Predictive capabilities of avalanche models (for solar flares)

Antoine Strugarek & Paul Charbonneau

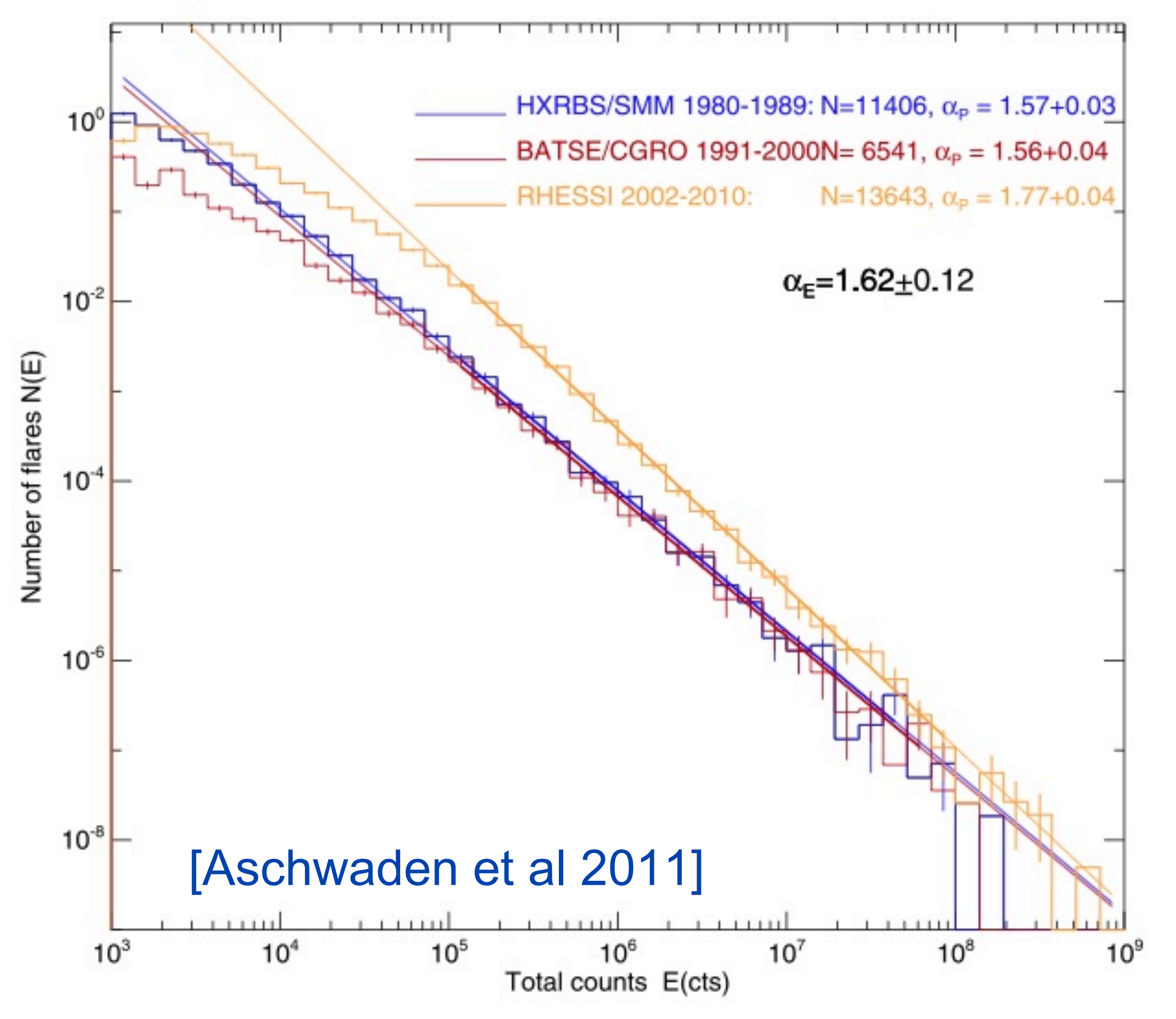
2nd ISSI Meeting
Self-Organized Criticality and Turbulence

Université de Montréal

Context: SOC models and solar flares

Observational evidences

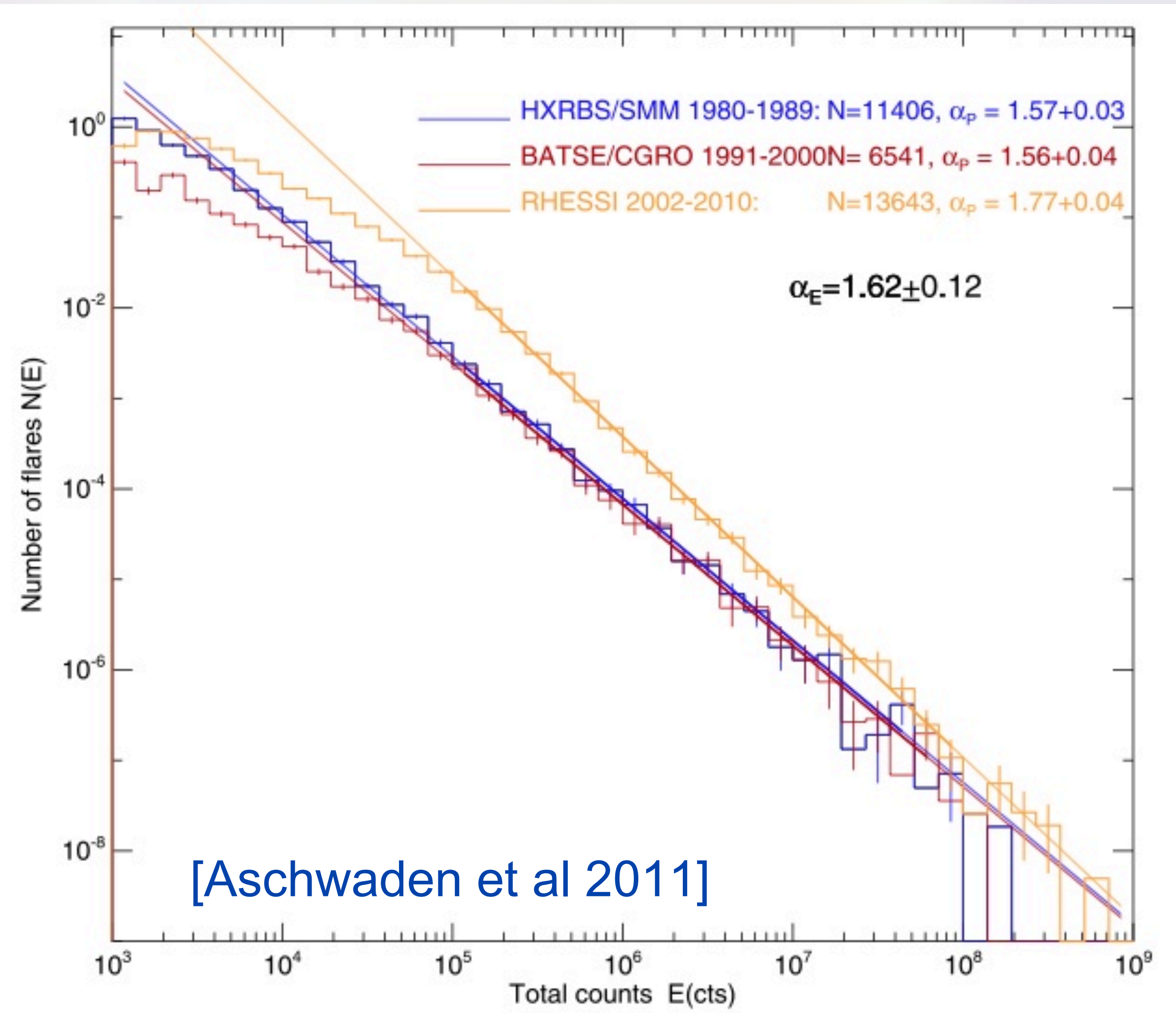
Power-law distribution functions



Context: SOC models and solar flares

Observational evidences

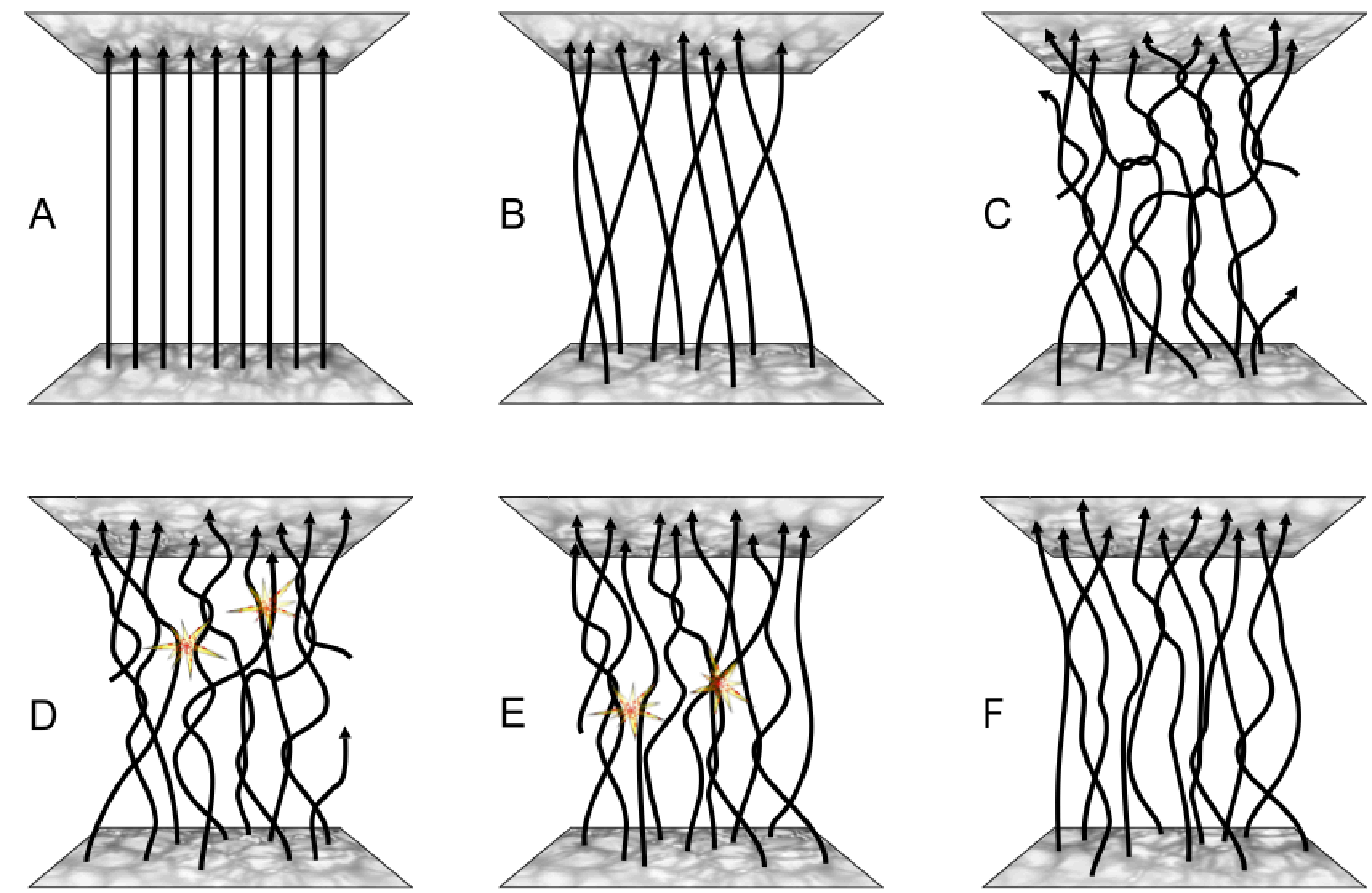
Power-law distribution functions



Physical interpretation(s)

(i) Open dissipative system

(ii) Slow driving by photospheric motions



Courtesy D. Passos

(iii) Self-limiting local threshold instability

Context: avalanche models and predictive capabilities

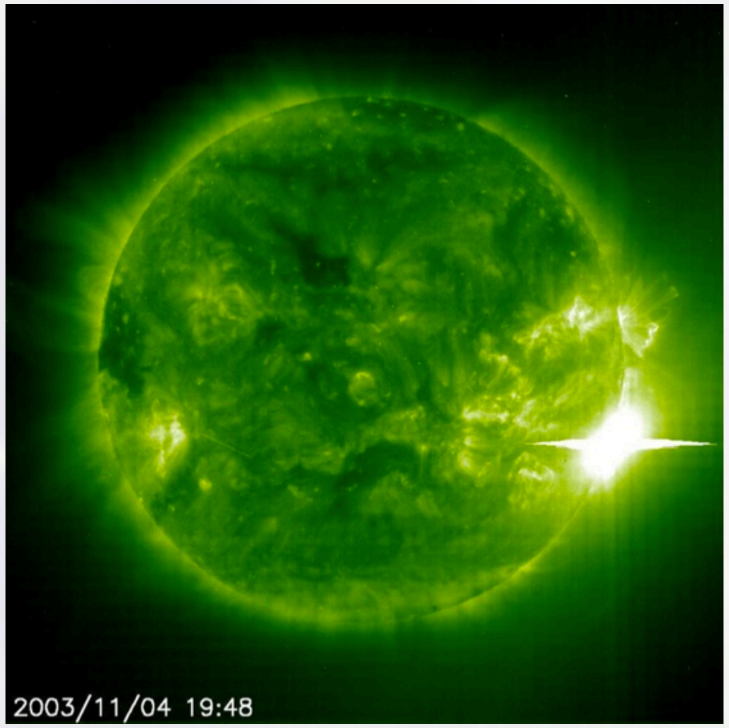
★ Goal: predict **large** and **rare** events.
Can SOC model provide us a way?

