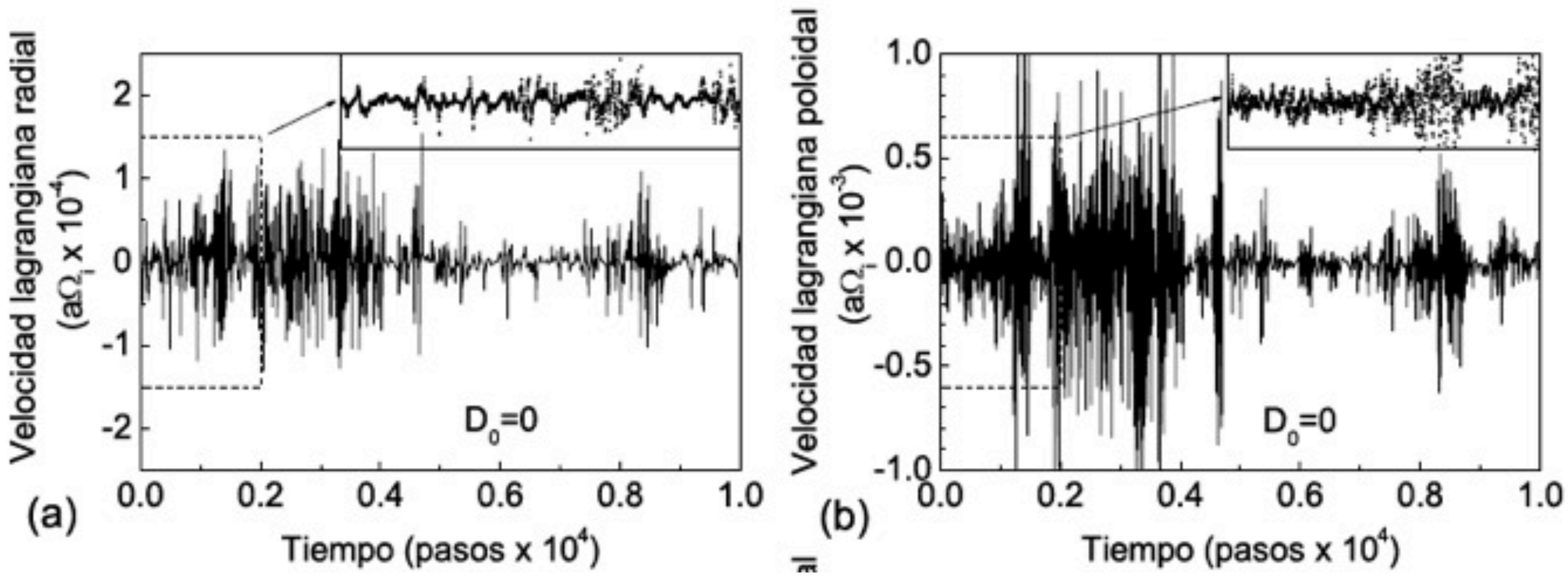
Universidad Carlos III de Madrid

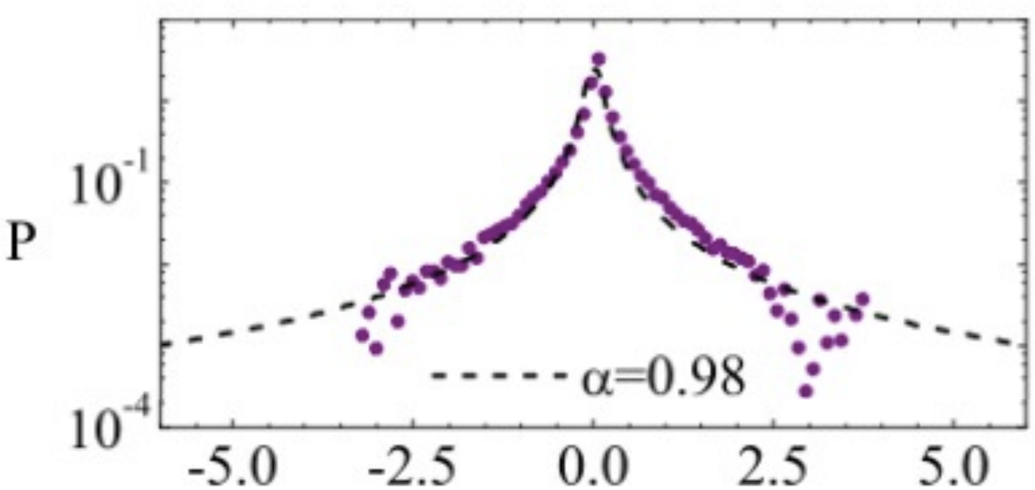
Dissipative trapped electron mode (DTEM) plasma turbulence in a periodic cylinder

