Preface

- These are the slides for my Tuesday talk with a few additional slides added for Wednesday's discussion slot (45, 46, 50)

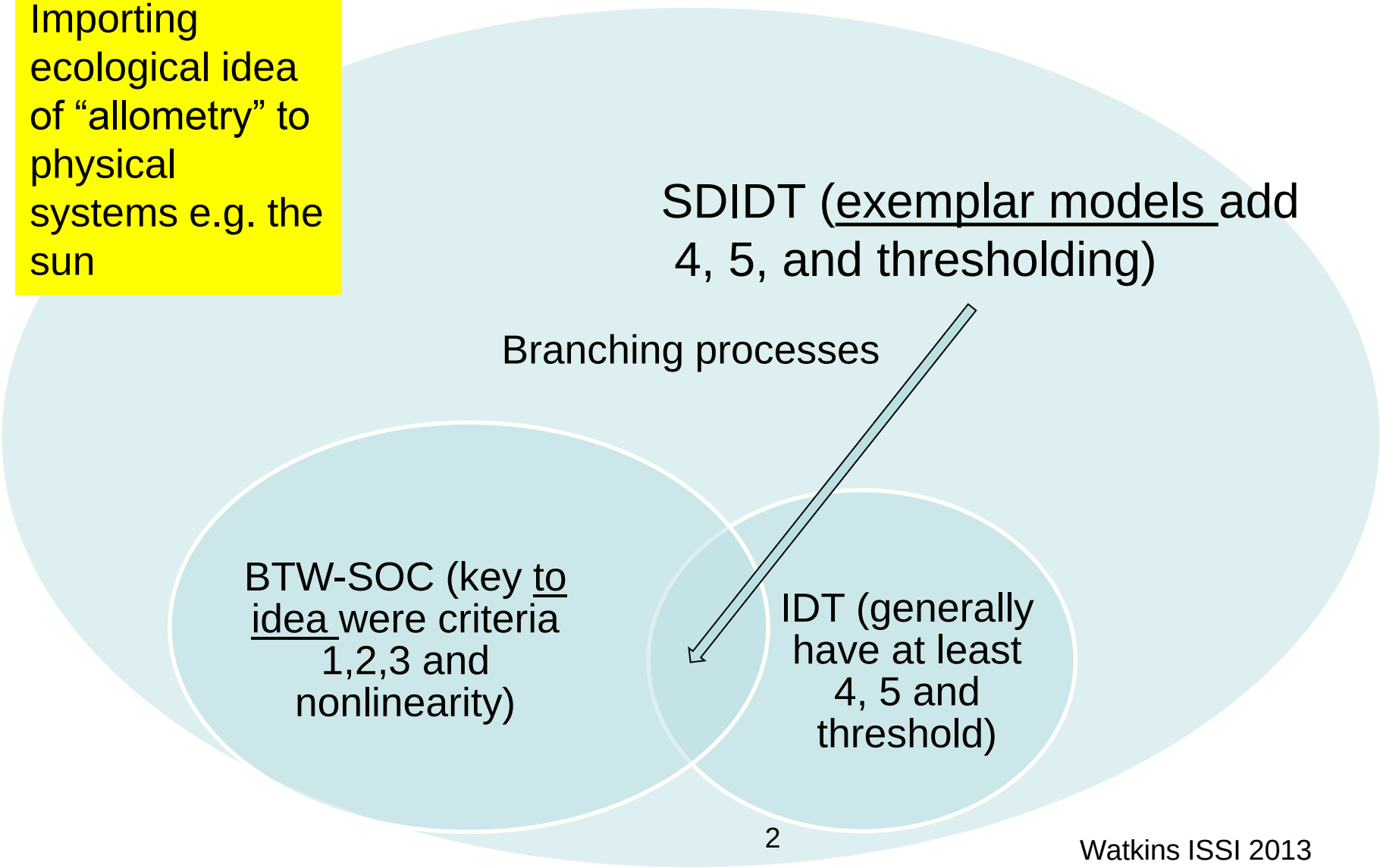
- And some corrections to Tues/Weds slides on phase transitions (pp 19-22) after discussions with Gunnar.

- And an annotated diagram based on my 5 minute summary Friday (p2), related to Gunnar's list (p 3) and the other contributions.

Friday summary diagram

"SOC in the wild"

Importing ecological idea of "allometry" to physical systems e.g. the sun



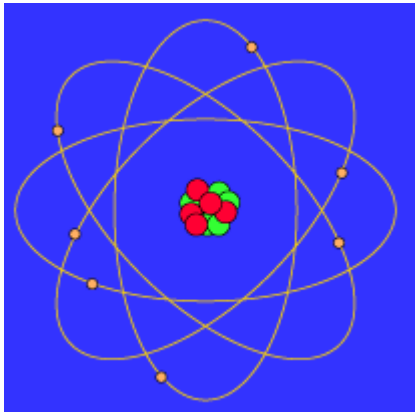
Gunnar's list of criteria

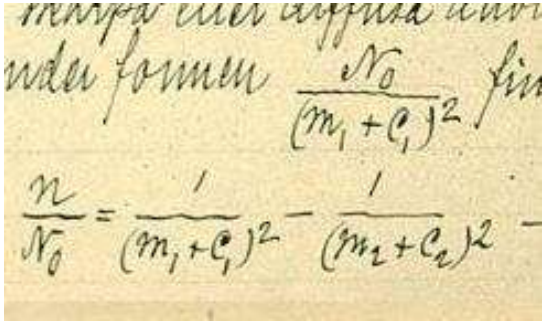
1. Non trivial scaling (finite size scaling – no control parameter)
2. Spatiotemporal correlations
3. **Apparent self tuning (underlying 2nd order phase transition ?)**
4. Separation of time scales
5. Avalanching (intermittency)
6. [non linear (thresholds) interactions] (supposedly required by 1)

One question I am unsure of the answer to is: "has any model, or, importantly, any data set, ever demonstrated all of 1-3 together without using 4, 5, and thresholding ?"

SOC and the Bohr Atom

- "The Bohr model of the atom ... was wrong, yet it turned out to be fruitful."
  - Gene Stanley, quoted by Mark Buchanan in *Nature, 2008*





Rydberg formula

SOC and turbulence in astrophysical plasmas: Some ways forward ?

