Note:

Slides given as springboard for discussion session at ISSI Bern on Friday 18th October, 2012. An additional point which came up and which we will probably return to in future meetings was the double meaning of the English word “critical”.

1. This is used in the phrase “Self Organised Criticality” by analogy with critical phenomena, where it refers to the long range spatial correlations in the system, see also Henrik’s slides from Thursday discussion.

2. It is also used in the phrase “critical threshold”, where it refers to the necessary threshold that a system needs to exceed locally for transport to occur. This usage might be compared with e.g. a pencil is said to be “critically balanced “ on its end.

Both concepts are important to BTW’s idea, but it might still be a good idea to use a different word for the second. The former usage is well established e.g http://en.wikipedia.org/wiki/Critical_phenomena

SOC++

Where (and how) next ?

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ISSI 2012

Many thanks to Markus, Stefan and you all for the 5th SOC and turbulence in astrophysics meeting that I have attended.

2000 RAS London

2001 BAS Cambridge

2002 Venice

2007 Geilo

2012 Bern

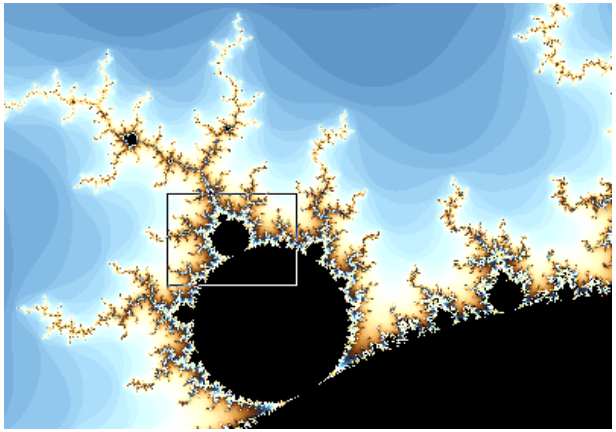
What was original physical motivation for SOC ?

In "The physics of fractals" [Physica D, Vol. 38, # 1-3 (1989)], Bak and Chen gave beautifully clear statement of original SOC idea; first introducing puzzle of spatial fractals …

"The importance of Mandelbrot's discovery that fractals occur widespread in nature can hardly be exaggerated. Many things which we used to think of as messy and structureless are in fact characterized by well-defined power-law spatial correlation functions. By now, we are so used to seeing fractals that we are tempted to feel that we understand them. But do we simply have to accept their existence as "God-given" without further explanation or is it possible to construct

a dynamical theory of the

physics of fractals?"



… then that of fractals in time- “1/f noise” …

Weatherwax et al,
GRL, 2000

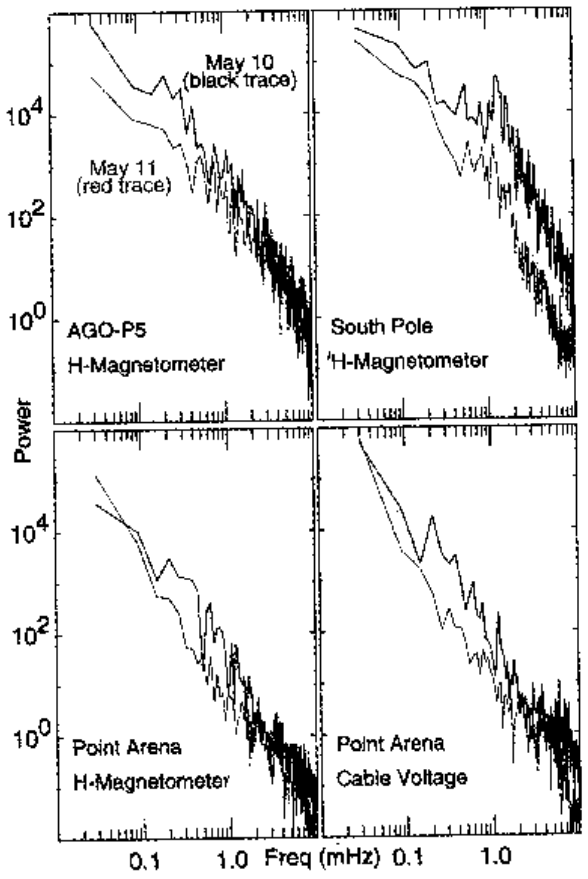


Figure 5. Geomagnetic and geopotential power spectra on May 10 and 11, 1999. Power units are in $(nT)^2/Hz$ and V^2/Hz $(\times 10^{-4})$, respectively.

