Implications of Generalized Rankine-Hugoniot Conditions for the PUI Population at the Voyager 2 Termination Shock

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Abstract. The Rankine-Hugoniot (R-H) jump conditions at the heliospheric termination shock provide a means of knitting together the *in situ* measurements from Voyager 2 (VGR2) with the remote sensing of the heliosheath plasma via energetic neutral atom (ENA) imaging by IBEX and Cassini/INCA. The VGR2 instrument suite has a gap (~1-30 keV) in the ion measurements. While the ENA images (0.2-6 keV and 5-55 keV) fill the VGR2 gap in the pixel containing the VGR2 spacecraft, they do so only in the sense that they provide the ion intensity integrated along the radial line of sight throughout the entire heliosheath. The synthesis we attempt is further complicated by the observational results from all three spacecraft that the non-thermal component of the ion pressure dominates that of the thermal component. We therefore have developed (and applied) a generalized formulation of the R-H conditions that does not invoke an equation of state, but rather can directly ingest the instrumentally-measured non-thermal spectrum. The result is an estimate that the ratio (upstream/downstream) of the non-thermal pressure is ~43%, confirming anew that the termination shock (at least at VGR2) is strongly mediated by non-thermal ions.

Keywords: Heliospheric termination shock, Rankine-Hugoniot conditions, Non-thermal plasma
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INTRODUCTION

We argue in this paper that the Voyager 2 (VGR2) measurements at its multiple termination shock crossings are of particular importance, because their substitution into generalized Rankine-Hugoniot (R-H) jump conditions can place a strong constraint on non-thermal plasma pressures and energy densities. Conventional formulations of the R-H conditions utilize an equation of state plus a "closure" (adiabatic) condition. This is not appropriate for VGR2 (because the non-thermal pressure dominates), but we can still write down useful R-H conditions in terms of the conserved quantities (the flow of mass, momentum and energy across the shock) if we can generalize the R-H conditions to the actual measurements made by the instruments.

Marvelous as they are, the Voyager spacecraft (VGR1/2) suffer a gap in their instrumentation for measurements of ions ~1-30 keV. The gap has become critical now that VGR1/2 are both immersed in the heliosheath (HS) beyond the solar wind termination shock (TS), because we know that non-thermal ions provide the dominant pressure in that region in the energy range ~1-30 keV of heated pickup ions (PUIs). Fortunately, the recently published all-sky images of the HS from the energetic neutral atom (ENA) cameras IBEX [1] and Cassini/MIMI/INCA [2] may help fill that energy gap. The ENA technique images singly-charged energetic ions that undergo charge-exchange collisions when they are immersed within a neutral gas medium. In the case of the heliosheath, the dominant energetic ion species is protons, and the gas consists predominantly of relatively cold hydrogen atoms (T~10,000K). However ENA imaging has one fundamental limitation: the ENA intensity (j_{ENA}) within an imaging pixel is produced by the integration of the energetic ion intensity (j_{ion}) multiplied by the the neutral H-atom density (n_H) along the entire line of sight (LOS) of that pixel. Although there is some spatial variation in n_H the heliosheath, the dominant variation is expected in the energetic ion intensity (j_{ion}). The ambiguity of the LOS integration has so far allowed many explanations of the remarkable IBEX "ribbon" and the INCA "belt" to be put forward that invoke a variety of spatial and/or temporal variations. However, the VGR1/2 observations provide *in situ* "ground truth" for the ENA images, albeit within only two pixels out of hundreds in the ENA images. Nonetheless, these two pixels are critical to the interpretation of the global images. The Voyager pixels are rather strategically located: VGR1 lies on the northern edge of the belt but outside the ribbon, whereas VGR2 lies in heart of the belt and on the southern edge of the ribbon.