Nick Watkins

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17th September 2013, ISSI, Bern.

Revised Abstract: I think our last meeting played a useful role for participants. Contrary to some still common misconceptions it showed that SOC was in fact quite tightly defined: first by its creators, and thereafter in mathematical physics. It also showed that its status-both observationally and theoretically-remains controversial in the theoretical physics community, even 25 years on [e.g. **Gunnar's book, *Self-Organised Criticality: Theory, Models and Characterisation,* CUP, 2012 and its predecessor from Henrik in 1998**].

We also saw, however, that considerable observational evidence existed in solar physics and elsewhere for various properties quite like to those which inspired the SOC idea [e.g. **Markus' book & ISSI talk, Norma's and others**]. In addition, several speakers including **Raul and Sandra** reminded us of the fertile role that ideas drawn from SOC continue to play in space & lab plasma transport and turbulence.

In this talk I will offer some thoughts on how these two views can be reconciled, and how SOC-inspired ideas can continue to play a useful role in space physics. I will also talk about the other types of physical process that have been studied as models for heavy tails and long-range dependence, having roots in Mandelbrot's work.

Summary of Talk

- Revisit BTW's motivation:
- Recap BTW's postulate and their definition of SOC
- Why is SOC still controversial ?
- "SOC in the wild": What in astroplasmas resembles SOC ?
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- Desire to unify heavy tails and LRD 20 years older than SOC
- What paper(s) could we write ?

BTW's Stated Motivation

- Spatial fractals
- Temporal fractals: 1/f noise
- Perceived need to unify them
- Claimed absence of existing way of doing this

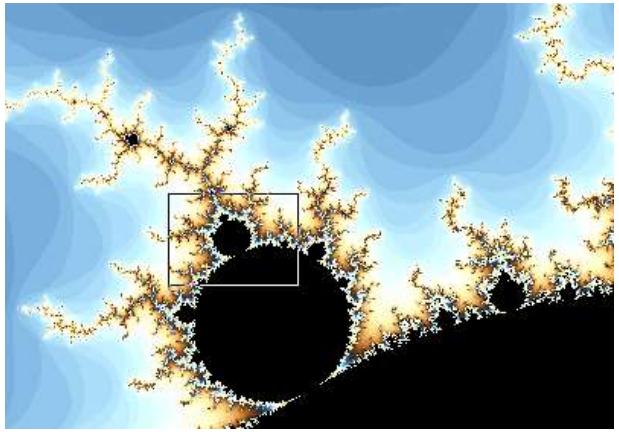
Spatial Fractals

In "The physics of fractals" [Physica D, Vol. 38, # 1-3 (1989)], Bak and Chen gave what I believe to be their clearest statement of SOC idea

"The importance of Mandelbrot's discovery that fractals occur widespread in nature can hardly be exaggerated. Many things which we used to think of as messy and structureless are in fact characterized by well-defined power-law spatial correlation functions. By now, we are so used to seeing fractals that we are tempted to feel that we understand them. But do we simply have to accept their existence as "God-given" without further explanation or is it possible to construct

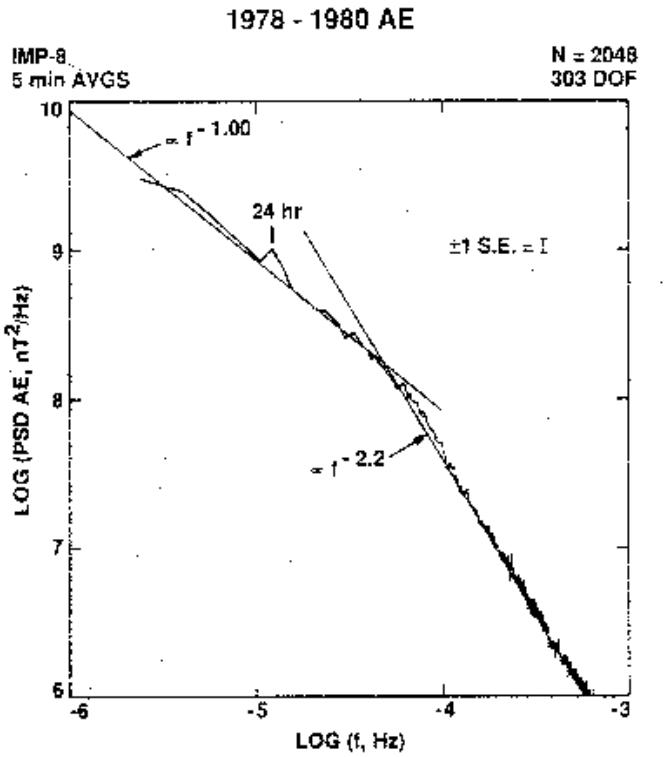
a dynamical theory of the

physics of fractals?"

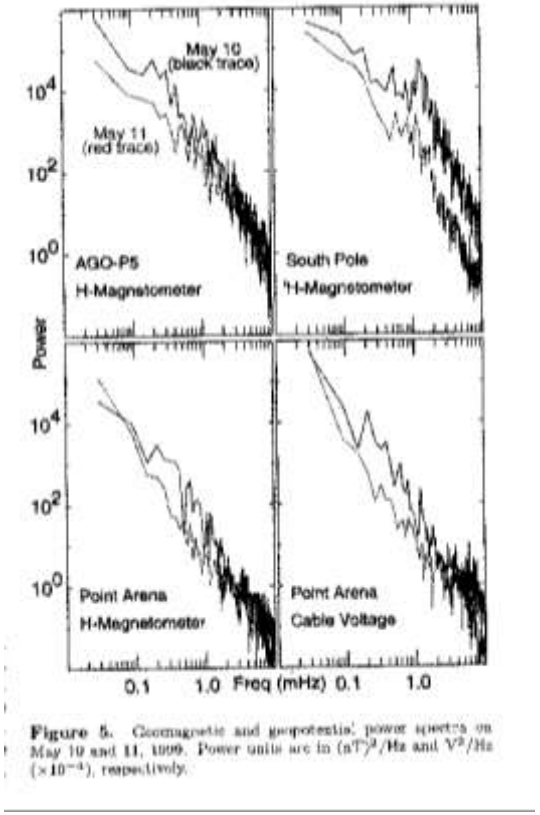


Fractals in time- 1/f noise

"There is another ubiquitous phenomenon which has defied explanation for decades. The signal (water, electrical current, light, prices, …) from a variety of sources has a power spectrum decaying with an exponent near unity at low frequencies .... This type of behavior is known as "1/ f" noise, or flicker noise." -**Bak and Chen, ibid.**



Tsurutani et al, GRL, 1991



Weatherwax et al, GRL, 2000

Perceived need to unify them and a claimed absence of existing ways to do so

“Strangely enough, just as those working on fractal phenomena in nature never seem to be interested in the temporal aspects of the phenomenon, … those working on ``1/ f'' noise never bother with the spatial structure of the source of the signal.