★ If yes, couple them to data assimilation techniques

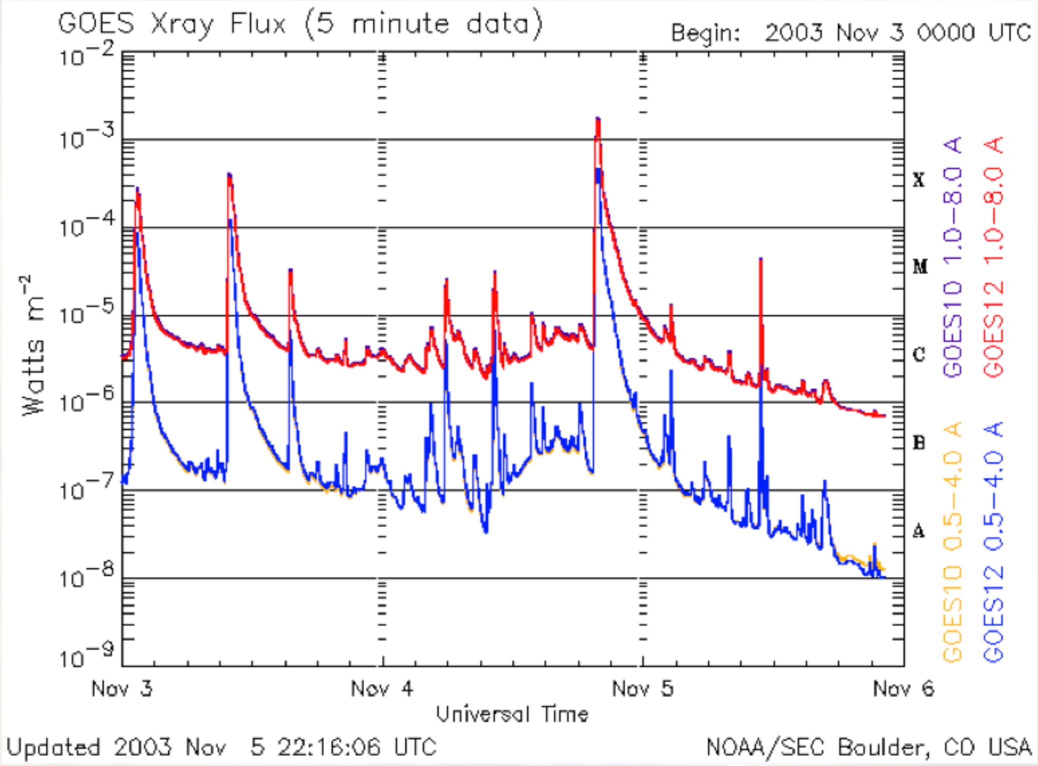
[Bélanger et al. 2007]

★ Two «conflicting» aspects of SOC models:

- ❖ Stochastic component
- ❖ Stress pattern from past history



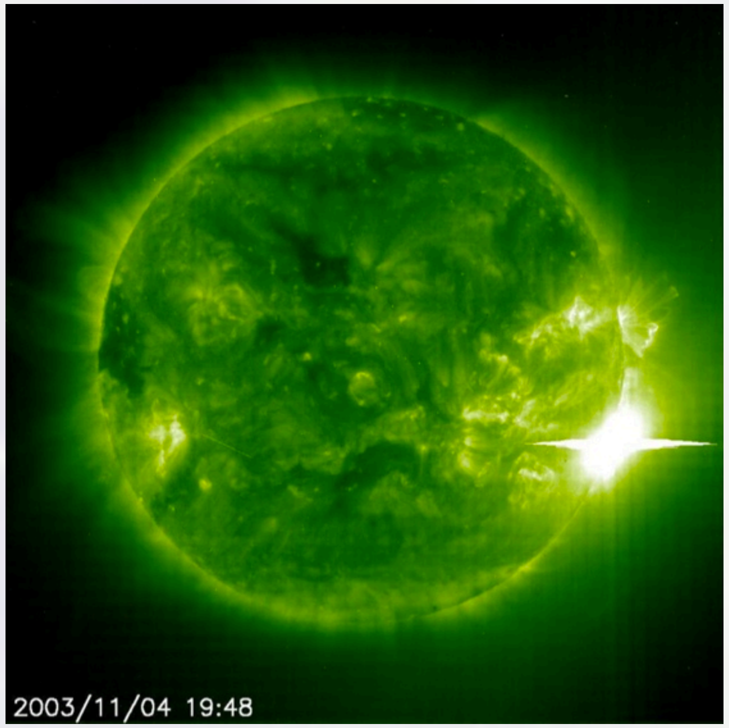
SOHO/EIT



GOES

Context: avalanche models and predictive capabilities

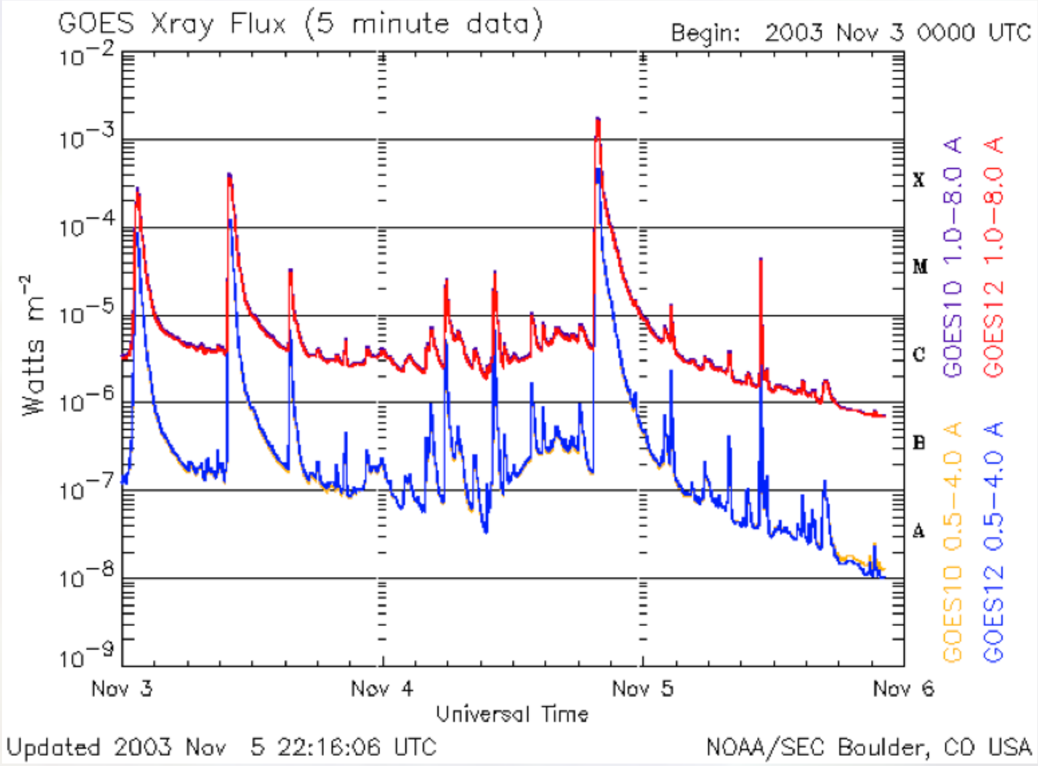
★ Goal: predict **large** and **rare** events.
Can SOC model provide us a way?



SOHO/EIT

★ If yes, couple them to data assimilation techniques

[Bélanger et al. 2007]



GOES

★ Two «conflicting» aspects of SOC models:

❖ Stochastic component
❖ Stress pattern from past history

Is it possible to predict something at all?

Outline

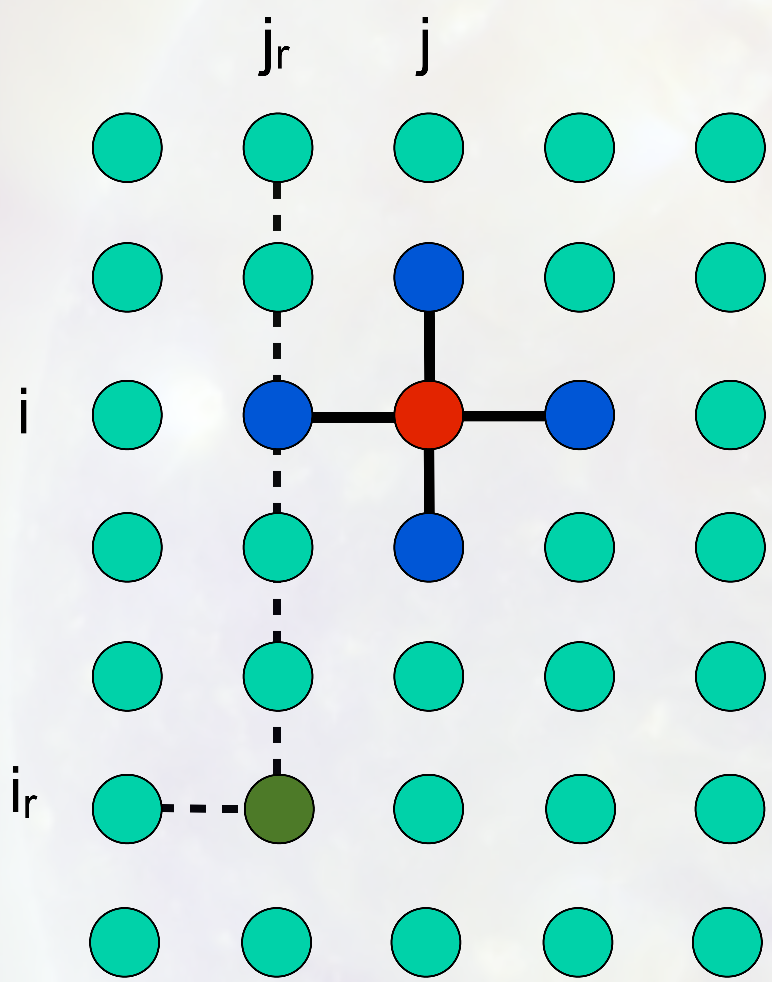
★ Avalanche models: variation on the driving scheme

- ❖ The Lu & Hamilton model
- ❖ The « Georgoulis & Vlahos » model
- ❖ A deterministic model

★ Defining ensemble averages and predictive capabilities

★ Can we really predict something, or do we only model a statistical distribution?

The Lu & Hamilton model



★ Model characteristics:

- ✤ **Purely random** driving on one node

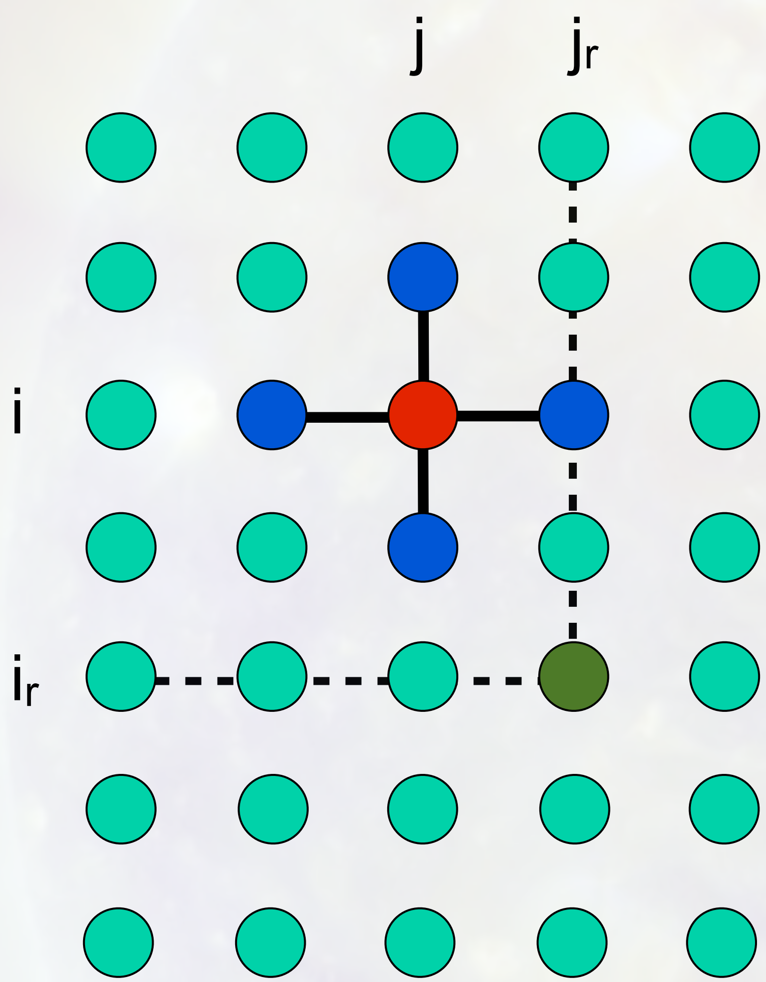
$$B_{i_r,j_r} \mathrel{+}= \delta B_r \ (\in [\sigma_1, \sigma_2])$$

- ✤ **Conservative** redistribution rule
- ✤ **Fixed** threshold

$$(Z_{i,j} > Z_c) \rightarrow \begin{cases} B_{i,j} & -= & 4\delta B \\ B_{i\pm1,j\pm1} & += & \delta B \end{cases}$$

[Lu & Hamilton '91,'93]

The «Georgoulis & Vlahos» model



★ Model characteristics:

- ✤ **Power-law random** driving on one node

$$B_{i_r,j_r} \mathrel{+}= \delta B_r \quad \left(P(\delta B_r) \propto \delta B_r^{-\alpha}\right)$$

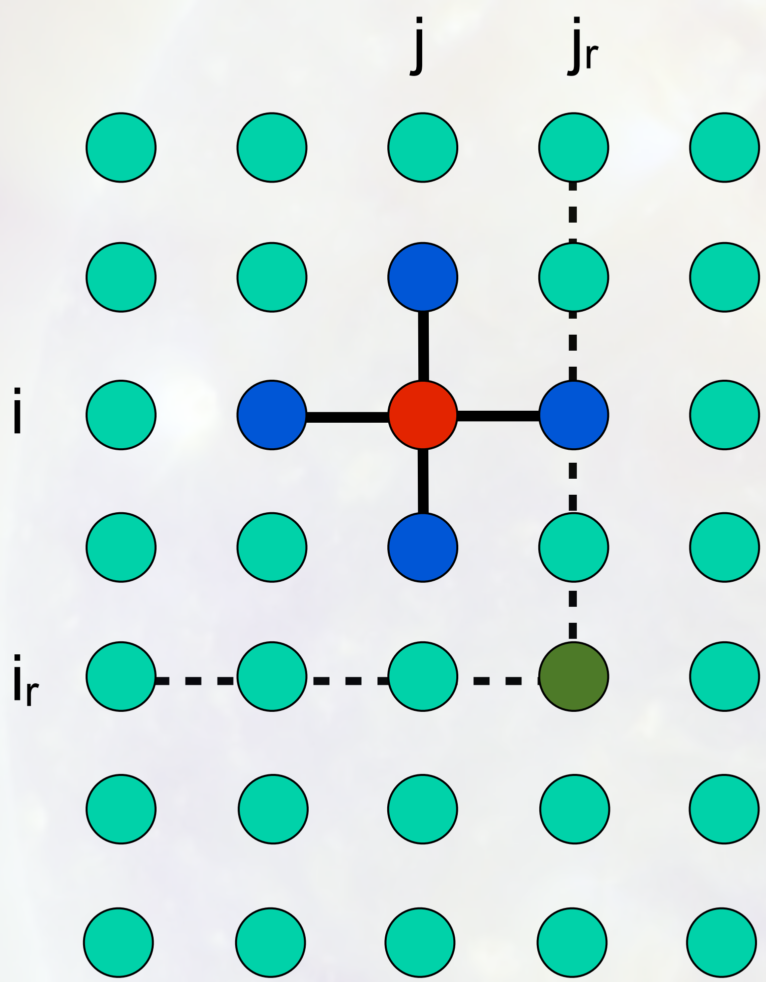
- ✤ **Conservative** redistribution rule

