

Κέντρο Ερευνών Αστρονομίας
και Εφαρμοσμένων Μαθηματικών
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Hybrid Modeling of Pulsar Magnetospheres: a **personal** view

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Research Center for Astronomy
and Applied Mathematics

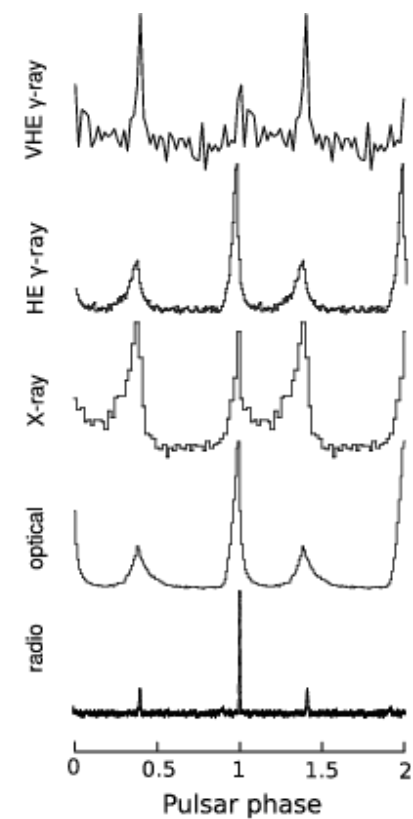
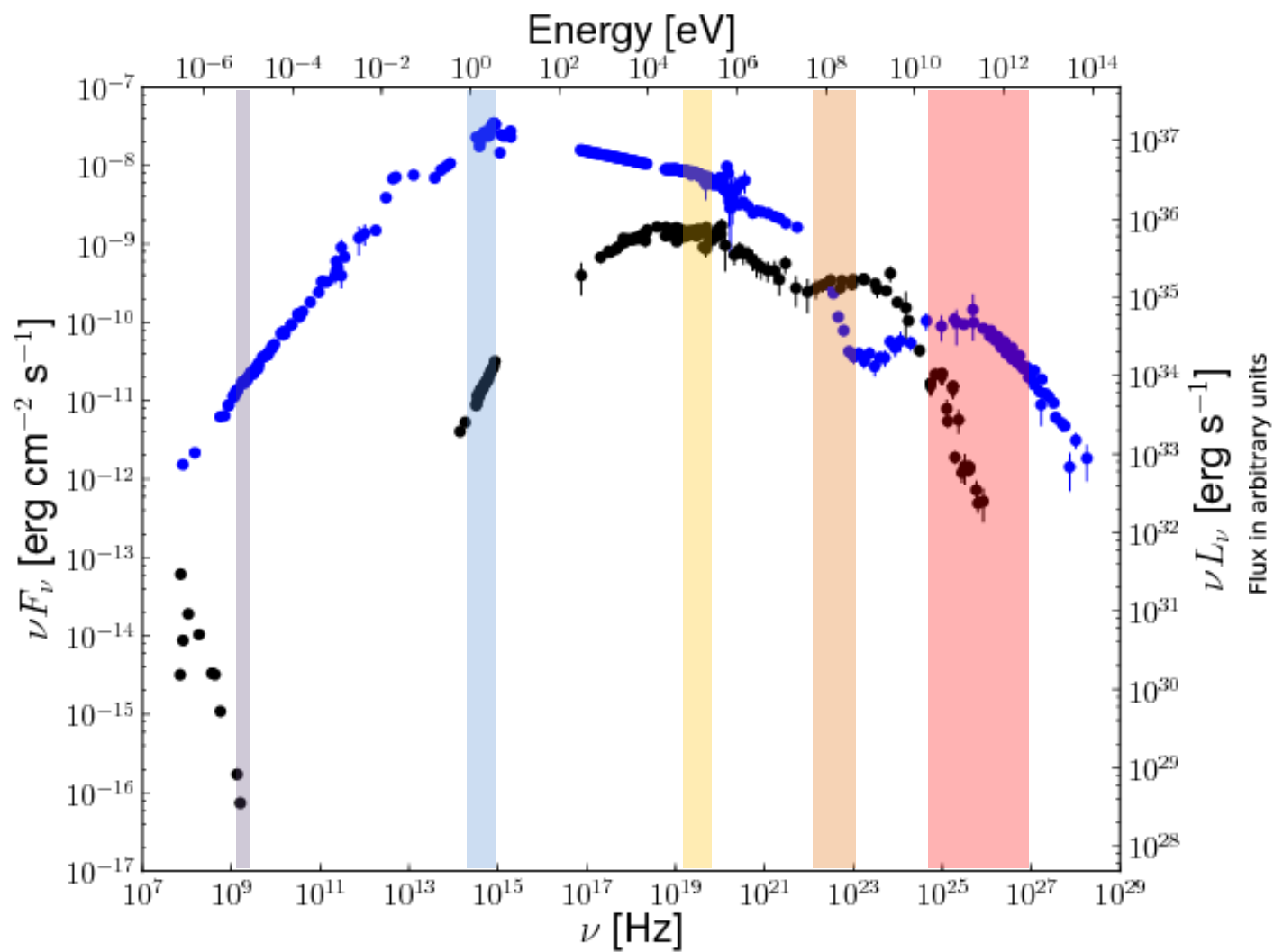
Academy of Athens

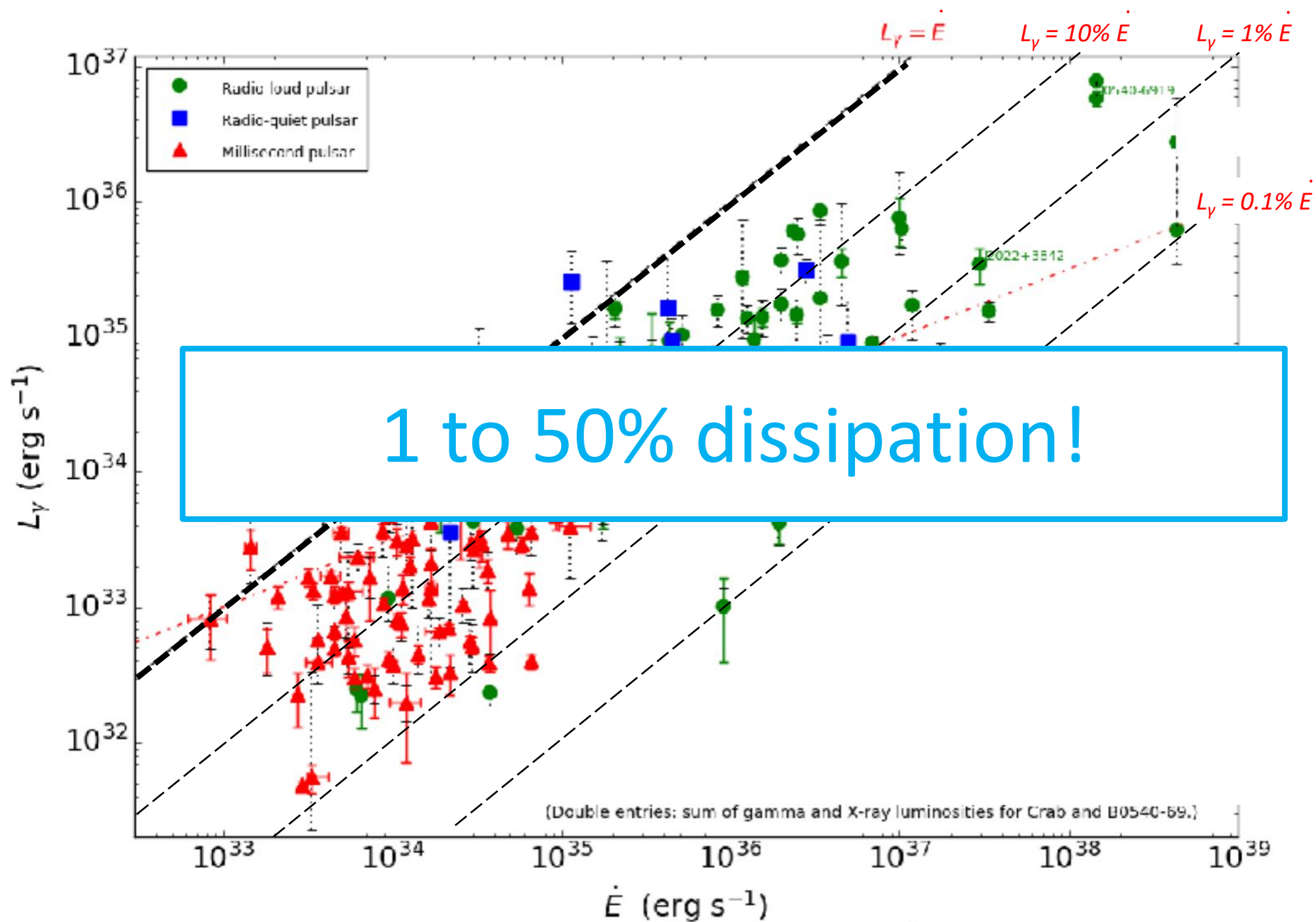
A little bit of...

- History
- Critique
- FFE everywhere + AE equatorial CS
- Particle trajectories
- (VHE γ -ray spectra)

Current collaborators:

Jérôme Pétri (Strasbourg), Petros Stefanou (Athens→Alicante)





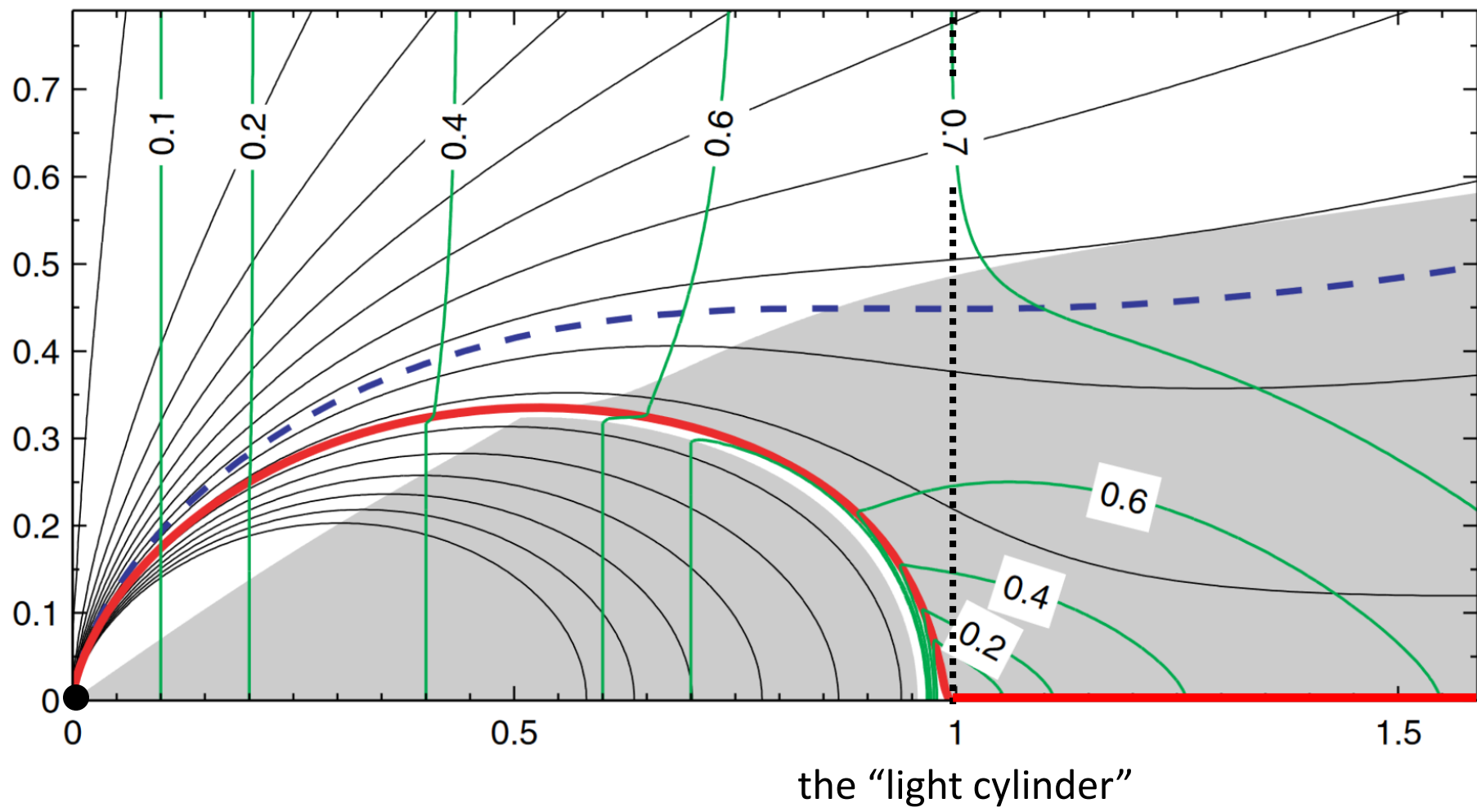
A little bit of history...

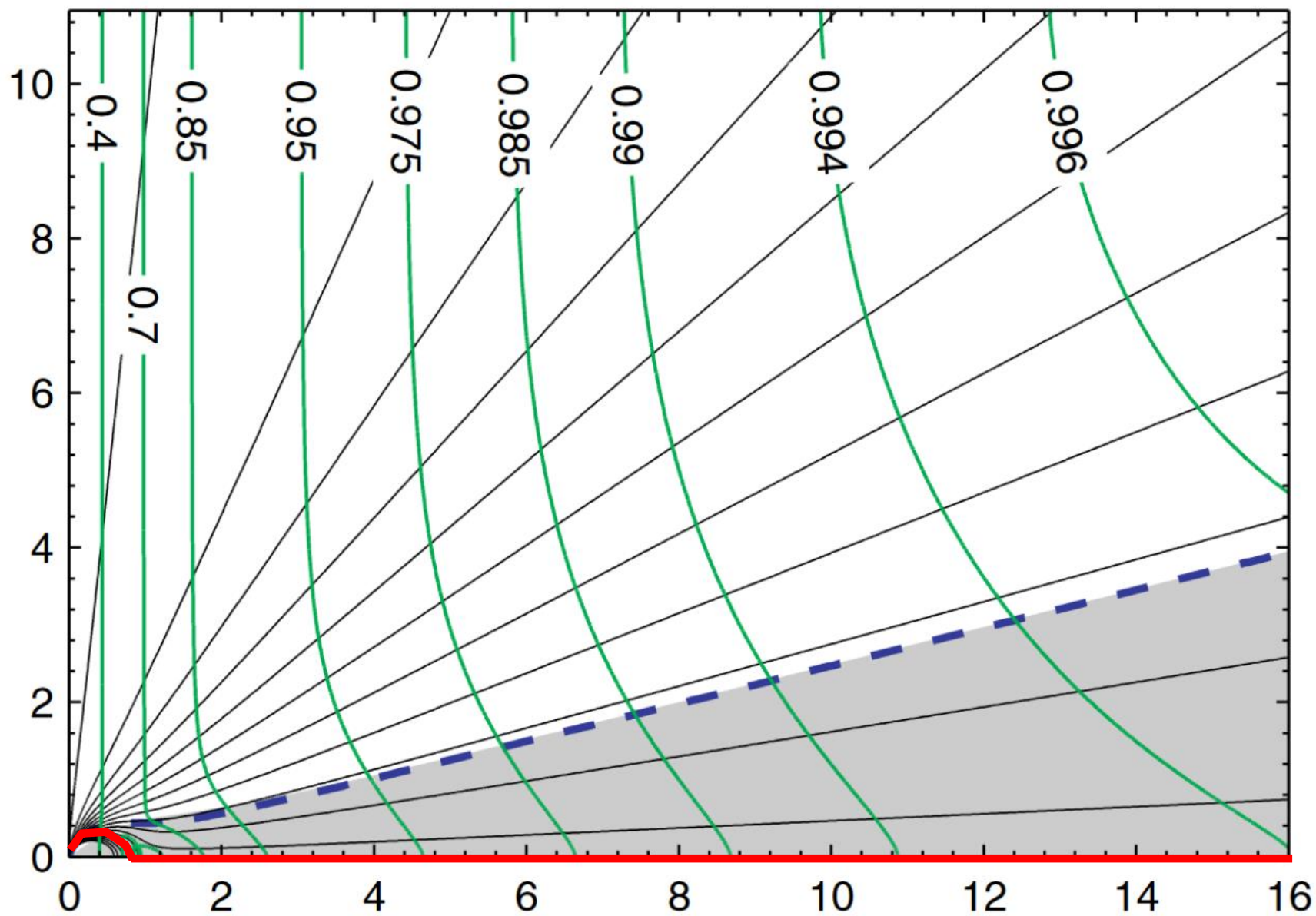
2D steady-state ideal
(force-free $\rightarrow$ pulsar equation)

1967 (Pulsar discovery: Jocelyn Bell)

1969 (Ideal force-free: Goldreich & Julian)

1999 (Contopoulos, Kazanas & Fendt)





A little bit of history...

Current Closure is key!