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BTW's Postulate and their Definition of SOC

- Postulate existence of a self-organized critical state
- Argument that spatial and temporal fractals must be linked
- Introduce criticality in analogy with equilibrium phase transitions
- Emphasised that standard criticality was "tuned"
- Argued that to be universal a self-organised criticality was needed
- Note: 2 contrasting meanings of word "critical"

BTW led to postulate the idea of SOC

“Strangely enough, just as those working on fractal phenomena in nature never seem to be interested in the temporal aspects of the phenomenon, … those working on ``1/ f'' noise never bother with the spatial structure of the source of the signal.

We believe that those two phenomena are often two sides of the same coin: they are the spatial and temporal manifestations of a self-organized critical state. **-Bak and Chen, ibid.**

They then argued that spatial & temporal criticality must be intrinsically linked.

"Actually, for those (like us) who are brought up as condensed matter physicists it is hard to believe that long-range spatial and temporal correlations can exist independently.

A local signal cannot be ``robust'' and remain coherent over long times in the presence of any amount of noise, unless stabilized by the interactions with its environment. And a large, coherent spatial structure cannot disappear (or be created) instantly …

-Bak and Chen, ibid.

Criticality was invoked used by direct analogy with equilibrium phase transitions.

"In fact, there is one area of physics where the relation between spatial and temporal power-law behaviour is well-established. At the critical point for continuous phase transitions, the correlation function for the order parameter decays spatially as $r^{2-d-\eta}$ *and temporally as* $t^{-d/z}$.

But this was a tuned criticality

But in order to arrive at the critical point, one has to fine-tune an ***external*** *control parameter such as the temperature or pressure, in contrast to the phenomena above, which occur universally without any fine-tuning.*

BTW argued that to be universal a self-organised criticality was needed

But in order to arrive at the critical point, one has to fine-tune an ***external*** *control parameter such as the temperature or pressure, in contrast to the phenomena above, which occur universally without any fine-tuning.*

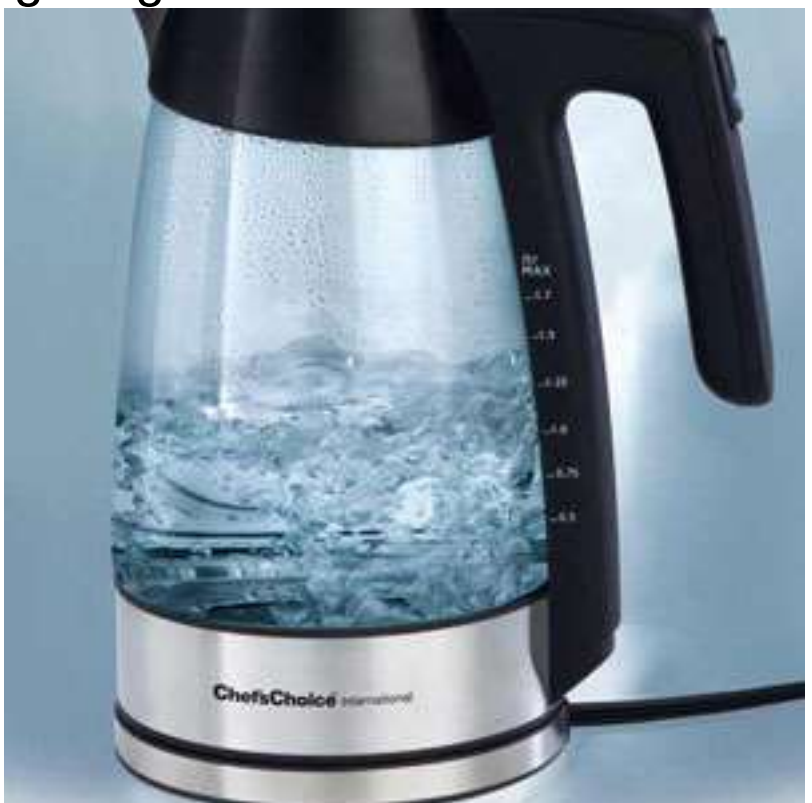
The explanation is that open, extended, dissipative dynamical systems may go automatically to the critical state as long as they are driven slowly: the critical state is self-organised. We see fractals as snapshots of systems operating at the self-organised critical state" **-Bak and Chen, ibid.**

What did BTW mean by "critical" ?

As we found last year, some confusion continues to arise from the double meaning of the English word "critical" in the BTW SOC papers.

1st meaning: is used in the phrase "Self Organised Criticality" by analogy with critical phenomena.

Bak and Chen explicitly referred to the long range spatial correlations seen in systems undergoing continuous phase transitions at a "critical point".



Discontinuous phase transition is, for example, liquid to gas.

What did BTW mean by "critical" ?

SOC concept was intended to explain such correlations, but BTW wanted it to do so without any tuning.

This would be unlike the tuning seen in the phase transitions previously known … such as raising the temperature of water in a kettle to 100 degrees C …

What did BTW mean by "critical" ?

… or bringing the temperature in a ferromagnet to temperature Tc

… or when T approaches Tc in Ising model [See Java demo at

http://www.pha.jhu.edu/~javalab/ising/ or one of the other excellent ones available].

Unlike the boiling water, the above 2 examples are continuous phase transformations. In these we have the long-range correlation behavior that Bak alluded to:-

Correlation function obeys $C(r) \sim r^{2-d-\eta}$

Critical Opalescence

What did BTW mean by "critical" ?