- ✤ **Fixed** threshold

$$(Z_{i,j} > Z_c) \rightarrow \left\{ \begin{array}{lll} B_{i,j} & -= & 4\delta B \\ B_{i\pm1,j\pm1} & += & \delta B \end{array} \right.$$

[Georgoulis & Vlahos '96,'98]

The «Georgoulis & Vlahos» model



★ Model characteristics:

- ✤ **Power-law random** driving on one node

$$B_{i_r,j_r} \mathrel{+}= \delta B_r \quad \left(P(\delta B_r) \propto \delta B_r^{-\alpha}\right)$$

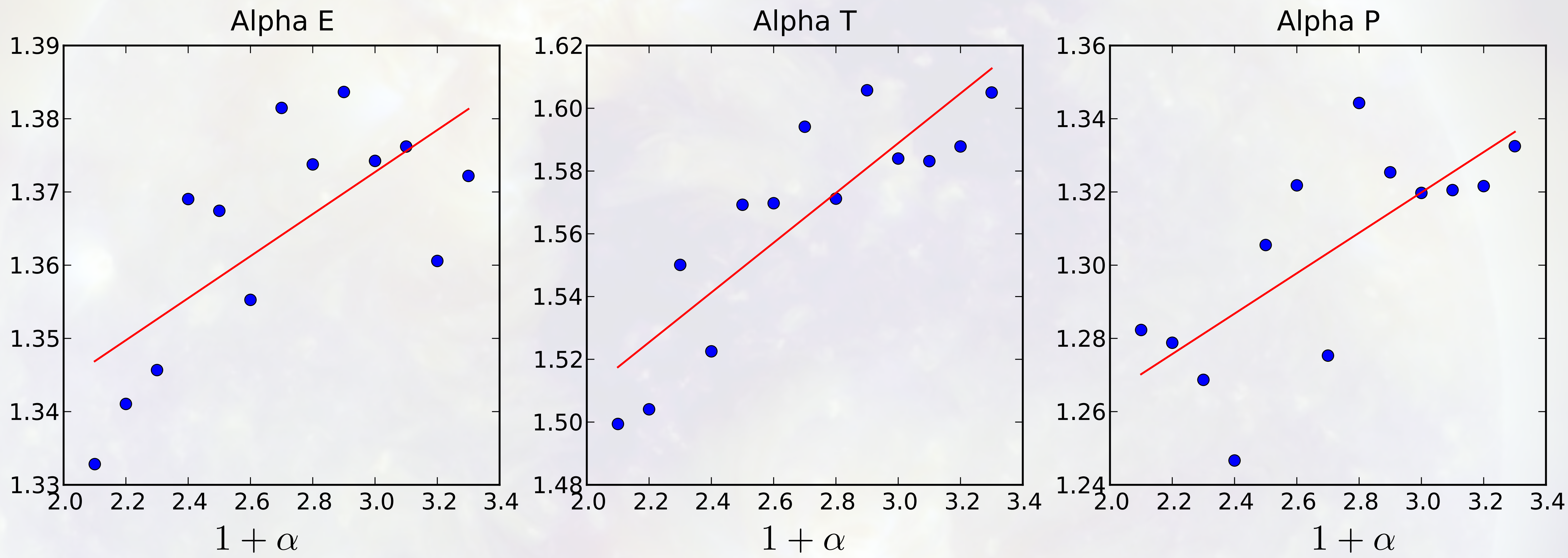
- ✤ **Conservative** redistribution rule

- ✤ **Fixed** threshold

$$(Z_{i,j} > Z_c) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B \\ B_{i\pm1,j\pm1} & += \quad \delta B \end{cases}$$

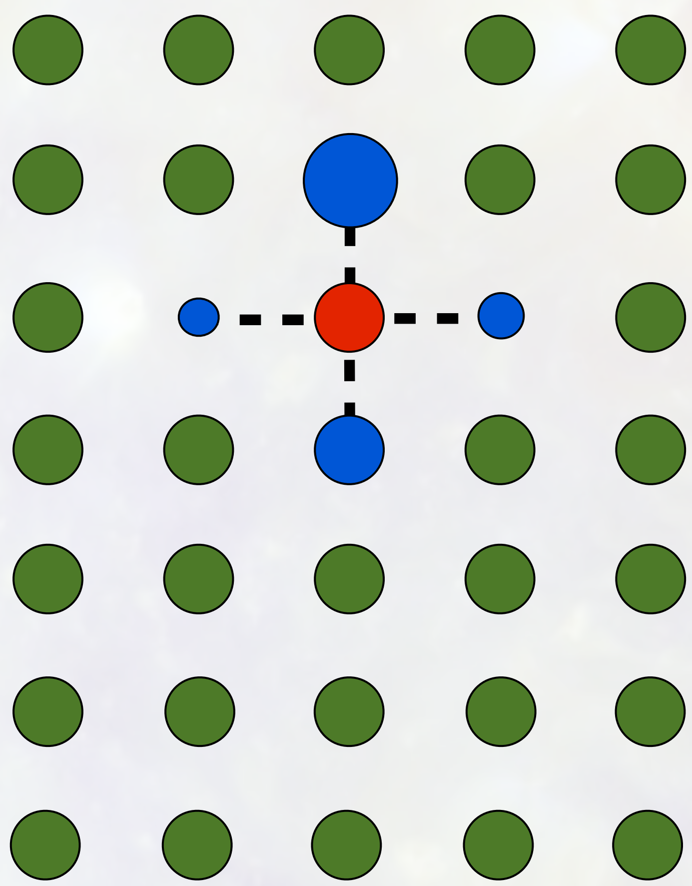
[Georgoulis & Vlahos '96,'98]

The «Georgoulis & Vlahos» model (cont’d)



Characteristic slopes of avalanche properties **linearly** depend on the power-law exponent of the random driver

A deterministically-driven model



★ Model characteristics:

- **Deterministic** driving on **all nodes**

$$B_{i,j} = B_{i,j} \cdot (1+\epsilon) \quad \forall (i,j), \ \epsilon \ll 1$$

- **Conservative** redistribution rule

- **Random** process in extraction, redistribution and/or threshold

$$(Z_{i,j} > Z_c^r) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B_r \\ B_{i\pm1,j\pm1} & += \quad \frac{r_k}{R}\delta B_r \end{cases}$$

$$r_k \text{ random deviate } \in [0,1] \ (k \in \{1,4\}) \qquad \sum_k r_k = R$$

Physical interpretation of deterministic driving

$$\mathbf{B}(\varpi, \phi, t) = \boldsymbol{\nabla} \times (A_z(\varpi, \phi, t)\mathbf{z}) + B_z\mathbf{z}$$

$$A^{n+1}_{i,j} = A^{n}_{i,j} \times (1 + \varepsilon)\ , \quad \varepsilon \ll 1\, , \quad \forall\, (i, j)$$

$\boldsymbol{\nabla} \times (A_z\mathbf{z})$ is primarily in the φ-direction

↔

Twist of the flux tube!

[Bareford et al 2013]

Physical interpretation of deterministic driving

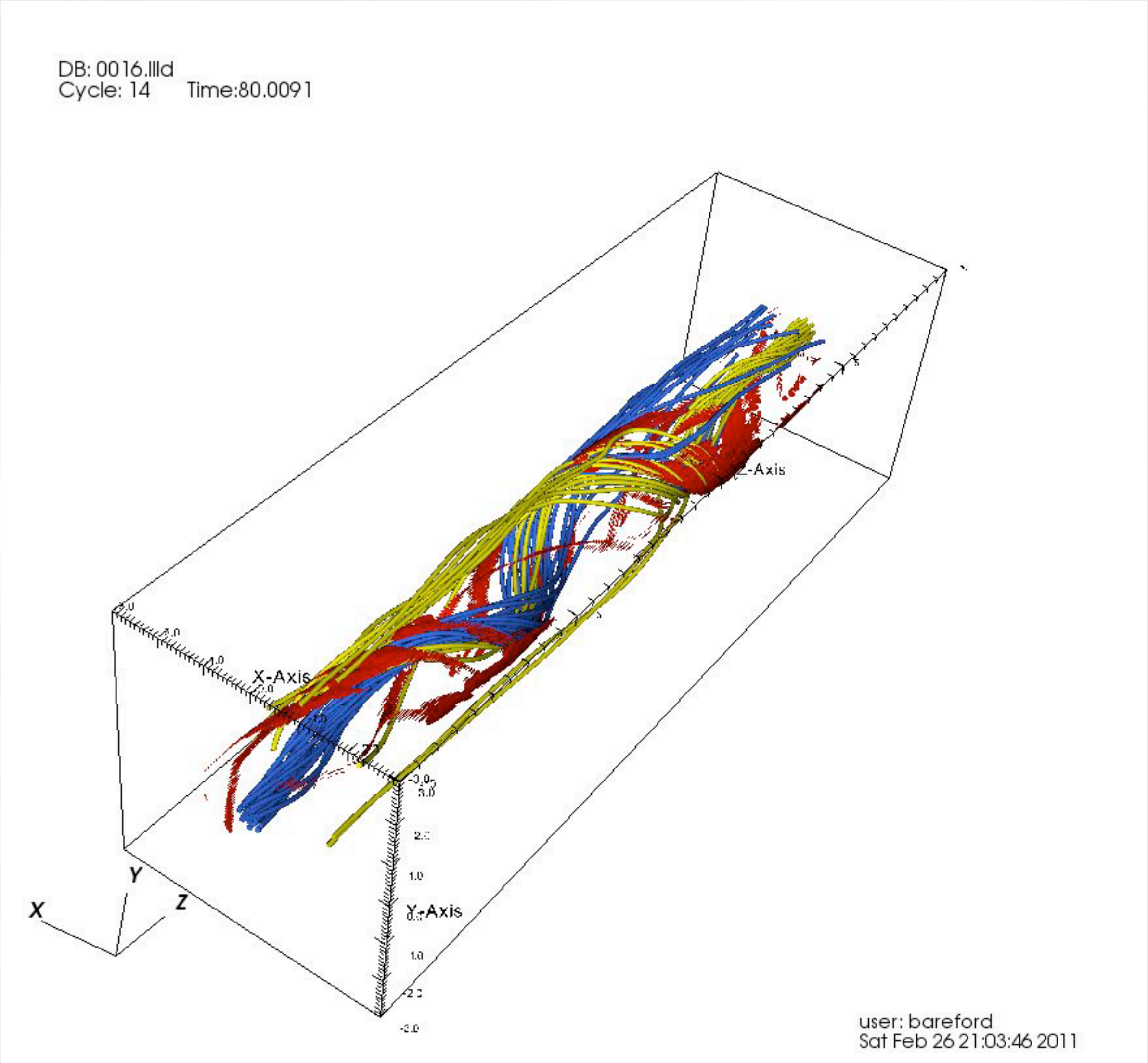
$$\mathbf{B}(\varpi, \phi, t) = \boldsymbol{\nabla} \times (A_z(\varpi, \phi, t)\mathbf{z}) + B_z\mathbf{z}$$

$$A_{i,j}^{n+1} = A_{i,j}^{n} \times (1+\varepsilon)\ ,\ \ \varepsilon \ll 1\ ,\ \ \forall\,(i,j)$$

$\boldsymbol{\nabla} \times (A_z\mathbf{z})$ is primarily in the φ-direction

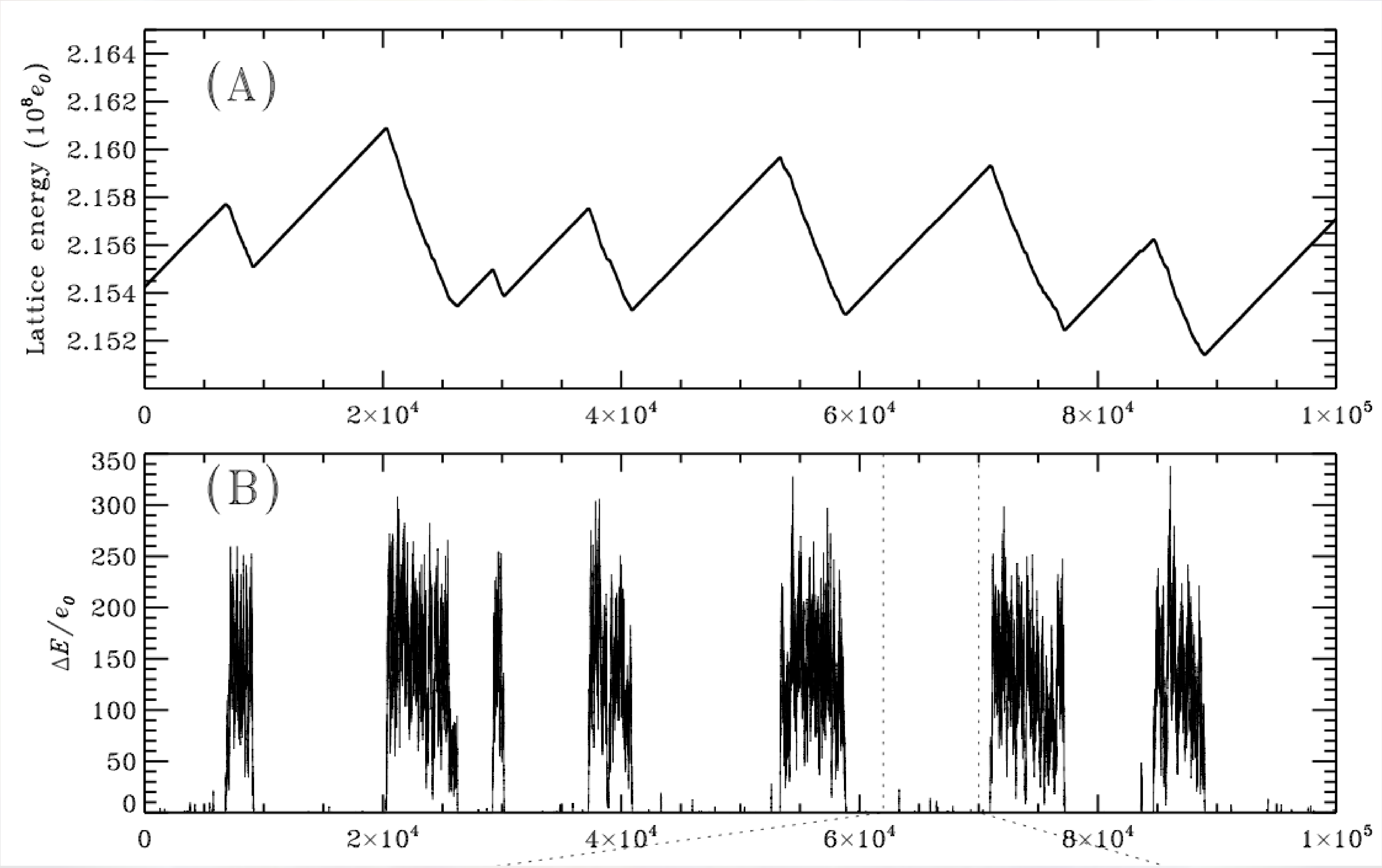
⟷

Twist of the flux tube!