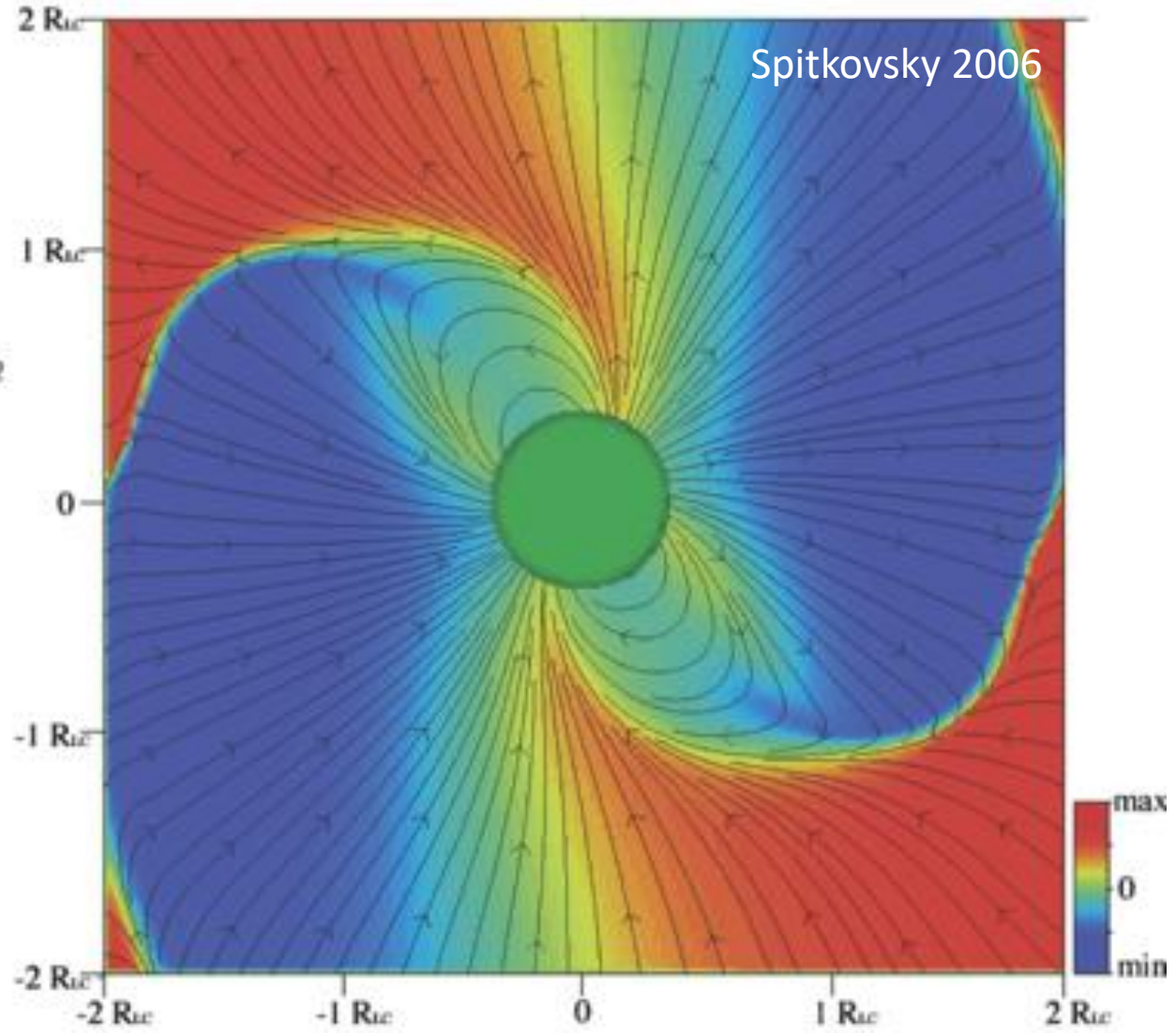
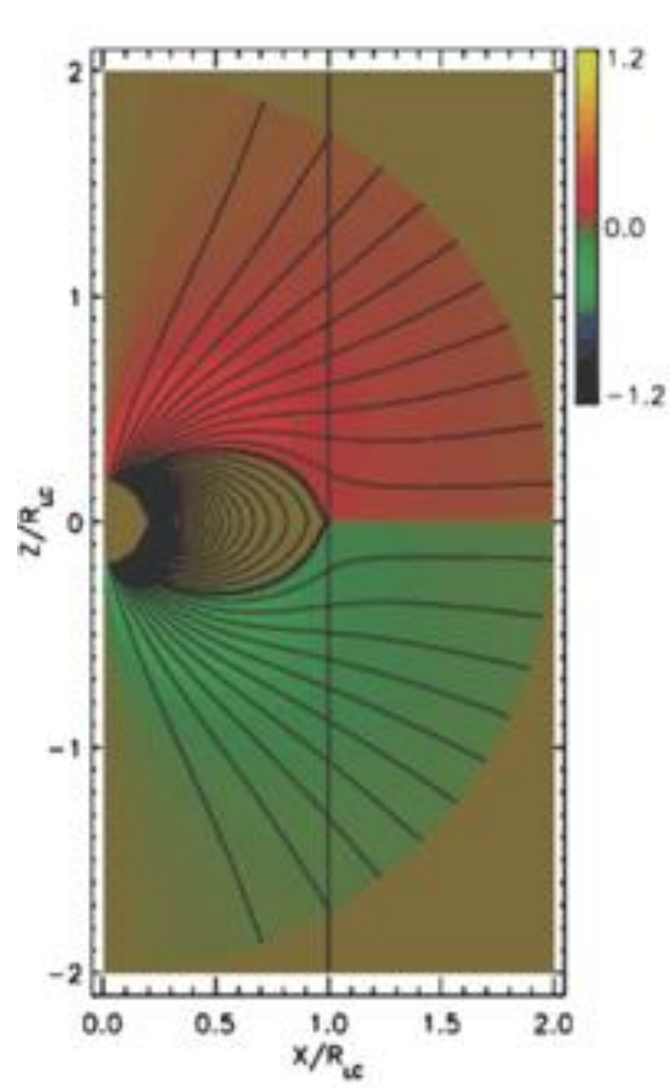
A little bit of history...

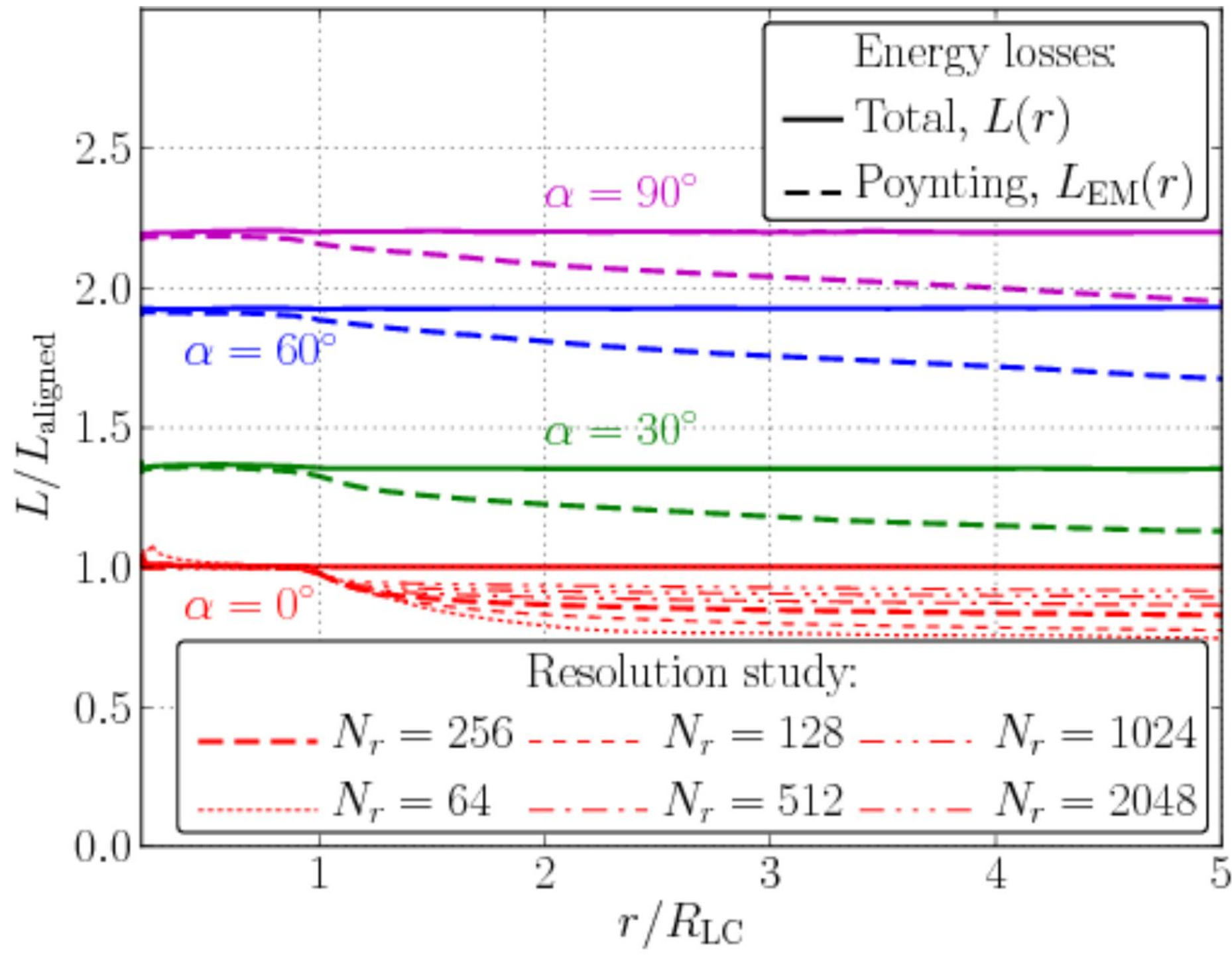
3D steady-state ideal (FFE, MHD)

2006 (FFE, Spitkovsky)

2009 (FFE, Contopoulos & Kalapotharakos)

2013 (MHD, Spitkovsky, Tchekhovskoy & Li)





A little bit of critique...

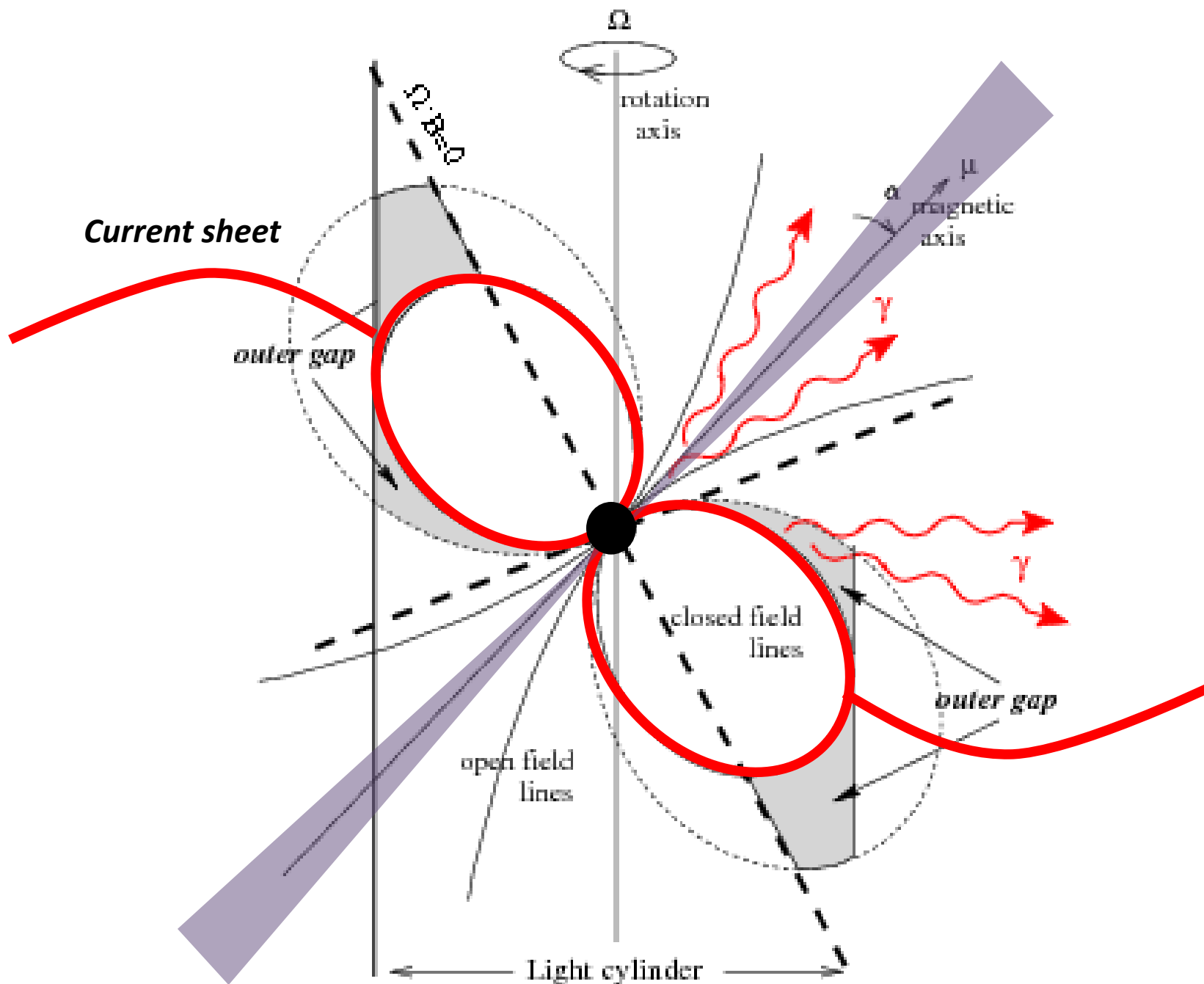
Ideal simulations are
non-ideal numerically...

A little bit of history...

Non-ideal magnetospheres (non-ideal prescriptions...)

2012 (Spitkovsky et al.)

2012 (Contopoulos, Kalapotharakos et al.)



2. NON-IDEAL PRESCRIPTIONS

In the FFE description of pulsar magnetospheres, Spitkovsky (2006) and Kalapotharakos & Contopoulos (2009) solved numerically the time-dependent Maxwell equations

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} \quad (1)$$

$$\frac{\partial \mathbf{E}}{\partial t} = c \nabla \times \mathbf{B} - 4\pi \mathbf{J} \quad (2)$$

under ideal force-free conditions

$$\mathbf{E} \cdot \mathbf{B} = 0, \quad \rho \mathbf{E} + \frac{1}{c} \mathbf{J} \times \mathbf{B} = 0,$$

where $\rho = \nabla \cdot \mathbf{E}/(4\pi)$. The evolution of these equations in time requires in addition an expression for the current density $\mathbf{J}$ as a function of $\mathbf{E}$ and $\mathbf{B}$. This is given by

$$\mathbf{J} = c\rho \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{c}{4\pi} \frac{\mathbf{B} \cdot \nabla \times \mathbf{B} - \mathbf{E} \cdot \nabla \times \mathbf{E}}{B^2} \mathbf{B} \quad (3)$$

(A) The above implementation of the ideal condition hints at an easy generalization that leads to non-ideal solutions: one can evolve Equations (1) and (2), using only the first term of the FFE current density (Equation (3)), and at each time step keep only a certain fraction b of the $\mathbf{E}_{\parallel}$ developed during this time instead of forcing it to be zero. In general, the portion b of the remaining $\mathbf{E}_{\parallel}$ can be either the same everywhere or variable (locally) depending on some other quantity (e.g., ρ , J). As b goes from 0 to 1, the corresponding solution goes from FFE to vacuum. In this case, an expression for the electric current density is not given a priori, and $\mathbf{J}$ can be obtained indirectly from the expression

$$\mathbf{J} = \frac{1}{4\pi} \left(c \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \right). \quad (4)$$

(B) Another way of controlling $\mathbf{E}_{\parallel}$ is to introduce a finite conductivity σ . In this case, we replace the second term in Equation (3) by $\sigma \mathbf{E}_{\parallel}$ and the current density reads

$$\mathbf{J} = c\rho \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \sigma \mathbf{E}_{\parallel} \quad (5)$$

Note that Equation (5) is related to but is not quite equivalent to Ohm's law, which is defined in the frame of the fluid. Others

non-dissipative. The current density expression in the so-called strong field electrodynamics (hereafter SFE) reads

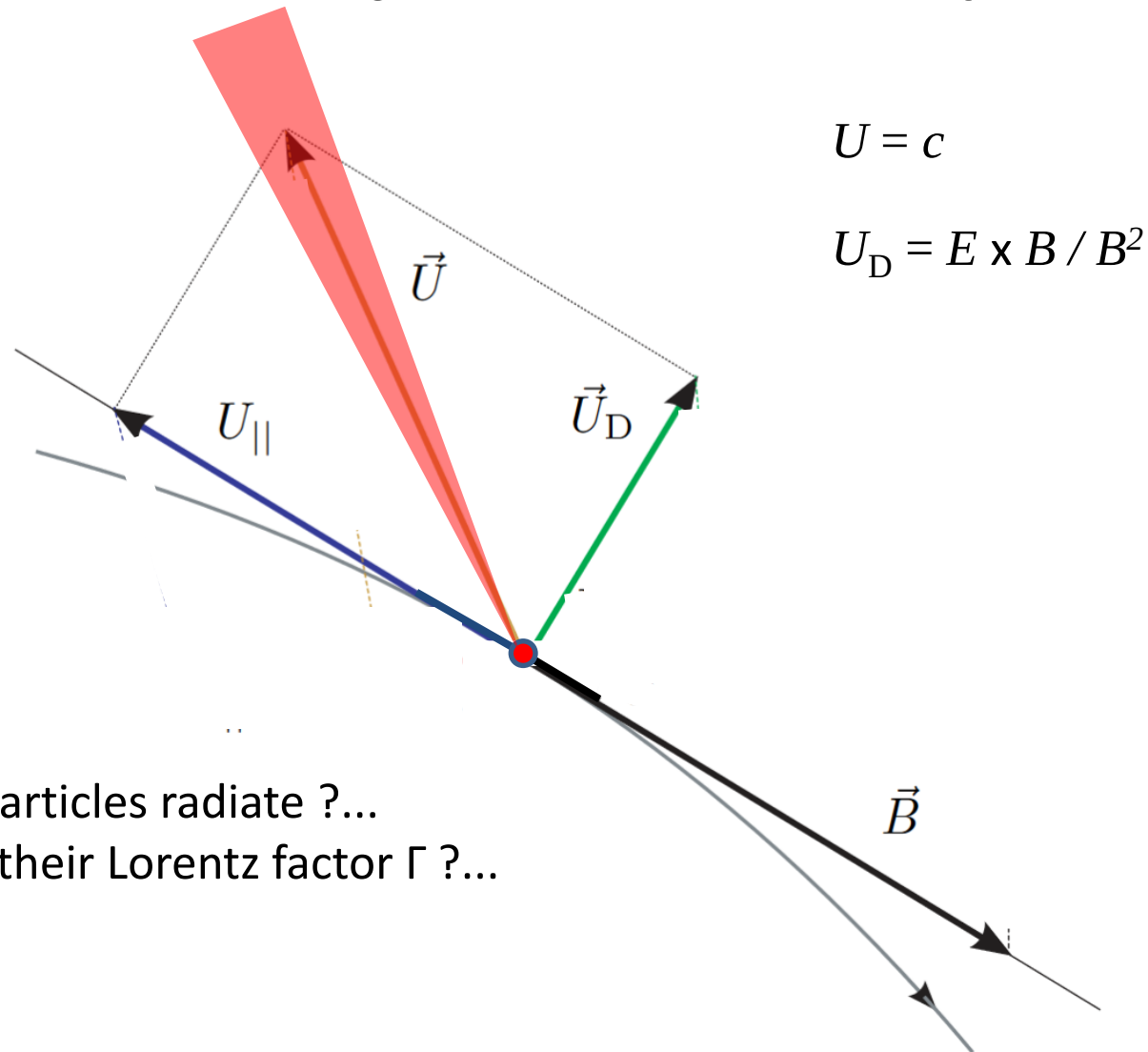
$$\mathbf{J} = \frac{c\rho \mathbf{E} \times \mathbf{B} + (c^2 \rho^2 + \gamma^2 \sigma^2 E_0^2)^{1/2} (B_0 \mathbf{B} + E_0 \mathbf{E})}{B^2 + E_0^2}, \quad (6)$$

