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Sunspot tilt angles revisited: Dependence on the solar cycle strength

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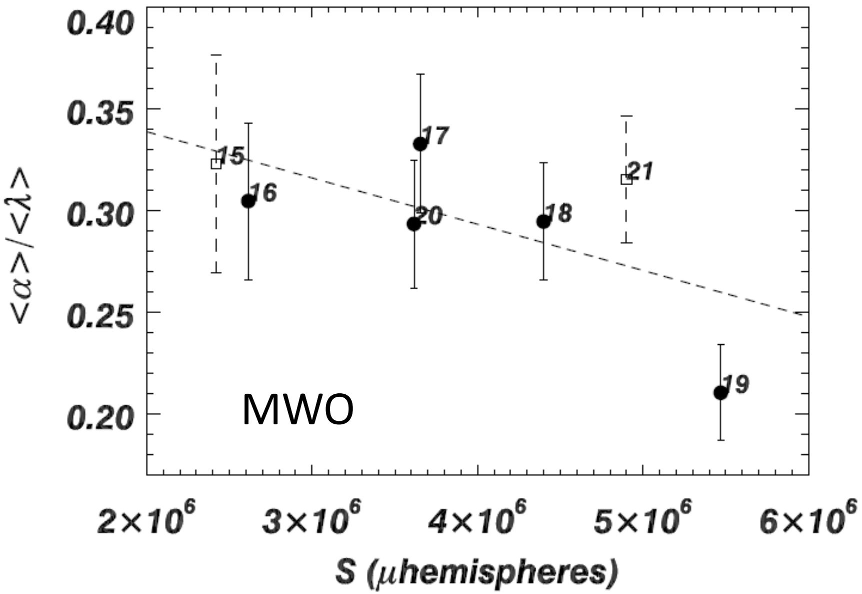
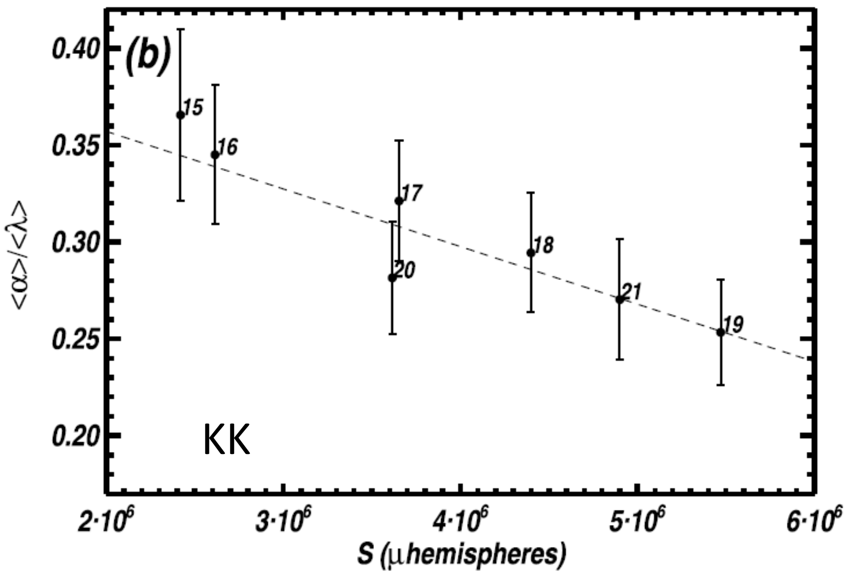
The story starts from ……





Dasi-Espuig et al. (2010, 2013), citations based on ADS: 140+

Summary of the controversial statistics on the cycle dependence of the tilt coefficient

Reference	Dataset	Years	Method [a]	r	p
Dasi2010	MW	1917-1985	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.79	0.10
Dasi2013	KK	1906-1987	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.93	0.02
Ivanov2012	MW	1917-1985	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.65	0.09
	KK	1906-1987	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.91	0.02
			m-values, $b_0 = 0$	−0.62	0.10
Mc2013	MW	1917-1985	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.75	0.05
			$\langle\alpha\rangle$	−0.16	0.67
Wang2014	MW/WL	1923-1986	m-values	−0.10	0.81
			$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.42	0.30
	MW/MAG	1974-2008	m-values	−0.29	0.62
			$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.37	0.52
	DPD	1974-2008	m-values	−0.99	0.08
			$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.99	0.08
Tlatova2018	MW	1917-2018	m-values	−0.40	0.21
Isik2018	Kandilli($\Delta s > 3°$)	1954-2017	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.57	0.23
	Kandilli($\Delta s > 2.5°$)			−0.75	0.08
Jiang2020	MW KK	1913-1986	$\langle\alpha\rangle/\langle\lvert\lambda\rvert\rangle$	−0.70	0.06