*“There is another ubiquitous phenomenon which has defied explanation for decades. The signal (water, electrical current, light, prices, …) from a variety of sources has a power spectrum decaying with an exponent near unity at low frequencies .... This type of behavior is known as “1/ f" noise, or flicker noise.” -***Bak and Chen, ibid.**

… then proposing that they should often be unified …

“Strangely enough, just as those working on fractal phenomena in nature never seem to be interested in the temporal aspects of the phenomenon, … those working on ``1/ f'' noise never bother with the spatial structure of the source of the signal. We believe that those two phenomena are often two sides of the same coin: they are the spatial and temporal manifestations of a self-organized critical state. **-Bak and Chen, ibid.**

… through the dynamics of a self-organised dynamical state of the system itself. These dynamics are the "mechanism" of SOC …

"Actually, for those (like us) who are brought up as condensed matter physicists it is hard to believe that long-range spatial and temporal correlations can exist independently. A local signal cannot be ``robust'' and remain coherent over long times in the presence of any amount of noise, unless stabilized by the interactions with its environment. And a large, coherent spatial structure cannot disappear (or be created) instantly …

-Bak and Chen, ibid.

… and are spatiotemporally "critical" by direct analogy with equilibrium phase transitions …

"In fact, there is one area of physics where the relation between spatial and temporal power-law behaviour is well-established. At the critical point for continuous phase transitions, the correlation function for the order parameter decays spatially as $r^{2-d-\eta}$ *and temporally as* $t^{-d/z}$*. But in order to arrive at the critical point, one has to fine-tune an* ***external*** *control parameter such as the temperature or pressure, in contrast to the phenomena above which occur universally without any fine-tuning. The explanation is that open, extended, dissipative dynamical systems may go automatically to the critical state as long as they are driven slowly: the critical state is self-organised. We see fractals as snapshots of systems operating at the self-organised critical state"* **-Bak and Chen, ibid.**

Spatial avalanches "grown" by BTW (and subsequent) SOC models-some also produce temporal "1/f" spectra.

Bak et al, PRA, 1988

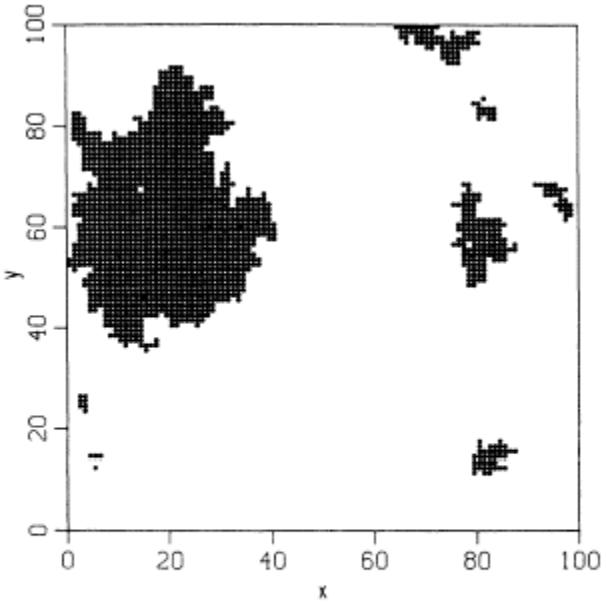


FIG. 2. Typical domain structures resulted from several local perturbations for a 100×100 array. Each cluster is triggered by a single perturbation.

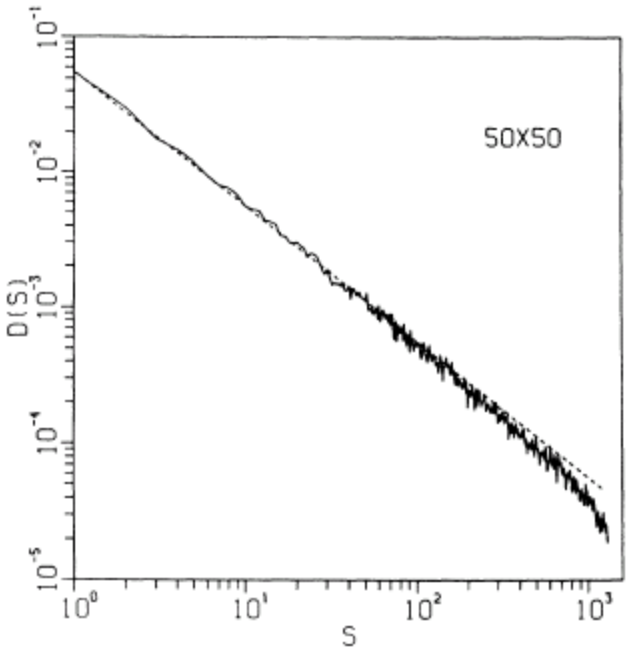


FIG. 5. Cluster size distribution for system built up from scratch according to rules (3.2) and (3.8) for a 50×50 array. The curve is indistinguishable from that in Fig. 3(a). For this system the system is in a stationary critical state and it is self-averaging. Rule (3.8) has been applied 100 000 times to the stationary critical state to obtain this curve.