where

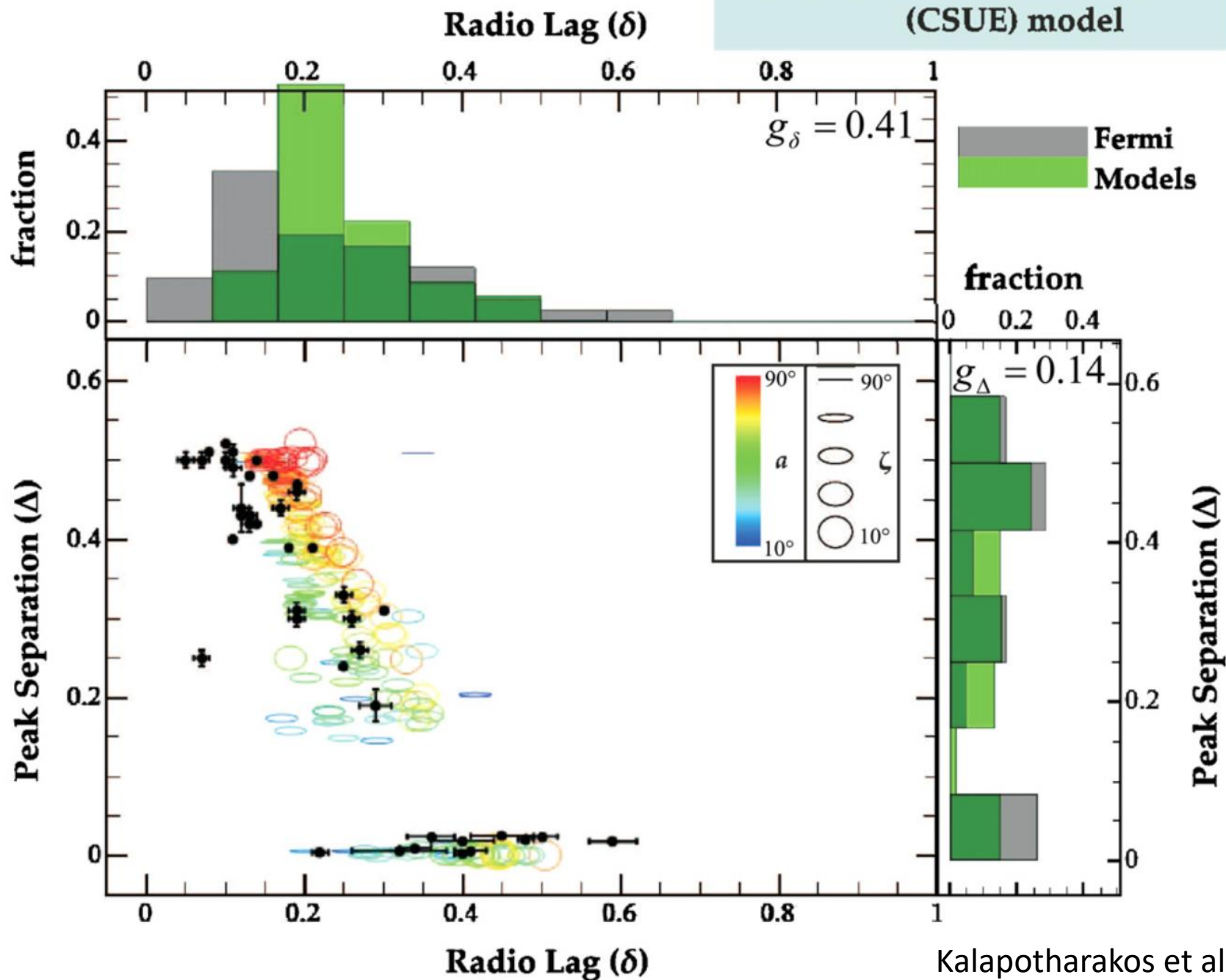
$$B_0^2 - E_0^2 = \mathbf{B}^2 - \mathbf{E}^2, \quad B_0 E_0 = \mathbf{E} \cdot \mathbf{B}, \quad E_0 \geq 0, \quad (7)$$

$$\gamma^2 = \frac{B^2 + E_0^2}{B_0^2 + E_0^2}, \quad (8)$$

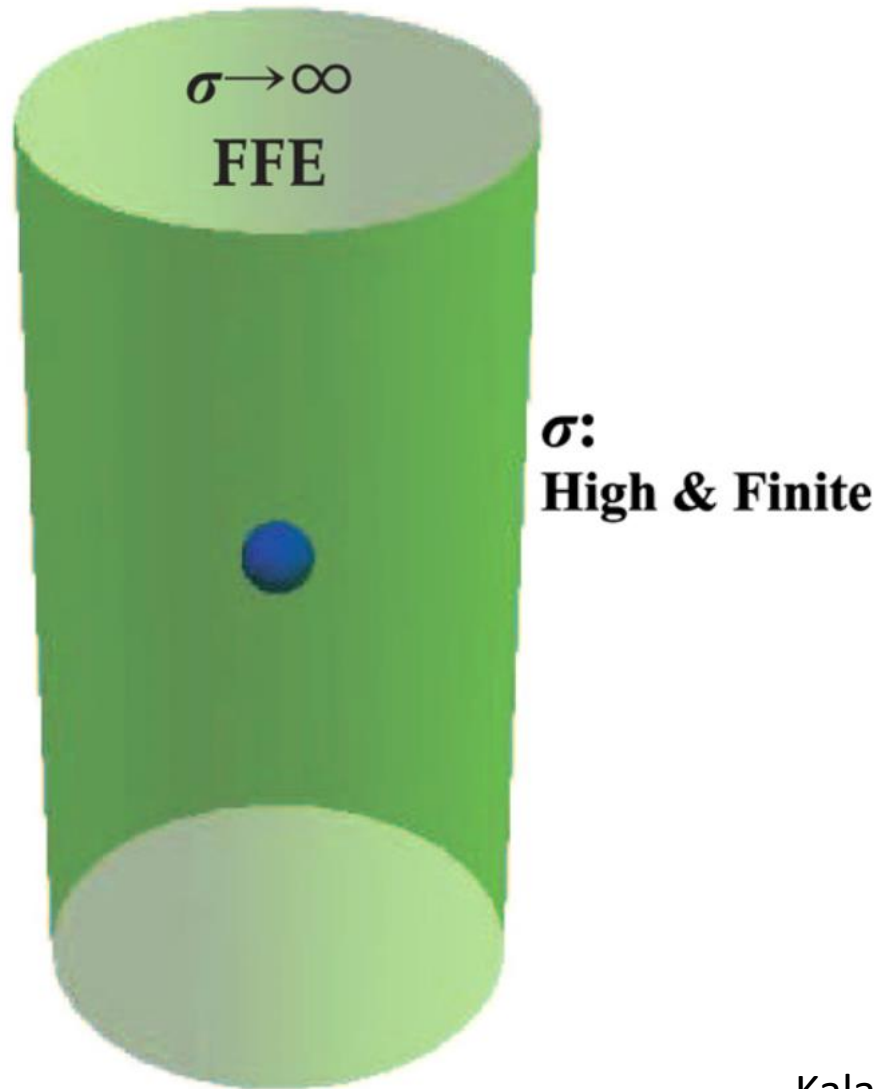
Radiation from particle trajectories with $\beta=1$...



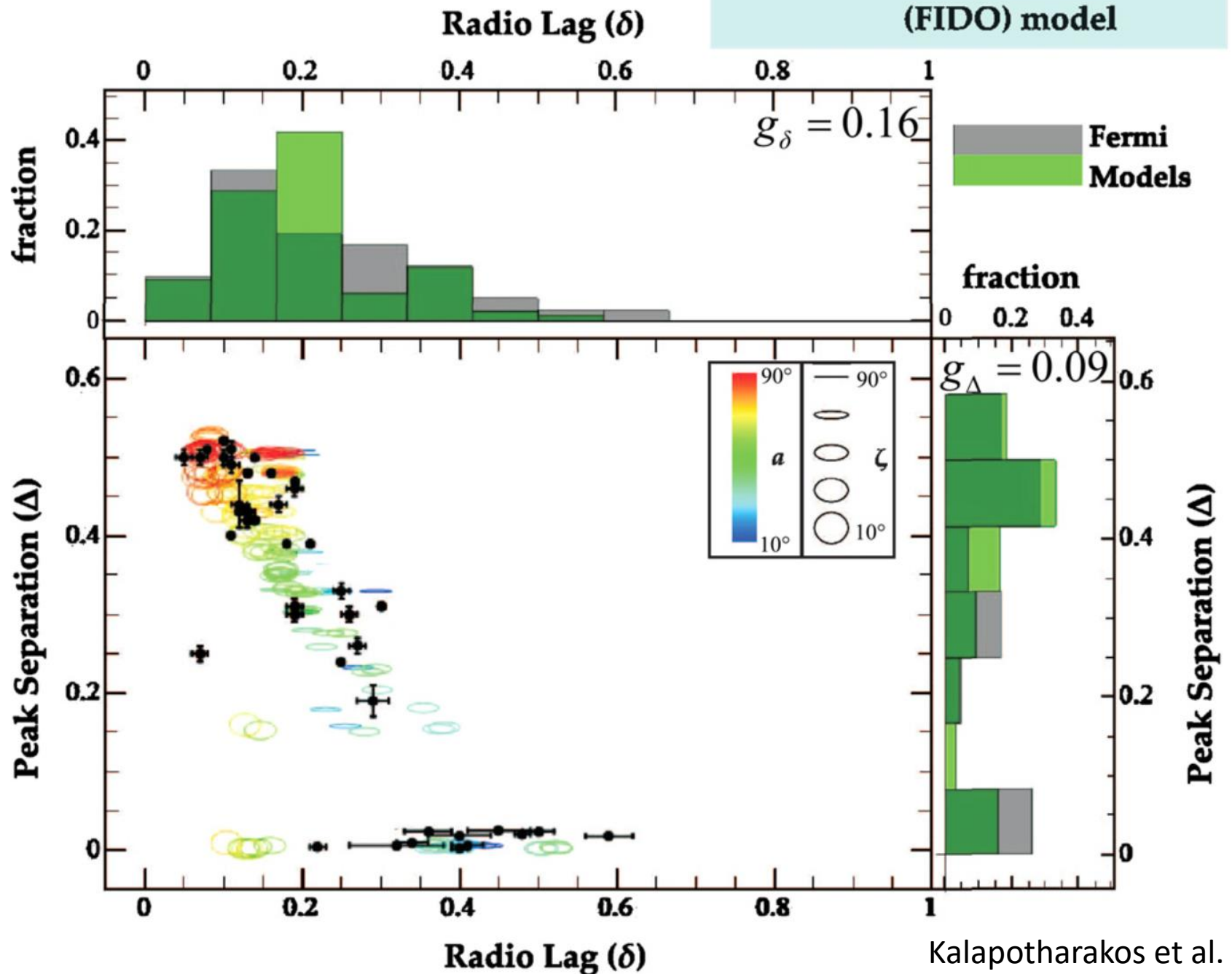
Current Sheet Uniform Emission
(CSUE) model



Force-Free Inside, Dissipative Outside



FFE Inside Dissipative Outside (FIDO) model

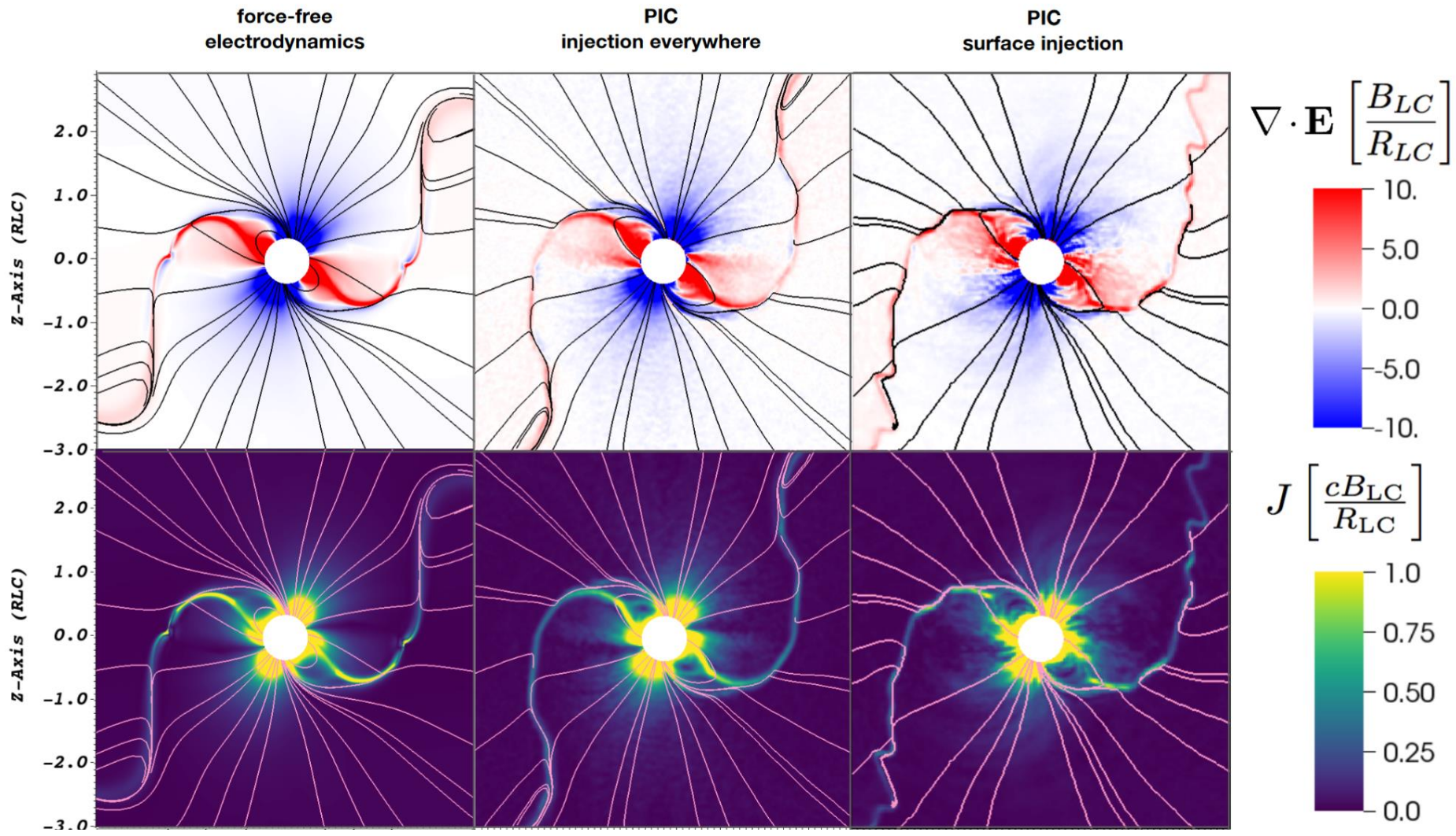


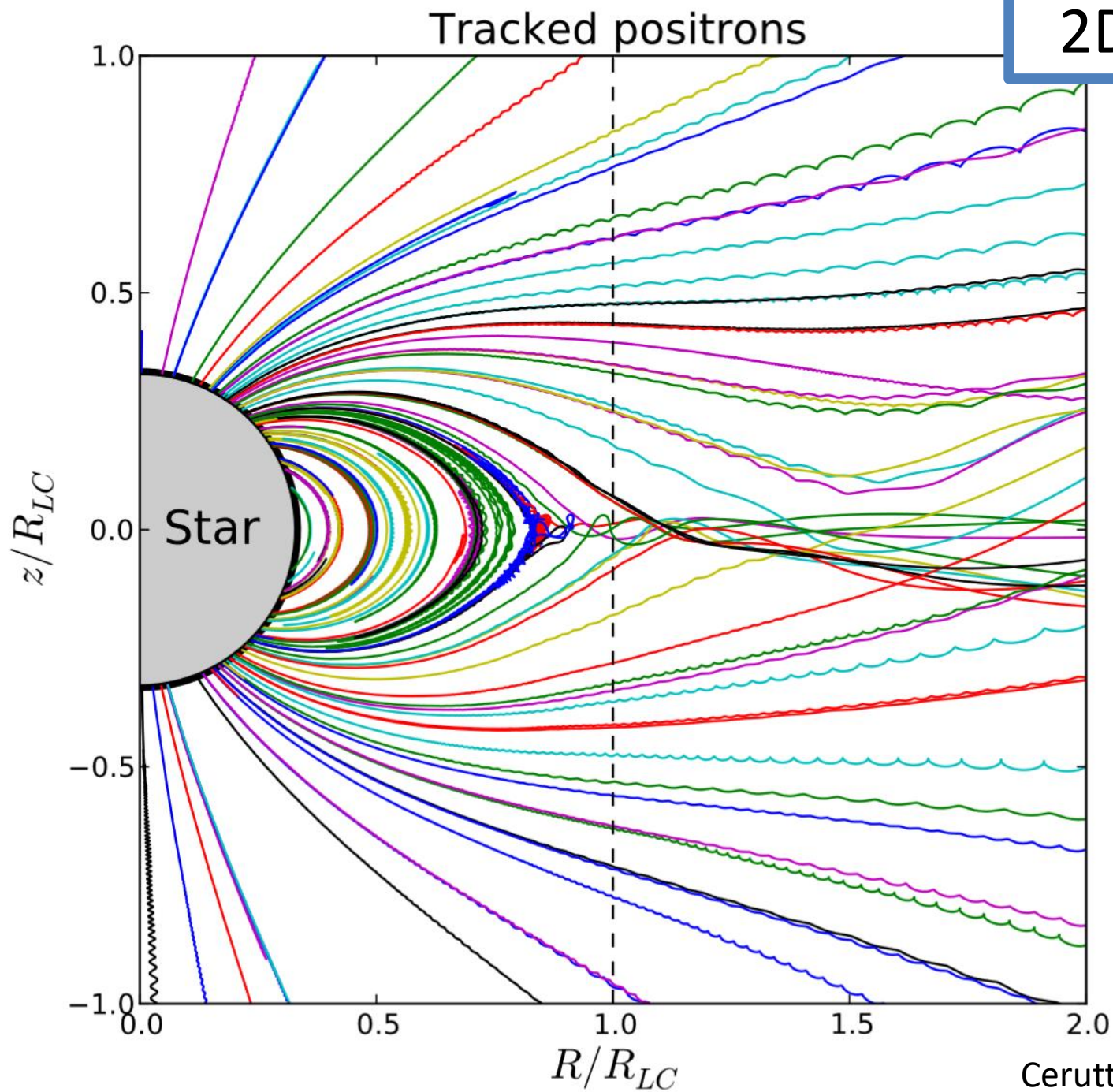
A little bit of history...

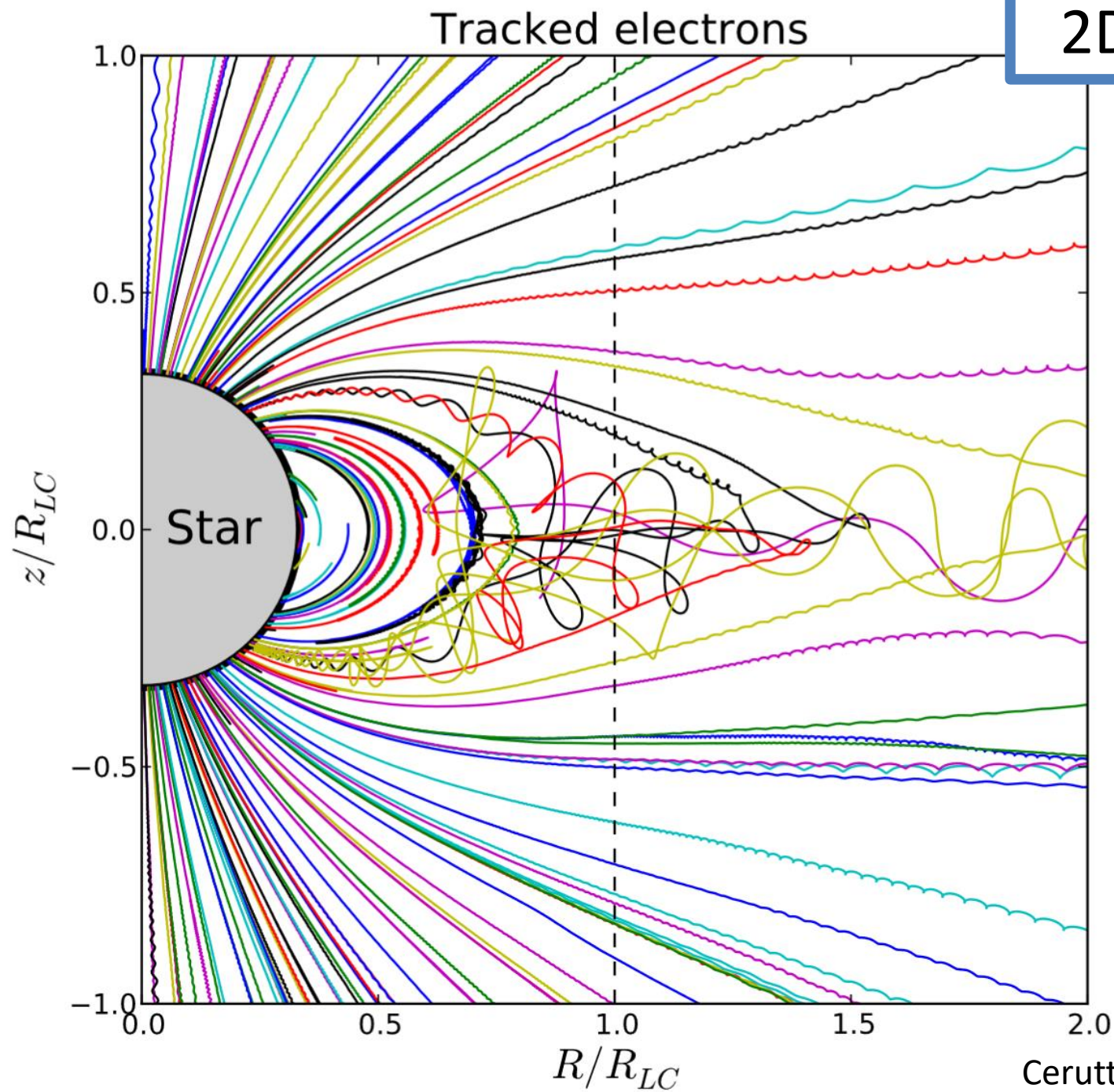
“Ab initio” numerical simulations (global PIC)

2014 (Spitkovsky, Sironi, Cerutti et al.)

