SELF-ORGANIZED CRITICALITY vs. LOW-**DIMENSIONAL CHAOS FOR THE UNDERSTANDING OF FUSION EDGE PLASMA TURBULENCE**



453

453

Time (ms)

453.5

453.5

454

454

y Tecnológicas

0.1

 0.0^{4}

-0.05

100

452

Freq (kHz)

452.5

452.5

Data



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2nd ISSI-Self-Organized Criticality Workshop September 16 - 20, 2013, Bern, SWITZERLAND



SELF-ORGANIZED CRITICALITY



<u>SELF-ORGANIZED CRITICALITY</u>: concept introduced by P. Bak in 1987 to attempt to explain 1/f noise in many natural systems.

- •Typically, requires the existence of a threshold that separates a quiescent phase from an active phase in which fast transport/redistribution takes place.
- It also requires a certain randomness/unpredictability in the system.

Bak's paradigmatic toy-model: the sandpile.

 Grains of sand dropped on a sandpile whose cells can go critical and turn when a certain local threshold condition is overcome.

Bak's original model was driven infinitely slow, with sand-addition being stopped as soon as an avalanche starts, and restarted when it stops.



Steady-state (SOC state) exhibits self-similarity, long-term correlations and other properties typical of equilibrium critical states but without the need of external tuning.

DIRECTED RUNNING SANDPILE



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The directed running sandpile (DRS), using as the critical threshold the local slope, is much closer in spirit to the real situation that turbulent fusion plasmas experience. It does not stop the external drive while avalanches take place.

SOC-state exhibits self-similar, scale-free statistics of avalanche sizes and energies

It can maintain a net non-zero outflux in spite of being submarginal on average

Intermittent transport is strongly correlated in time via shaping of the height profile

The shape of the height profile at the SOCstate is rather insensitive to the location of the source as long as the system is not overdriven

Avalanche overlapping is a real issue (invoked to explain the 1/f spectrum by HK)



[See: T. Hwa and M. Kardar, Phys. Rev. A 45, 7002 (1992)]



tim

FUSION TOROIDAL PLASMAS



No magnetic field





Helical magnetic fields can be used to confine hot plasmas long enough to produce energy





Turbulence dominates radial losses of energy and particles



GYRO, General Atomics

First EVIDENCE FOR SOC: Power spectra







1/f regions in power spectra such as those exhibited by DRS were considered a trademark of SOC then.

1/f regions were sought for in fusion experiments, both tokamaks and stellarators.

Mainly, using edge fluctuation data measured with Langmuir probes. From these, time series of the turbulent fluctuations and turbulent fluxes can be obtained at a single radial location.

Self-similarity of power-spectra and power-laws close to 1/f were indeed reported.

[See: M.A. Pedrosa et al, Phys. Rev. Lett. 82, 3621 (1999)]

L-MODE TOKAMAK PLASMAS



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SOC hypothesis suggested as an explanation?

NEAR-MARGINAL TURBULENCE





SOC AND TOKAMAK PLASMAS



An analogy between HK's sandpile an a magnetically confined toroidal fusion plasma can be easily done if it is near-marginal conditions.







TABLE I. Analogies between the sandpile transport model and a turbulent transport model.

Turbulent transport in toroidal plasmas	Sandpile model	
Localized fluctuation (eddy)	Grid site (cell)	
Local turbulence mechanism:	Automata rules:	
Critical gradient for local instability	Critical sandpile slope (Z _{crit})	
Local eddy-induced transport	Number of grains moved if unstable (N_f)	
Total energy/particle content	Total number of grains (total mass)	
Heating noise/background fluctuations	Random rain of grains	
Energy/particle flux	Sand flux	
Mean temperature/density profiles	Average slope of sandpile	
Transport event	Avalanche	
Sheared electric field	Sheared flow (sheared wind)	

[See: D.E. Newman et al, Phys. Plasmas 3, 1858 (1996)]

radius

Lots of activity (not comprehensive...)