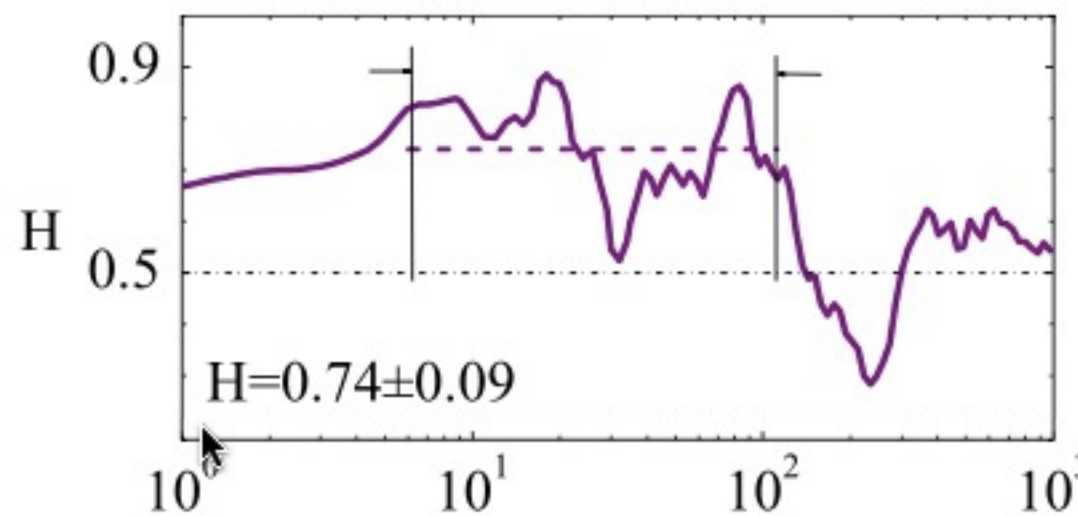


Lagrangian information from DTEM TURBULENCE SIMULATION

Universidad Carlos III de Madrid







Lack of typical scales

Spatio-temporal correlations

[See: J.A. Mier, R. Sanchez, L. Garcia, D.E. Newman and B.A. Carreras, *Phys. Rev. Lett.* **101**,165001 (2008)]

$$R / s \propto \tau^H$$

Effect of adding a subdominant diffusive transport channel

Universidad Carlos III de Madrid

Competition with subdominant, random channel may reduce the strength of SOC dynamics and change effective transport model.