The first indication that a new paradigm might be required for the heliosheath came with the surprising VGR2 plasma observation (the plasma instrument on VGR1 failed after its Jupiter flyby) that the thermal plasma remained *supersonic* beyond the termination shock (TS) [3]. This immediately implied that the downstream suprathermal plasma flow had to be *subsonic* and that the bulk of the downstream pressure had to be produced by the suprathermal plasma. Both the IBEX and INCA ENA spectra in the VGR pixels were much harder than a simple extrapolation of the VGR thermal spectrum, suggesting that the bulk of the downstream pressure was generated by 0.2-30 keV ions [1,2,4]. In addition, the LECP observations [5] from both VGR1 and VGR2 for ions >30keV revealed non-thermal high-energy power-law "tails" (E^{-k}) with k~1.5. The pressure from such a hard spectrum would diverge logarithmically with energy if it extended to very high energies, so even over the finite energy range 30-3500 keV the ion tails make a significant pressure contributions [2,5]. Consequently, there is certainly no simple "equation of state" for the downstream heliosheath plasma, *e.g.,* no single-temperature Maxwellian or kappa-function could describe the complicated ion phase space density. Therefore, if we wish to extract the information contained in the R-H jump conditions at the TS, we must re-formulate them more generally without invoking any equation of state. We will go back to very basic concepts in order to establish the generality of our results.

BASIC TRANSPORT EQUATION AND MOMENTS

$$\partial f/\partial t + \mathbf{v}\cdot\nabla f + (Ze)\,(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\partial f/\partial\mathbf{p} = Q \qquad (1)$$

This "fine-grained" equation describes the evolution of the phase-space density f($\mathbf{r}$,$\mathbf{p}$,t) for a *single species* of charge (Ze) and mass (m) with $\mathbf{p}$=m$\mathbf{v}$ using conventional notation; species subscripts are suppressed for clarity. The electric and magnetic fields ($\mathbf{E}$,$\mathbf{B}$) are considered to be self-consistent. We indicate the electric field as the vector ($\mathbf{E}$), whereas we will reserve the scalar symbol (E) for its customary use as a particle energy. The “source/sink” function (Q) is a “catch-all” for all other physical processes, including mass loading, momentum transfer (Boltzmann-like), and energy change due to “collisions” with other species of particles. The general equation (Eq. 1) can be converted into a set of coupled relations among momentum moments over phase space of ascending order [6] ; the first three moment equations are summarized in Table 1 and the moments ($N,\mathbf{K},\boldsymbol{\Psi},\boldsymbol{\Pi},\ldots$) are defined in Table 2. In a first step away from dependence upon an equation of state, the moments may be re-cast in terms of the directly measureable uni-directional differential particle intensity $j(\mathbf{r},\mathbf{u}_p,E,t)$ through the general relationships $f=j(E)/p^2$ and $dE=vdp$, where $\mathbf{u}_p=\mathbf{p}/p=\mathbf{v}/v$ is the unit momentum (or velocity) vector, and E is the kinetic energy. See Table 2. The remaining quantities ($Q_0,\mathbf{Q}_1,Q_2$) in the moment equations describe the exchanges of mass, momentum and energy, respectively.

TABLE 1. Moment equations (individual species) for conserved quantities.

$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) = Q_0$	Mass density ($\rho=mN$)
$\partial\mathbf{K}/\partial t + \nabla\cdot\boldsymbol{\Pi} - ZeN(\boldsymbol{E}+\mathbf{V}\times\mathbf{B}) = \mathbf{Q}_1$	Momentum density ($\mathbf{K}=\rho\mathbf{V}$)
$\partial\varepsilon/\partial t + \nabla\cdot\boldsymbol{\Psi} + Ze\mathbf{E}\cdot(N\mathbf{V}) = Q_2$	Energy density (ε)

The closure of the hierarchy of moments, *i.e.,* the familiar terms of MHD equations, is usually achieved by imposing an equation of state, *e.g.,* $PV=NkT$ and invoking a thermodynamic constraint, *e.g.,* $PN^{-\gamma}$=constant. However, we have no need of such closure, because we can measure directly the moments themselves. Nor need we invoke an equation of state (except for the lowest energy plasma, and that only for notational convenience), or any thermodynamic constraint. Instead, we express the moments directly in terms of the measured particle intensity (j). We write $\langle xj\rangle=(4\pi)^{-1}\int d\Omega_p xj$ for the average over all momentum directions (at a fixed energy) of the quantity (xj). The first three intensity directional moments (integrated over momentum space holding E=constant) are assigned the notations $j_0(E)=\langle j\rangle$, $\mathbf{j}_1(E)=\langle\mathbf{u}_p j\rangle$, and $\boldsymbol{j}_2(E)=\langle\mathbf{u}_p\mathbf{u}_p j\rangle$. The total moment of the quantity (X) for a single species is the integral over all momentum space of (xf). The preceding definitions yield the moments summarized in Table 2.

$$X = {}_0\int^\infty dp p^2\, {}_{4\pi}\int d\Omega_p xf = {}_0\int^\infty dp 4\pi\langle xj\rangle = 4\pi\, {}_0\int^\infty dE(1/v)\langle xj\rangle \qquad (2)$$

TABLE 2. Momentum-space moments for individual particle species in terms of uni-directional intensities.