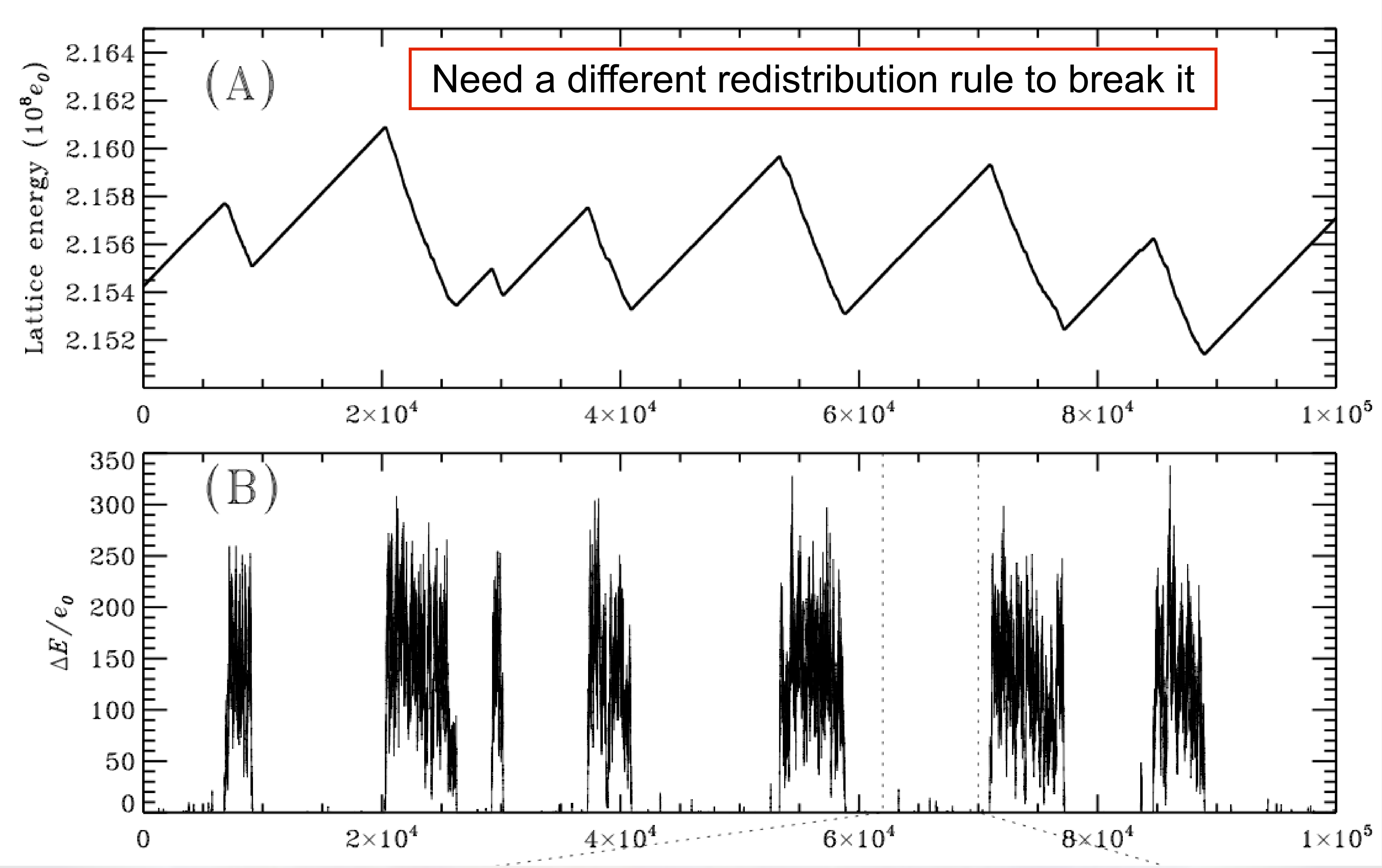


[Bareford et al 2013]

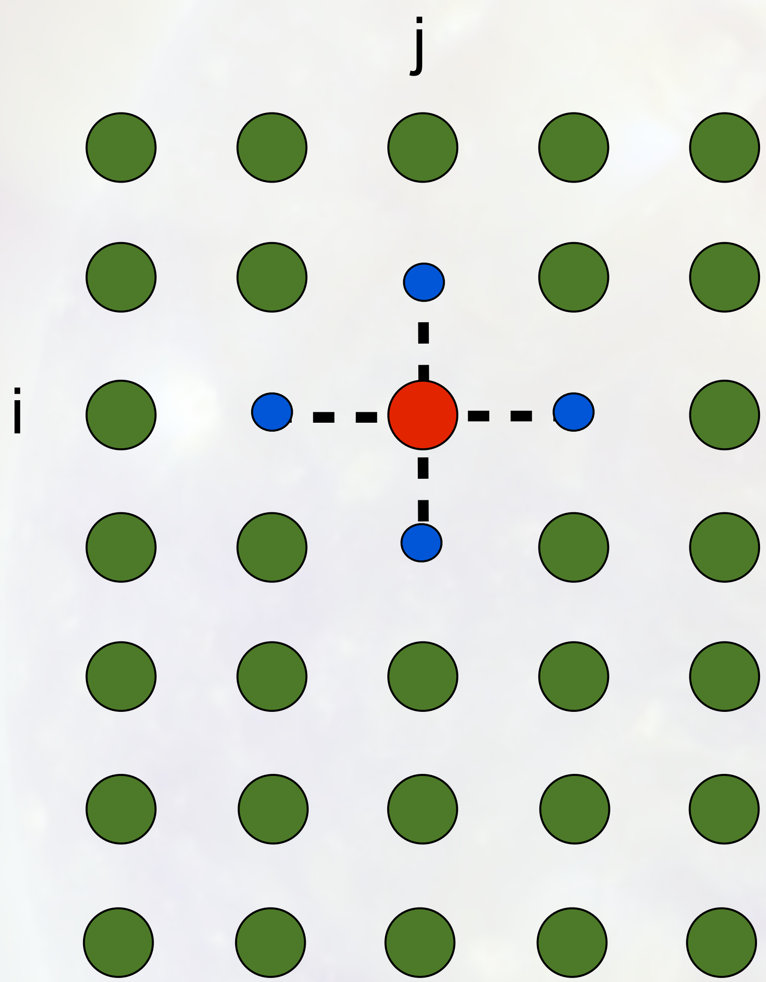
Deterministically-driven and conservative models: loading/unloading cycles



Deterministically-driven and conservative models: loading/unloading cycles



A deterministically-driven non-conservative model



★ Model characteristics:

- **Deterministic** driving on **all nodes**

$$B_{i,j} = B_{i,j} \cdot (1+\epsilon) \quad \forall (i,j), \;\; \epsilon \ll 1$$

- **Non-conservative** redistribution rule

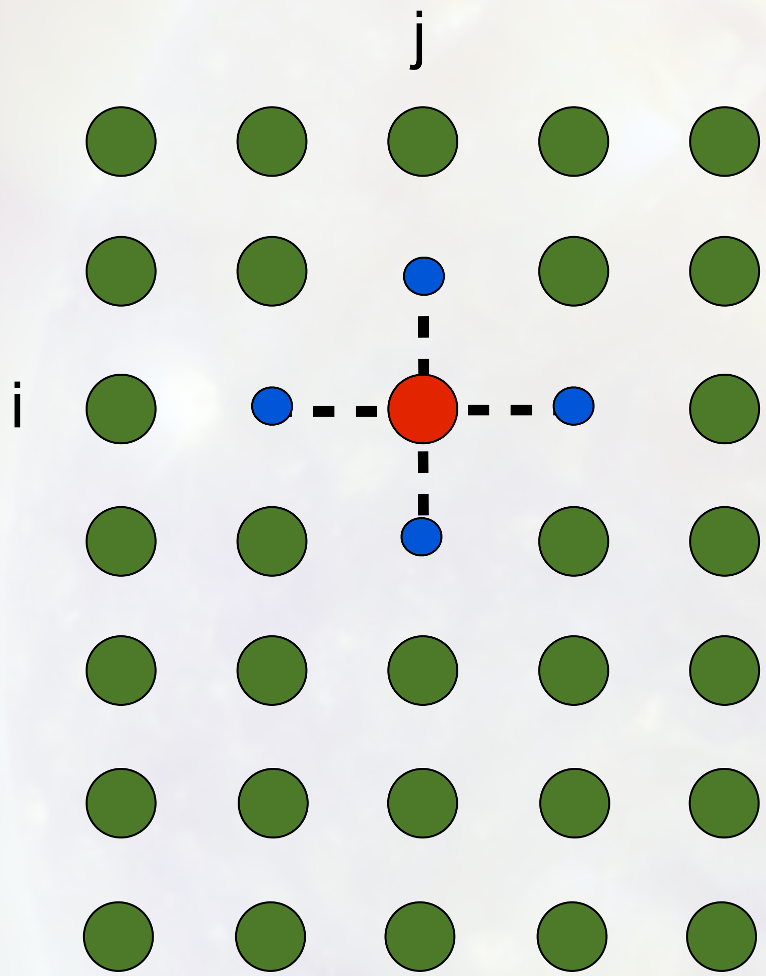
$$(Z_{i,j} > Z_c) \rightarrow \left\{ \begin{array}{lcr} B_{i,j} & -= & 4\delta B \\ B_{i\pm 1, j\pm 1} & += & r_0 \delta B \end{array} \right.$$

$$\text{random } r_0 \in [D, 1]$$

- **Fixed** threshold

[Olami et al '92; Liu et al '06]

A deterministically-driven non-conservative model



★ Model characteristics:

- ✤ **Deterministic** driving on **all nodes**

$$B_{i,j} = B_{i,j} \cdot (1+\epsilon) \quad \forall (i,j), \epsilon \ll 1$$

- ✤ **Non-conservative** redistribution rule

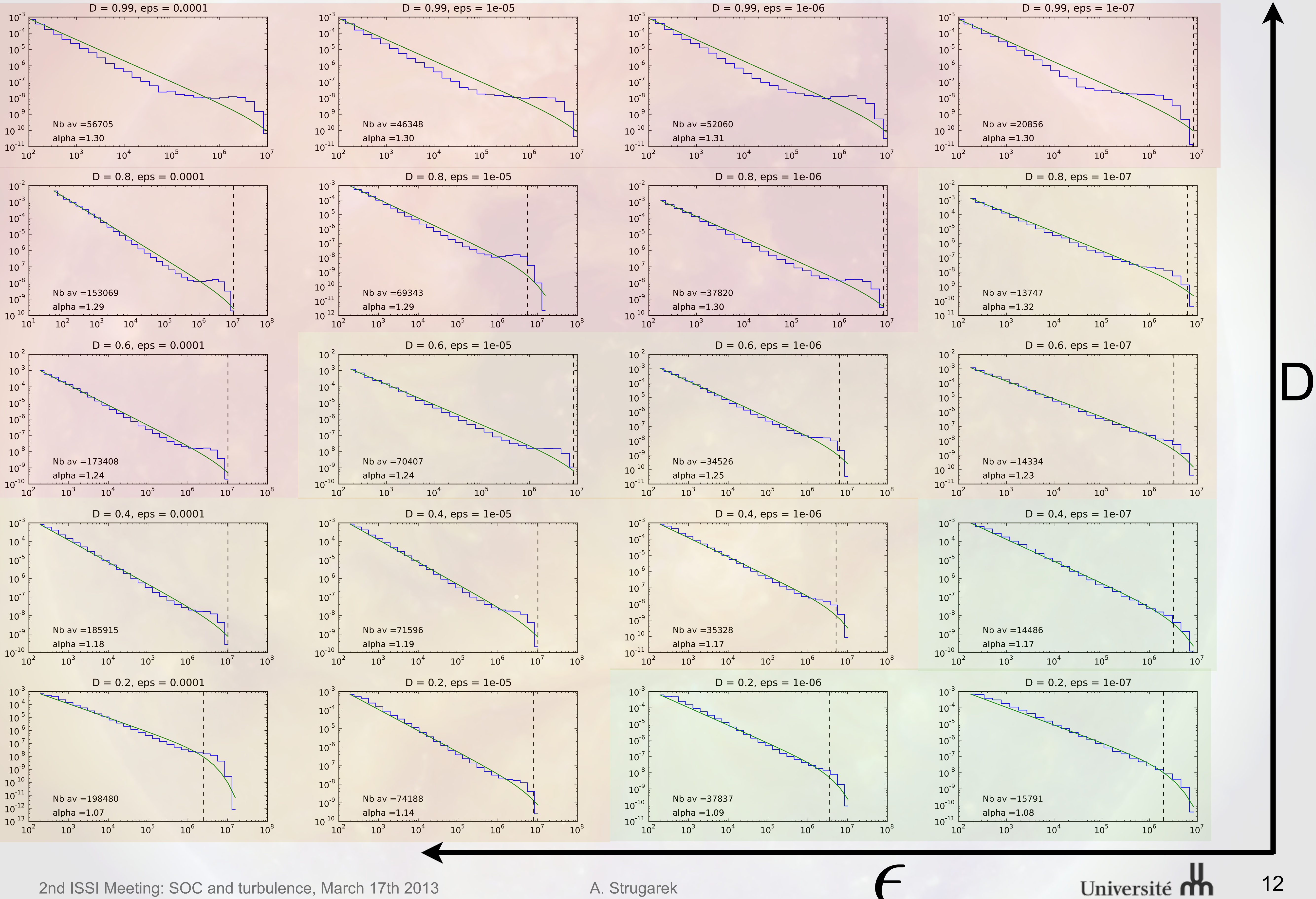
$$(Z_{i,j} > Z_c) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B \\ B_{i\pm1,j\pm1} & += \quad r_0 \delta B \end{cases}$$