Notes. [a] $\langle\alpha\rangle/\langle|\lambda|\rangle$: tilt coefficient designated by the normalization method; m-values: slope of linear binned fitting method $\alpha = m|\lambda| + b_0$.
References. Dasi2010: Dasi-Espuig et al. (2010); Dasi2013: Dasi-Espuig et al. (2013); Ivanov2012: Ivanov (2012); Mc2013: McClintock & Norton (2013); Wang2014: Wang et al. (2014); Tlatova2018: Tlatova et al. (2018); Isik2018: Işık et al. (2018a); Jiang2020: Jiang (2020).

van Driel-Gesztelyi & Green (2015):

The history of Joy's law studies are studded with confusing and controversial results.

Our objective is NOT to add more confusing and controversial 'evidence', but try to disentangle from the previous controversial stage.

Sources of the divergence

- Different datasets
- Different Criteria for the data selection: 1. remove invalid data (e.g., zero tilt; very large separation, etc); 2. angular separation; 3. different phases of sunspots
- Different Methods for the fitting: normalization method ($\langle\alpha\rangle/\langle\lambda\rangle$ $\langle\alpha_\omega\rangle/\langle\lambda\rangle$); binned fitting method; unbinned fitting method
- Different Functions for Joy's law: linear versus square-root functions
- Other details: how to separate different cycles? how to do the weighted fits?

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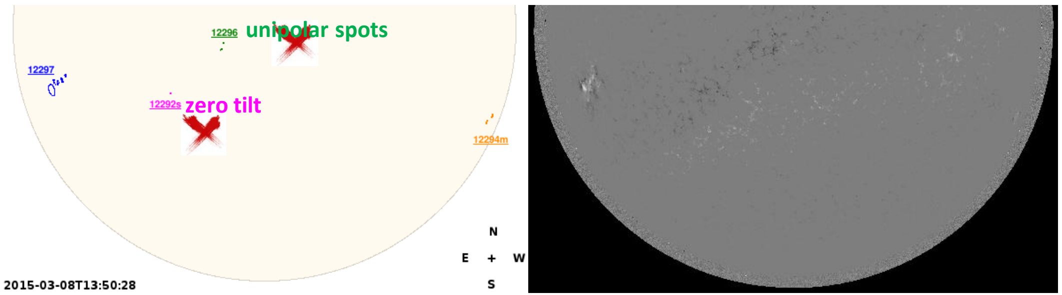
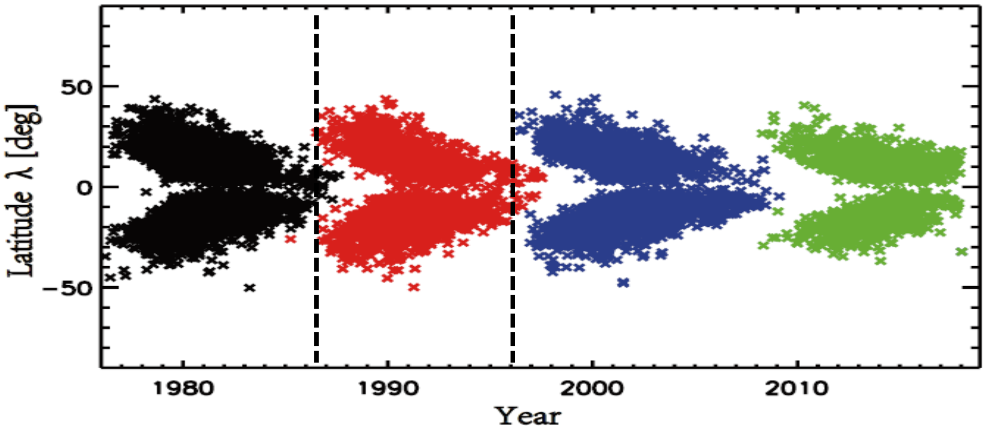
→ How these differences affect the result?
→ Which one is more reasonable?