$N = 4\pi\, {}_0\int^\infty dE(1/v)j_0(E)$	$j_0(E) = \langle j(E,\mathbf{u}_p)\rangle$	number density ($x=v^0$)
$\rho = 4\pi\, {}_0\int^\infty dE(m/v)j_0(E)$	$j_0(E) = \langle j(E,\mathbf{u}_p)\rangle$	mass density ($x=mv^0$)
$N\mathbf{V} = 4\pi\, {}_0\int^\infty dE\mathbf{j}_1(E)$	$\mathbf{j}_1(E) = \langle\mathbf{u}_p j(E,\mathbf{u}_p)\rangle$	number flux ($x=\mathbf{v}$)
$\mathbf{K} = 4\pi\, {}_0\int^\infty dEm\mathbf{j}_1(E)$	$\mathbf{j}_1(E) = \langle\mathbf{u}_p j(E,\mathbf{u}_p)\rangle$	momentum density ($x=\mathbf{p}$)
$\boldsymbol{\Psi} = 4\pi\, {}_0\int^\infty dEE\mathbf{j}_1(E)$	$\mathbf{j}_1(E) = \langle\mathbf{u}_p j(E,\mathbf{u}_p)\rangle$	energy flux density ($x=\mathbf{v}E$)
$\boldsymbol{\Pi} = 4\pi\, {}_0\int^\infty dEp\boldsymbol{j}_2(E)$	$\boldsymbol{j}_2(T) = \langle\mathbf{u}_p\mathbf{u}_p j(E,\mathbf{u}_p)\rangle$	particle stress tensor ($x=\mathbf{pv}$)

In order to examine the meaningful physical quantities in the R-K conditions, in Table 3 we express the moments in the shock frame (unprimed quantities) in terms of their values in reference frames upstream and downstream of the shock (primed quantities) for each individual species. The trace of the particle stress tensor is twice the energy density (ε'), while if the stress tensor is isotropic (diagonal), its trace also equals three times the (isotropic) pressure (P): $Tr(\boldsymbol{\Pi}')=2\varepsilon'=3P'$. Thus the relation $\varepsilon'=(3/2)P'$ follows from the basic definition of the stress tensor and not from any thermodynamic considerations.

TABLE 3. Transformations from shock frame into plasma frames (upstream or downstream).

$N = \int d^3p f(\mathbf{p})$	$= \int d^3p' f'(\mathbf{p}') = N'$	$N = N'$
$\mathbf{K} = \int d^3p \mathbf{p} f(\mathbf{p})$	$= \int d^3p'(\mathbf{p}'+m\mathbf{V})f'(\mathbf{p}')$	$\mathbf{K} = \mathbf{K}'+\rho\mathbf{V}$
$\boldsymbol{\Psi} = \int d^3p\ \mathbf{v}(mv^2/2)f(\mathbf{p})$	$=(m/2)\int d^3p'(\mathbf{v}'+\mathbf{V})(v'^2+2\mathbf{V}\cdot\mathbf{v}'+V^2)f'(\mathbf{p}')$	$\boldsymbol{\Psi}=\boldsymbol{\Psi}'+\mathbf{V}\varepsilon'+\mathbf{V}\cdot\boldsymbol{\Pi}'+\mathbf{V}\mathbf{V}\cdot\mathbf{K}'+(V^2/2)\mathbf{K}'+(\rho V^2/2)\mathbf{V}$
$\boldsymbol{\Pi} = \int d^3p \mathbf{p}\mathbf{v} f(\mathbf{p})$	$= \int d^3p'(\mathbf{p}'+m\mathbf{V})(\mathbf{v}'+\mathbf{V})f'(\mathbf{p}')$	$\boldsymbol{\Pi} = \boldsymbol{\Pi}'+\mathbf{V}\mathbf{K}'+\mathbf{K}'\mathbf{V}+\rho\mathbf{V}\mathbf{V}$
$\mathbf{S} = (1/\mu_0)\mathbf{E}\times\mathbf{B}$	$= \mathbf{S}'+(1/\mu_0)(-\mathbf{V}\times\mathbf{B})\times\mathbf{B}$	$\mathbf{S} = \mathbf{S}'+(1/\mu_0)(\mathbf{V}B^2-\mathbf{B}\mathbf{B}\cdot\mathbf{V})$