$$\text{random } r_0 \in [D, 1]$$

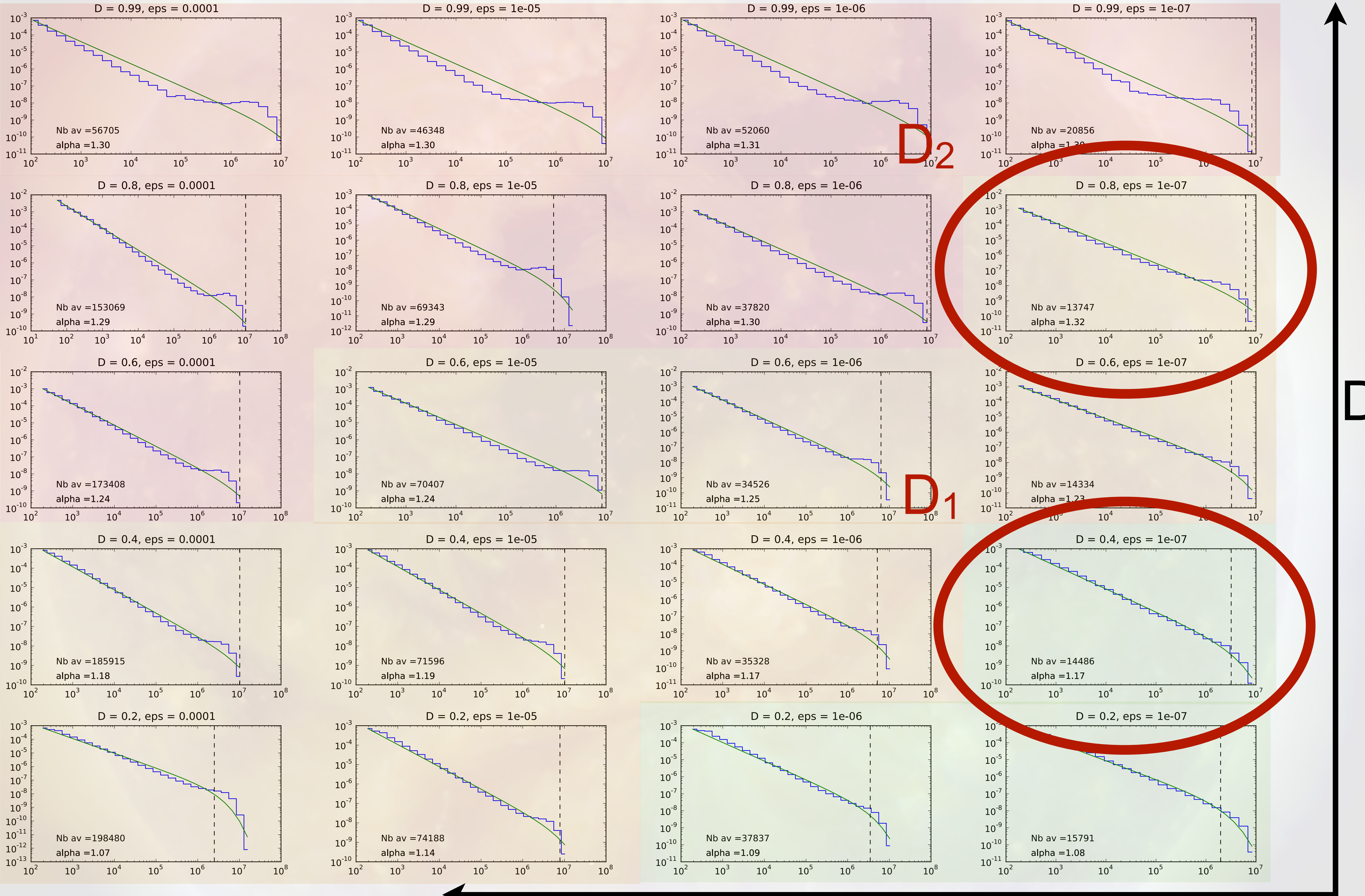
- ✤ **Fixed** threshold

[Olami et al ’92; Liu et al ’06]

A deterministically-driven non-conservative model (cont'd)



A deterministically-driven non-conservative model (cont'd)



Statistical properties of the models we considered

★ Lattice 48x48

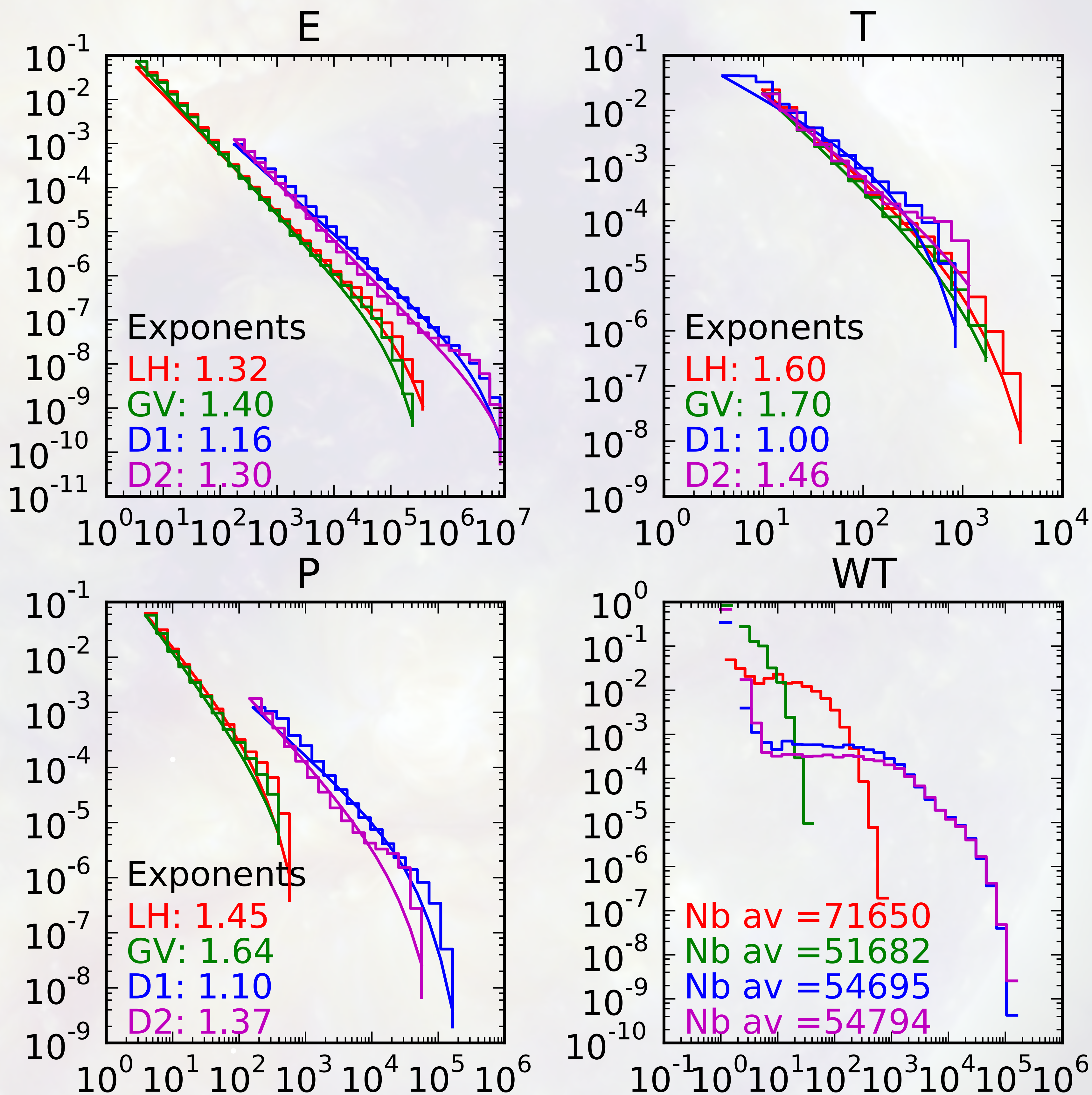
★ GV: alpha=1.6

★ D1: D=0.4

★ D2: D=0.8

Statistical properties of the models we considered

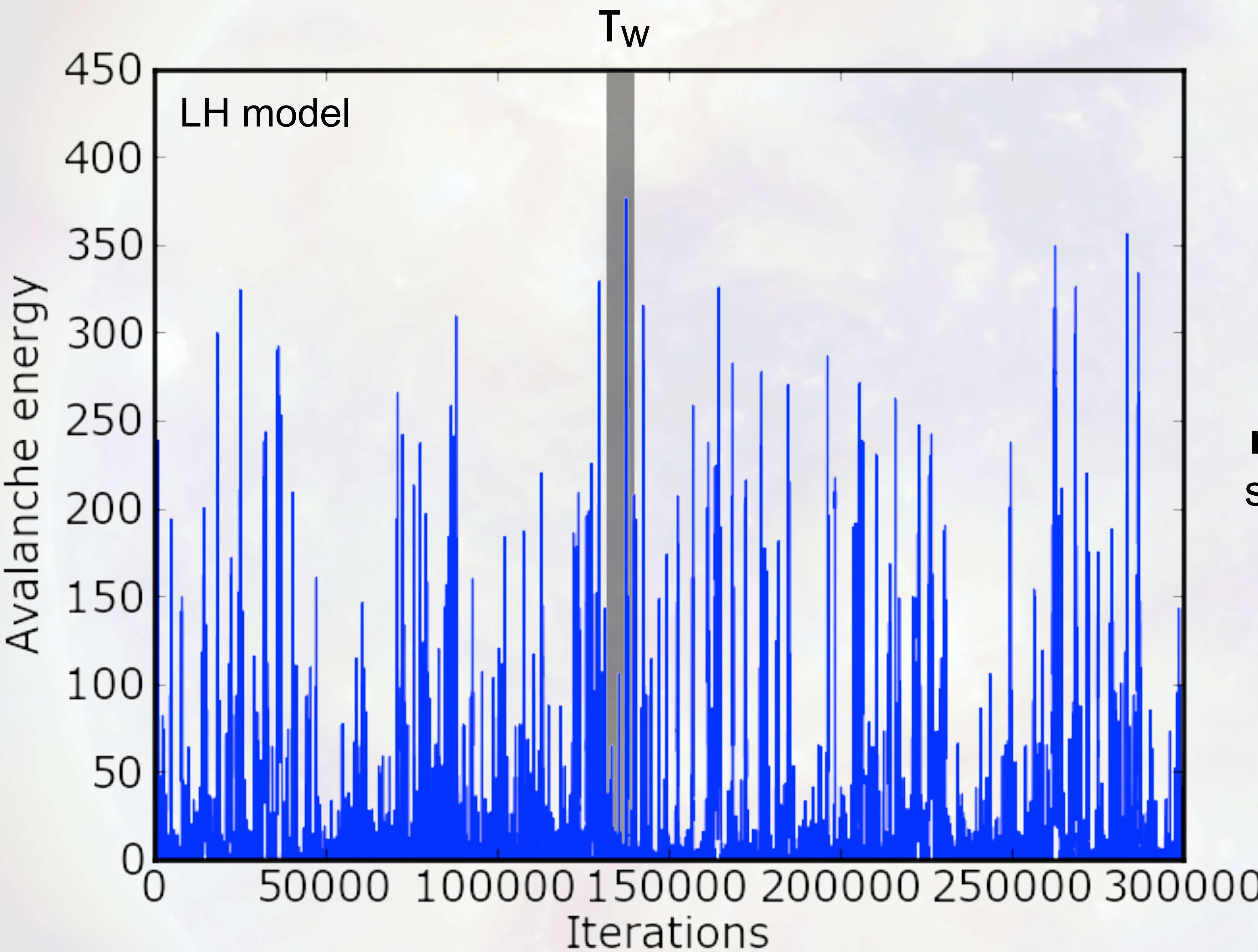
- ★ Lattice 48x48
- ★ GV: alpha=1.6
- ★ D1: D=0.4
- ★ D2: D=0.8



How to «predict» from an avalanche model?

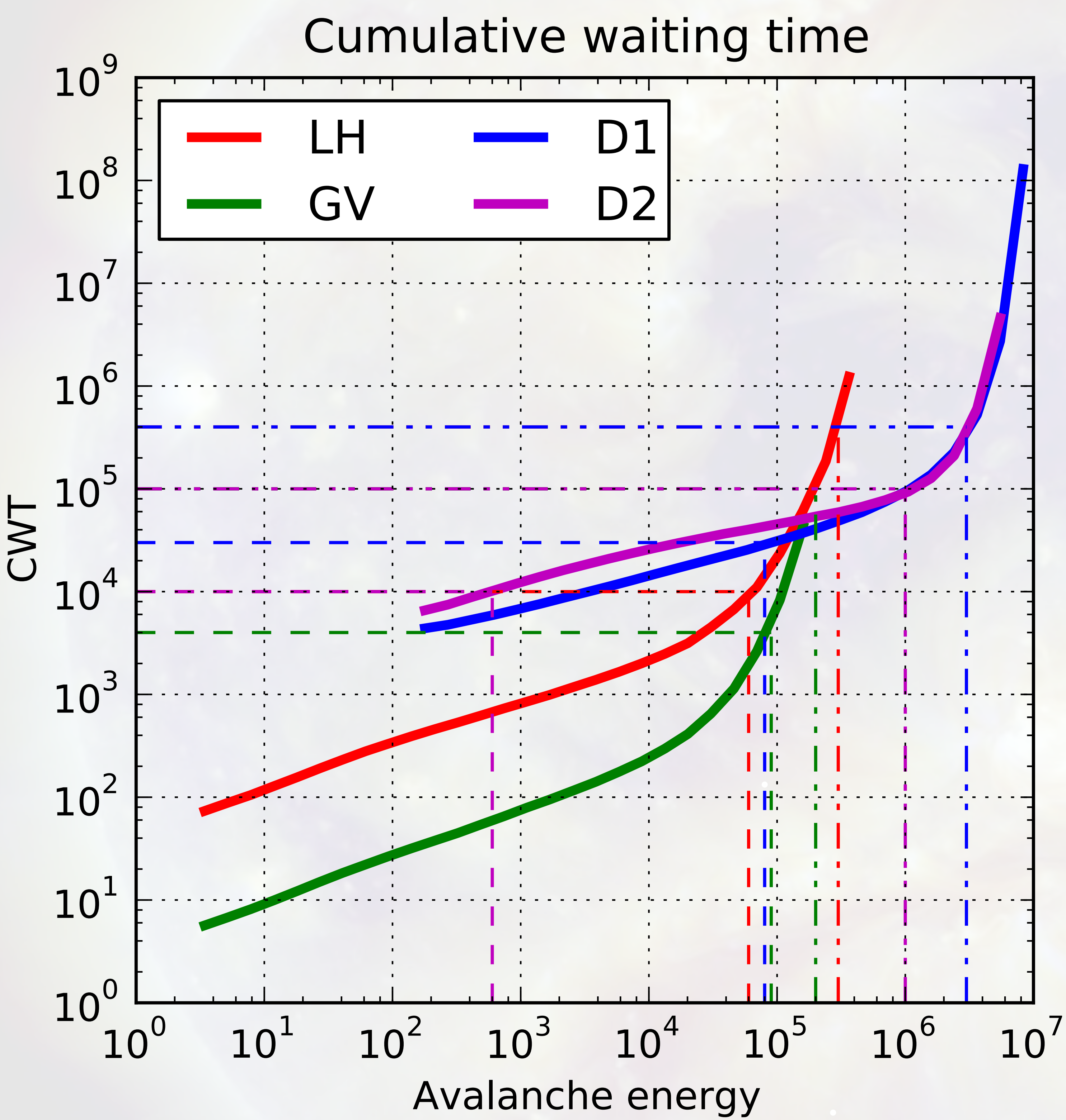
What is the maximum avalanche energy occuring in the next Δt?