2nd meaning. Word is also used in the phrase "critical threshold", where it refers to the necessary threshold that a system needs to exceed locally for transport to occur. Note, this is first link to sandpile models ! Sand or rice grain on a slope ….

Both meanings were important to BTW's idea. The first is standard, "well known" and importantly predates SOC.

To reduce confusion might still be a good idea to use a different word for the second. Can/should we explore this ?

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So why then is SOC still controversial ?

- A few reasons: **Gunnar's** book treats many more in detail.
- One reason: confusion of the symptoms with the disease …
- Another was that sandpile models not ideal exemplars of SOC
- Many are spatially fractal… or roughly fractal …
- But many don't produce 1/f noise **…[Jensen et al]**
- Often thought that SOC all about explaining all power laws
- Now a thread of work on driven criticality **[Henrik's talk]**
- Another reason is human-**BTW** ignored much relevant work
- Yet another is ongoing arguments about how much space and time fractality actually exists in nature … and how often space & time effects coexist … i.e. is there really a problem to solve.

Many people, quite understandably, were unclear as to what SOC was trying to explain (ubiquitous spatiotemporal fractals), and what the explanation was supposed to be (the dynamics of SOC processes):

"Fractals give rise to power laws
I have seen a power law
Therefore my power law was caused by a fractal"

"SOC is a theory which explains fractals
I have seen a fractal
Therefore my fractal was caused by SOC".

See Wikipedia: "the fallacy of the undistributed middle …".

In the theory literature spatial criticality was, from the outset, tested for by scaling collapse

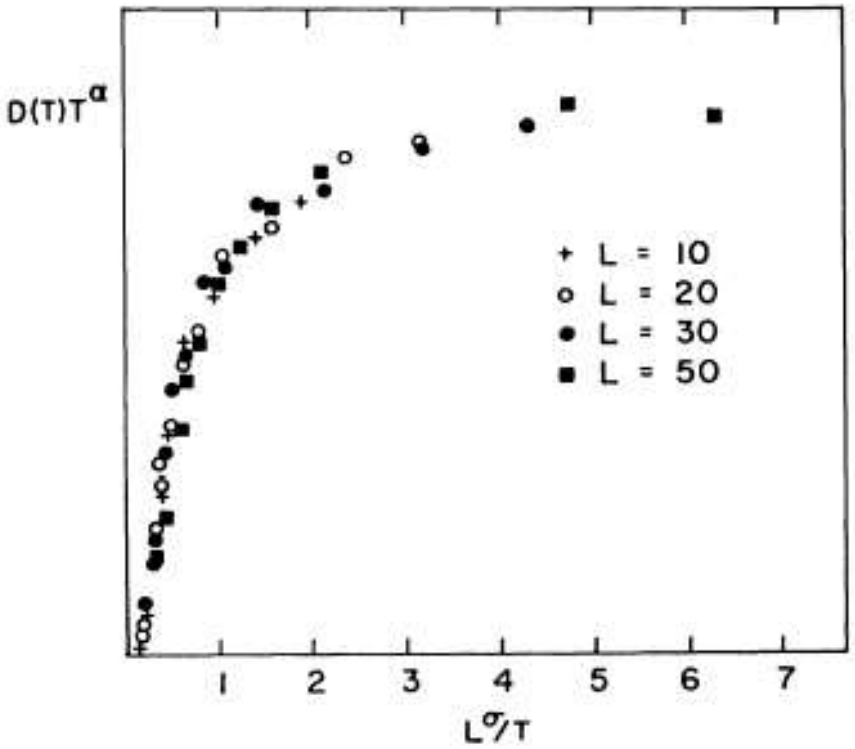

FIG. 11. Finite-size-scaling plot.

Bak et al, Phys Rev A, 1988

Precision of these methods and size of simulations has Improved over the years -raised many questions e.g. **Henrik's** & **Gunnar**'s books

Critical spatial avalanches were "grown" by BTW (& later) SOC models-but not all produced "1/f" spectra.

Bak et al, PRA, 1988

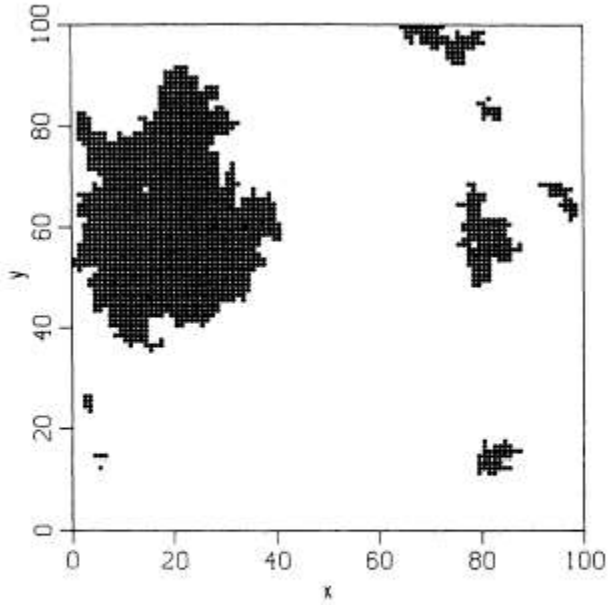

FIG. 2. Typical domain structures resulted from several local perturbations for a 100×100 array. Each cluster is triggered by a single perturbation.

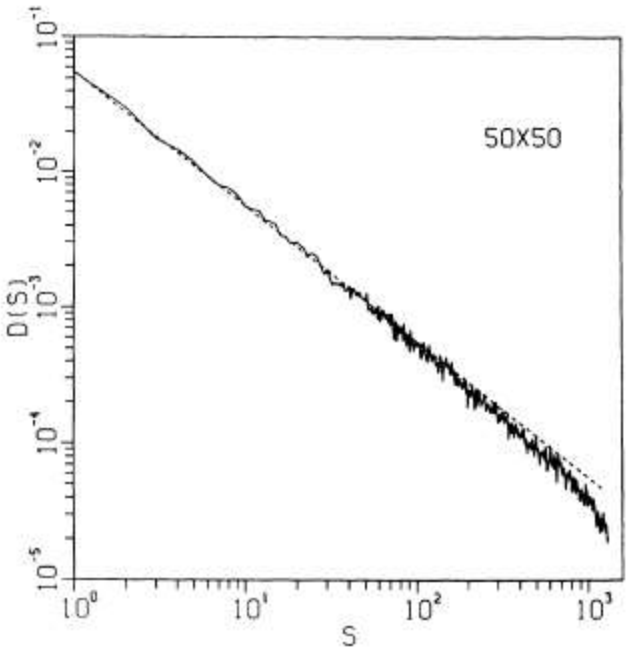

FIG. 5. Cluster size distribution for system built up from scratch according to rules (3.2) and (3.8) for a 50×50 array. The curve is indistinguishable from that in Fig. 3(a). For this system the system is in a stationary critical state and it is self-averaging. Rule (3.8) has been applied 100 000 times to the stationary critical state to obtain this curve.