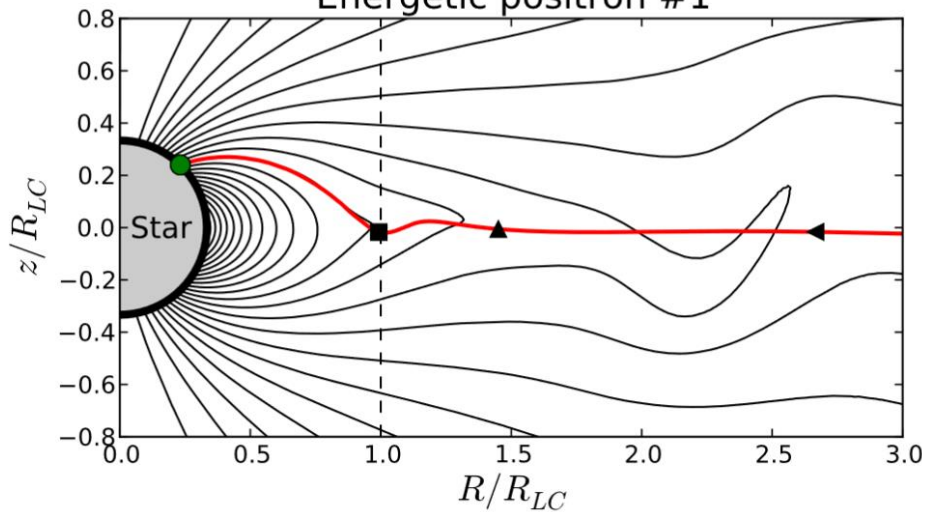
2016 (Kalapotharakos)



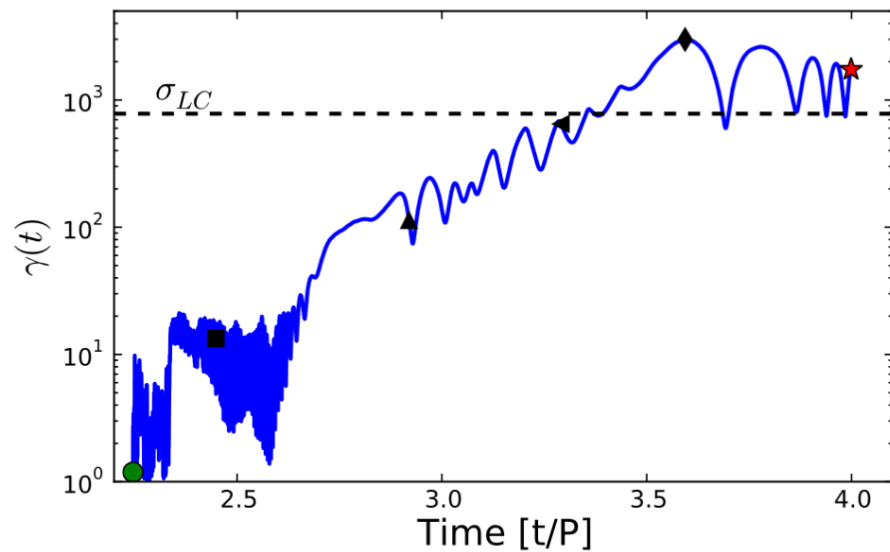
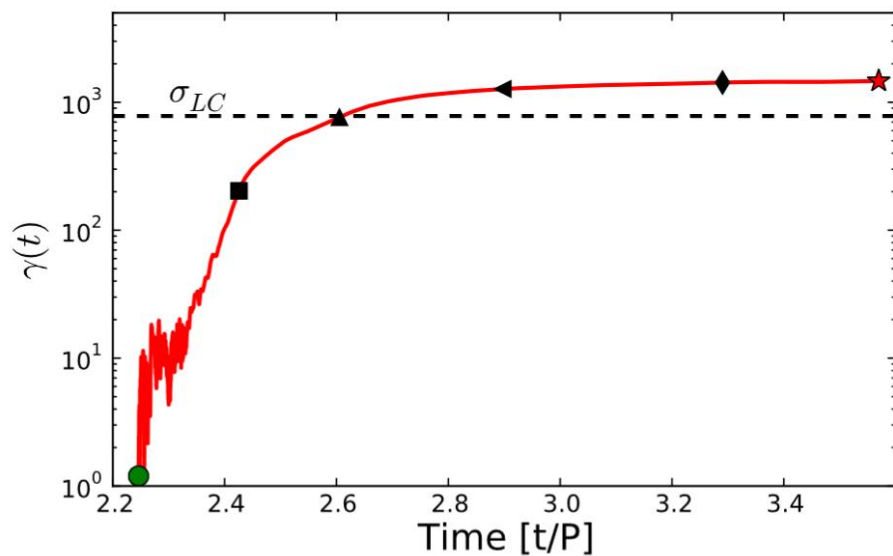
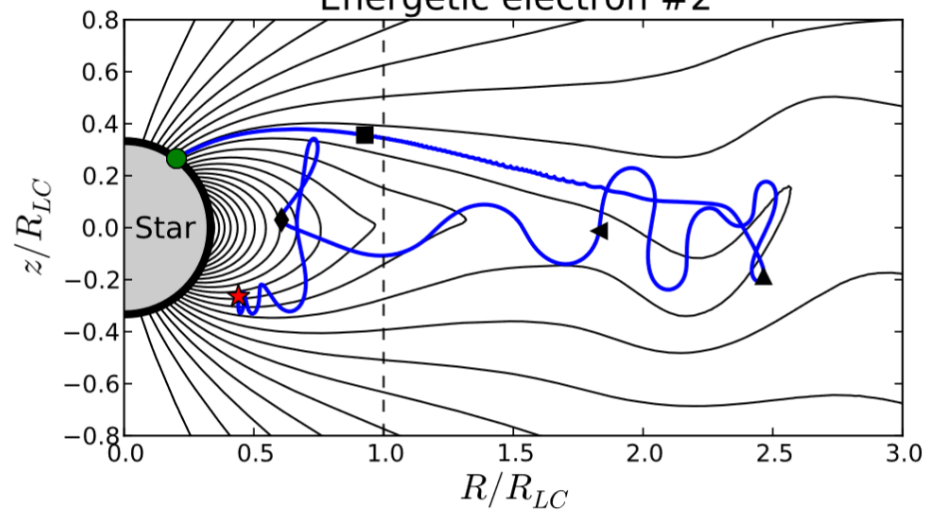




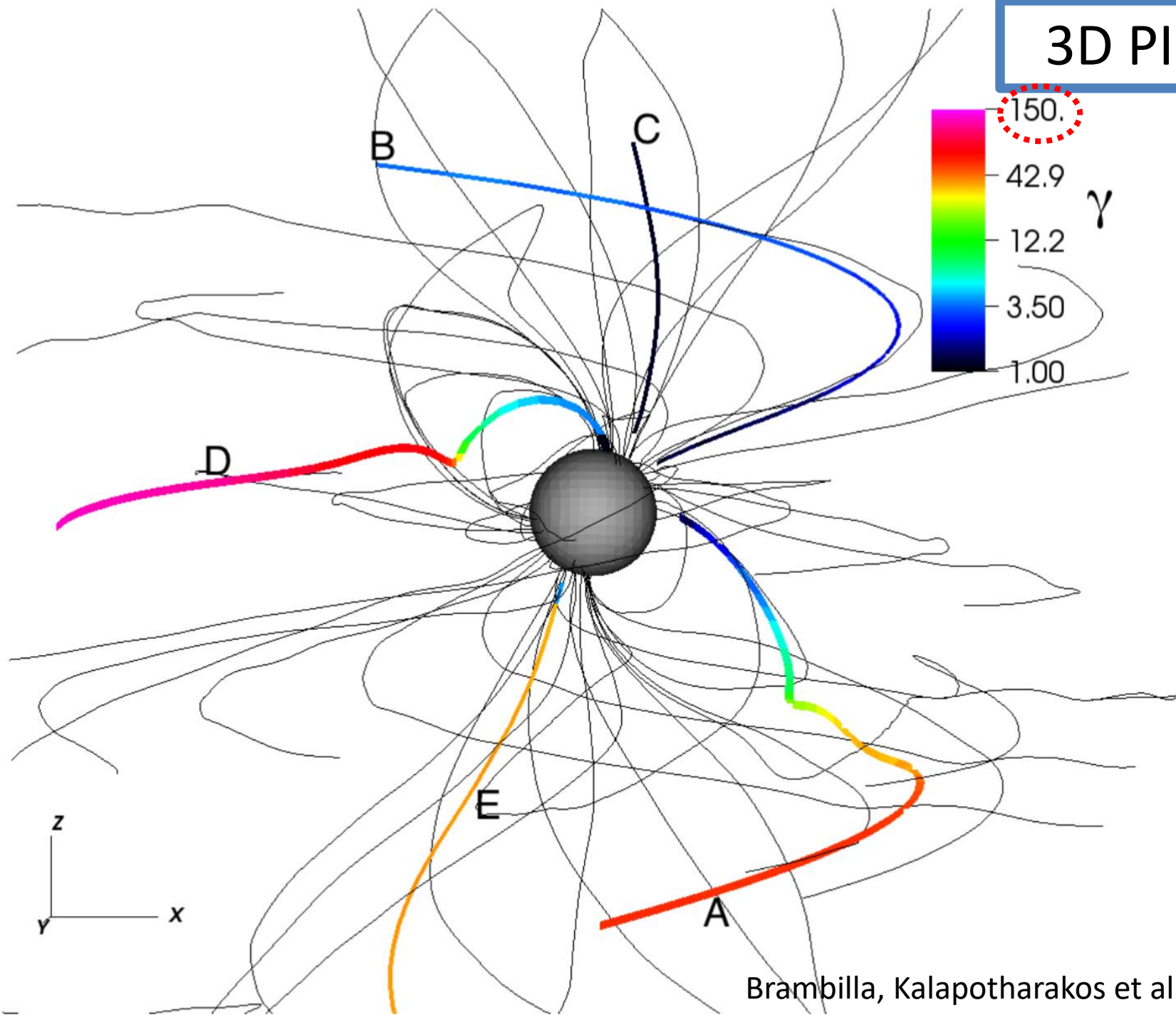
Energetic positron #1



Energetic electron #2



3D PIC

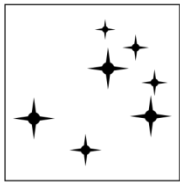


A little bit of critique...

“Ab initio” is too ambitious!
(is it hopeless?...)

A little bit of critique...

The key is Current Closure



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Ideal Force-Free everywhere, non-ideal non-force-free ECS

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Academy of Athens

1st Meeting of ISSI International Team, Bern, Tuesday, December 10 2019

FFE everywhere + electrostatic CS

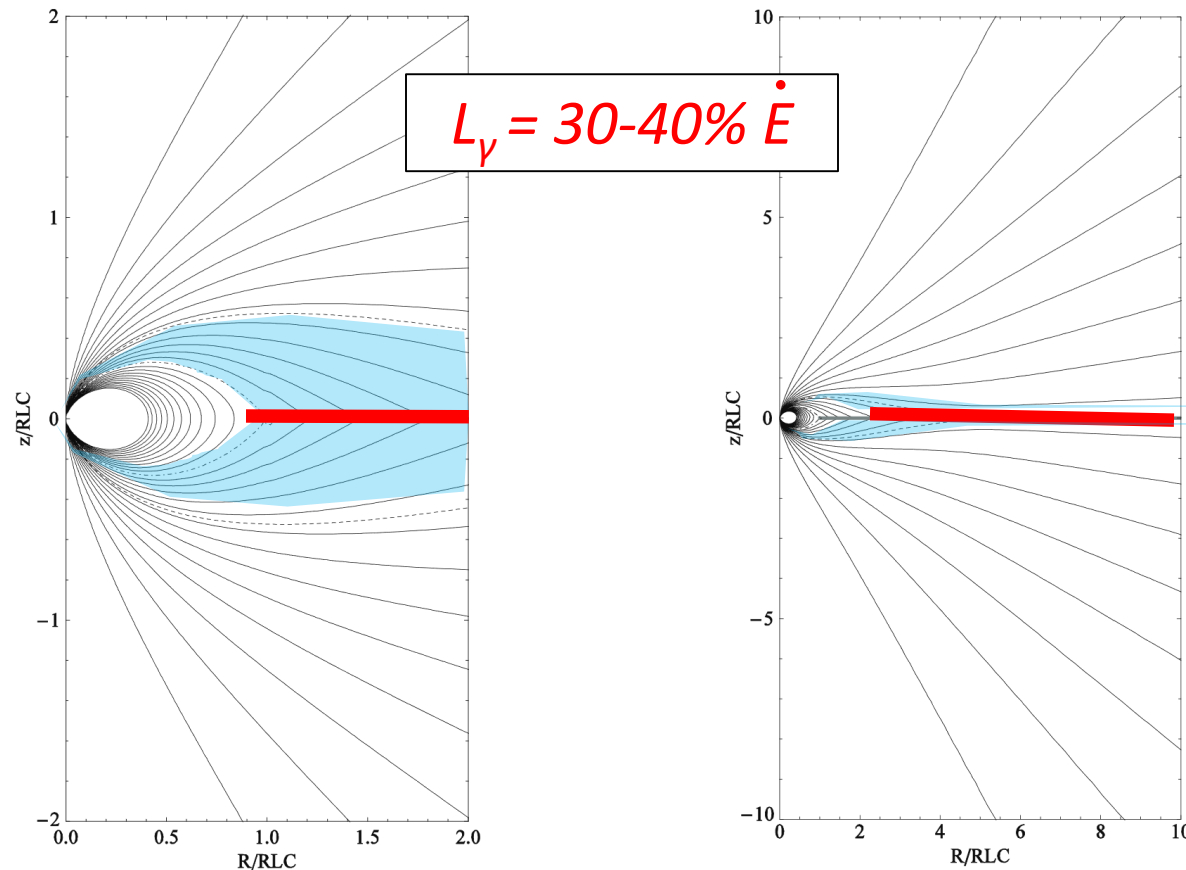
THE ASTROPHYSICAL JOURNAL, 781:46 (5pp), 2014 January 20

doi:[10.1088/0004-637X/781/1/46](https://doi.org/10.1088/0004-637X/781/1/46)

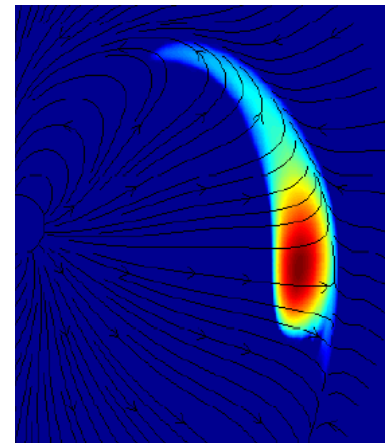
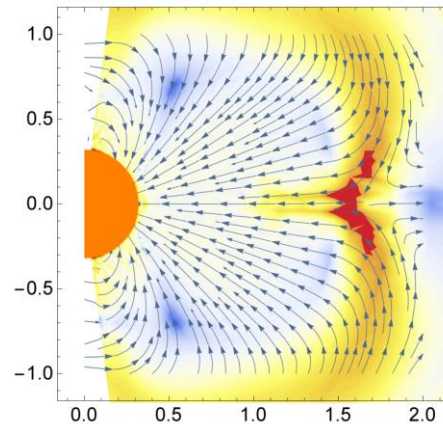
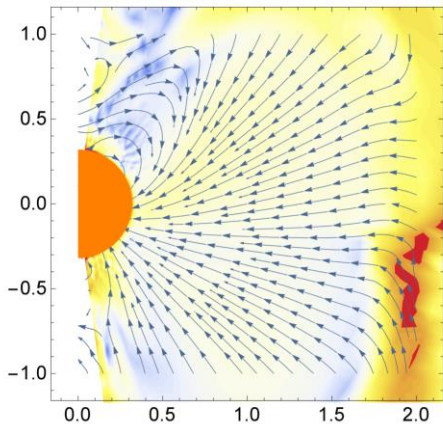
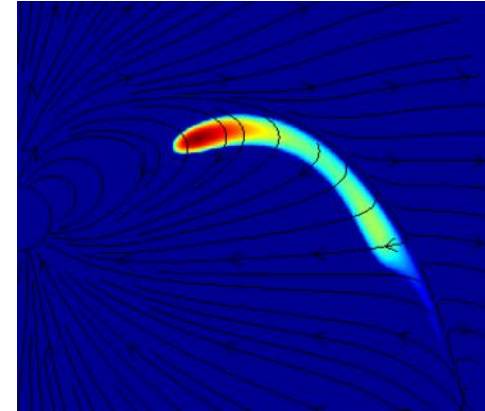
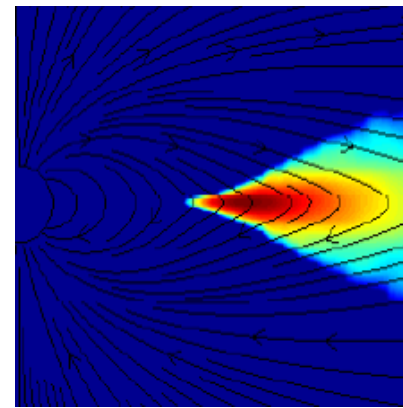
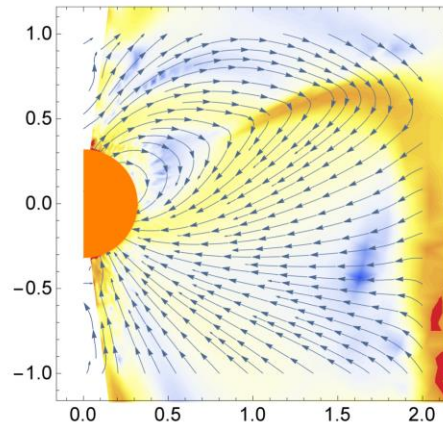
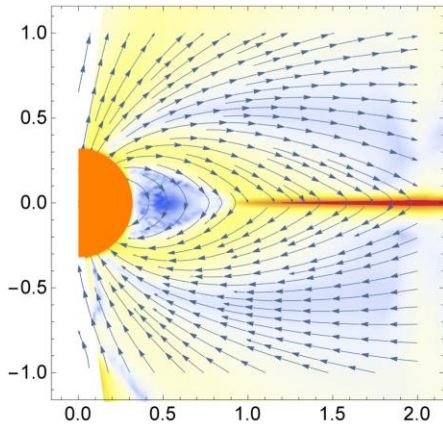
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A NEW STANDARD PULSAR MAGNETOSPHERE