Time window definition



Large events are **rare** **BUT** possess a statistical distribution

Time window definition (cont'd)



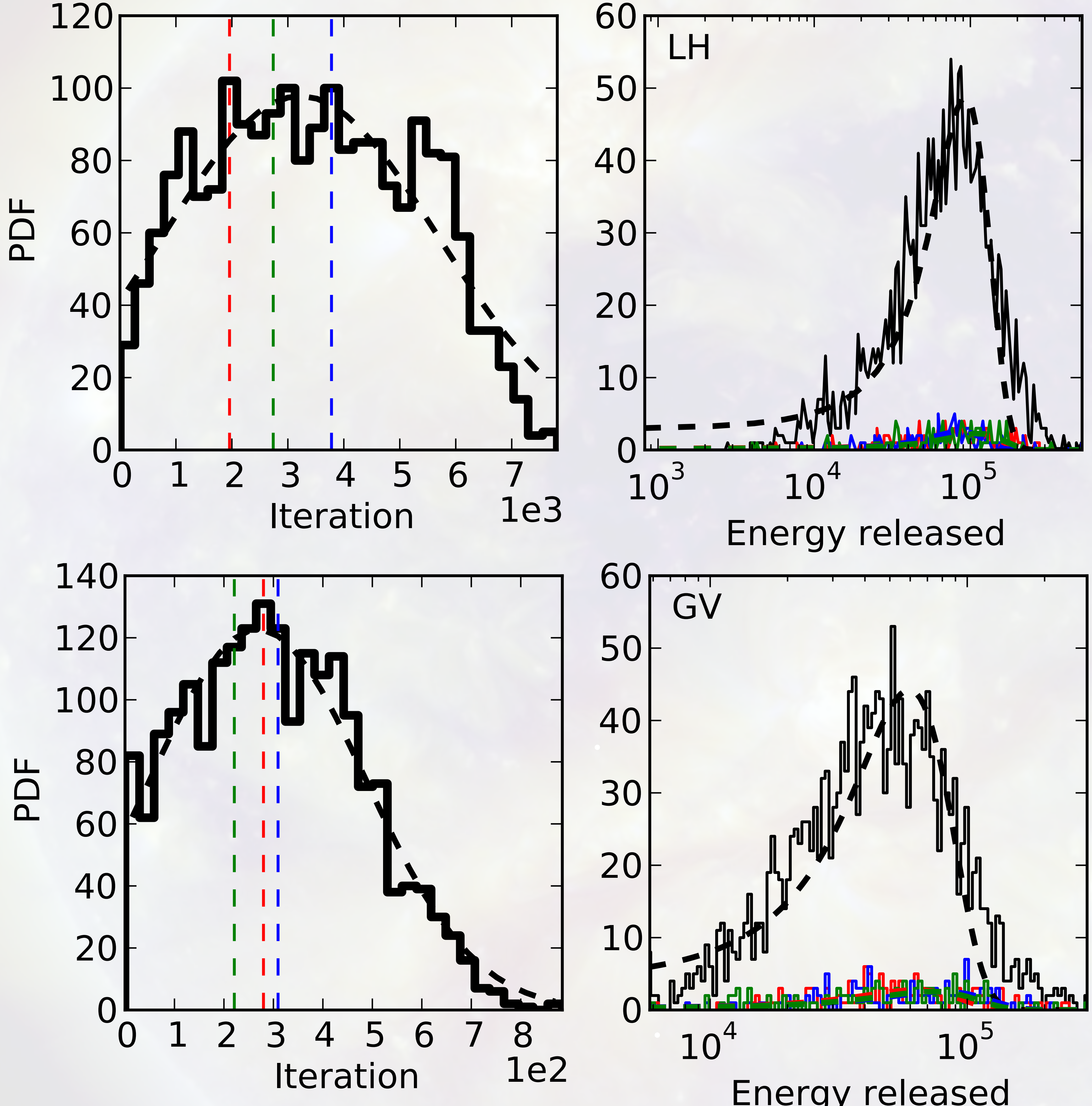
★ Cumulative WT τ_E = mean waiting time to the next avalanche bigger than x

★ Time window defined such that

$\tau_W < \tau_E / 10$

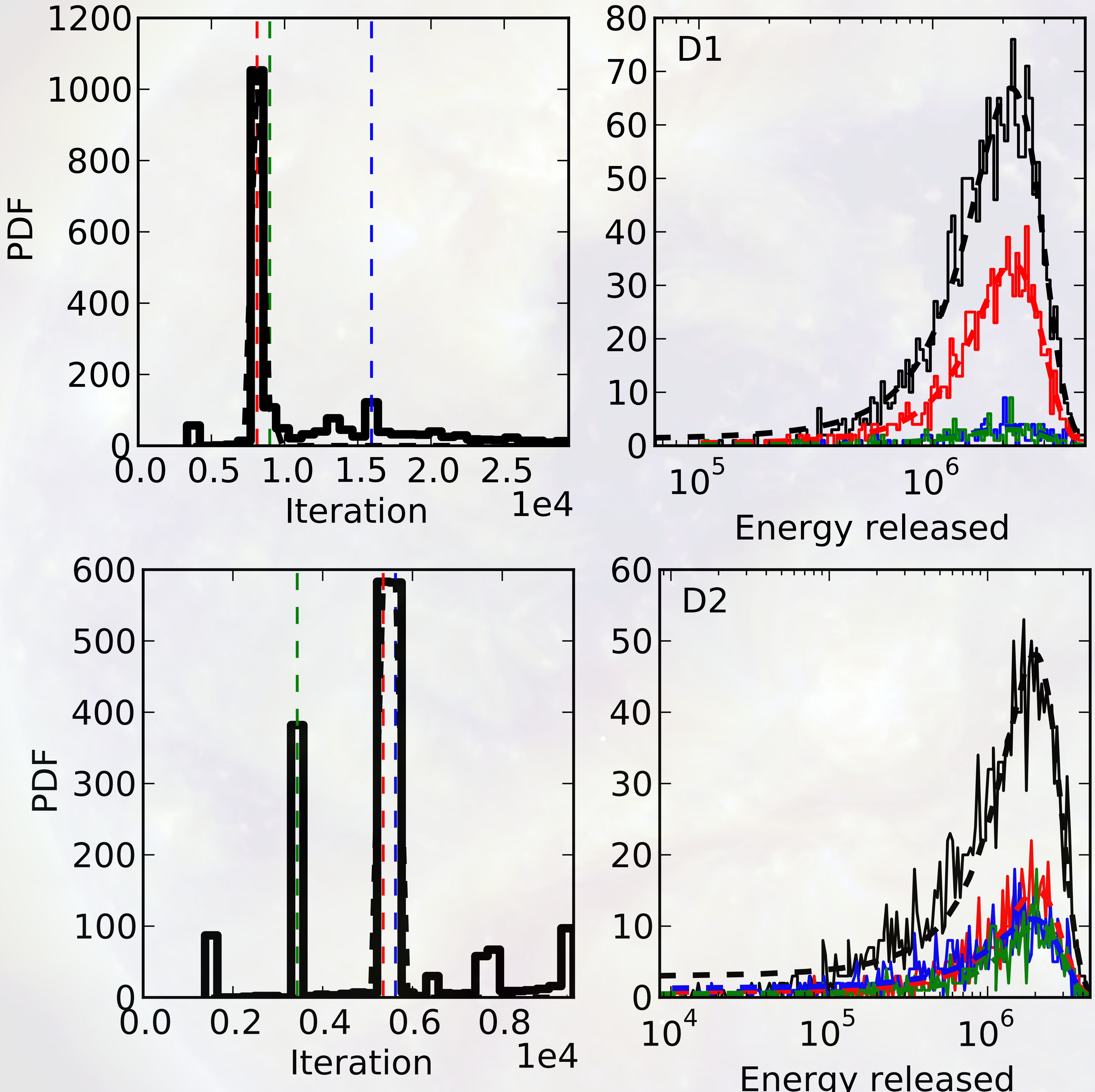
★ Equivalent results for all (τ_E, τ_W) tested in each model

Time and energy prediction: LH & GV models



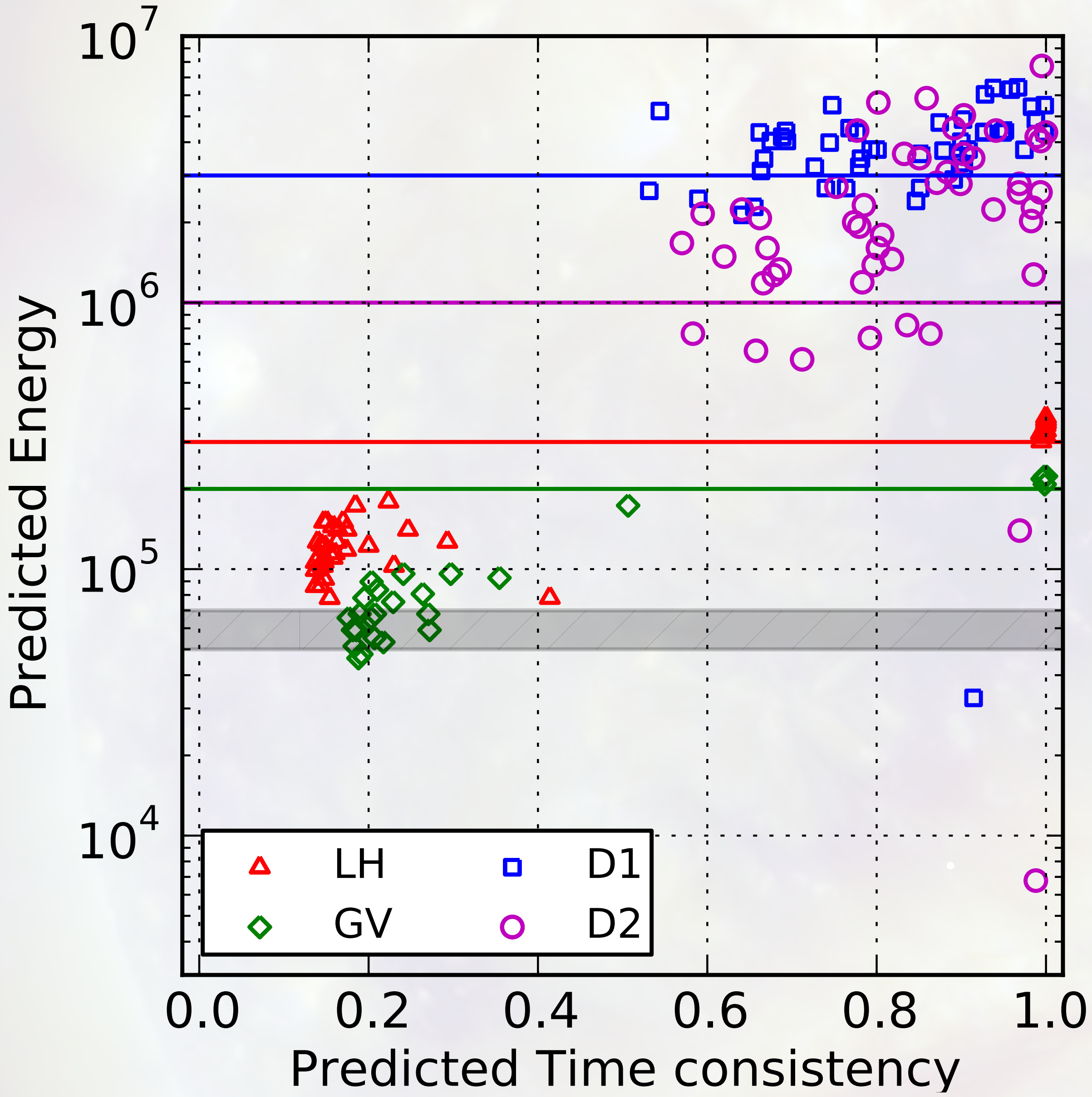
Statistics of the biggest avalanche over τ_W statistics for 2000 random number sequences

Time and energy prediction: LH & GV models



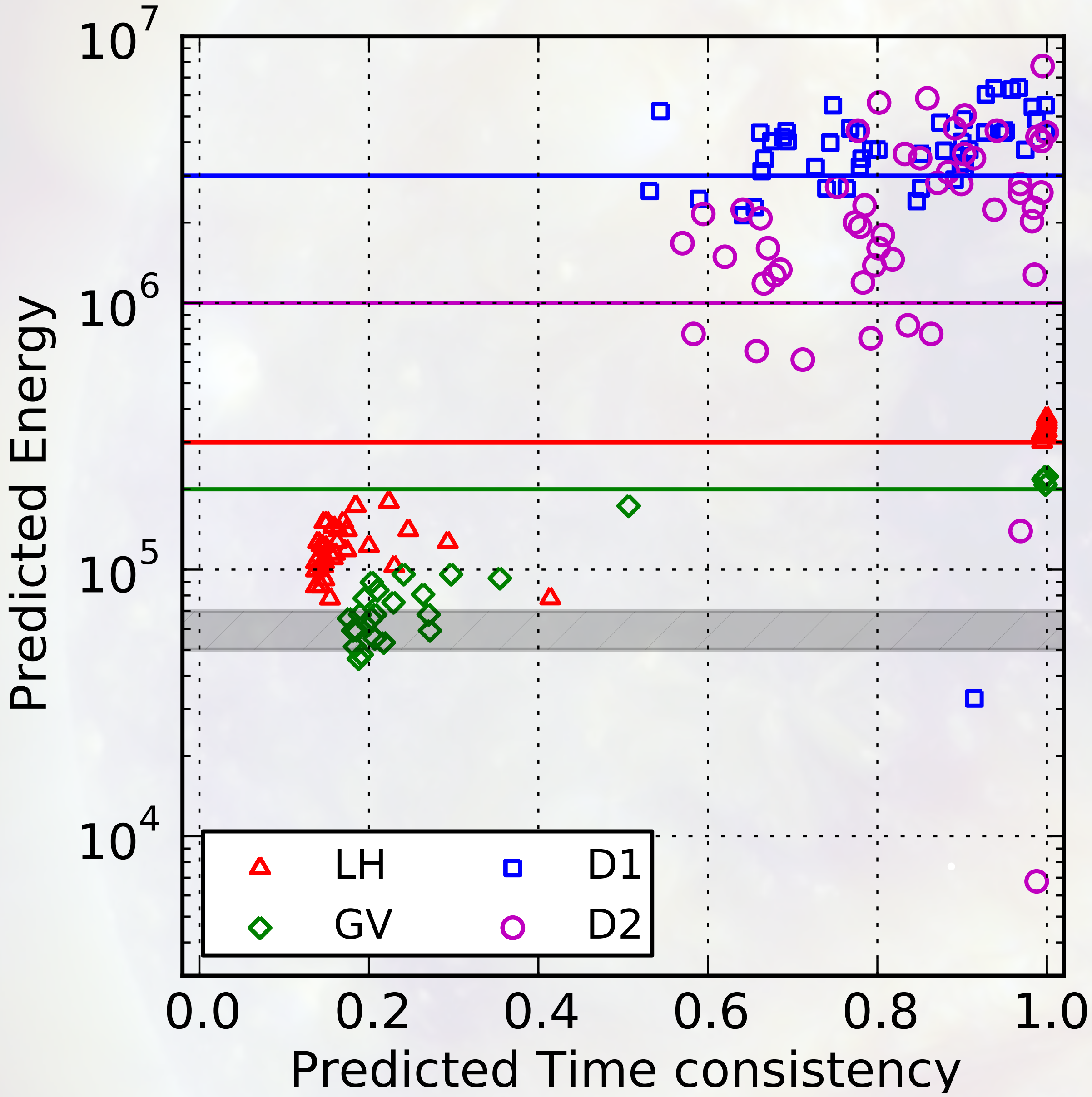
Statistics of the biggest avalanche over τ_w statistics for 2000 random number sequences