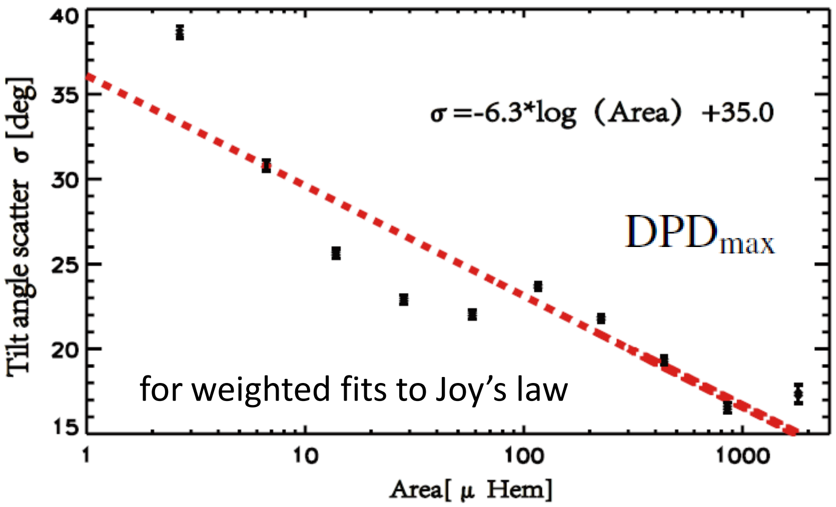
Courtesy of DPD web page



DPD (Gyori et al. 2011; Baranyi 2015; Baranyi et al. 2016; Gyori et al. 2016):

- **January 1974 -- January 2018: complete solar cycles 21-24 (weakest cycle in 100 years!)**
- **completeness: in some cases the DPD data are derived from cooperative ground-based observatories and satellite-borne imagery; the continuous records of each sunspot group once per day →DPD_{all} ,DPD_{max}**
- **The magnetogram information is referenced while grouping the sunspot groups.**

		DPD_{all}	DPD_{max}
$\Delta s \geq 0°$	$\overline{\alpha}$	$5\overset{\circ}{.}77$	$5\overset{\circ}{.}99$
	σ_α	$27\overset{\circ}{.}03$	$24\overset{\circ}{.}67$
$\Delta s \geq 2\overset{\circ}{.}5$	$\overline{\alpha}$	$6\overset{\circ}{.}55$	$5\overset{\circ}{.}94$
	σ_α	$22\overset{\circ}{.}63$	$18\overset{\circ}{.}81$



Removing the unipolar groups by the filter Δs>2.5 deg can bring the tilt angle measurements based on white-light images and magnetograms into a good agreement.

The data with Δs>2.5 deg has a smaller σ_α compared with the data with Δs>0 deg.

MW: 1917–1985 KK: 1906–1987

Groups were tracked two consecutive days.

Table 4. Comparisons among different data during 1974 to 1985.

		DPD_{all}	DPD_{max}	MW	KK
$\Delta s \geq 0°$	Amount	21707	3588	4085	4196
	$\overline{\alpha}$	5°.01	5°.45	4°.66	4°.67
	σ_α	28°.60	26°.51	29°.51	30°.84
$\Delta s \geq 2°.5$	Amount	16272	2756	2436	2307
	$\overline{\alpha}$	5°.79	5°.06	5°.72	7°.01
	σ_α	24°.67	20°.79	20°.39	21°.38

25% 25% 45% 40% unipolar spots

A larger σ_α decrease for the KK and MW data sets. The DPD data are more stable and less affected by selection conditions

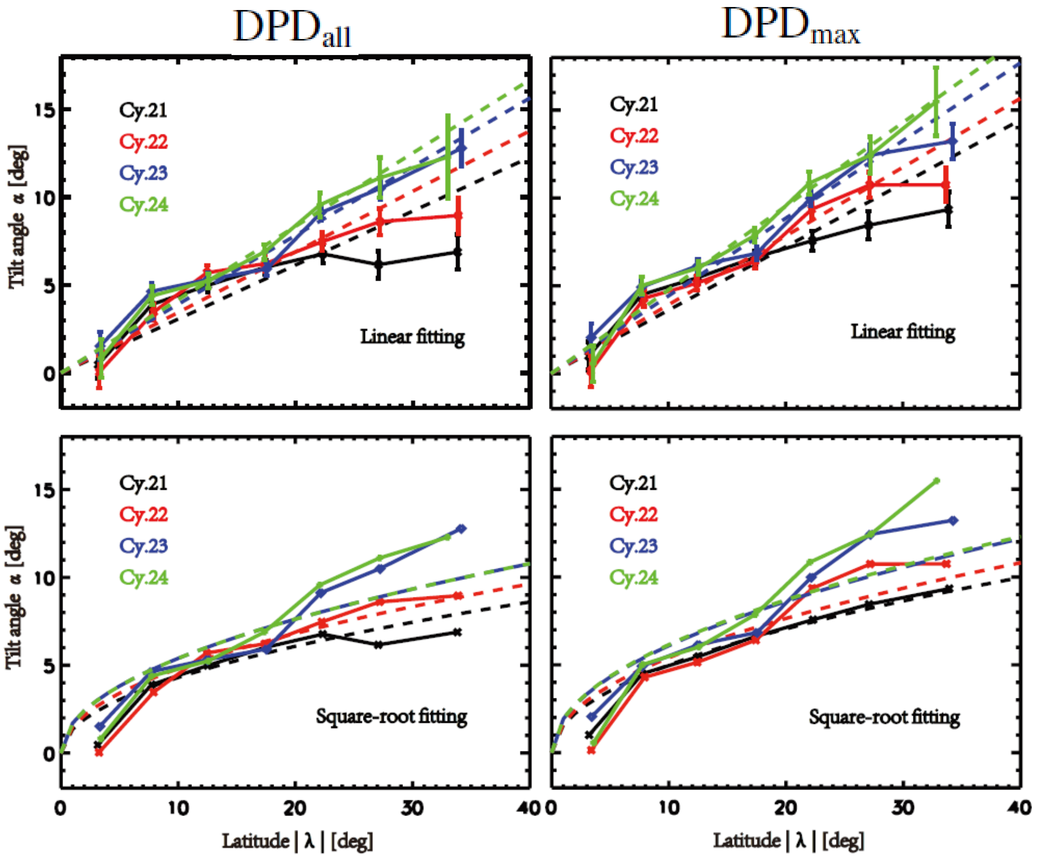
Two Functions for Joy’s law: $\alpha(\lambda) = T_{\text{lin}}|\lambda|$ $\alpha(\lambda) = T_{\text{sqr}}\sqrt{|\lambda|}$