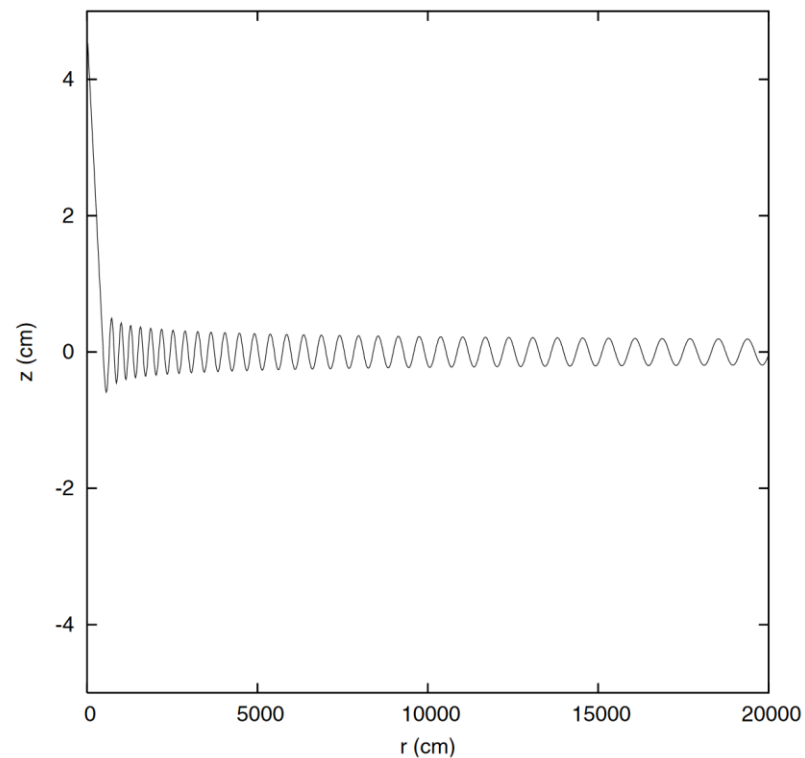
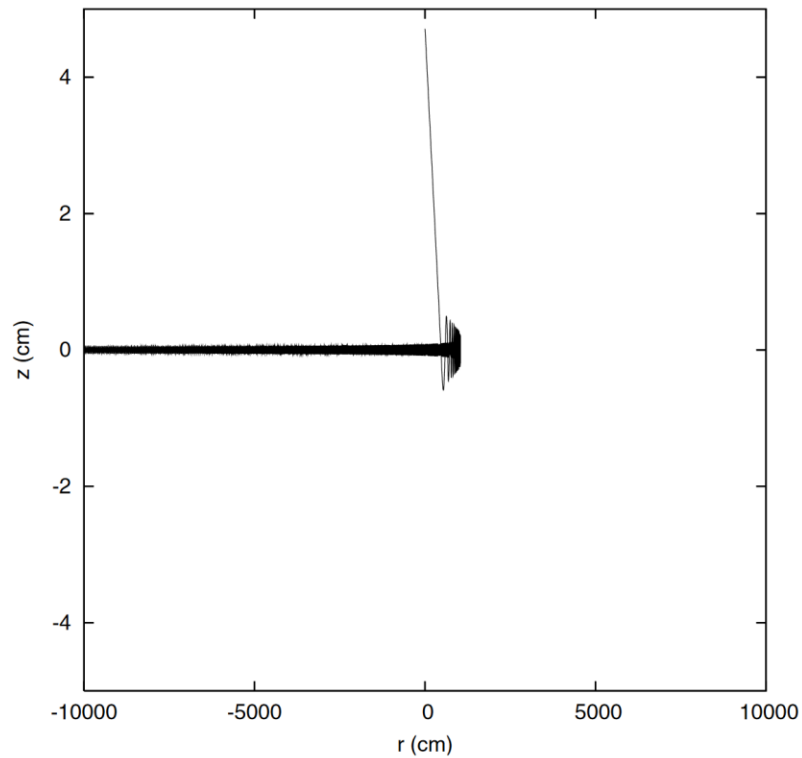
IOANNIS CONTOPOULOS¹, CONSTANTINOS KALAPOTHARAKOS^{2,3}, AND DEMOSTHENES KAZANAS³



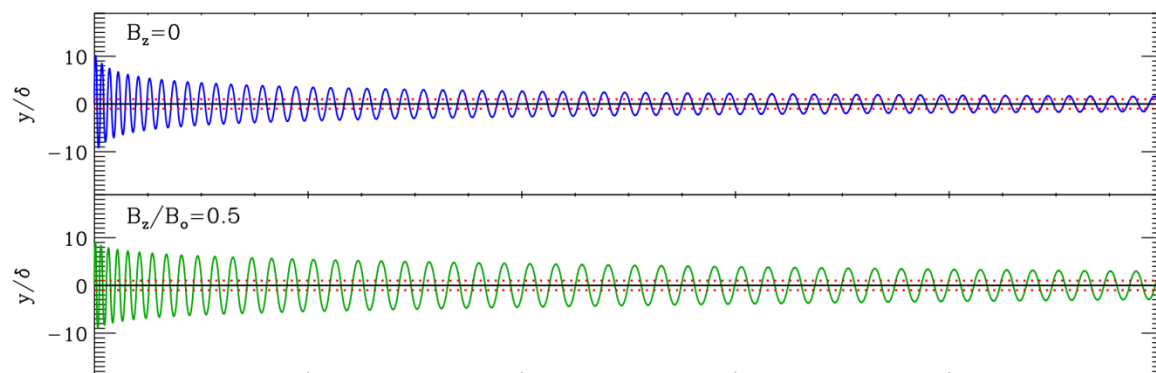
FFE everywhere + Aristotelian CS



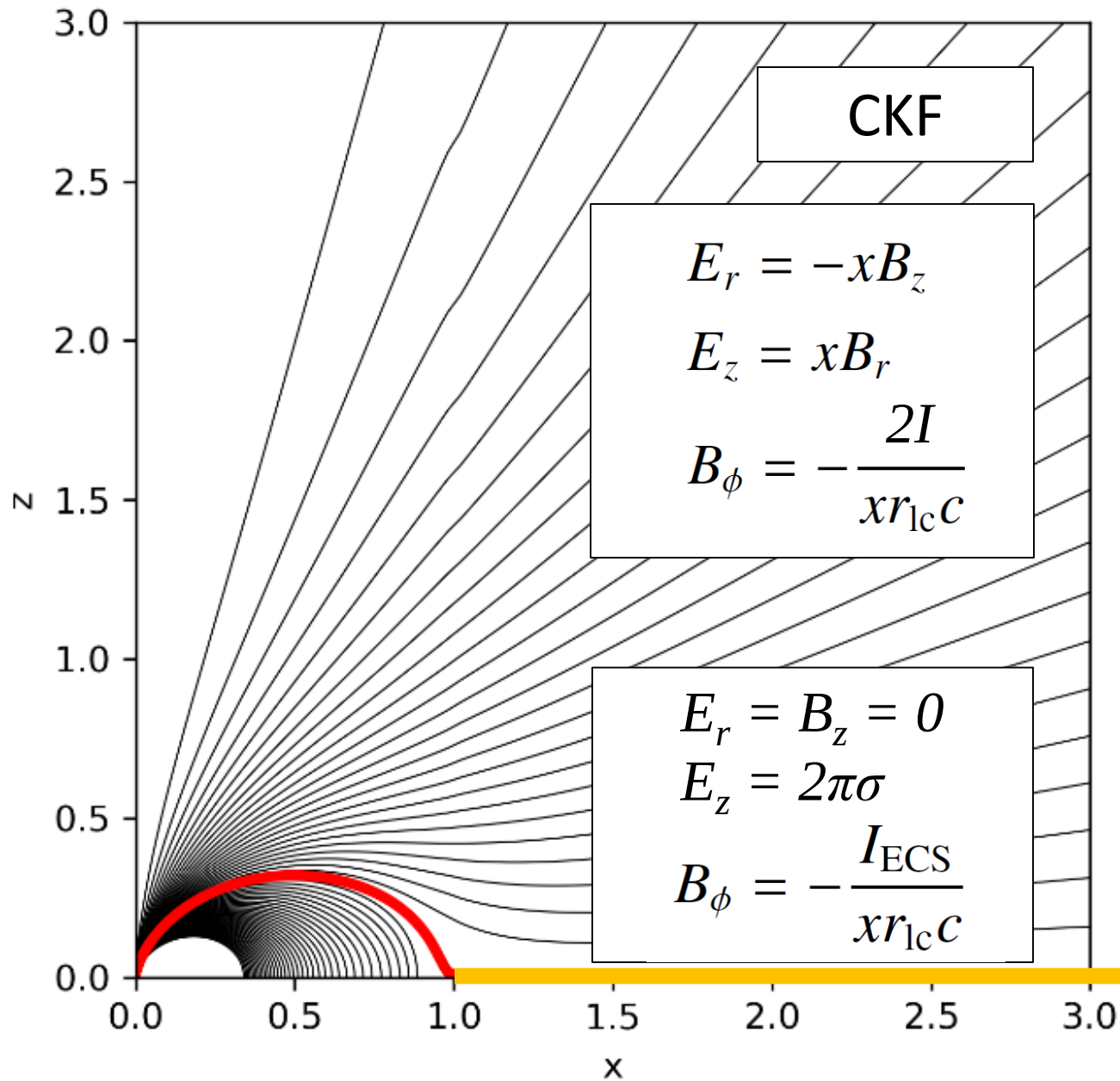
Aristotelian $\rightarrow$ Electrostatic for $\kappa=0$



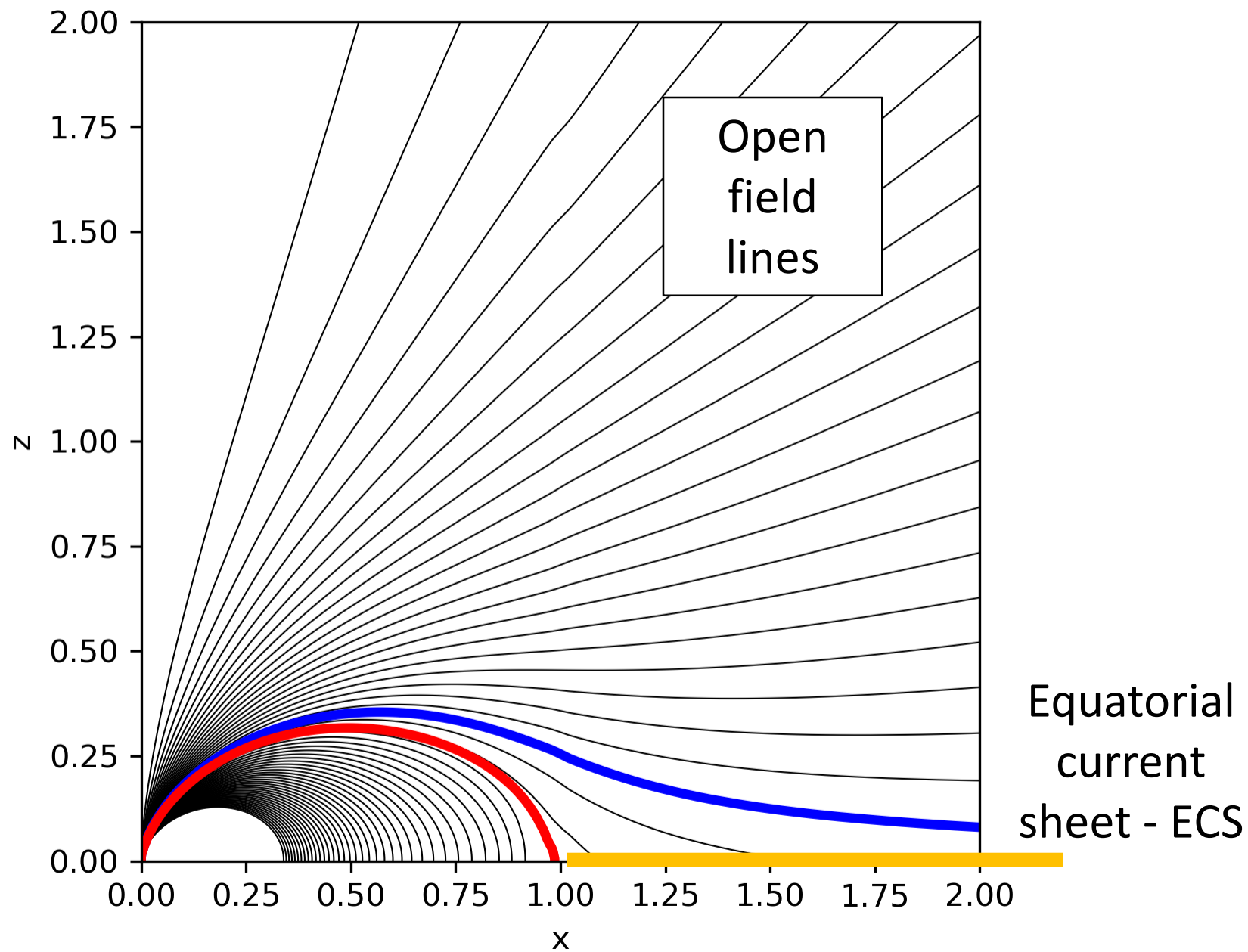
Speiser orbits

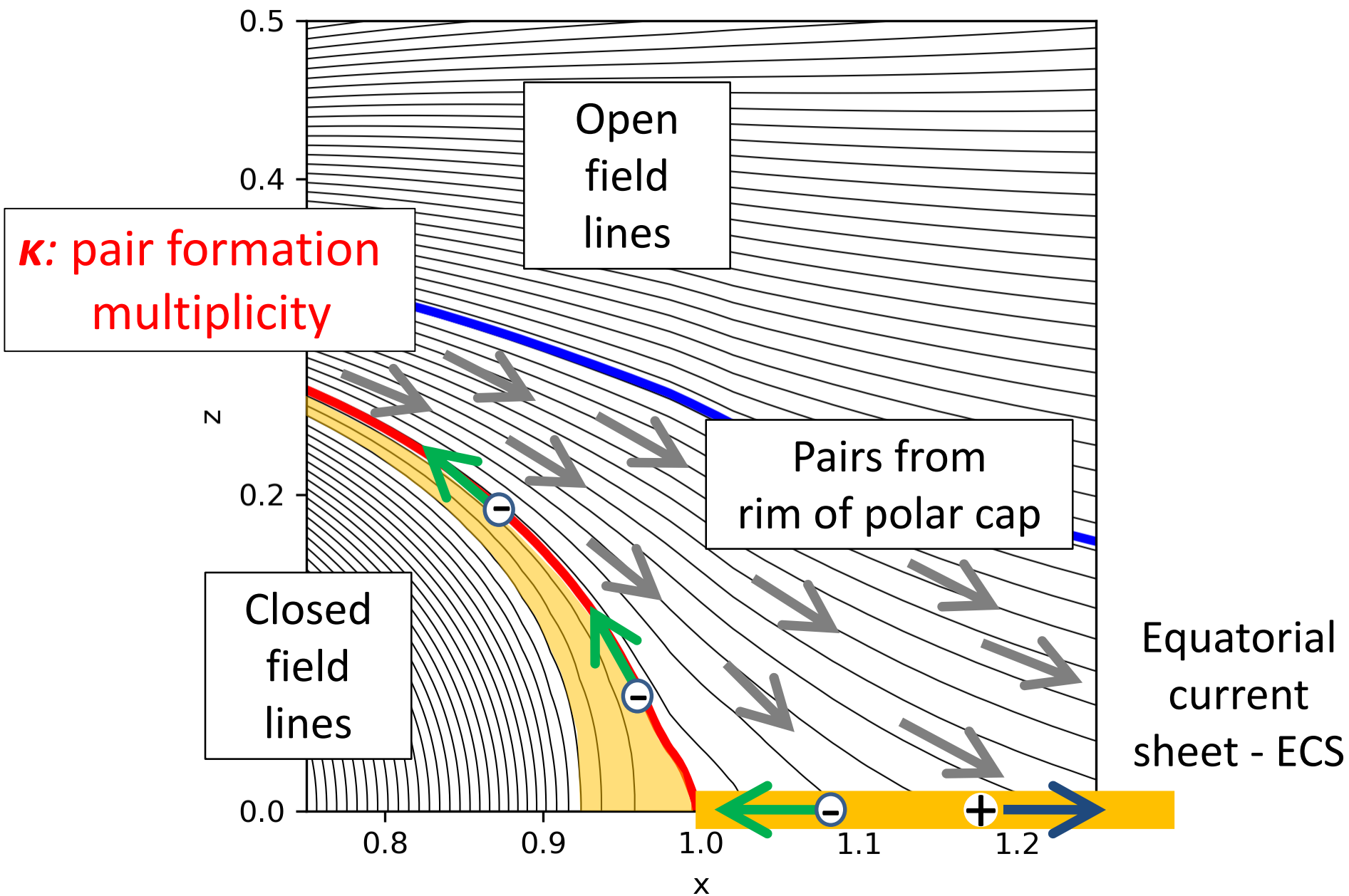


Contopoulos 2007; Cerutti et al. 2012, 2015



Equatorial
current
sheet - ECS



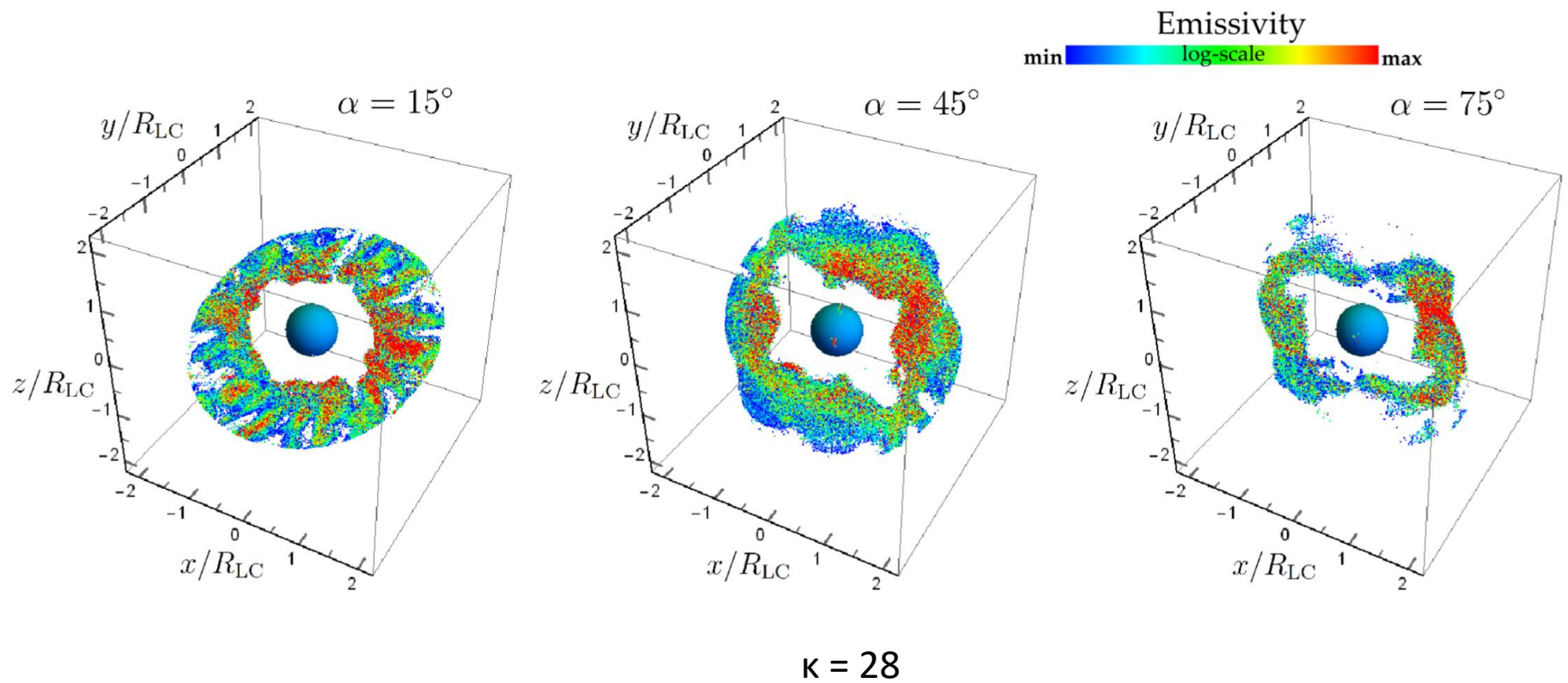


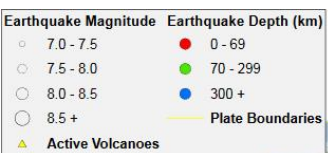
A “ring of fire” in the pulsar magnetosphere