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P.H. Diamond et al, Phys. Plasmas 2, 3640 (1995) D.E. Newman et al, Phys. Plasmas 3, 1858 (1996) B.A. Carreras et al, Phys. Plasmas 3, 2904 (1996) R. Dendy et al, Pl. Phys. Contr. Fusion 39, 1947(1997) X. Garbet et al, Phys. Plasmas 5, 2836 (1998) Y. Sarazin et al, Phys. Plasmas 5, 4214 (1998) T. Rhodes et al., Phys. Lett. A 253, 181 (1998) B.A. Carreras et al., Phys. Rev. Lett. 80, 4438 (1998) M.A. Pedrosa et al., Phys. Rev. Lett. 82, 3621 (1999) B.A. Carreras et al., Phys. Rev. Lett. 83, 3653 (1999) P. Beyer et al, Pl. Phys. Contr. Fusion 41, A757 (1999) B. A. Carreras et al, Phys. Rev. E 60, 4770 (1999) B. A. Carreras et al., Phys. Plasmas 8, 5096 (2001) R. Sanchez et al., Nucl. Fusion 41, 247 (2001) S. Chapman et al, Phys. Rev. Lett. 86, 2814 (2001) E. Spada et al, Phys. Rev. Lett. 86, 3032 (2001) R. Sanchez et al, Phys. Rev. Lett. 88, 068302 (2002) P.A. Politzer et al, Phys. Plasmas 9, 1962 (2002) L. García et al, Phys. Plasmas 9, 841 (2002) R. Sanchez et al, Phys. Rev. Lett. 90, 185005 (2003) I Gruzinov et al, Phys. Plasmas 10, 569 (2003) V. Tangri et al, Phys. Rev. Lett. 91, 025011 (2003) Y Xu et al, Phys. Plasmas 11, 5413 (2004) B.Ph. van Milligen et al, Phys. Plasmas 11, 2272 (2004) D. del-Castillo-Negrete et al, Phys. Plasmas (2004) B. Dudson et al, Pl. Phys. Contr. Fusion 47, 885 (2005) L. Marrelli et al, Phys. Plasmas 12, 030701 (2005) R. Sanchez et al, Phys. Rev. E 74, 016305 (2006) J.A. Mier et al, Phys. Plasmas 13, 102308 (2006) F. Sattin et al, Phys. Rev. Lett. 96, 105005 (2006) J. A. Mier et al, Phys. Rev. Lett. 101, 165001 (2008) Y. Sarazin et al, Nucl. Fusion 50, 054004 (2010) H. Isliker et al, Phys. Plasmas 17, 082303 (2010) J. Maggs et al, Phys. Rev. Lett. 107, 185003 (2011) S. Tokunaga et al, Phys. Plasmas 19, 092303 (2012) B. van Milligen et al, Phys. Rev. Lett. 109, 105001 (2012) GZ dos Santos et al, Phys. Lett. A 376, 753 (2012)

- Theory, PDE
- Sandpile, near-marginal dyns, shear flow effects
- Simulation (Interch. fluid), near marginal, avalanches
- Sandpile, near-marginal dyns
- Simulation (gyrofluid, DTEM), near marginal, flux-driven
- Simulation (gyrofluid, ITG), near marginal, flux-driven
- Experimental, tokamak, fluctuation statistics
- Experimental, Hurst exponents, edge fluctuations
- Experimental, power spectra, edge fluctuations
- Experimental, self-similarity, edge fluctuations
- Experimental, tokamak, edge fluctuations
- Sandpile, Effective Tr. Model (CTRWs)
- Simulation (Interch., fluid), Eff. Tr. Model (CTRWs)
- Sandpile, diffusion effects, ELMs
- Sandpile, ELMs
- Experimental (against) W. times, No s-similarity
- Sandpile, Wait. times
- Experimental, avalanche viz., ECE, tokamak
- Theory, PDE (continuum sandpile)
- Experimental W. times, Response to Spada's PRL
- Sandpile, ELMs
- Theory, PDE (continuum sandpile)
- Experimental, Tokamak, edge fluctuations
- Theory, Effective Tr. Model (CTRWs/Master Eqs.)
- Simulation (Interch., fluid), Eff. Tr. Model (Fract. Diff. Eqs)
- Experimental, spherical tokamak
- Experimental, reverse field pinch
- Theory, Eff. Tr. Models (FBM/FLM -> FDEs)
- Simulation (DTEM, fluid), effect of diffusion
- Sandpile, include WTs
- Simulation (DTEM, fluid), Eff. Tr. Model (Lagr. Diag.)
- Simulation (ITG, gyrokinetics), near-marg., avalanches
- Sandpile, ITG-like
- Experimental (against) Lorenzian, random pulses
- Experimental, tokamak, enhanced modes
- Experimental Response to Magg's PRL
- Experimental, tokamak