three major methods for obtaining the tilt coefficients:

Method 1: normalization method:

$$\overline{T_{\text{lin}}} = \frac{\sum_j A_j \alpha_j}{\sum_j A_j |\lambda_j|} \qquad \overline{T_{\text{sqr}}} = \frac{\sum_j \sqrt{A_j}\alpha_j}{\sum_j \sqrt{A_j}\sqrt{|\lambda_j|}}$$

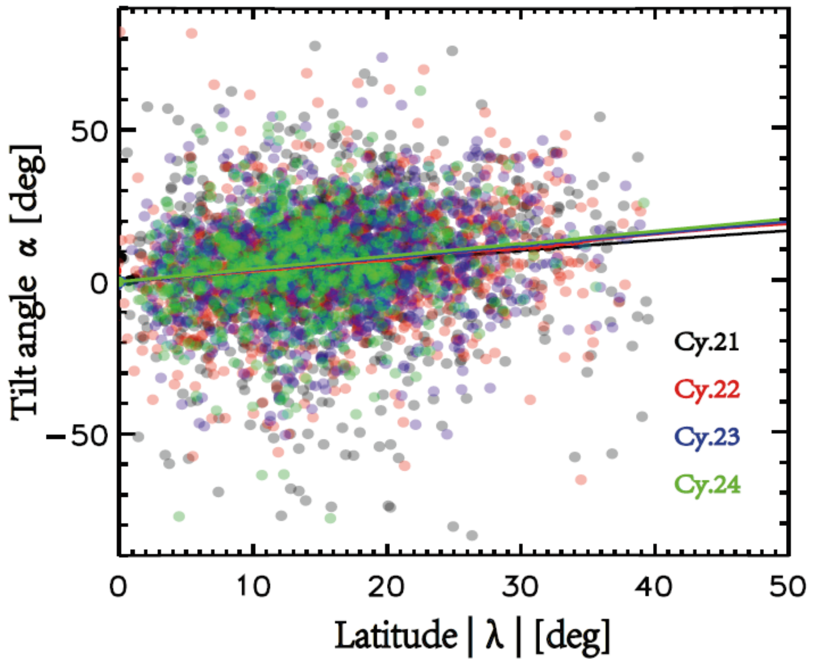
Method 2: binned fitting method



- Most widely used method
- **Intrinsic problem** with the method
- Past studies have shown that even with the same data, different ways to deal with binning cause significantly different tilt coefficients.

The linear fits to the data of weaker cycles have smaller χ^2-value and the square-root fits to the data of stronger cycles have smaller χ^2-value.

Method 3: unbinned fitting method



People usually perform a binned fit to data because the trend of the data can be better seen when the fitted form is unknown.

We add $1/\sigma_{\alpha}^{2}$ **to each data point as a weight.**

Still square-root (linear) fitting to Joy's law of stronger (weaker) cycles tends to have smaller χ^2-values. But the difference of χ^2-values between the strong and weak cycles is much smaller than that derived using the binned fitting method.