Contopoulos 2019

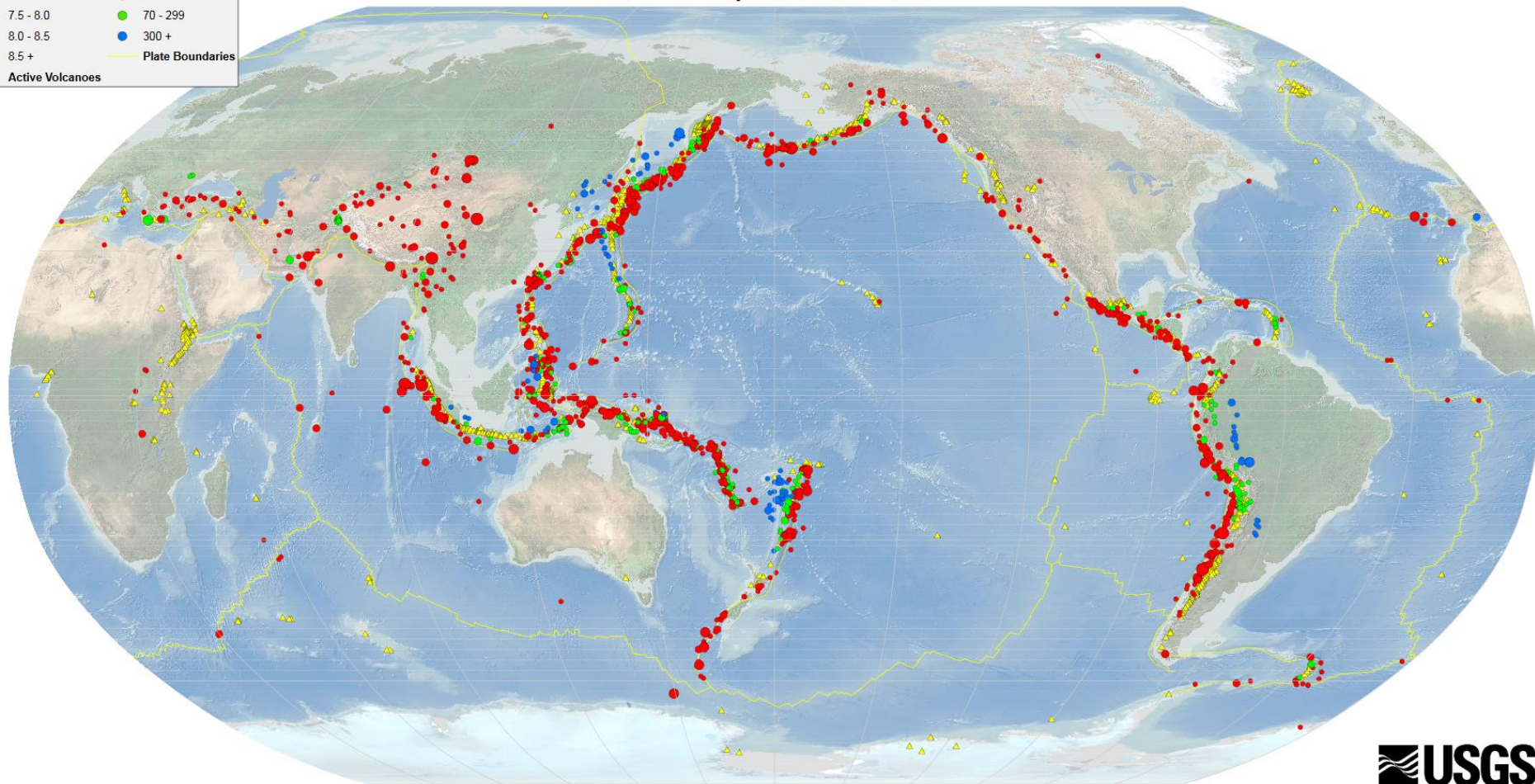
Contopoulos & Stefanou 2019

Contopoulos, Petri & Stefanou 2019

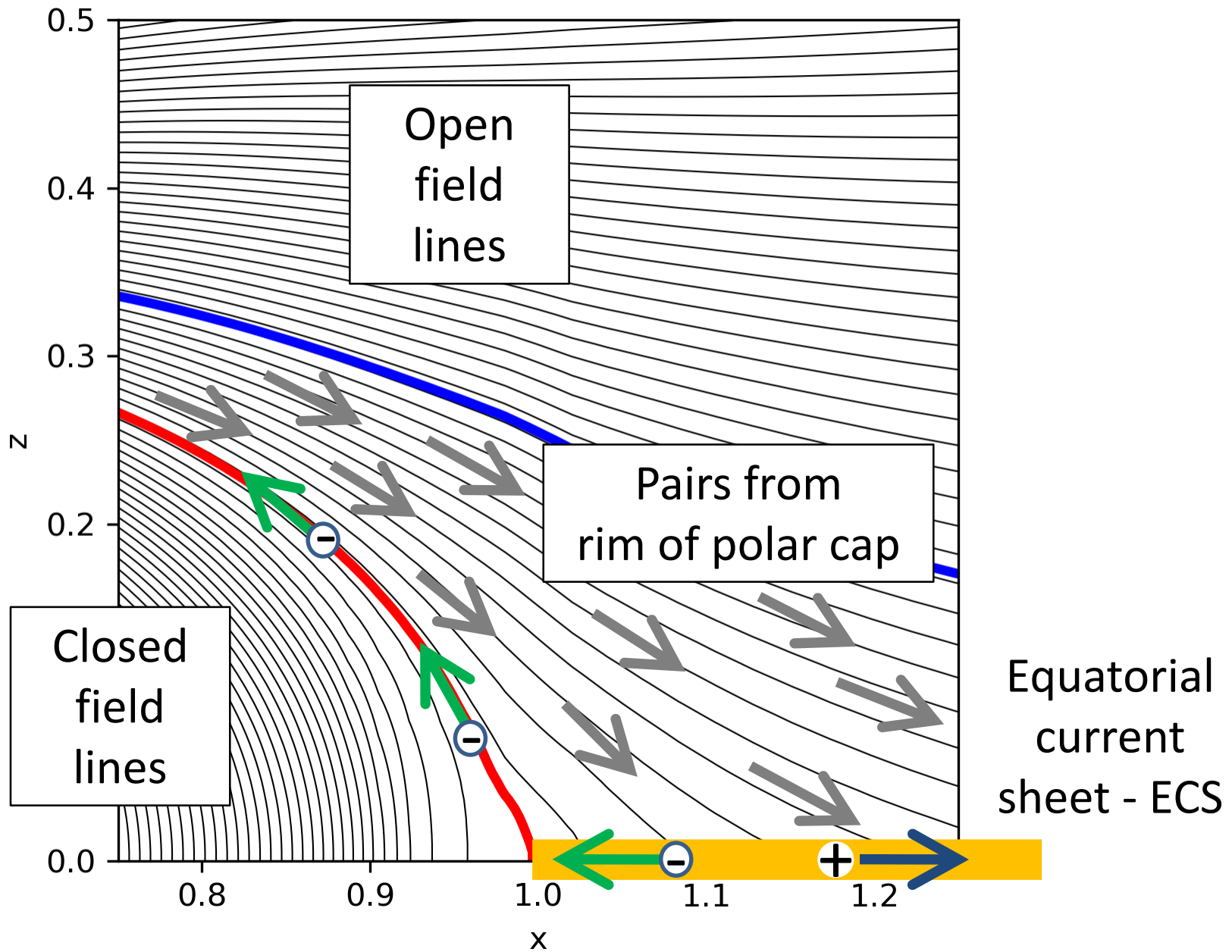




Global Earthquakes 1900 - 2013



$$\begin{aligned}
\sigma &= \sigma_+ + \sigma_- \\
&= 2e \left\{ \frac{\int_{r_{lc}}^r 2\pi r' dr' n_{\text{pairs}} |v_z|}{2\pi r v_{r+}} + \frac{-\int_r^\infty 2\pi r' dr' n_{\text{pairs}} |v_z|}{-2\pi r v_{r-}} \right\} \\
&\approx \frac{2e}{r|v_r|} \left\{ \int_{r_{lc}}^r r' dr' n_{\text{pairs}} |v_z| - \int_r^\infty r' dr' n_{\text{pairs}} |v_z| \right\} \\
&= \frac{2e}{r|v_r|} \left\{ 2 \int_{r_{lc}}^r r' dr' n_{\text{pairs}} |v_z| - \int_{r_{lc}}^\infty r' dr' n_{\text{pairs}} |v_z| \right\} .
\end{aligned}$$



$$\begin{aligned}
\sigma &= \sigma_+ + \sigma_- + \frac{I_{\text{ECS separatrix}}}{2\pi r |v_r|} \\
&= \frac{4e}{r |v_r|} \int_{r_{lc}}^r r' dr' n_{\text{pairs}} |v_z| , \\
I_{\text{ECS}} &\approx 2\pi r |v_r| (\sigma_+ - \sigma_-) + I_{\text{ECS separatrix}} \\
&= 4e \int_{r_{lc}}^{\infty} 2\pi r' dr' n_{\text{pairs}} |v_z| \\
&= 4e \int_{r_{lc}}^{\infty} 2\pi r' dr' \left(\frac{n_{\text{pairs}} v_p}{B_p} \right) |B_z| \\
&= 4e \frac{\kappa \Omega}{2\pi e} \int_{r_{lc}}^{\infty} 2\pi r' dr' B_z \\
&\equiv \frac{2\kappa \Omega}{\pi} \Psi_{\text{ECS}} .
\end{aligned}$$

κ : pair formation multiplicity

$$\begin{aligned}
\frac{d}{dr}(r|v_r|\sigma) &= \\
\frac{d}{dx}(\sigma \sqrt{x^2 - 1}c) &= 4ern_{\text{pairs}}|v_z| = 4ern_{\text{pairs}}v_p(|B_z|/B_p) \\
&= \frac{2\kappa\Omega r}{\pi}|B_z|. \tag{19}
\end{aligned}$$

Solving for the distribution of B_z along the dissipation layer, and remembering that $\sigma = E_z/(2\pi) = xB_r/(2\pi)$ yields

$$B_z = -\frac{1}{4\kappa x} \frac{d}{dx}(x \sqrt{x^2 - 1} B_r). \tag{20}$$

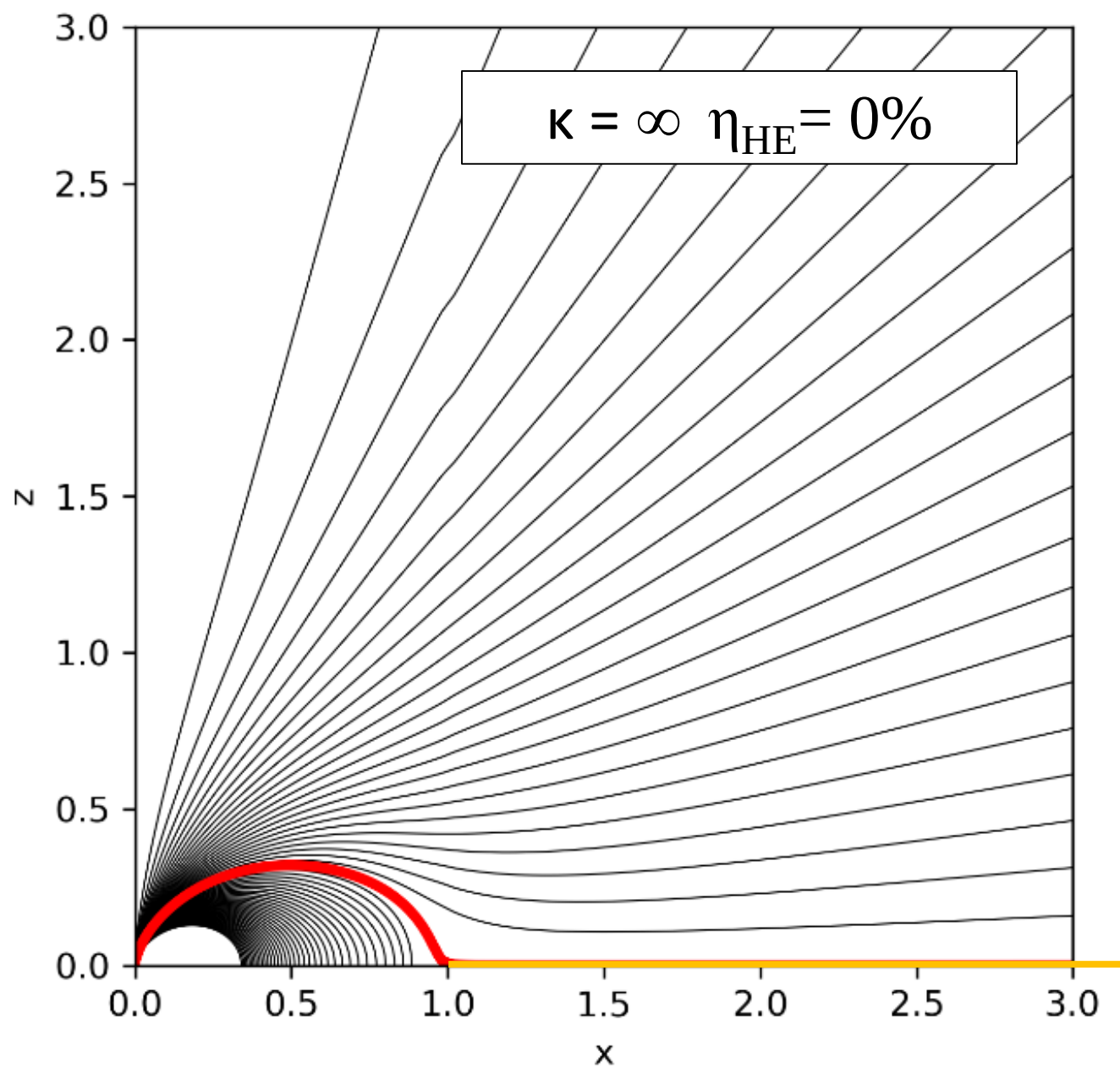
$$\begin{aligned}
\sigma &= \sigma_+ + \sigma_- + \frac{I_{\text{ECS separatrix}}}{2\pi r|v_r|} + 2 \frac{(I(r) - I(r_{\text{lc}}))}{2\pi r|v_r|} \\
&= \frac{2\kappa\Omega}{\pi r|v_r|} \int_{r_{\text{lc}}}^r r' dr' |B_z| + \frac{2}{r|v_r|} \int_{r_{\text{lc}}}^r r' dr' |B_z| \left| \frac{dI}{d\Psi} \right|, \quad (29)
\end{aligned}$$

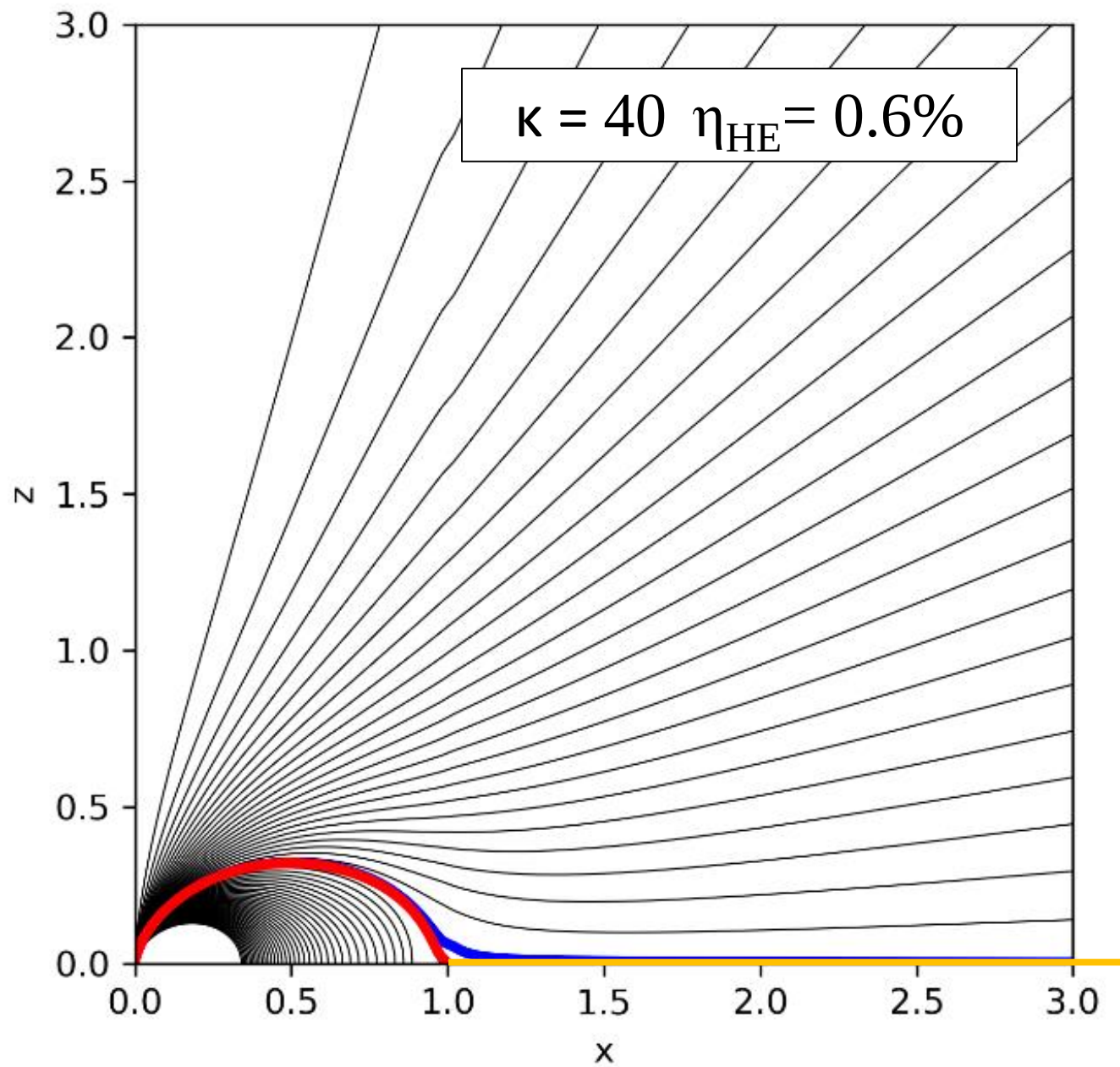
and consequently, the equatorial boundary condition of eq. (20) becomes