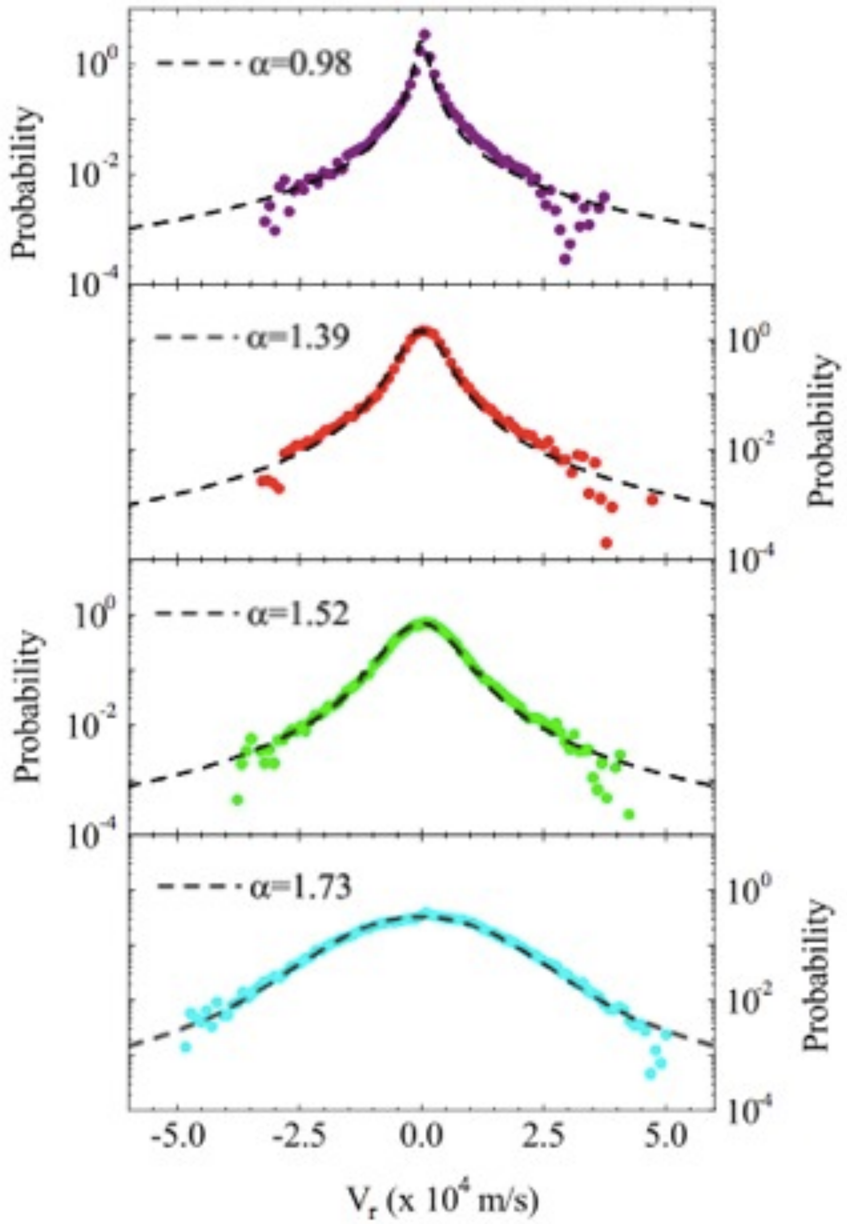


FIG. 2 (color online). Probability density functions of Lagrangian velocities for various diffusivities (top: $D_0 = 0$; second: $D_0 = 0.07$ m^2/s; third: $D_0 = 0.18$ m^2/s; bottom: $D_0 = 1.19$ m^2/s). Best Lévy fit shown in dashed black lines.