Image:

Yann-Arthus Bertrand

Power laws

- Explaining any and all naturally occurring power laws is not the main point of SOC, as is so often thought. This belief was unfortunately nourished by otherwise excellent books like **How Nature Works**, where relevant (e.g flare) and irrelevant (e.g Zipf) power laws were used rather indiscriminately.

- Instead, to BTW, the power law distribution in avalanche sizes and durations were proxies for the power law spatiotemporal correlation functions that they saw as a crucial aspect of the unification of spacetime fractals that they were trying to achieve.

Do we really need criticality to be self-organised ?

- Over time, many workers concluded that there was an implicit dimensionless tuning parameter in SOC-the ratio of fuelling to unloading rates. See in particular **Zapperi and colleagues**' papers, and **Sandra's** talk last year.
- This raised the questions, as discussed by **Henrik's** talks, of whether one might as well just look for a naturally occurring but tuned criticality, and diagnose the spatiotemporal correlation functions as directly as possible. **Ole Peters'** papers, **Henrik's** etc..
- SOC would then be seen primarily as having been an inspiration for looking more widely in nature for criticality than one would have done without it.

Bak tended to minimise much relevant work

“Strangely enough, just as those working on fractal phenomena in nature never seem to be interested in the temporal aspects of the phenomenon, … those working on ``1/ f'' noise never bother with the spatial structure of the source of the signal. **Bak and Chen, ibid**

- Above is a bit disingenuous:
- laboratory critical phenomena already linked space & time
- but also so do multifractal turbulent cascades.
- also there was work on the linking of space and time fractality by Mandelbrot himself, about 20 years earlier, prior to his work on cascades

Space and time fractality are not always linked

"Actually, for those (like us) who are brought up as condensed matter physicists it is hard to believe that long-range spatial and temporal correlations can exist independently.

A local signal cannot be ``robust'' and remain coherent over long times in the presence of any amount of noise, unless stabilized by the interactions with its environment. And a large, coherent spatial structure cannot disappear (or be created) instantly …

-Bak and Chen, ibid.

Despite apparent reasonableness of above argument, fact is that space and time fractality are not always linked in nature (or even in many SOC models)

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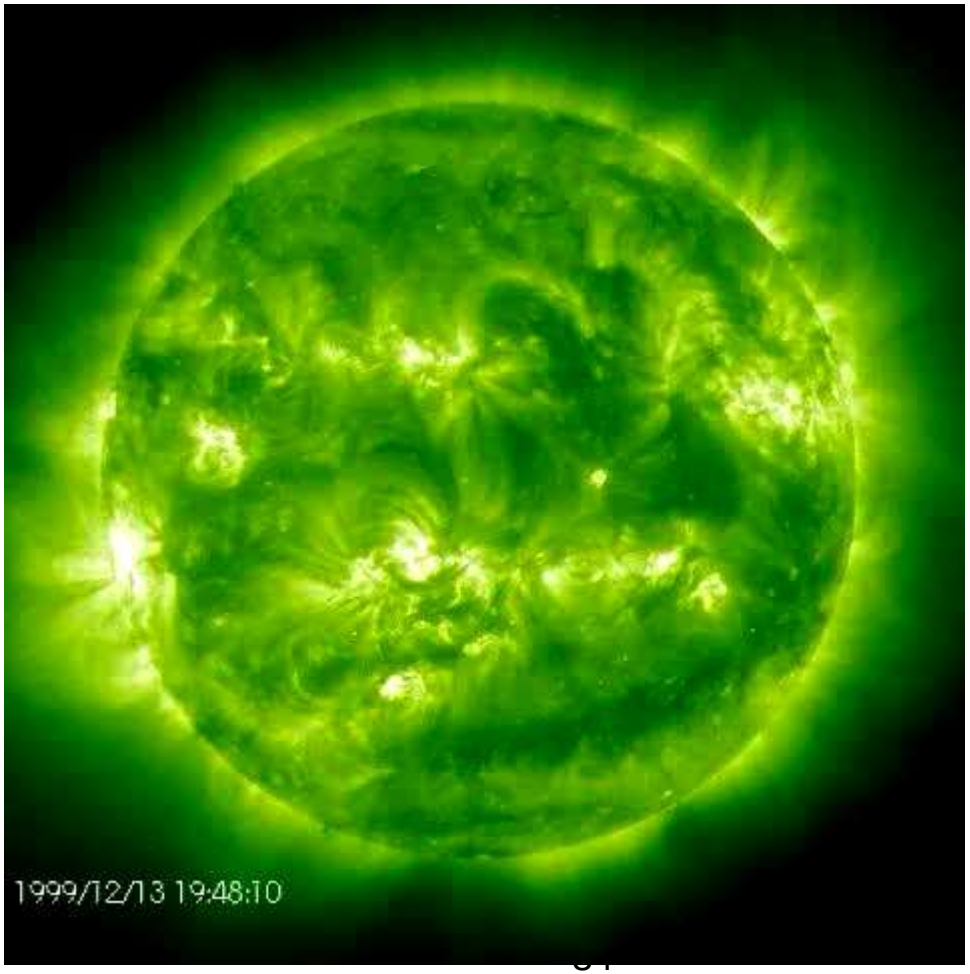
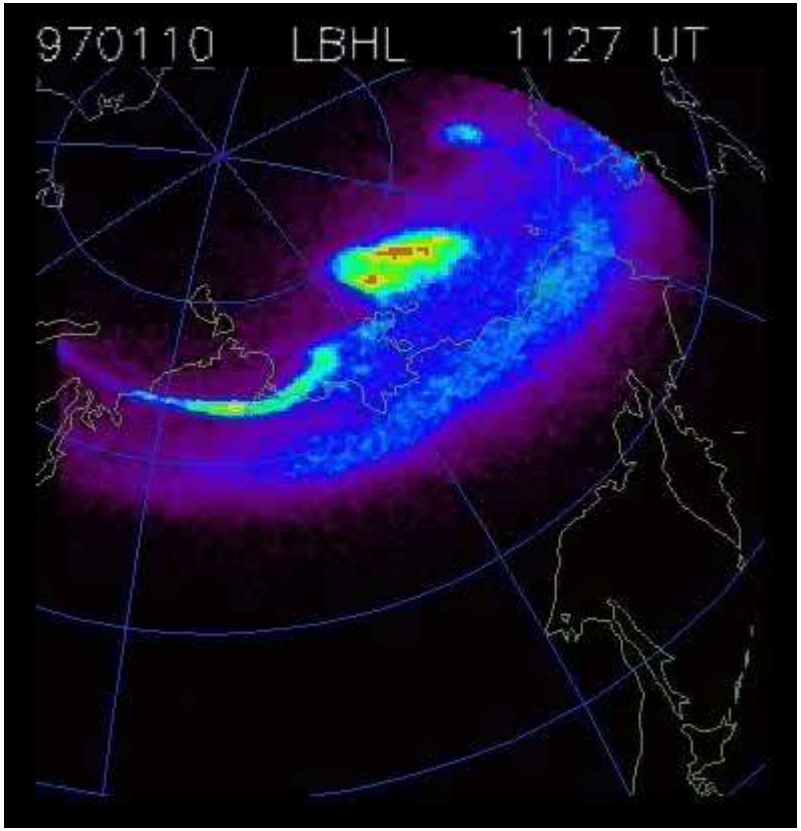
“SOC in the wild”: what in astroplasmas looks like it