Representation of moments in terms of instrumental measurements

Suppose we have a set of instruments that cover all energies $0<E<\infty$ in a set of energy ranges $0<E<E_1$, $E_1<E<E_2,\ldots$, $E_k<E<\infty$ for each species of interest. For a first example, consider how the number flux ($N\mathbf{V}$) can be calculated directly from the measurements.

$$N\mathbf{V}/4\pi = {}_0\!\int^{E1} dE\,\mathbf{j}_1(E) + {}_{E1}\!\int^{E2} dE\,\mathbf{j}_1(E) + \ldots + {}_{Ek}\!\int^{\infty} dE\,\mathbf{j}_1(E) \qquad (3)$$

If the first integral, *i.e.,* the measurement from the thermal plasma instrument over the lowest energy range ($0<E<E_1$), dominates all the rest, then it alone $(N\mathbf{V})_{th}$ provides a good estimate of the total number flux $N\mathbf{V}$ and also of the bulk velocity for the species ($\mathbf{V}$) because $N\sim N_{th}$.

For a second example, the scalar pressure (P) is proportional to the trace of the particle stress tensor ($\boldsymbol{\Pi}$). The total particle pressure can be written as the sum of partial pressures measured by all the instruments: $P=\Delta P_{th}+\Delta P_1+\ldots+\Delta P_k$. Henceforward we suppress the prime (′) notation on the partial pressures (ΔP), since pressures are always calculated from intensities (j') transformed into the plasma frame.

$$3P/4\pi = {}_0\!\int^{E1} dE\,p\,j_0(E) + {}_{E1}\!\int^{E2} dE\,p\,j_0(E) + \ldots + {}_{Ek}\!\int^{\infty} dE\,p\,j_0(E) \qquad (4)$$

The first partial pressure (lowest energy) may be usefully parametrized in terms of a temperature as $\Delta P_{th}=N_{th}kT_{th}$, because $N_{th}\cong N$ (most of the density is in the thermal range). This is not really invoking an equation of state for the low-energy "thermal" plasma; it is done for convenience because the Voyager plasma results have been reported in terms of a temperature (T_{th}) derived from the plasma momentum moments. The important observational result from VGR2 is that the thermal partial pressure is smaller than the non-thermal partial pressures in the higher energy instruments ($\Delta P_{th}<<\Delta P_1+\ldots+\Delta P_k$), and that these higher energy distributions do not exhibit "thermal" spectra.

Summation over all species for conserved quantities

The symbol ($\sum$) indicates a summation operation over *all particle species* (ions, electrons *and* neutrals) for *all mathematical terms to the right of the symbol.* The electrons should be included in general. The neutrals will not contribute to the jump conditions because their momentum, energy flux and pressure are continuous at the shock. There are five scalar jump conditions (the $\mathbf{u}\cdot\sum\boldsymbol{\Pi}$ equation has three components). The jump conditions for a given species at a given energy are too complicated to write down here, since they involve the other species. Only when we sum over *all* species do the conservation laws give us simple jump conditions. When the moment equations for individual species in Table 2 are summed over all species (including neutrals) in Table 4, exchanges of mass, momentum and energy cancel out: $\sum Q_0$, $\sum\mathbf{Q}_1=0$, $\sum Q_2=0$. Also, we identify the following sums over all species: $\sum mN=\sum\rho$ (total mass density), $\sum ZeN=0$ (total charge neutrality), and $\sum ZeN\mathbf{V}=\mathbf{J}$ (electric current intensity).

TABLE 4. Total moment equations summed over all species (including neutrals).