PRL 107, 185003 (2011) PHYSICAL REVIEW LETTERS

week ending 28 OCTOBER 2011

Generality of Deterministic Chaos, Exponential Spectra, and Lorentzian Pulses in Magnetically Confined Plasmas

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The dynamics of transport at the edge of magnetized plasmas is deterministic chaos. The connection is made by a previous survey [M. A. Pedrosa *et al.*, Phys. Rev. Lett. **82**, 3621 (1999)] of measurements of fluctuations that is shown to exhibit power spectra with exponential frequency dependence over a broad range, which is the signature of deterministic chaos. The exponential character arises from Lorentzian pulses. The results suggest that the generalization to complex times used in studies of deterministic chaos is a representation of Lorentzian pulses emerging from the chaotic dynamics.

DOI: 10.1103/PhysRevLett.107.185003

PACS numbers: 52.25.Fi, 05.45.Ac, 52.25.Gj, 52.25.Xz

Recently, it has been proposed that edge turbulent fluctuations could be better described as an uncorrelated superposition of Lorentzian pulses with a narrow distribution of widths.

Original data from the LAPD linear device (*Pace et al, 2008*). Then, from cold plasmas inside the TJ-K stellarator (*Hornung et al, 2011*).

If correct, fluctuations would exhibit an exponential power spectrum, that of the Lorentzian pulse.

Secondly, a connection with low-dimensional chaotic dynamics might exist, in contrast to previous interpretations from edge data from tokamaks and stellarators based on near-marginal, correlated dynamics (*Carreras et al, 1998, Pedrosa et al, 1999, Carreras et al, 2000, Sanchez et al, 2003*).

How universal are the LAPD/TJK results and their conclusions?



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Outline



Our plasmas (in both tokamaks and stellarators) are quite different from LAPD and TJ-K:

1. closed magnetic surfaces, not open field lines like in LAPD





2. very hot plasmas (~0.5 KeV). TJ-K and LAPD are very cold (~10-20 eV).









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So, the question is whether the Lorentzian model interpretation describes our data or not. In particular, we want to answer the questions?

- 1. Are the power spectra of our tokamak/stellarator data exponential?
- 2. Are the pulses Lorentzians in shape, and their widths narrowly-distributed?
- 3. Are the pulses randomly triggered or correlated in time?

We have reexamined two sets of data, one from the edge of the JET tokamak, one from the edge of the W7-AS stellarator, just inside the LCFS.



See: B.Ph. van Milligen, R. Sanchez and C. Hidalgo, Phys. Rev. Lett. 109, 105001 (2012)



Are our spectra exponential?



Spectral shape analysis

Are spectra truly exponential? To test this, we fit an actual spectrum (W7-AS) to various alternative shapes.



Case	Expression	R ²
a	$A\exp(-f/f_1)$	0.9146
b	$A \exp(-f/f_2)/(1 + (f/f_1)^{\beta})$	0.9716
С	three connected power-law lines	0.9716

At least in this case, the exponential fit is worse than alternatives based on power laws.





Are our pulses Lorentzian?



Pulse shape from conditional averaging

Depending on threshold, the pulse shape varies (W7-AS data). I.e., it is difficult to determine "the" shape of the pulses.







The Hurst exponent was introduced by Hurst (1952) to quantify correlation in time series.

He assumed that any stationary signal can be thought of as the ordered sequence of displacements of a particle. If the signal is random, the motion of such a particle will be that of a random walk, and the distance from its initial position will grow on average as $t^{1/2}$.

If the signal contains positive correlations between successive displacement, such distance will grow with a stronger exponent, t^{H} , H > 1/2. Similarly, if negative correlations exist, it will grow with H < 1/2.

H, the Hurst exponent, can be determined in many ways. One of the most popular ones is the R/S method, which is one of the more un-sensitive to noises.



Are our pulses uncorrelated?