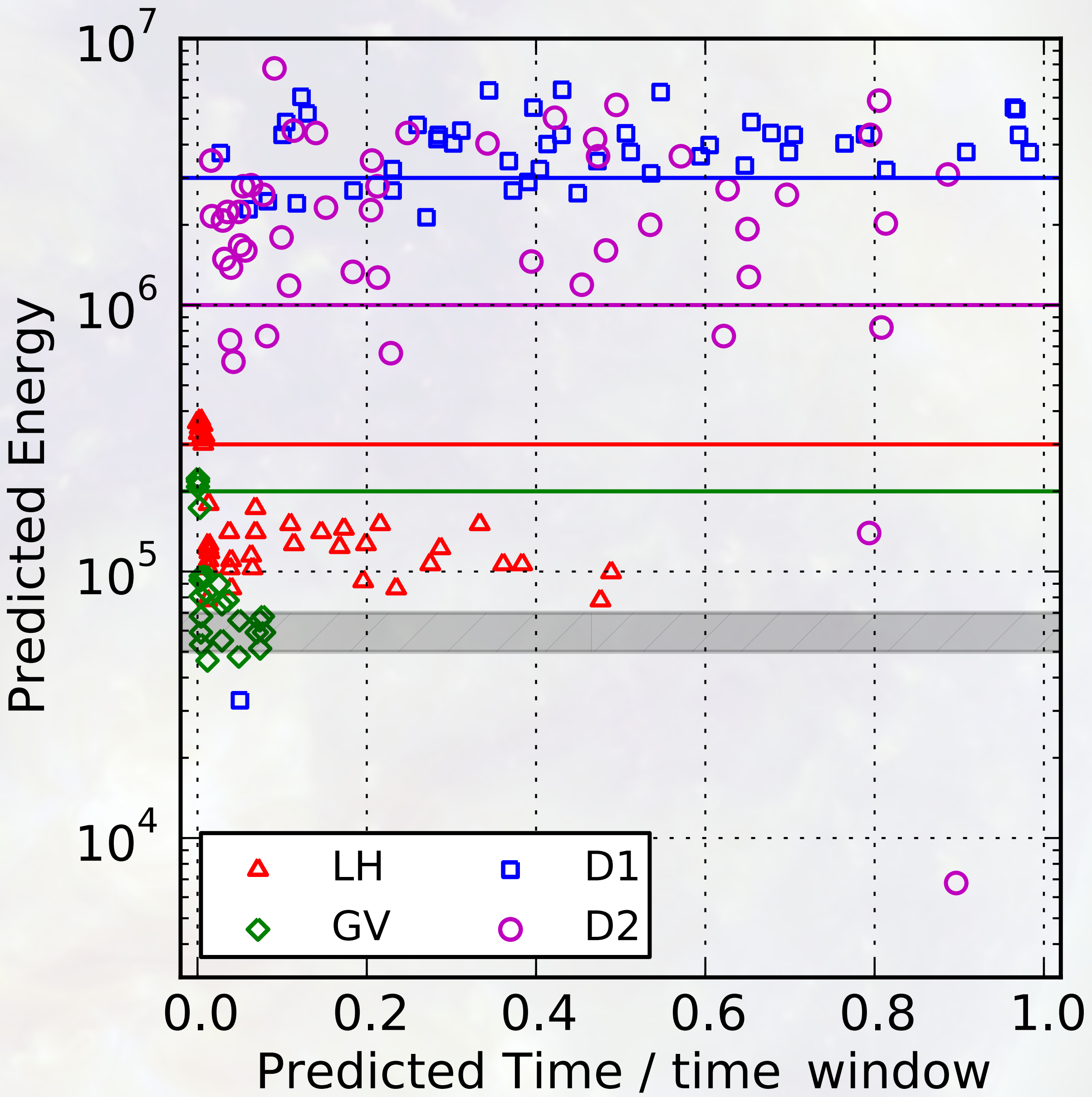
Predictive capabilities



50 large events for each model

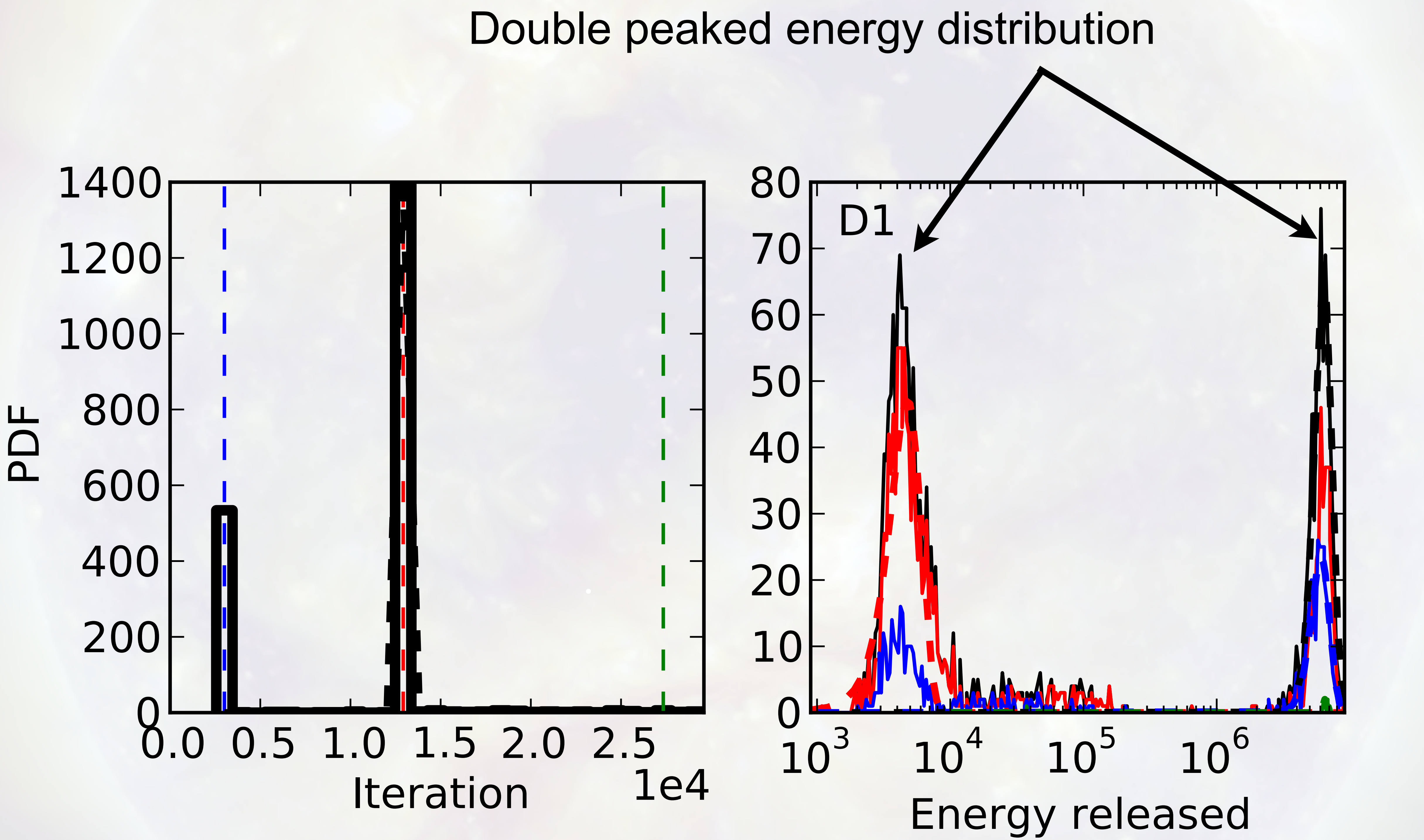
Predictive capabilities





50 large events for each model

«Ambiguous» predictions



Université de Montréal

Conclusions

★ By analyzing different avalanche models based on the original LH model, we obtained **very different predictive capabilities**

★ The classical LH model **is not** well suited for practical prediction of solar flares

★ The new deterministic model we developed possess very strong predictive capabilities

Conclusions

★ By analyzing different avalanche models based on the original LH model, we obtained **very different predictive capabilities**

★ The classical LH model **is not** well suited for practical prediction of solar flares

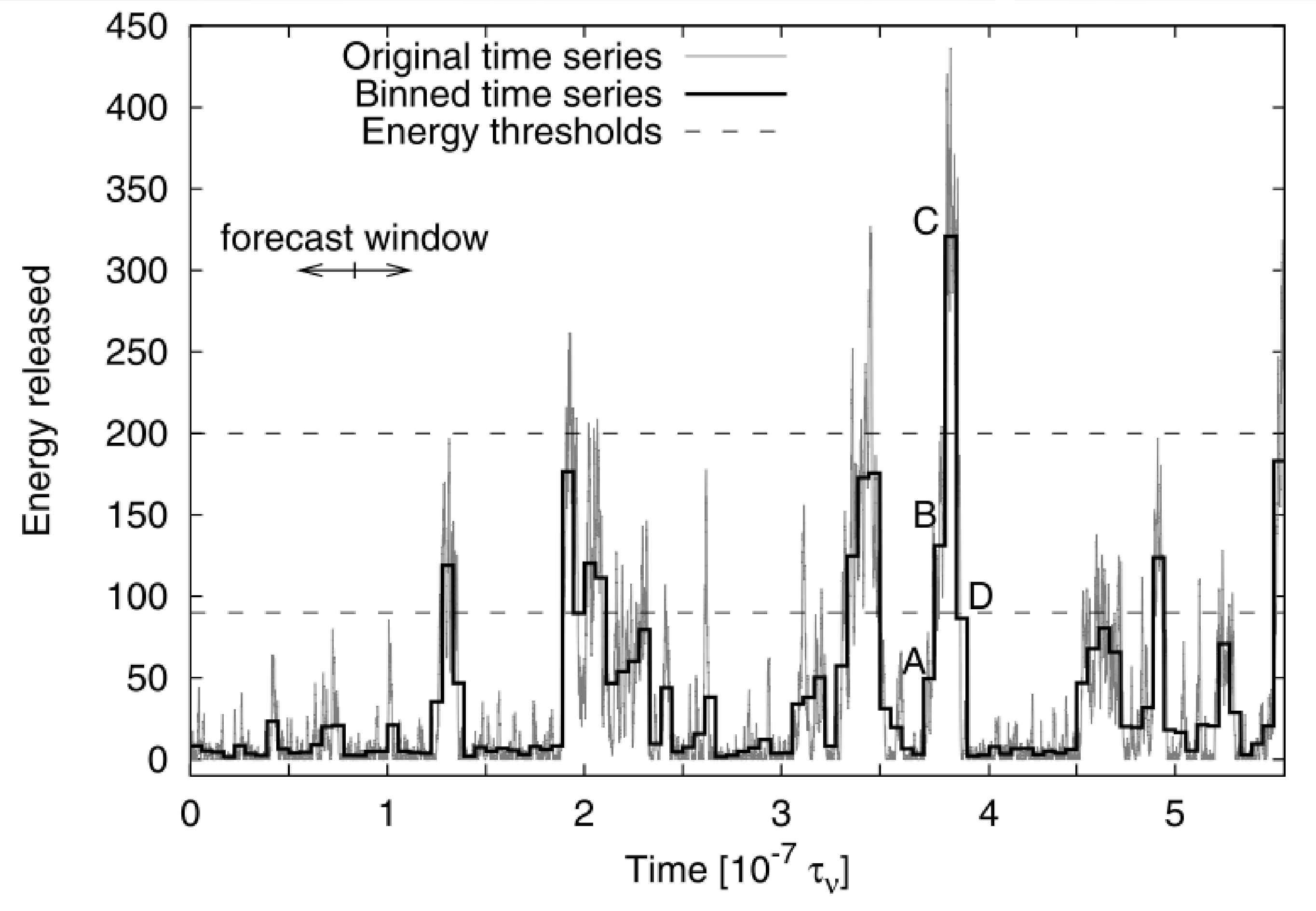
★ The new deterministic model we developed possess very strong predictive capabilities