$\Delta s \geq 0°$

Method	T	Cy21		Cy22		Cy23		Cy24		c-value	Correlation	
		$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2		r	p
Mean	$\overline{\alpha_{\rm all}}$	5.°18 ± 0.°198	…	5.°86 ± 0.°210	…	6.°42 ± 0.°189	…	6.°21 ± 0.°235	…	6.56	−0.71	0.16
	$\overline{\alpha_{\rm max}}$	5.°65 ± 0.°454	…	6.°25 ± 0.°479	…	6.°33 ± 0.°446	…	6.°54 ± 0.°535	…	1.66	−0.83	0.10
Normalization	$\overline{T^{\rm all}_{\rm lin}}$	0.39 ± 0.015	…	0.35 ± 0.013	…	0.43 ± 0.013	…	0.45 ± 0.018	…	5.56	−0.78	0.12
	$\overline{T^{\rm max}_{\rm lin}}$	0.38 ± 0.032	…	0.35 ± 0.028	…	0.40 ± 0.030	…	0.43 ± 0.037	…	2.16	−0.87	0.08
	$\overline{T^{\rm all}_{\rm sqr}}$	1.59 ± 0.065	…	1.46 ± 0.055	…	1.74 ± 0.054	…	1.74 ± 0.070	…	4.00	−0.73	0.15
	$\overline{T^{\rm max}_{\rm sqr}}$	1.55 ± 0.130	…	1.48 ± 0.120	…	1.64 ± 0.122	…	1.71 ± 0.148	…	1.55	−0.86	0.09
Binned fitting	$T^{\rm all}_{\rm lin}$	0.31 ± 0.012	7.30	0.34 ± 0.012	3.97	0.39 ± 0.011	3.55	0.42 ± 0.015	1.07	7.33	−0.94	0.06
	$T^{\rm max}_{\rm lin}$	0.34 ± 0.026	2.09	0.37 ± 0.027	0.76	0.40 ± 0.025	0.10	0.44 ± 0.033	0.33	3.03	−0.97	0.05
	$T^{\rm all}_{\rm sqr}$	1.36 ± 0.050	1.74	1.52 ± 0.052	2.24	1.70 ± 0.047	6.21	1.71 ± 0.061	3.14	5.74	−0.83	0.10
	$T^{\rm max}_{\rm sqr}$	1.48 ± 0.115	1.66	1.65 ± 0.119	1.15	1.72 ± 0.110	1.46	1.81 ± 0.137	1.12	2.41	−0.91	0.07
Unbinned fitting	$T^{\rm all}_{\rm lin}$	0.33 ± 0.010	1.15	0.34 ± 0.010	1.13	0.40 ± 0.003	0.96	0.43 ± 0.014	0.89	7.14	−0.94	0.06
	$T^{\rm max}_{\rm lin}$	0.34 ± 0.024	1.06	0.36 ± 0.025	0.99	0.40 ± 0.026	0.89	0.43 ± 0.027	0.75	3.33	−0.98	0.05
	$T^{\rm all}_{\rm sqr}$	1.48 ± 0.043	1.09	1.51 ± 0.045	1.06	1.74 ± 0.044	0.96	1.76 ± 0.055	0.90	5.09	−0.87	0.08
	$T^{\rm max}_{\rm sqr}$	1.52 ± 0.105	1.06	1.61 ± 0.106	1.00	1.72 ± 0.000	0.89	1.80 ± 0.005	0.75	2.64	−0.96	0.06

$\Delta s \geq 2.°5$

Method	T	Cy21		Cy22		Cy23		Cy24		c-value	Correlation	
		$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2		r	p
Mean	$\overline{\alpha_{\rm all}}$	5.°99 ± 0.°197	…	6.°52 ± 0.°204	…	7.°23 ± 0.°180	…	7.°03 ± 0.°221	…	5.61	−0.75	0.14
	$\overline{\alpha_{\rm max}}$	5.°36 ± 0.°407	…	6.°31 ± 0.°401	…	6.°50 ± 0.°386	…	6.°47 ± 0.°464	…	2.46	−0.81	0.10
Normalization	$\overline{T^{\rm all}_{\rm lin}}$	0.44 ± 0.015	…	0.38 ± 0.012	…	0.46 ± 0.012	…	0.48 ± 0.016	…	6.25	−0.71	0.15
	$\overline{T^{\rm max}_{\rm lin}}$	0.37 ± 0.030	…	0.38 ± 0.026	…	0.42 ± 0.027	…	0.42 ± 0.032	…	1.56	−0.86	0.09
	$\overline{T^{\rm all}_{\rm sqr}}$	1.76 ± 0.061	…	1.58 ± 0.053	…	1.86 ± 0.050	…	1.87 ± 0.064	…	4.53	−0.63	0.21
	$\overline{T^{\rm max}_{\rm sqr}}$	1.73 ± 0.130	…	1.57 ± 0.115	…	1.76 ± 0.113	…	1.80 ± 0.142	…	1.62	−0.62	0.22
Binned fitting	$T^{\rm all}_{\rm lin}$	0.36 ± 0.012	5.70	0.39 ± 0.011	3.10	0.44 ± 0.010	4.32	0.48 ± 0.014	1.52	8.57	−0.96	0.06
	$T^{\rm max}_{\rm lin}$	0.33 ± 0.026	1.57	0.39 ± 0.025	0.39	0.39 ± 0.023	0.53	0.42 ± 0.030	0.61	3.00	−0.87	0.08
	$T^{\rm all}_{\rm sqr}$	1.58 ± 0.050	0.94	1.71 ± 0.050	5.71	1.93 ± 0.045	7.20	1.94 ± 0.057	5.15	6.32	−0.87	0.08
	$T^{\rm max}_{\rm sqr}$	1.52 ± 0.121	2.20	1.59 ± 0.110	2.85	1.71 ± 0.110	1.20	1.64 ± 0.128	0.79	1.48	−0.80	0.11
Unbinned fitting	$T^{\rm all}_{\rm lin}$	0.37 ± 0.007	1.09	0.37 ± 0.01	1.03	0.44 ± 0.007	0.93	0.47 ± 0.014	0.85	7.14	−0.95	0.06
	$T^{\rm max}_{\rm lin}$	0.34 ± 0.015	1.06	0.38 ± 0.000	0.85	0.40 ± 0.023	0.87	0.42 ± 0.031	0.78	2.58	−0.88	0.08
	$T^{\rm all}_{\rm sqr}$	1.67 ± 0.043	1.09	1.67 ± 0.005	1.03	1.91 ± 0.044	0.93	1.94 ± 0.058	0.85	4.66	−0.87	0.08
	$T^{\rm max}_{\rm sqr}$	1.49 ± 0.066	1.06	1.68 ± 0.071	0.86	1.75 ± 0.099	0.88	1.73 ± 0.131	0.78	1.98	−0.70	0.16