"SOC in the wild": what in astroplasmas looks like it

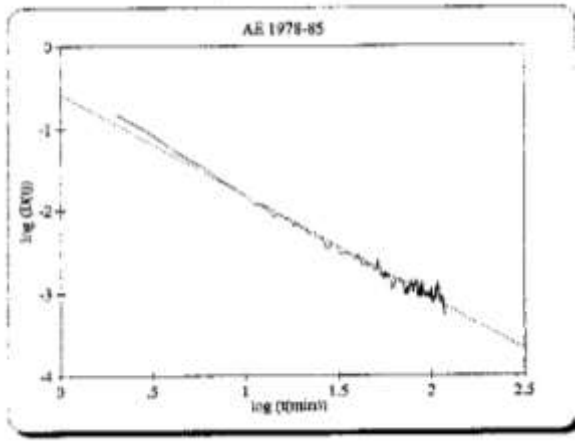
- Several problems in astroplasma physics resemble SOC
- One clear example is the wideband distribution of flare energies noted by **Markus, Norma and others-**studied extensively
- Like Gutenberg-Richter law this observation pre-dates SOC
- Another case is the presence of 1/f regions in several plasma diagnostics-examples include **Tsurutani et al** paper on Ae index and solar wind; accretion disks; …
- These phenomena would still need explanation even if SOC paradigm didn't exist.

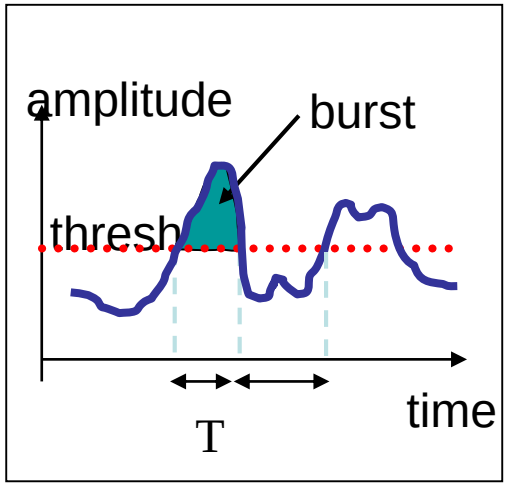
“SOC in the wild”: new STP data analyses



"SOC in the wild": new STP data analyses

- Also paradigm has inspired analysis of STP data in new ways:
- Examples: **Consolini 1998 & Takalo 1993** burst measures for AE
- **Lui and Chapman** blob distributions in UVI data
- **Uritsky et al** UVI burst measures and spreading exponents
-





These and other newer observations again need explaining, with or without SOC.