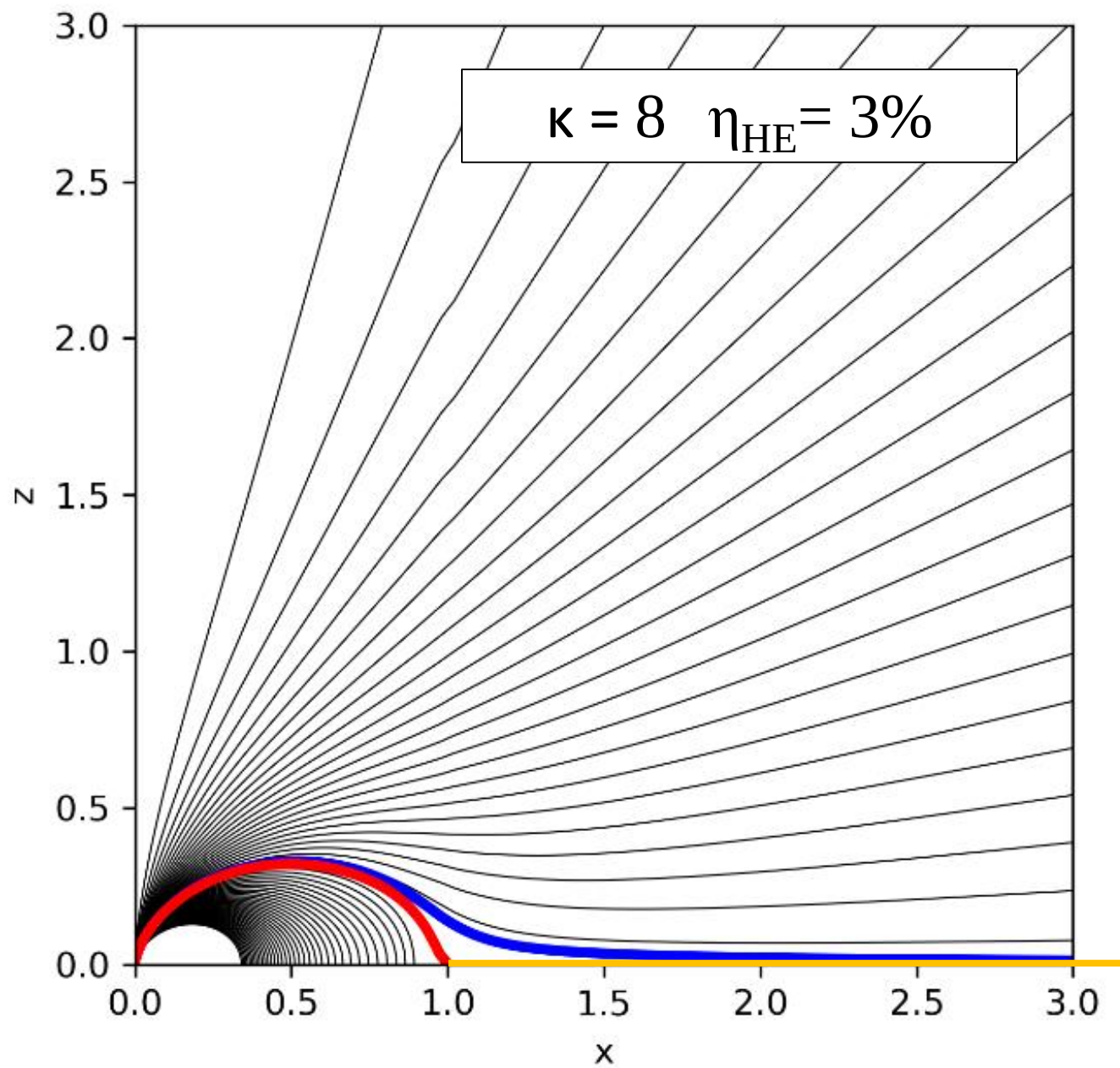
$$B_z = - \left(4\kappa + \frac{4\pi}{\Omega} \left| \frac{dI}{d\Psi} \right| \right)^{-1} \frac{1}{x} \frac{d}{dx} (x \sqrt{x^2 - 1} B_r). \quad (30)$$

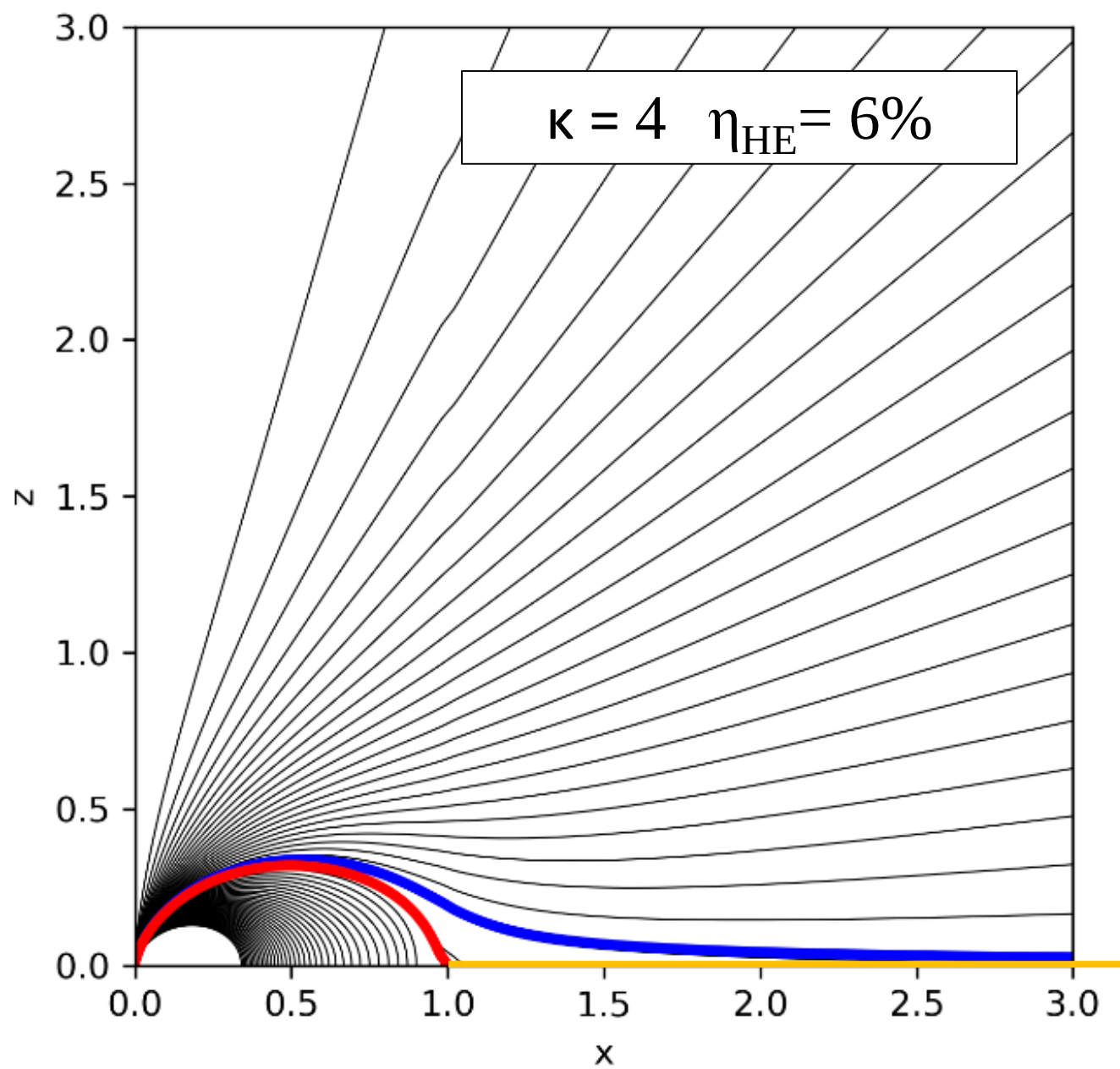
Here, $I(r)$ is the magnetospheric electric current contained inside radius r of the ECS. Eq. (30) allows us to extend figure 4 to very low κ values.

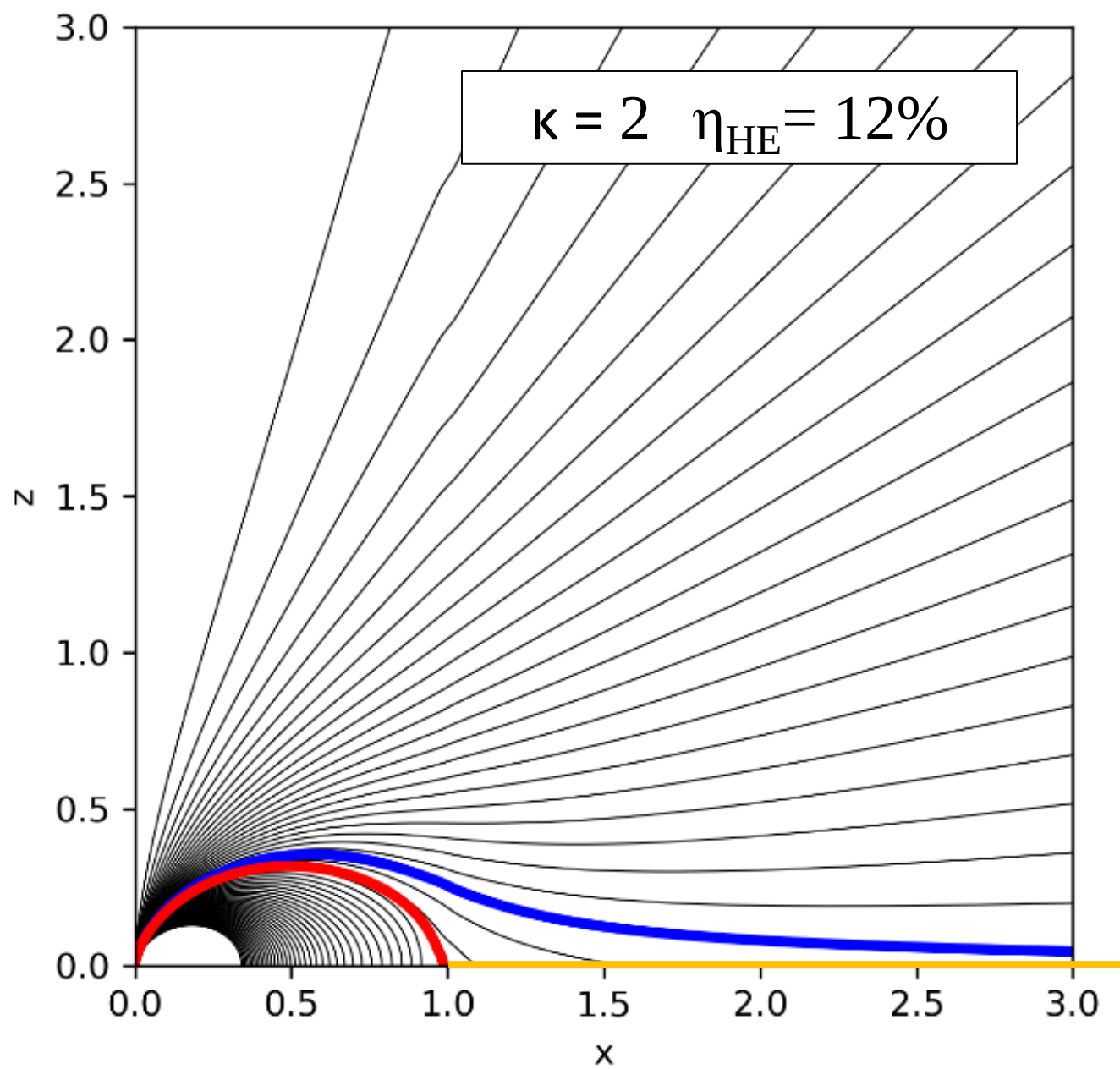
Aristotelian $\rightarrow$ Electrostatic for $\kappa=0$

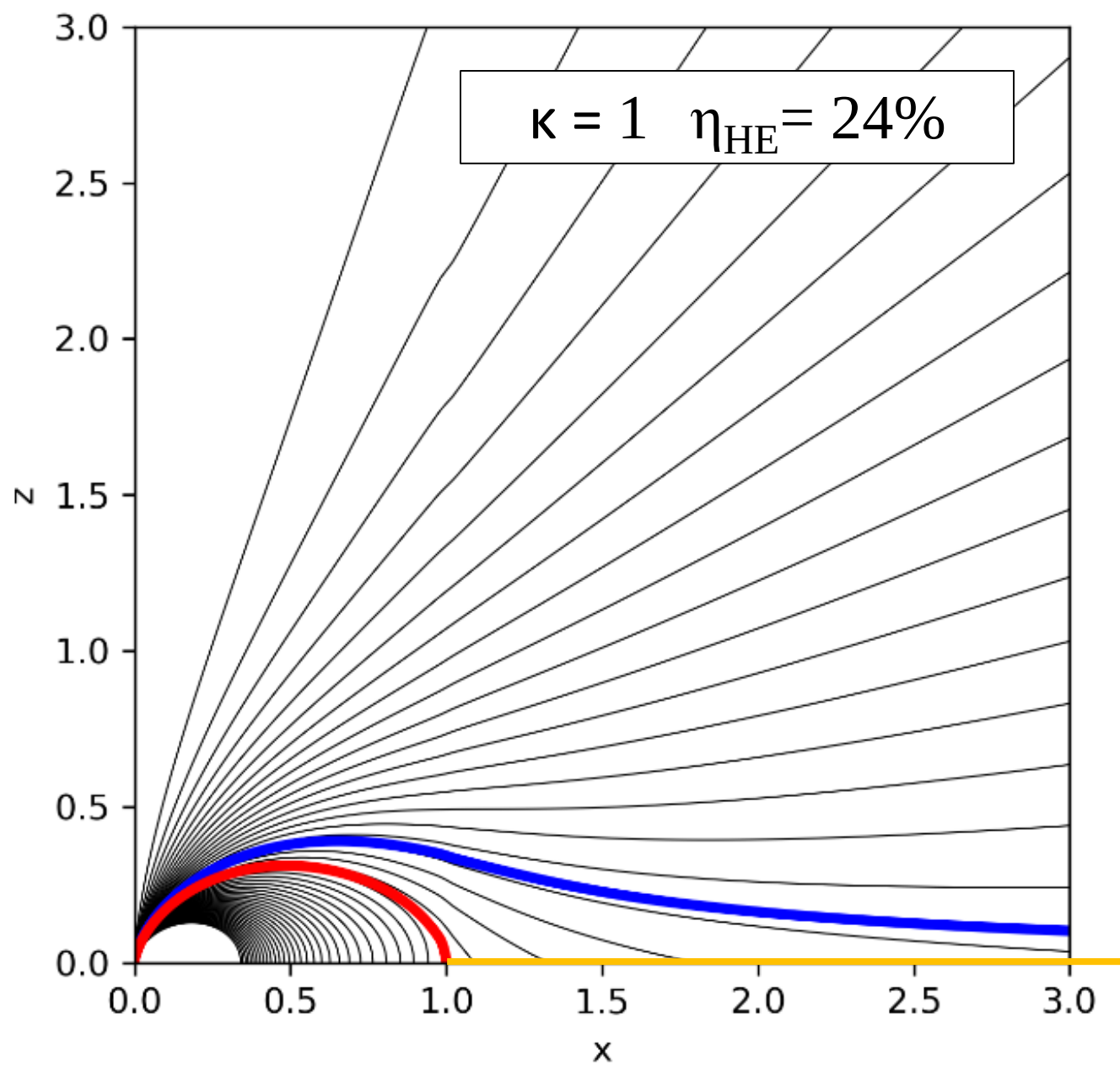




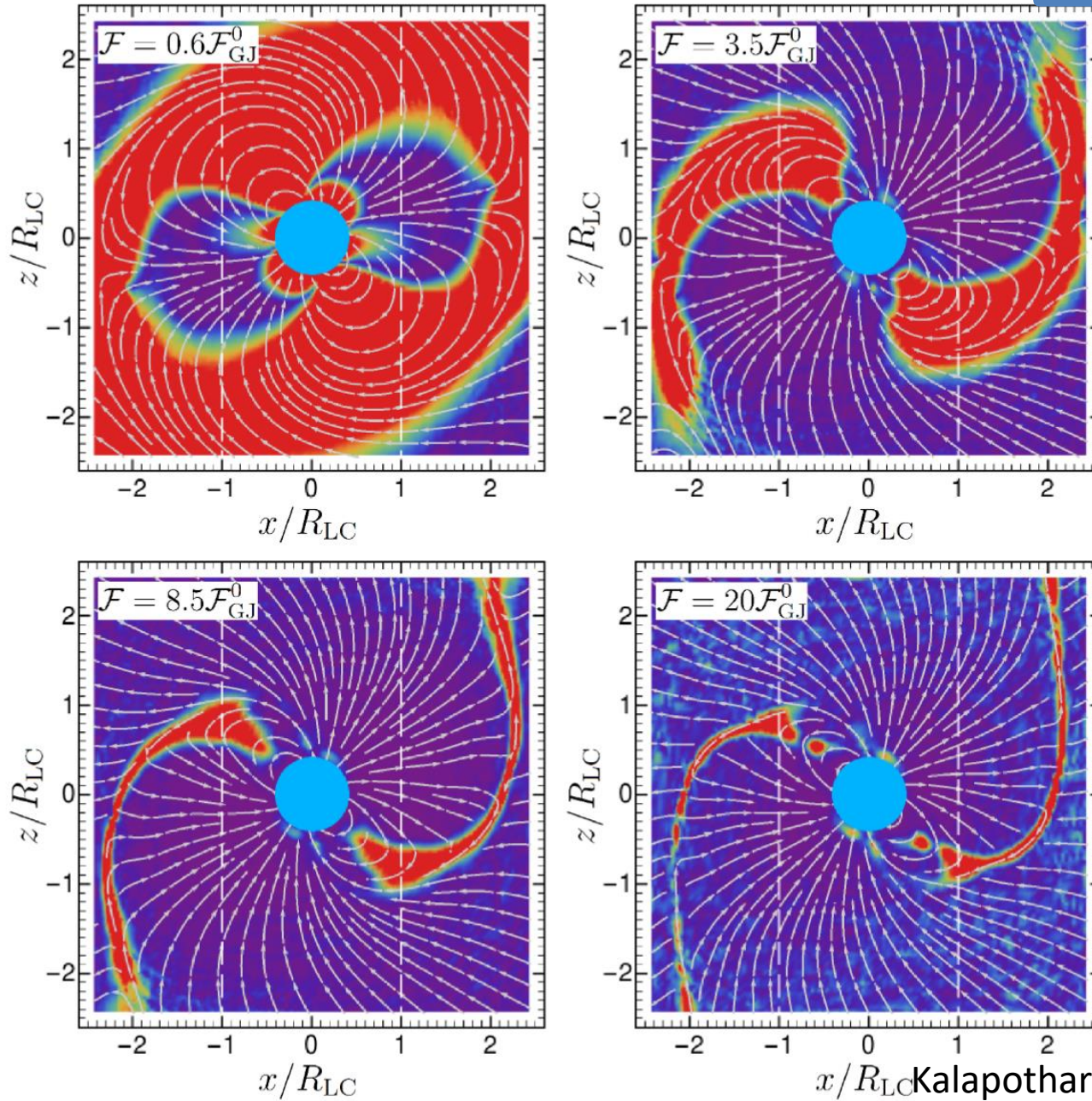








$$\alpha = 45^\circ$$



$$B_r \approx \frac{1}{x^2} \left(1 - \frac{1}{x^2}\right)^{0.7} B_{\text{lc dipole}} . \quad (22)$$

Here, $B_{\text{lc dipole}} \equiv B_* r_*^3 / (2r_{\text{lc}}^3)$ is the equatorial value of the vacuum dipole magnetic field at the light cylinder. Therefore, according to eqs. (2) and (20)

$$B_z \approx -\frac{3}{5\kappa x^4} \left(1 - \frac{1}{x^2}\right)^{0.2} B_{\text{lc dipole}} , \quad (23)$$