$\partial\sum\rho/\partial t + \nabla\cdot\sum\mathbf{K} = 0$	$\partial\sum\rho/\partial t + \nabla\cdot\sum\mathbf{K} = 0$
$\partial\sum\mathbf{K}/\partial t + \nabla\cdot\sum\boldsymbol{\Pi} - \mathbf{J}\times\mathbf{B} = 0$	$\partial(\sum\mathbf{K}+\mathbf{S}/c^2)\partial t + \nabla\cdot(\boldsymbol{\Pi}-\mathbf{BB}/\mu_0+\boldsymbol{I}B^2/2\mu_0) = 0$
$\partial\sum\varepsilon/\partial t + \nabla\cdot\sum\boldsymbol{\Psi} - \mathbf{E}\cdot\mathbf{J} = 0$	$\partial/\partial t(\sum\varepsilon+B^2/2\mu_0) + \nabla\cdot(\sum\boldsymbol{\Psi}+\mathbf{S}) = 0$

In Table 4, we have replaced the electro-magnetic terms ($\mathbf{J}\times\mathbf{B}$ and $\mathbf{E}\cdot\mathbf{J}$) with terms involving Maxwell stresses and Poynting's vector $\mathbf{S}=(1/\mu_0)\mathbf{E}\times\mathbf{B}$. The electric field stress tensor and energy density is neglected, because they are $O(V/c)^2$ compared to those of the magnetic field. These equations are now in a "conservation law" format. Since they have been formulated in the shock frame, the time derivatives vanish. Then Gauss's law, applied to a "pillbox" containing the termination shock with a unit normal vector ($\mathbf{u}$) yields the Rankine-Hugoniot jump conditions for a plasma shock [7]:

$$\mathbf{u}\cdot[\sum\mathbf{K}]=0 \qquad \mathbf{u}\cdot[\boldsymbol{\Pi}-\mathbf{BB}/\mu_0+\boldsymbol{I}B^2/2\mu_0]=0 \qquad \mathbf{u}\cdot[\sum\boldsymbol{\Psi}+\mathbf{S}]=0 \tag{5}$$

where the notation $\mathbf{u}\cdot[x]$ indicates the jump in the quantity x projected along the shock normal.

SIMPLIFIED GENERAL RANKINE-HUGONIOT CONDITIONS

For simplicity, we will now assume isotropic pressures (P') and neglect residual momentum intensity ($\mathbf{K}'$) and heat flux ($\boldsymbol{\Psi}'$) in the upstream/downstream frames. Also, $\mathbf{S}'=0$ in the plasma frames because $\mathbf{E}'=0$, so $\mathbf{u}\cdot\mathbf{S}=\mathbf{u}\cdot(\mathbf{V}B^2-\mathbf{BB}\cdot\mathbf{V})/\mu_0\cong(B^2/\mu_0)(\mathbf{u}\cdot\mathbf{V})$ for a quasi-perpendicular shock. Please refer to Table 3. Conservation of mass density becomes continuity of total momentum (mass flux) across the shock, while conservation of energy density becomes continuity of total energy density flux across the shock.

$$\mathbf{u}\cdot[\sum\rho\mathbf{V}]\rightarrow[\sum\rho\mathbf{u}\cdot\mathbf{V}] = 0 \qquad \mathbf{u}\cdot[\sum\boldsymbol{\Psi}+\mathbf{S}]\rightarrow[\sum(\varepsilon'+P'+\rho V^2/2+ B^2/\mu_0)\mathbf{u}\cdot\mathbf{V}]=0 \tag{6}$$

Note the natural appearance of the *specific enthalpy* ($\varepsilon'+P'$) in the total energy density flux, representing the energy necessary to "put the gas together", even though no thermodynamic concepts have been invoked explicitly. This quantity is more easily recognized in its familiar representation as the *enthalpy* of classical thermodynamics if it is multiplied by a volume V, since $\varepsilon' V=U'$ is the internal energy within that volume while

$P'V$ is the work done against a pressure reservoir (P'=constant) while creating that volume of gas.