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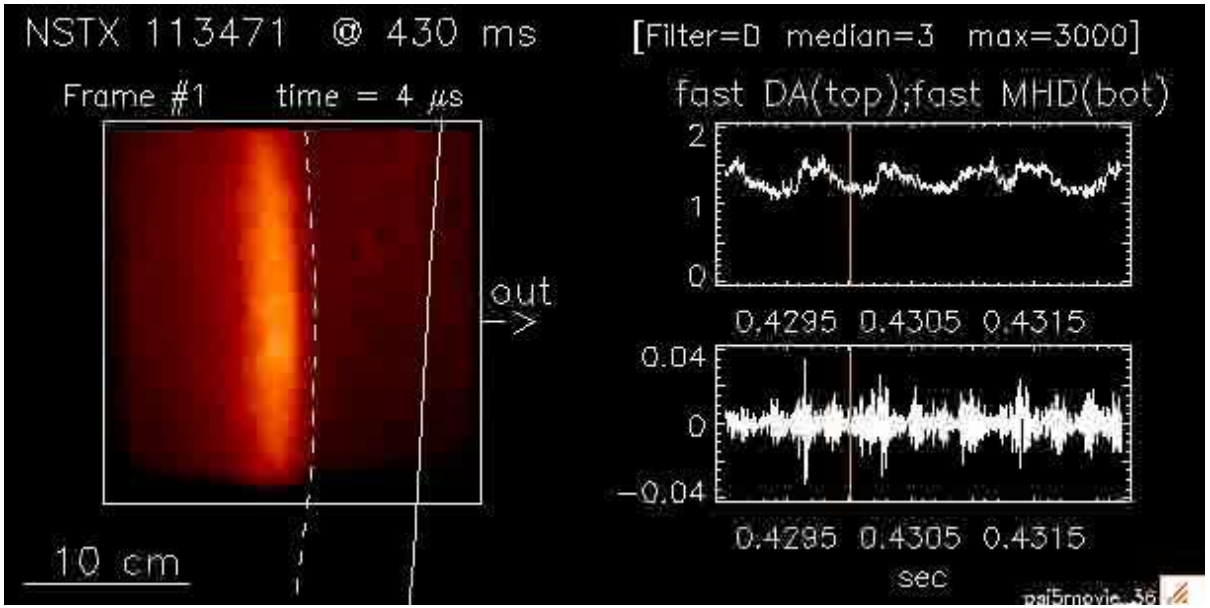
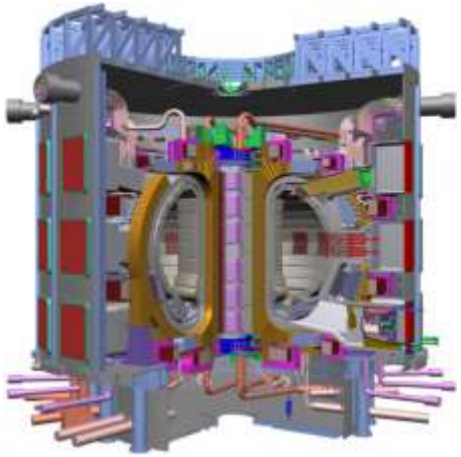
How has SOC stimulated lab plasma physics ?



How has SOC stimulated lab plasma physics ?

- Paradigm has also inspired lab community working on broader **problem of driven, nonlinear, multiscale, plasma instabilities.**
- Particularly interest in SOC as sub-threshold transport **[Diamond & Hahm; Sanchez, Newman, Carreras et al].**
- Has inspired interest in modified sandpile models **[Newman and colleagues; Chapman, Dendy and colleagues**] that resemble Tokamak physics more clearly
- & models that attempt to make the link to MHD

  more explicit …
- & cross-disciplinary conversations



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Idea of scaling much older and broader than SOC

- **Markus'** work (presented at last meeting) using dimensional arguments to study how instabilities might be aggregated seems to me to relate to a much older & broader question than SOC:
- **Problem of aggregation of spatiotemporal processes in nature.**
- **informed by current knowledge about scaling, and fractals, but also by the other relevant physics of the problem, which may well not be critical.**

Geophysical predecessor: Main & Burton 1984

Bulletin of the Seismological Society of America, Vol. 74, No. 4, pp. 1409–1426, August 1984

INFORMATION THEORY AND THE EARTHQUAKE FREQUENCY-MAGNITUDE DISTRIBUTION

BY IAN G. MAIN AND PAUL W. BURTON

ABSTRACT

A new frequency-magnitude relation consistent with an average magnitude $\langle m \rangle$ and an average seismic moment $\langle M_0 \rangle$ in the magnitude range (m_c, ω) is derived using the principles of information theory. The resulting density distribution $n(m)\,dm = C \exp\{-\lambda_1 m - \lambda_2 M_0(m)\}\,dm$ can be interpreted as a Boltzmann distribution of possible energy transitions scaled by a geometric factor, depending on how such transitions may occur on a fault plane. It gives a better fit to available frequency data on the Central Mediterranean area than other distributions which can only successfully model part of the magnitude range. The

Geometric fault model: Main & Burton 1984

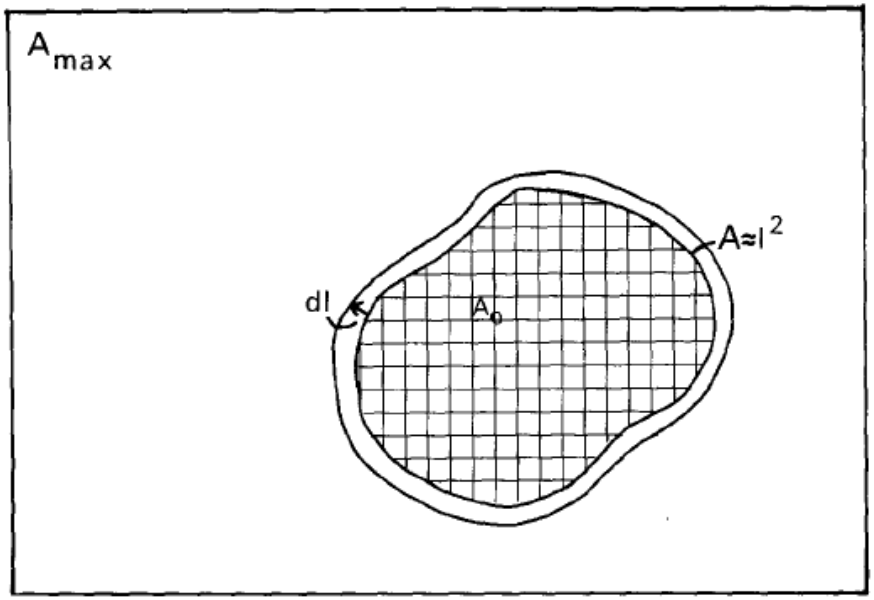

FIG. 1. *A geometric fault model.* The area A can fit into A_{max} in A_{max}/A ways. A_0 represents the physical lower limit to seismic energy release and depends on the spacing of inhomogeneities in the earth. The density of degenerate states if A_0 is assumed to be very small is $D(l)\,dl = \{A_{max}/l^2\} - \{A_{max}/(l + dl)^2\} = 2A_{max}/l^3\,dl$.

to unity. If moment is the relevant parameter

$$n(M_0)\,dM_0 = \text{const}\ M_0^{-5/3} e^{-\beta M_0}\,dM_0 \qquad (17)$$

where the geometric term $M_0^{-5/3}$ follows from (14) with $M_0 \propto l^3$. The form of this distribution can then be interpreted as a Boltzmann distribution of energy transitions via $\exp(-\beta M_0)$, multiplied by a geometric factor $M_0^{-5/3}$ which results in another exponential if magnitude is the relevant parameter.