Image:

Yann-Arthus

Bertrand

There are sandtraps for the unwary in testing for SOC by looking for fractals, 1/f noise or power laws …

"Fractals give rise to power laws
I have seen a power law
Therefore my power law was caused by a fractal"

"SOC is a theory which explains fractals
I have seen a fractal
Therefore my fractal was caused by SOC".

See Wikipedia on "the fallacy of the undistributed middle …" and Sornette Chapters 14 and 15, and his online talk slides on this topic.

Spatial fractals were, from the outset, carefully tested for by scaling collapse …

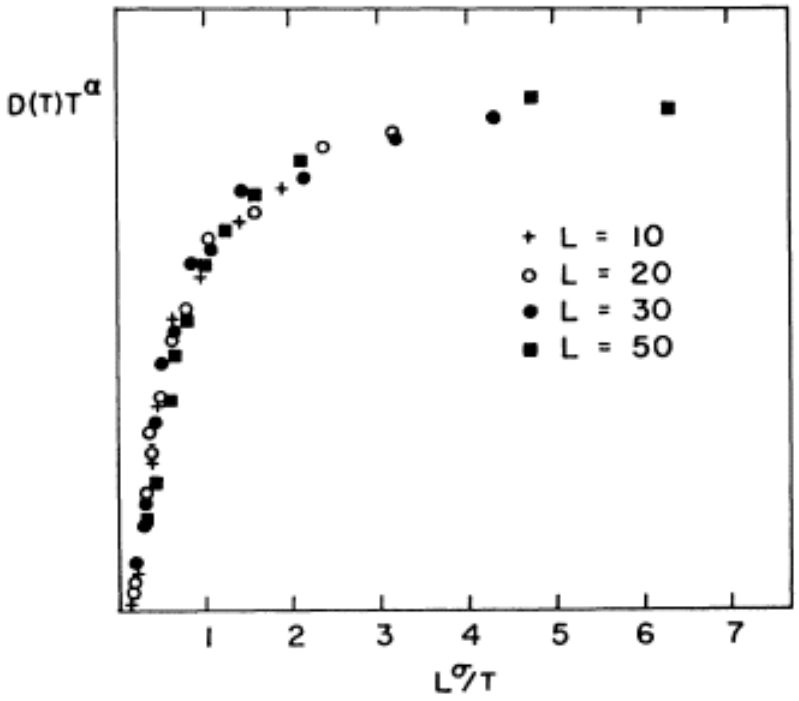


FIG. 11. Finite-size-scaling plot.

Bak et al, PRA, 1988

Precision of these methods and size of simulations has Improved over the years -raised many questions e.g. **Gunnar Pruessner'**s book

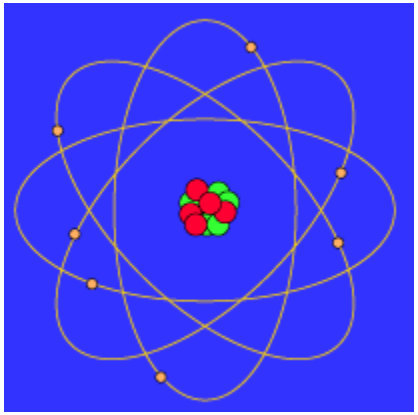
25 years on, can see at least 4 threads in “SOC”

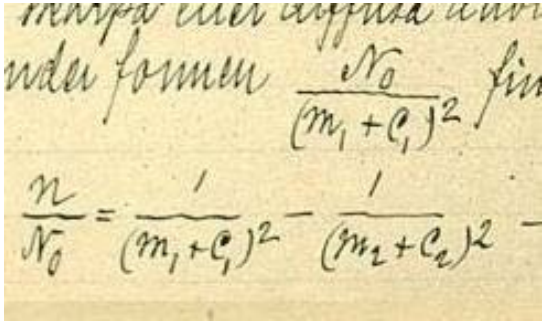
- The phenomena Bak was trying to explain and unify (Michaela’s “CV”)
- The physical process he proposed - SOC
- The model he used to typify it – sandpiles

- **The problem of driven nonlinear multiscale plasma instabilities**
- **The broader question of aggregation of processes in nature, informed by current knowledge about scaling, and fractals, and the other relevant physics of the problem.**

The Bohr Atom

- "The Bohr model of the atom ... was wrong, yet it turned out to be fruitful."
  – Gene Stanley, quoted by Mark Buchanan in *Nature, 2008*





Rydberg formula