In Eq. (6), we can either write $P'=2\varepsilon'/3$ or $\varepsilon'=3P'/2$ and consider the term $\rho V^2/2$ to be either the kinetic energy of the bulk flow or half the total ram pressure. The units can either be those of energy (pJ/m^3) or pressure (pPa) because 1J=1N-m and 1Pa=1N/m^2.

$$[(5/3\sum\varepsilon'+\sum\rho V^2/2+B^2/\mu_0)\mathbf{u}\cdot\mathbf{V}]=0 \quad \text{or} \quad [(5/2\sum P'+\sum\rho V^2/2+B^2/\mu_0)\mathbf{u}\cdot\mathbf{V}]=0 \tag{7}$$

In this jump condition the total energy density (ε') is weighted by a relative factor (5/3) with respect to the kinetic energy of the bulk flow ($\sum\rho V^2/2$), or equivalently, the total pressure ($\sum P'$) is weighted by a relative factor (5) with respect to the total ram pressure ($\sum\rho V^2$). We now can see that is not particularly helpful to make separate upstream/downstream comparisons of the energy density or the pressure, because they are combined in the specific enthalpy. The energy density (or the ram pressure) of the bulk flow and the magnetic pressure must also be included in the conserved quantities (but with the proper weighting factors, none of which is unity). The remaining vector jump condition involving the stress tensors determines the deflection of the bulk velocity (**V**) across the shock. However, the VGR2 shock crossings were consistent with a quasi-perpendicular shock [8] because $[B/N]\cong 0$, so that $\mathbf{u}\cdot\mathbf{V}\cong V$ and, after clearing fractions, we are left with the simple expressions

$$5[V\sum\varepsilon']+(3/2)[V\sum\rho V^2]+3[VB^2/\mu_0]=0 \quad \text{or} \quad 5[V\sum P']+[V\sum\rho V^2]+2[VB^2/\mu_0]=0 \tag{8}$$

Separation of Thermal from Non-Thermal Components

We now separately identify the thermal and non-thermal components. Also, from here on, we'll use the pressure formulation (because the equivalent expressions in terms of energy density are simply related to them through $\varepsilon'=3P'/2$). Eq. (8) becomes

$$V_{up}\sum(NkT')_{up}+V_{up}\sum\Delta P'_{up}+(V_{up}/5)\sum(\rho V^2)_{up}+2V_{up}B_{up}^2/\mu_0$$
$$= V_{dn}\sum(NkT')_{dn}+V_{dn}\sum\Delta P'_{dn}+(V_{dn}/5)\sum(\rho V^2)_{dn}+2V_{dn}B_{dn}^2/\mu_0 \tag{9}$$

where $\Delta P'$ now will refer only to the non-thermal pressure (because we have written the thermal pressure explicitly as nkT'). Because the jump condition on total momentum $[\sum\rho V]=0$ is dominated by the thermal plasma, we may define a compression ratio $R=\rho_{dn}/\rho_{up}=V_{up}/V_{dn}$ wherein ρ and V now indicate the values for the thermal plasma alone. This allows us to write $\sum(\rho V^2)_{dn}\cong(1/R)\sum(\rho V^2)_{up}$, which is useful because the upstream values are better defined in the data than those downstream. From the Voyager 2 thermal plasma observations [3], we have $R\cong 300\text{kms}^{-1}/(150)\text{kms}^{-1}\cong 2$ and R-$1/R=1.5$. From the magnetic field observations [8], if we ignore the fine structure (48s) of the shocks (*e.g.,* magnetic ramp and overshoot), typical values are $B_{up}\sim 0.05$ nT and $B_{dn}\sim 0.10$ nT. Finally, we divide Eqs. 8 and 9 by V_{dn} and then re-arrange to obtain the result from our (simplified) generalized Rankine-Hugoniot relations.

$$\sum\Delta P'_{dn}\text{-}R\sum\Delta P'_{up} \cong$$

$$(1/5)(R\text{-}1/R)\sum(\rho V^2)_{up}+R\sum(NkT'+2\ B_{up}^2/\mu_0)_{up}-\sum(NkT'+2\ B_{up}^2/\mu_0)_{dn} \qquad (10)$$

All relevant observational data for the VGR2 TS are summarized in Table 5 for the upstream and downstream regions. The contribution of electrons is included in the thermal pressures, but is negligible in the non-thermal range. Only non-thermal pressures appear on the LHS of Eq. 10, while all quantities on the RHS are evaluated from the measurements of the thermal component and the magnetic field. Substituting the values from Table 5 into the RHS of Eq. 10 yields

$$\sum\Delta P'_{dn}\text{-}2\sum\Delta P'_{up} \cong 0.033\ \text{pPa} \qquad (11)$$