- **Varied CCs from r = 0.98 (p = 0.05) to r = 0.62 (p = 0.22) depending on the methods. → confirm the controversial statistics**

$$c = \frac{max(T) - min(T)}{max(\sigma_T)}$$

We take c=3 as the critical value.
A larger c, a stronger statistical significance.

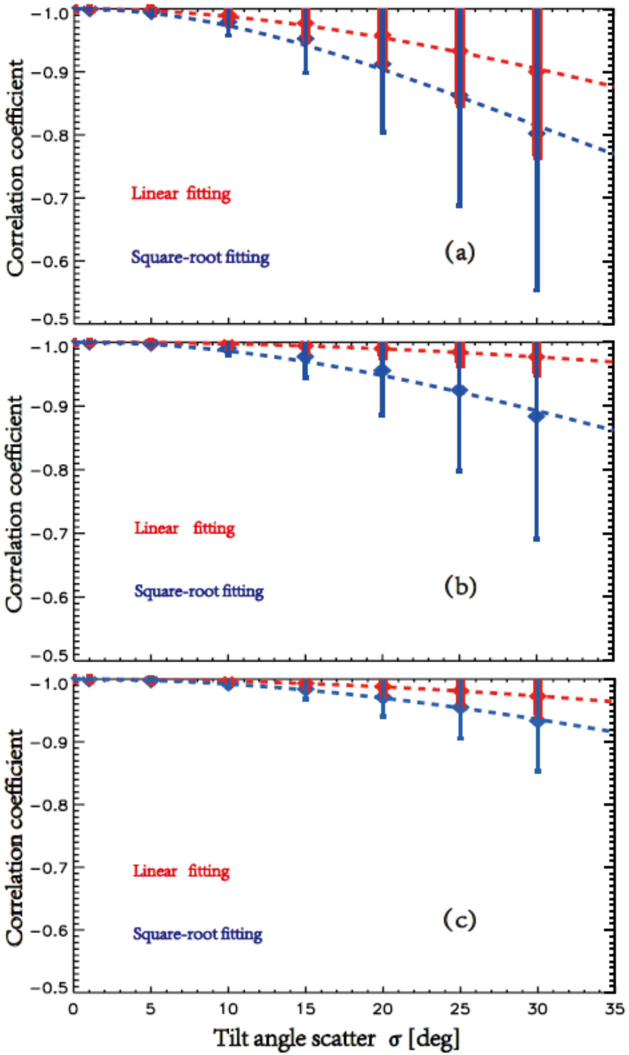
- **Prominent variation of σ_T**
  - **σ_T for the data with the filter Δs>2.5 deg is always smaller than those without the filter**
  - **the normalization method has the largest σ_T & the unbinned fitting method has the smallest σ_T**
    → unbinned linear fitting can minimize the uncertainty.
  - **the linear fit always has a larger c than that of the square-root fit**

To investigate how much the tilt scatter affects the relationship between the tilt coefficient and the cycle strength &
To evaluate which statistical method can minimize the uncertainty of the tilt coefficient