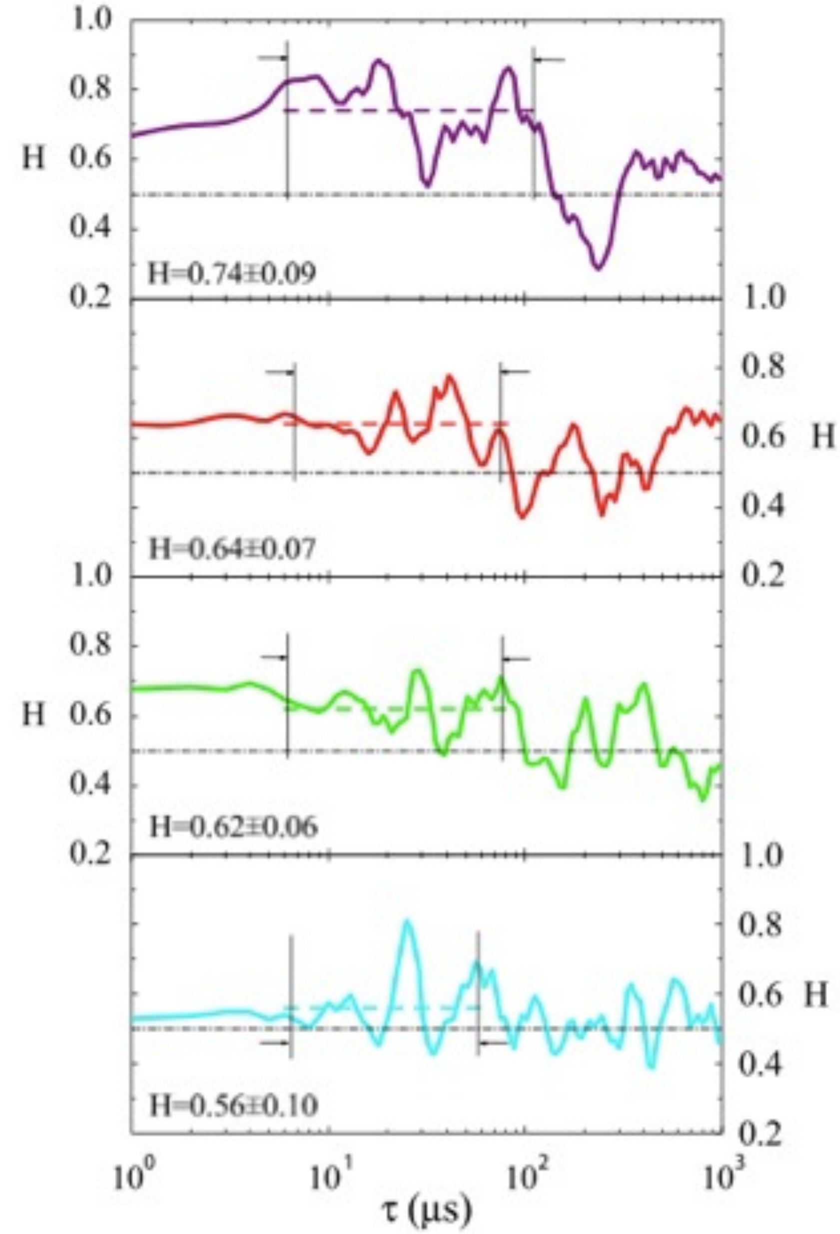


FIG. 3 (color online). α-Hurst exponent of Lagrangian velocity series versus elapsed time for various diffusivities (top: $D_0 = 0$; second: $D_0 = 0.07$m^2/s; third: $D_0 = 0.18$ m^2/s; bottom: $D_0 = 1.19$ m^2/s). The mesoscale is marked by arrows.

CONCLUSIONS

Universidad Carlos III de Madrid

• Turbulence involves many degrees of freedom and nonlinear interactions

→ breeding ground for *self-organized criticality* and *complexity*

• Toroidal fusion plasmas confined in tokamaks or stellarators are strongly turbulent and may exhibit complex behaviors: emergence of patterns and coherent structures, self-organization, memory, etc.

• *Near-marginal turbulence* is an example of a situation in which 'SOC' behaviors should be expected in fusion plasmas.

• Relevant from practical stand-point: even if mean energy outflux is fixed by external heating in steady-state, intermittent dynamics may yield large peak outfluxes that first wall components and/or divertors may not be able to withstand.

• Scaling of confinement properties with system size also quite different in SOC-dominated regimes.

• Tools, mathematical models and ideas from "*complexity theory*" are very useful to understand and characterize what is going on in SOC states.