TABLE 5. Summary of measured and estimated pressures (VGR2 termination shock)

Pressure types	Pressure Contributions	Up (pPa)	Down (pPa)
Ram (bulk flow) [3]	$\sum(\rho V^2)$	0.150	(~0.075)
Thermal [3]	$\sum NkT'$ (0-0.2keV)	$5.5\text{x}10^{-4}$	(~$5.2\text{x}10^{-3}$)
Magnetic [8]	$B^2/2\mu_0$	0.001	0.004
Non-thermal			
Heated PUIs	$\sum\Delta P'$ (0.2-6keV)	???	(0.150*, 0.121**)
Cassini/INCA[2]	$\sum\Delta P'$ (5-55keV)	???	0.077***
VGR2 LECP [5] *in situ*	$\sum\Delta P'$ (28-3500keV)	<<0.023	0.023
Total (without PUIs)	$\sum\Delta P'$ (no 0.2-6keV)	???	0.100
Estimates (PUIs)			
*IBEX (Lo+Hi) LOS [4]	$\int dr\sum\Delta P'$ (0.2-6keV) L=50AU	???	7.5 **pPa-AU**
**Simulation [9]	$\sum\Delta P'$ (0.2-6keV)	???	0.121
***Normed to LECP			

DISCUSSION OF THE NON-THERMAL PRESSURE

We can immediately conclude from Eq. 11 that the upstream non-thermal pressure is greater than half of the downstream non-thermal pressure. However, we would like to extract more information. The Cassini/INCA energetic ion spectrum over 5-55 keV was derived [2] by normalizing the ENA intensity (measured in the VGR2 pixel and divided by the charge-exchange cross section) to the VGR2 *in situ* ion intensity in the overlapping energy range 30-55 keV. The agreement of the spectral slope in the overlap was consistent with the downstream non-thermal pressure extending approximately uniformly from the vicinity of the TS outward into the heliosheath. On the other hand, the IBEX and INCA ENA images are insensitive to the upstream ion intensities, because they extend only ~1AU inward of the TS and thus contribute little to the ENA LOS integrals. This is why there are only (???) entries for them in Table 5.

There was a notable absence of the >30 keV ion intensity observed upstream of the TS during by VGR2 during the months before the crossing [5], but since the bulk of the INCA-derived pressure resides at the lower energies, we have no way of knowing what the upstream pressure was in the range 5-30 keV. However, we know of two estimates of the downstream heated PUIs in the range 0.2-6.0 keV (see Table 5). The first is from

IBEX [4] where the ENA intensity in the VGR2 pixel implies an ion pressure (integrated over the ENA LOS) of 7.5 pPa-AU. If we choose a LOS distance of 50AU from the TS out to the heliopause, then $\Delta P'_{dn}$~0.15 pPa. The second is from a simulation [9] with self-consistent magnetic turbulence based on the VGR2 plasma measurements. The computed proton spectrum implies a partial pressure $\Delta P'_{dn}$~0.121 pPa; unfortunately an upstream spectrum was not provided. If we add the average (~0.13 pPa) of these two estimates to the remaining non-thermal pressure (0.100 pPa) in Table 5 and rewrite Eq. 11, we obtain

$$\sum \Delta P'_{up} = (1/2)(\sum \Delta P'_{dn} \text{ -0.033 pPa}) = (1/2)(0.230 \text{ pPa} -0.033 \text{ pPa}) = 0.099 \text{ pPa} \qquad (12)$$

which in turn implies $\sum\Delta P'_{up}/\sum\Delta P'_{dn}$=(0.099 pPa/0.230 pPa)=43%. One can see that this ratio is not too sensitive to the exact values of our estimates for $\Delta P'_{up}$ in the 0.2-6.0 keV energy range (listed in Table 5).

We therefore conclude from our generalized Rankine-Hugoniot analysis that almost half of the downstream non-thermal pressure has to appear upstream of the termination shock. This further supports earlier arguments [10,11,12] that the termination shock (at least at VGR2) was strongly mediated by non-thermal pressure, presumably from heated pick-up ions.

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