Markus is proposing a new kind of “allometry” for physics ?

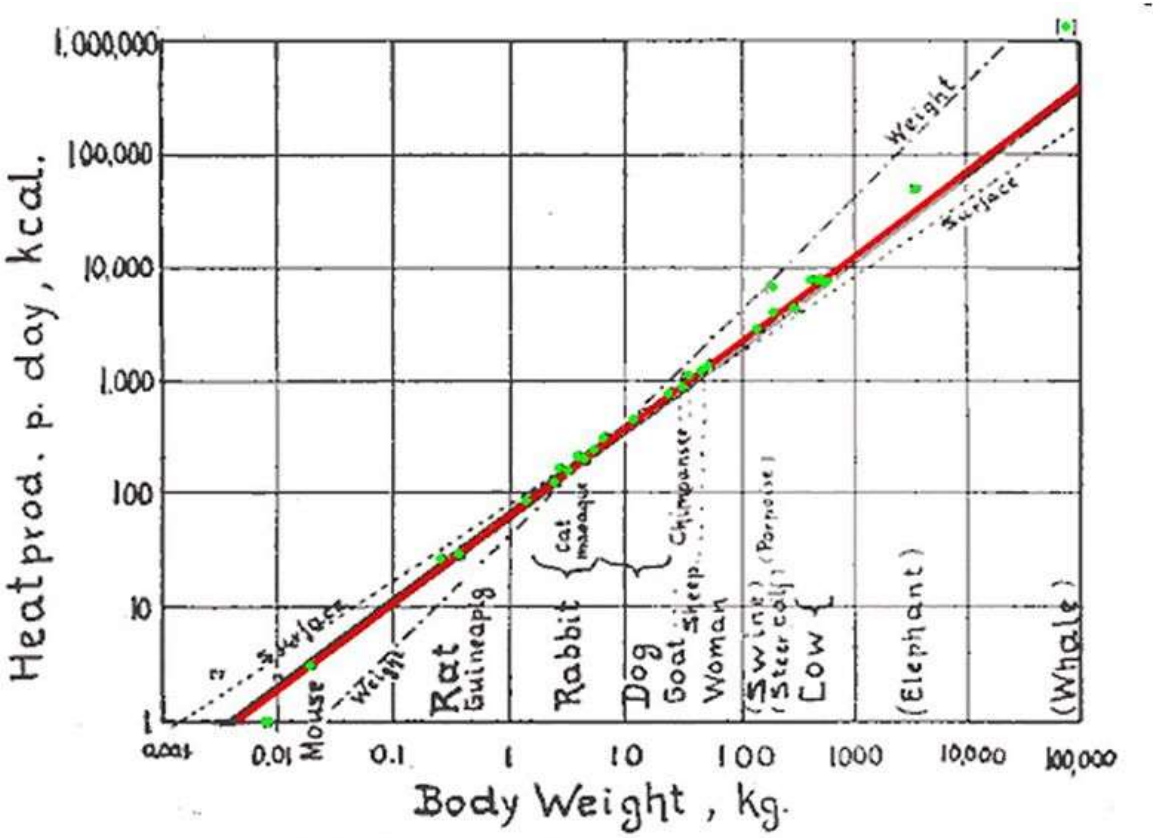

Fig. 1. Log. metabol. rate/log body weight

c.f. Kleiber’s law in ecology

Another interesting precursor: Craig 2001

A RECONNECTION MODEL FOR THE DISTRIBUTION OF FLARE ENERGIES

I. J. D. CRAIG
University of Waikato, Hamilton, New Zealand

(Received 2 January 2001; accepted 11 April 2001)

Abstract. A physically based explanation is given for the distribution of flare energies $N(E) \sim E^{-\alpha}$ where $\alpha \simeq 1.5$. In contrast to previous approaches, the present treatment is based on a physical theory of the flare reconnection site. The central assumption is that topological flare energy, although released explosively, is slowly accumulated over several hundred Alfvén timescales. When coupled to the geometric properties of the reconnective flare source, this assumption is shown to lead naturally to a deduction of the flare energy distribution. Current sheet models yield the exponent $\alpha = \frac{3}{2}$ whereas more compact current structures imply steeper spectra $\frac{3}{2} \leq \alpha \leq 2$.

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Desire to unify heavy tails and LRD ~20 years older than SOC

- **Mandelbrot** himself noticed heavy tailed pdfs in financial data. Explained them using alpha-stable ("Levy") distribution model in 1963.
- He then heard about Hurst effect, the anomalously fast growth of range of river Nile maximum heights. His initial belief was that it could be explained by heavy tails. However data not heavy tailed.
- Then with **van Ness, and Wallis**, used a long-range dependent model originally due to **Kolmogorov,** called fractional Brownian motion (1965-68), to explain the Hurst effect.
- By 1969 was asking himself how to produce a model that linked heavy tails with long range dependence.

Bunched black swans

- In 1969 **Mandelbrot** demonstrated a heavy-tailed long range dependent noise model ("fractional hyperbolic noise") that unified heavy tails with LRD.
- Now a mainstay of applied stochastics, under name of linear fractional stable noise (and related motion).
- Outstanding challenges include showing how diffusive models like LFSM relate to the Langevin formalism.
- Many of **Mandelbrot's** models are self-similar but don't have dissipation time scale.
- Another open challenge is improving understanding of how LFSM relates to fractional kinetic models that modify random walks-e.g. **Raul's talks**.

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Frontier Article

Bunched black (and grouped grey) swans: Dissipative and non-dissipative models of correlated extreme fluctuations in complex geosystems

N. W. Watkins[1,2,3,*]

Article first published online: 31 JAN 2013

DOI: 10.1002/grl.50103

Issue

Geophysical Research Letters

Volume 40, Issue 2, pages 402–410, 28 January 2013

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Summary of Talk

- Revisit BTW's motivation:
- Recap BTW's postulate and their definition of SOC
- Why is SOC still controversial ?
- "SOC in the wild": What in astroplasmas resembles SOC ?
- How has SOC stimulated plasma physics?
- Idea of scaling much older and more general than SOC
- Desire to unify heavy tails and LRD 20 years older than SOC
- What paper(s) could we write ?