Hurst analysis



Rescaled Range analysis for W7-AS I_{sat} data. Top curve (green circles): original data. Slope of fitted line: H=0.64 ± 0.02. Bottom curve (blue triangles): shuffled data, T_{shuf} =0.02 ms. Slope of fitted line: H=0.54 ± 0.02. Thus, the data contain significant temporal correlations.





Wavelet analysis: identification of pulses





Wavelet analysis

Define a "Difference of Lorentzians" mother wavelet Standard wavelet analysis follows:

wavelet

Transform and inverse transform:

$$a(t,\tau) = \int s(t')\psi_{\tau}(t-t')dt',$$

$$s'(t) = \frac{1}{C_{\psi}} \iint a(t',\tau)\psi_{\tau}^{*}(t-t')\frac{d\tau dt'}{\tau^{2}}$$





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Do pulses have narrowly distributed widths?



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For waiting time distribution, only consider "energetic" pulses

$$E = (a_{\max}/\tau_{\max})^2 > E_{\theta}$$

Conclusion: fusion edge data are power-law distributed



Waiting times in SOC systems



In the late-90s, it was suggested that the statistics of waiting-times between avalanches could be used as a test for SOC.

In the original Bak sandpile and in DRS, waiting-times followed Poisson (exponential) statistics, due to the randomness of the drive.

This statement must be made more precise, though. When sufficiently large avalanches are considered, correlations are indeed apparent, and power-laws appear.

Furthermore, for non-random drives, powerlaws may appear with no thresholding. These, however, are a reflection of drive correlations, not dynamical ones. Via thresholding, the SAME power-law can be made to appear as in the random drive case! [See: R. Sanchez et al, Phys. Rev. Lett.88, 068302 (2002)]



FIG. 2. Quiet times PDFs, for sandpiles with L = 100, 400, 2000, 4000, and 10000, respectively thresholded using $d_t = 30, 75, 200, 400$, and 800. The two upper and two lower PDFs have been, respectively, shifted up and down by a half and a full decade to allow for easier comparison. In the inset, the PDF is shown without thresholding.

CONCLUSIONS



PRL 109, 105001 (2012)

PHYSICAL REVIEW LETTERS

week ending 7 SEPTEMBER 2012

Relevance of Uncorrelated Lorentzian Pulses for the Interpretation of Turbulence in the Edge of Magnetically Confined Toroidal Plasmas

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Recently, it has been proposed that the turbulent fluctuations measured in a linear plasma device could be described as a superposition of uncorrelated Lorentzian pulses with a narrow distribution of durations, which would provide an explanation for the reported quasiexponential power spectra. Here, we study the applicability of this proposal to edge fluctuations in toroidal magnetic confinement fusion plasmas. For the purpose of this analysis, we introduce a novel wavelet-based pulse-detection technique that offers important advantages over existing techniques. This technique allows for extracting the properties of individual pulses from the experimental time series, and for quantifying the distribution of pulse duration and energy as well as temporal correlations. We apply the wavelet technique to edge turbulent fluctuation data from the W7-AS stellarator and the JET tokamak, and find that the pulses detected in the data do not have a narrow distribution of durations and are not uncorrelated. Instead, the distributions are of the power-law type, exhibiting temporal correlations over scales much longer than the typical pulse duration. These results suggest that turbulence in open and closed field line systems may be distinct, and cast doubts on the proposed ubiquity of exponential power spectra in this context.

DOI: 10.1103/PhysRevLett.109.105001

PACS numbers: 52.35.Ra, 52.25.Gj, 52.55.Fa, 52.55.Hc

• Edge fluctuation data at the edge of hot fusion plasmas from JET and W7-AS DO NOT follow the uncorrelated, narrowly-distributed width Lorentzian model.

• Instead, time correlations are present. Pulse durations and sizes are power-law distributed, their triggerings are correlated and their shape is difficult to determine uniquely.

• Power spectra of fluctuations ARE NOT well-fit by exponential law. Compound expressions containing power-laws over certain ranges of frequencies seem better.





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CONCLUSIONS



• Why LAPD/TJK data are so different from JET/W7AS data?

Critical dynamics requires near-marginality and that any subdominant transport mechanism competing with the near-marginal channel be weak.

Parallel transport at LAPD can disable critical dynamics due to open field lines

Low temperature at both LAPD and TJK can make collisional transport dominant; or not drive instabilities enough to keep profiles close to marginal.