→ **Monte Carlo experiments**

- **based on the DPD_{all} data with $\Delta s>2.5$ deg**

- **The tilt angle of each sunspot group is artificially synthesized.**
    - **assume that the tilt coefficient and the cycle strength are fully correlated.**
    - **consider both the linear and square-root Joy's law equations.**
    - **use the observed latitudes to obtain the ideal artificial tilt angles satisfying Joy's law**
    - **add random numbers to the ideal artificial tilt angle data**
    - **increase σ_α from 0 to 30 with 5 interval; generate 10,000 sets of artificial tilt angles for each σ_α**



normalization method

binned fitting method

unbinned fitting method

- **The CC between the tilt coefficient and cycle strength decreases with increasing σ_α**

- **The unbinned fitting method and linear form of Joy's law can effectively minimize the effects of the uncertainty.**

Extension of the data

- **Cycle dependence of the tilt coefficient for separate hemispheres of cycles 21–24**
- **Combining DPD data with MW and KK data**

Filter	Method	T	Cy21		Cy22		Cy23		Cy24		c-value	Correlation	
			$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2	$T \pm \sigma_T$	χ^2		r	p
$\Delta s \geq 0°$	Binned fitting	$T_{\text{lin}}N$	0.31 ± 0.017	3.91	0.35 ± 0.044	5.04	0.39 ± 0.015	5.48	0.40 ± 0.021	1.11	2.73	−0.73	0.04
		$T_{\text{lin}}S$	0.35 ± 0.017	4.24	0.34 ± 0.016	2.21	0.39 ± 0.014	1.29	0.43 ± 0.019	1.19			
		$T_{\text{sqr}}N$	1.36 ± 0.072	1.18	1.55 ± 0.076	3.49	1.70 ± 0.068	5.66	1.58 ± 0.088	2.48	5.23	−0.68	0.05
		$T_{\text{sqr}}S$	1.36 ± 0.070	1.33	1.50 ± 0.072	1.96	1.71 ± 0.065	3.70	1.82 ± 0.085	1.71			
	Unbinned fitting	$T_{\text{lin}}N$	0.35 ± 0.004	1.08	0.35 ± 0.001	1.07	0.41 ± 0.014	0.92	0.41 ± 0.020	0.89	6.00	−0.84	0.02
		$T_{\text{lin}}S$	0.31 ± 0.004	1.10	0.33 ± 0.000	1.05	0.39 ± 0.013	0.99	0.43 ± 0.019	0.90			
		$T_{\text{sqr}}N$	1.55 ± 0.018	1.08	1.59 ± 0.065	1.07	1.79 ± 0.002	0.92	1.66 ± 0.077	0.89	5.44	−0.68	0.05
		$T_{\text{sqr}}S$	1.41 ± 0.058	1.09	1.45 ± 0.043	1.05	1.71 ± 0.059	0.99	1.84 ± 0.079	0.90			
$\Delta s \geq 2.5°$	Binned fitting	$T_{\text{lin}}N$	0.38 ± 0.016	4.22	0.38 ± 0.017	4.58	0.44 ± 0.015	5.04	0.47 ± 0.021	3.71	5.24	−0.76	0.03
		$T_{\text{lin}}S$	0.34 ± 0.017	4.25	0.40 ± 0.016	1.41	0.44 ± 0.014	2.28	0.49 ± 0.019	2.59			
		$T_{\text{sqr}}N$	1.66 ± 0.070	1.78	1.67 ± 0.073	5.93	1.91 ± 0.065	4.68	1.81 ± 0.082	7.79	7.20	−0.54	0.13
		$T_{\text{sqr}}S$	1.50 ± 0.071	1.83	1.74 ± 0.069	3.22	1.94 ± 0.061	5.38	2.09 ± 0.079	1.83			
	Unbinned fitting	$T_{\text{lin}}N$	0.41 ± 0.014	1.05	0.37 ± 0.015	1.07	0.45 ± 0.000	0.92	0.45 ± 0.021	0.86	7.14	−0.81	0.02
		$T_{\text{lin}}S$	0.34 ± 0.014	1.13	0.38 ± 0.014	1.00	0.43 ± 0.014	0.94	0.49 ± 0.019	0.85			
		$T_{\text{sqr}}N$	1.81 ± 0.044	1.04	1.66 ± 0.067	1.07	1.96 ± 0.006	0.92	1.80 ± 0.000	0.86	6.79	−0.62	0.08
		$T_{\text{sqr}}S$	1.54 ± 0.061	1.13	1.67 ± 0.063	1.01	1.87 ± 0.060	0.95	2.07 ± 0.078	0.84			

- **Values of σ_T based on the unbinned fitting method are prominently smaller than the corresponding values based on the binned fitting method.**
- **The tilts tend to show a linear dependence on the latitudes for weak cycles and a square-root dependence on strong cycles.**
- **The CCs based on the linear form of Joy's law are larger than those based on the square-root form of Joy's law.**