Perspectives

★ Coupling to assimilation data technique using GOES Xray fluxes

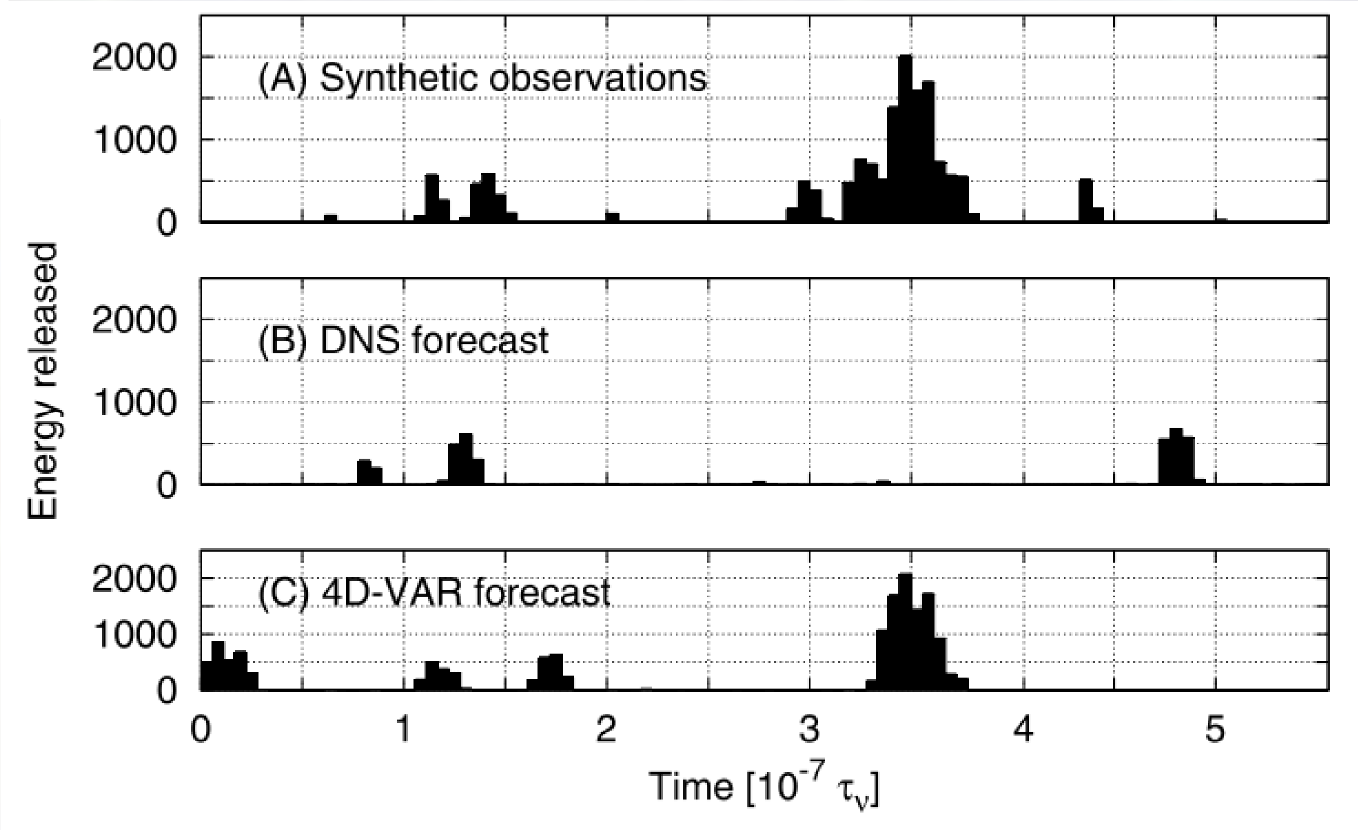
★ Estimates of Heidke and climatological skill scores of SOC models for large (M,X) flares prediction

Using data assimilation in SOC models



★ Create synthetic data from SOC model

★ 4DVAR method: use the adjoint formulation to modify the initial condition



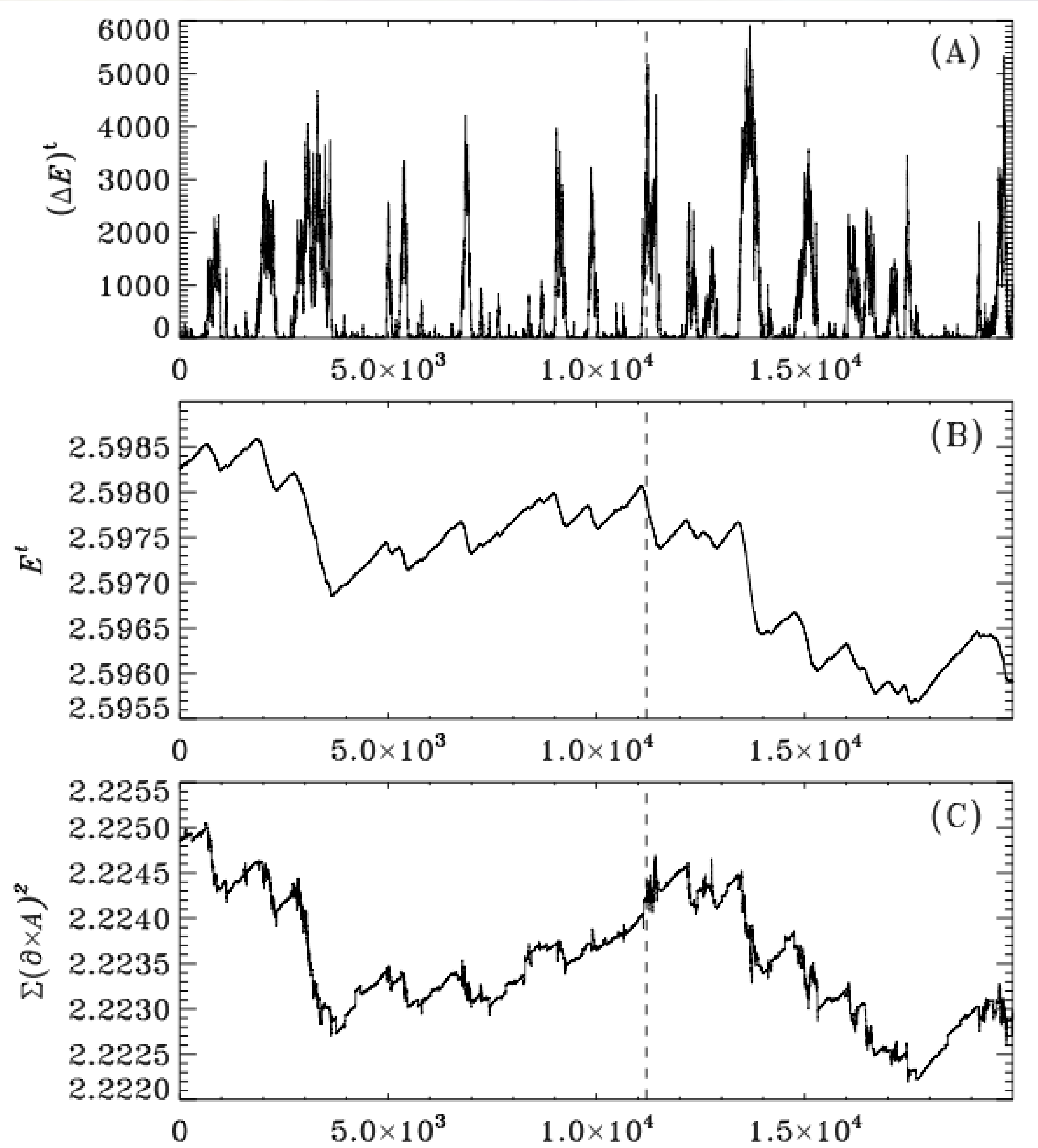
Assimilation technique done for **one** random number sequence

[Bélanger et al 2007]

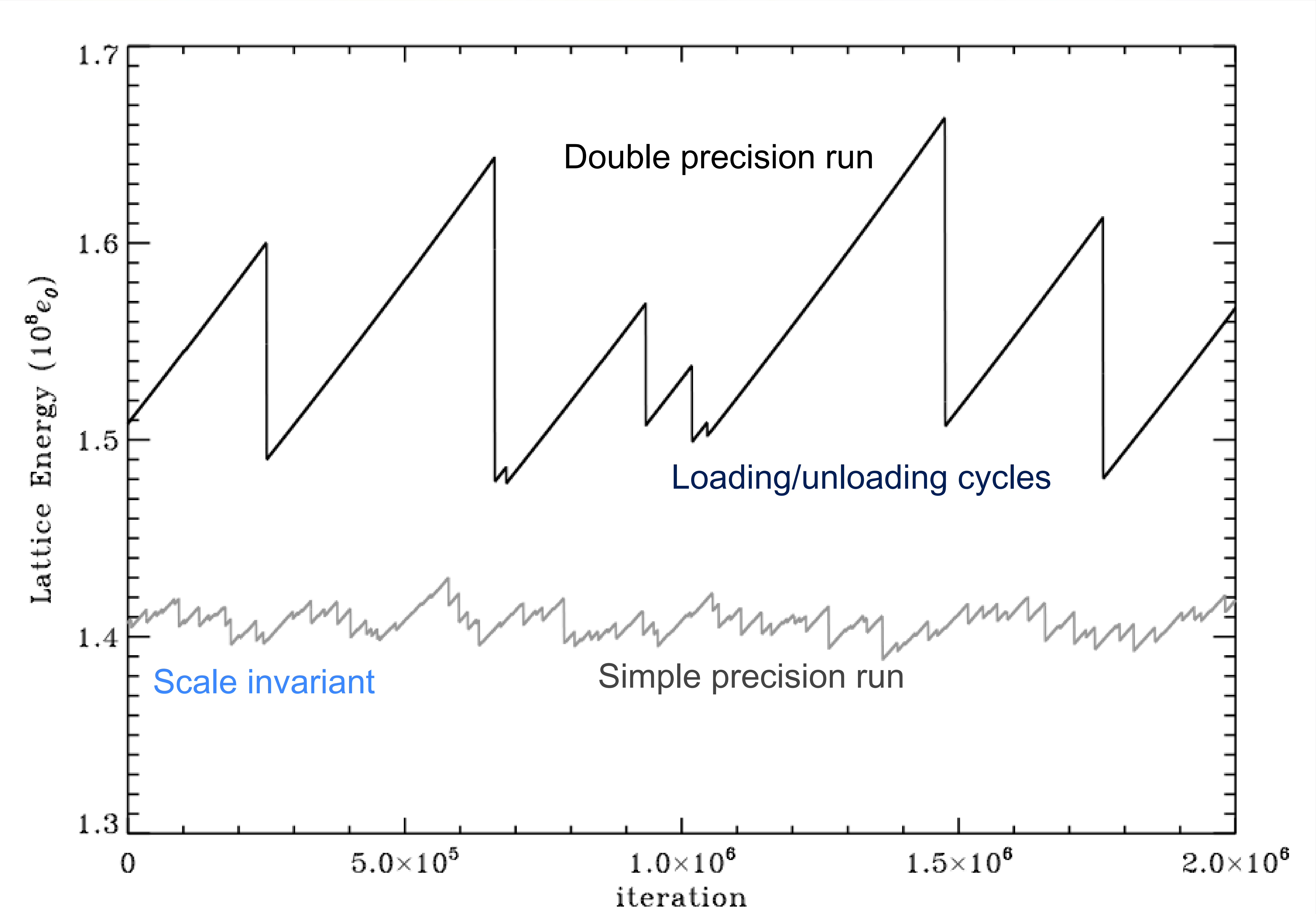
Supplemental material

On the physical interpreation of the nodal variable

[Charbonneau 2013;
chap 12 in new SOC book]



Intersting source of stochaticity



Variations of the deterministic model: conservative redistribution rules

★ Where to put the random process?

✤ Random **extraction**

$$(Z_{i,j} > Z_c) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B_r \\ B_{i\pm1,j\pm1} & += \quad \delta B_r \end{cases}$$

✤ Random **threshold**

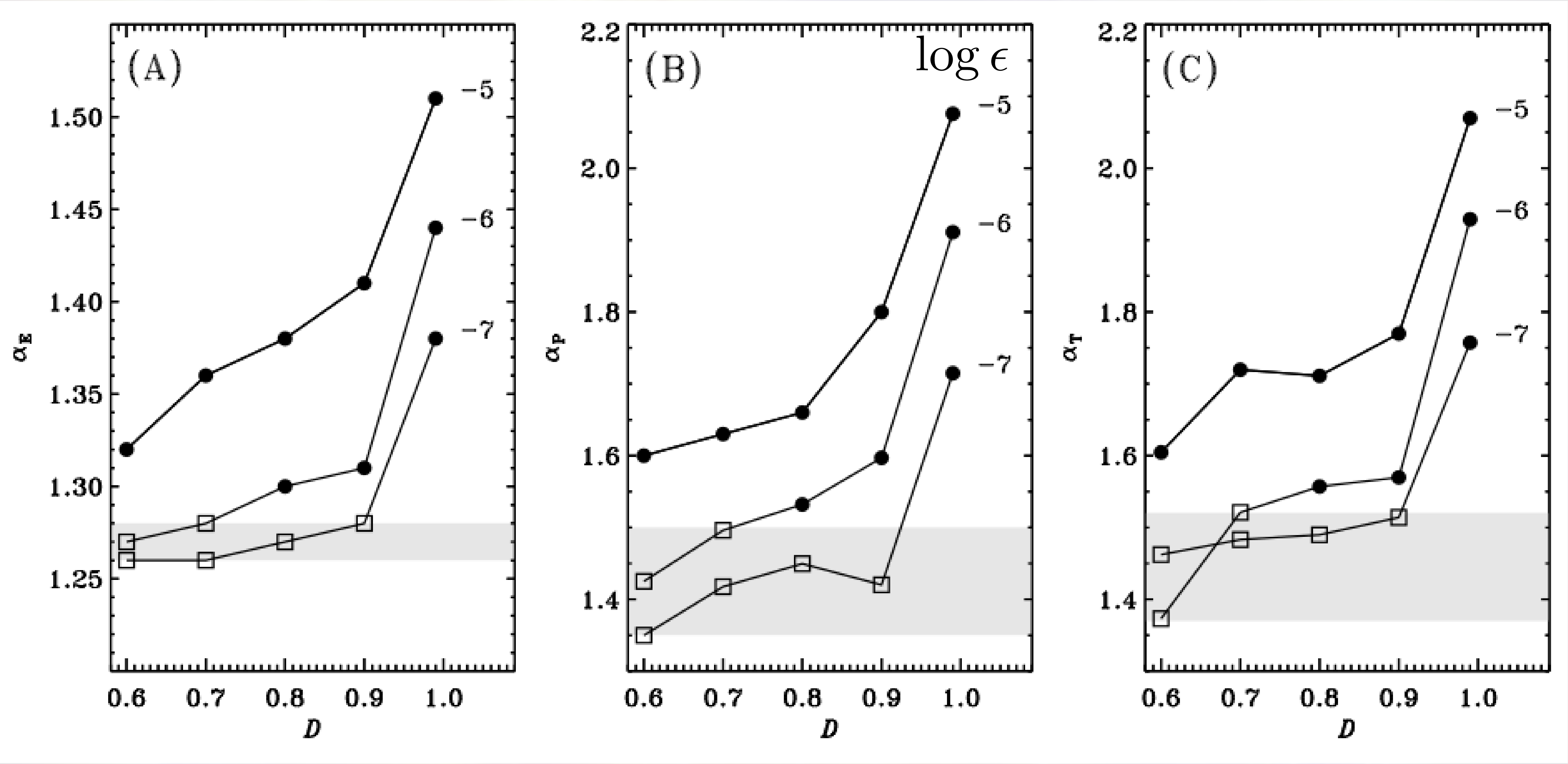
$$(Z_{i,j} > Z_c^r) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B \\ B_{i\pm1,j\pm1} & += \quad \delta B \end{cases}$$

✤ Random **redistribution**

$$(Z_{i,j} > Z_c) \rightarrow \begin{cases} B_{i,j} & -= \quad 4\delta B \\ B_{i\pm1,j\pm1} & += \quad \frac{r_k}{R}\delta B \end{cases}$$

$$r_k \text{ random deviate } \in [0,1] \; (k \in \{1,4\}) \qquad \sum_k r_k = R$$

Avalanches characteristics in the deterministic non-conservative model



4DVAR Algorithm