MW and KK data

the unbinned fitting method, weighted fits

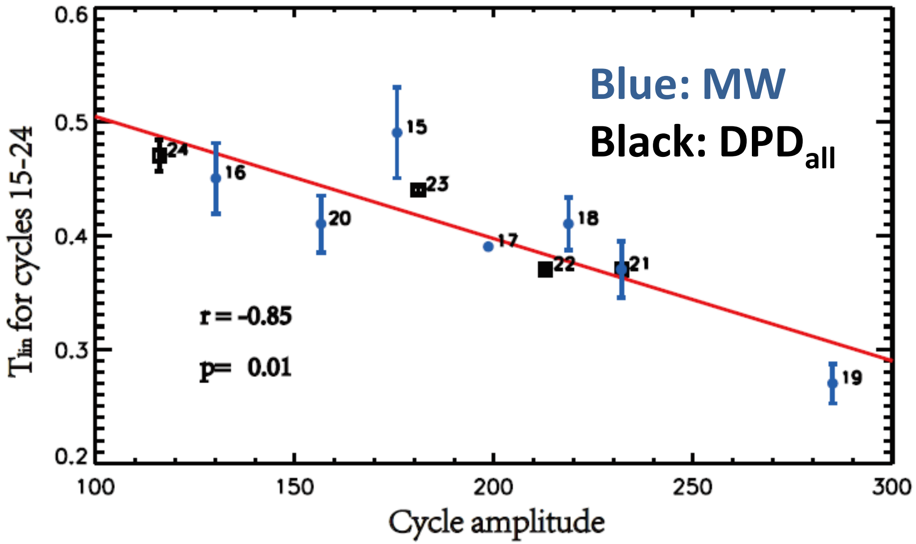
T	Data	Cy15 $T\pm\sigma_T$	Cy16 $T\pm\sigma_T$	Cy17 $T\pm\sigma_T$	Cy18 $T\pm\sigma_T$	Cy19 $T\pm\sigma_T$	Cy20 $T\pm\sigma_T$	Cy21 $T\pm\sigma_T$	c-value	Correlation(15-21) r	p	f-value
T_{lin}	MW	0.37±0.004	0.31±0.023	0.32±0.028	0.30±0.024	0.19±0.019	0.29±0.008	0.31±0.027	6.43	-0.65	0.08	1.06
	$\Delta s \geq 2°.5$	0.49±0.040	0.45±0.031	0.39±0.001	0.41±0.023	0.27±0.017	0.41±0.024	0.37±0.025	5.50	-0.84	0.03	1.00
	KK	0.38±0.028	0.34±0.021	0.34±0.013	0.26±0.019	0.24±0.000	0.31±0.018	0.26±0.019	5.00	-0.75	0.05	1.27
	$\Delta s \geq 2°.5$	0.46±0.034	0.39±0.027	0.37±0.007	0.38±0.007	0.30±0.002	0.41±0.007	0.40±0.024	4.70	-0.67	0.08	0.93
T_{sqr}	MW	1.47±0.005	1.29±0.134	1.44±0.118	1.36±0.031	0.91±0.086	1.23±0.108	1.35±0.114	4.91	-0.54	0.15	1.10
	$\Delta s \geq 2°.5$	1.91±0.110	1.86±0.004	1.58±0.114	1.78±0.097	1.21±0.082	1.64±0.101	1.58±0.105	6.14	-0.79	0.04	0.94
	KK	1.48±0.110	1.45±0.091	1.47±0.078	1.17±0.079	1.11±0.072	1.34±0.001	1.20±0.083	3.36	-0.80	0.03	1.39
	$\Delta s \geq 2°.5$	1.83±0.135	1.60±0.035	1.56±0.010	1.61±0.097	1.35±0.028	1.70±0.092	1.79±0.010	3.56	-0.49	0.19	0.93

- **Values of σ_T are several times lower than those of previous studies.**
- **The average tilt coefficients in the case of Δs>2.5 deg are much stronger, typically 30%, than that in the case of Δs>0.0 deg. The difference based on MW data is larger than that based on KK data.**
- **The square-root form of Joy's law show much weaker CCs.**
- ***f*: ratio between the cycle 21 tilt coefficient derived based on diff. methods for the KK and MW data sets and the corresponding value based on DPD_{all}.**

Combining DPD data with MW and KK data

$$T_{\rm lin} = -0.00107 * S_n + 0.61$$

- Clearly an anti-correlation between the tilt coefficient and the cycle strength with a significant confidence level.
- The improvements owe to the filter Δs>2.5 deg and the unbinned linear fitting method used in analyzing the data.



With the filter Δs>2.5 deg , the deviation of cycle 19 tilt coefficient from the fitted line is much less than before.