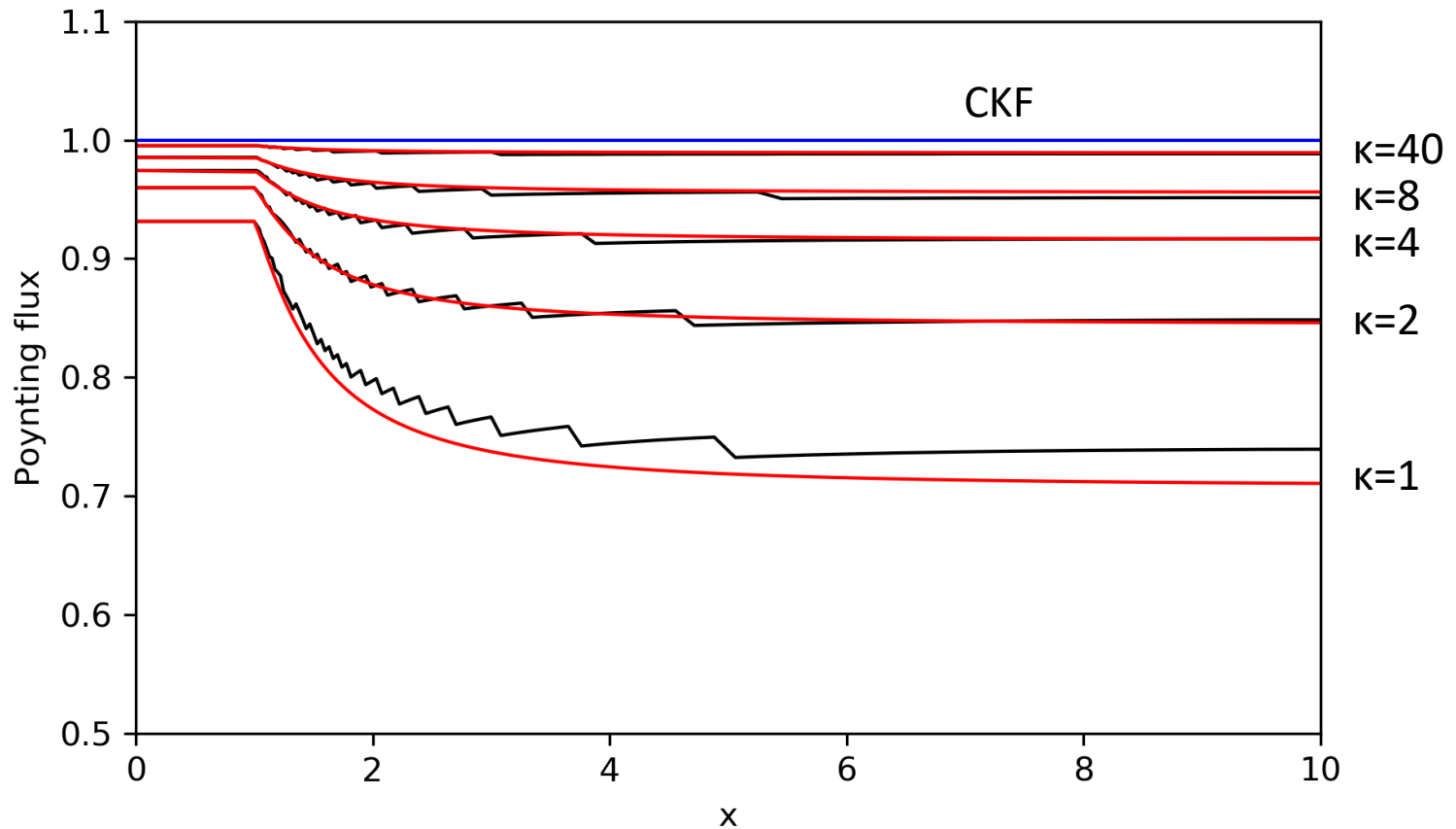
$$E_r \approx \frac{3}{5\kappa x^3} \left(1 - \frac{1}{x^2}\right)^{0.2} B_{\text{lc dipole}} . \quad (24)$$

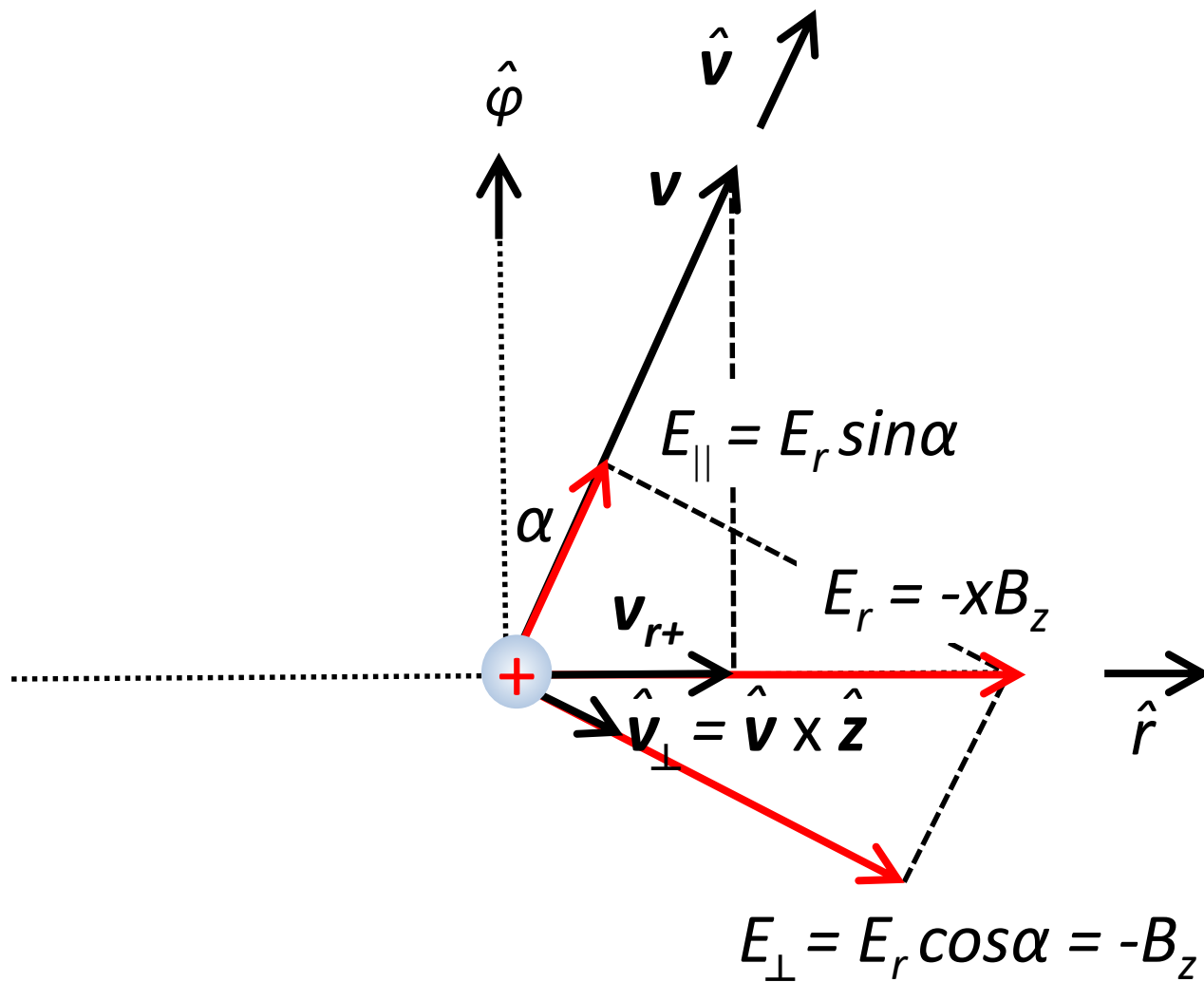
Notice the very sharp decrease of B_z and E_r with distance. Finally, let us also introduce

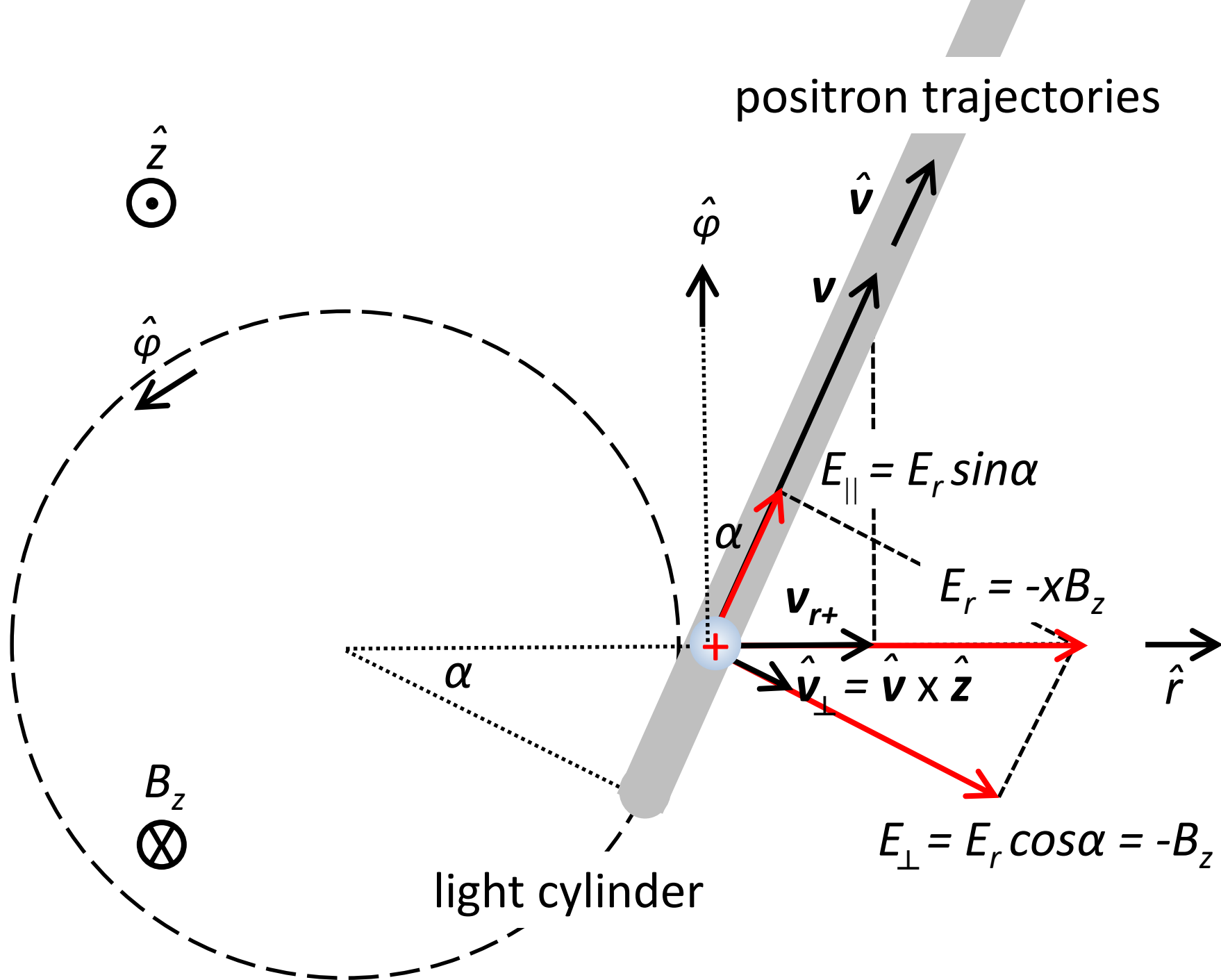
$$B_\phi = -\frac{I_{\text{ECS}}}{x r_{\text{lc}} c} = -\frac{\Omega r_{\text{pc dipole}}^2 B_*}{2x r_{\text{lc}} c} = -\frac{B_{\text{lc dipole}}}{x} . \quad (25)$$

$$\dot{E}_{\text{Poynting}}(x) = \dot{E} - \dot{E}_{\text{ECS}}(x)$$

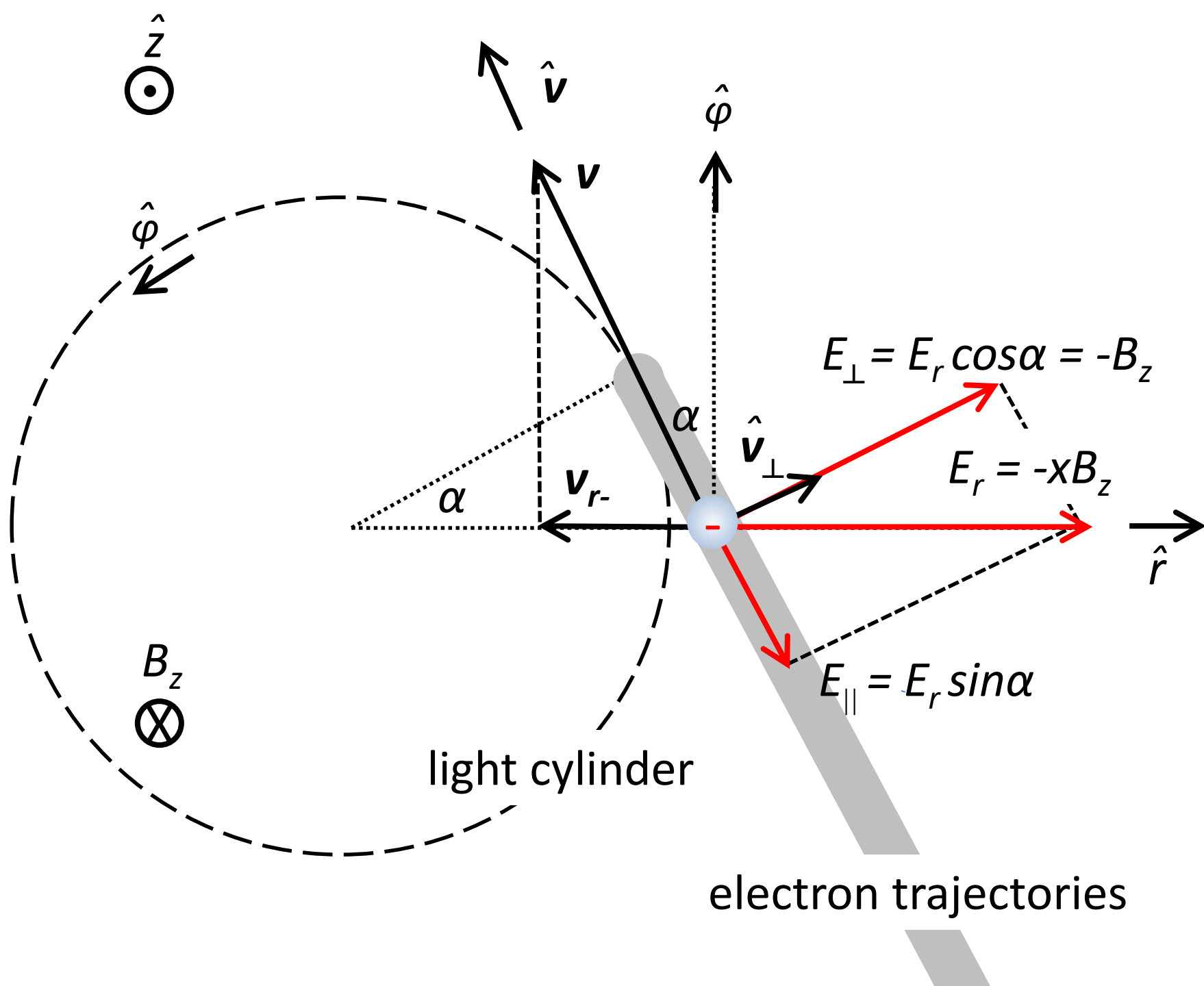
$$\approx \begin{cases} \dot{E} \left(1 - \frac{6}{25\kappa} \left(1 - \frac{1}{x^2} \right)^{1.2} \right) & \text{if } x \geq 1 \\ \dot{E} & \text{otherwise} \end{cases}$$







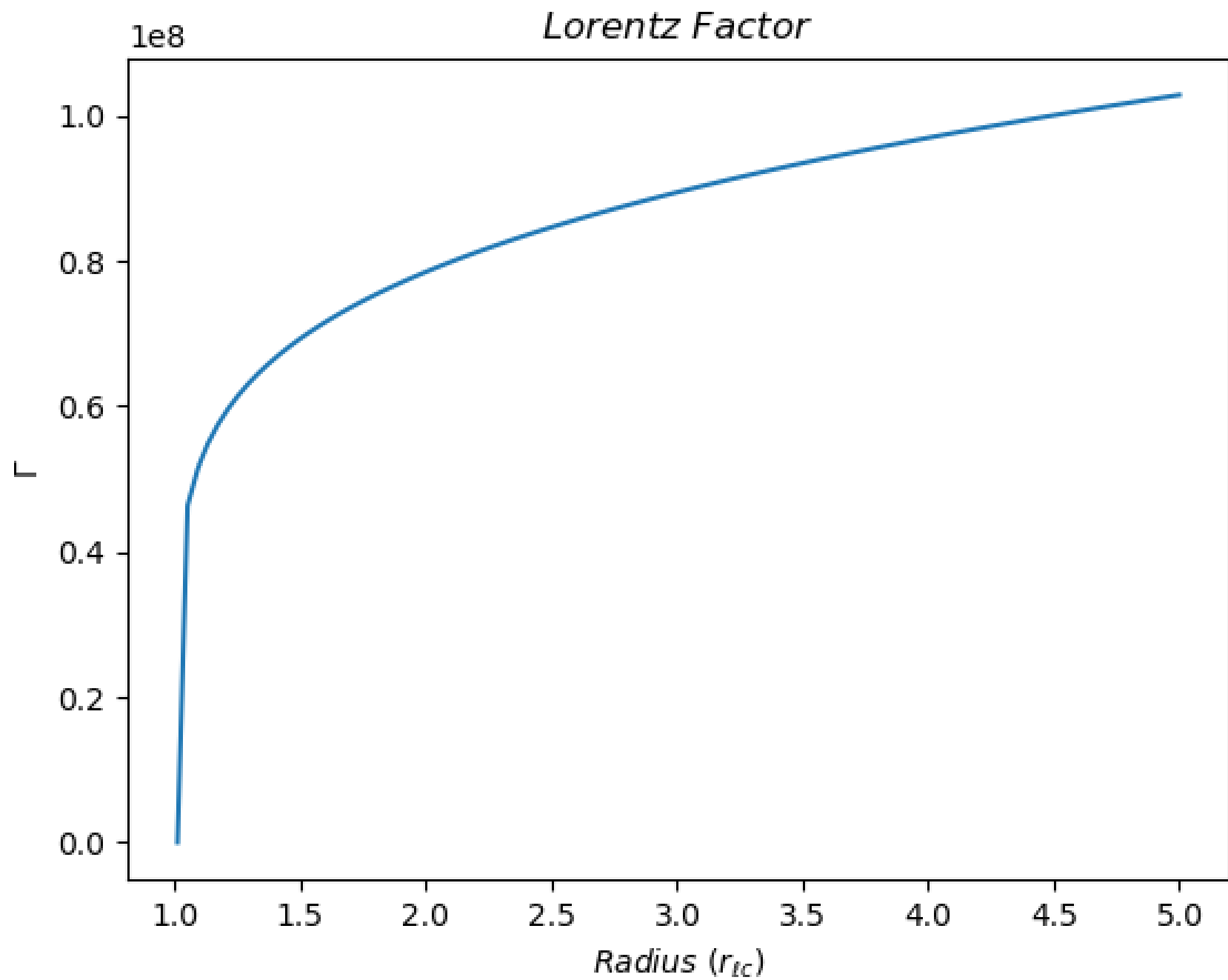




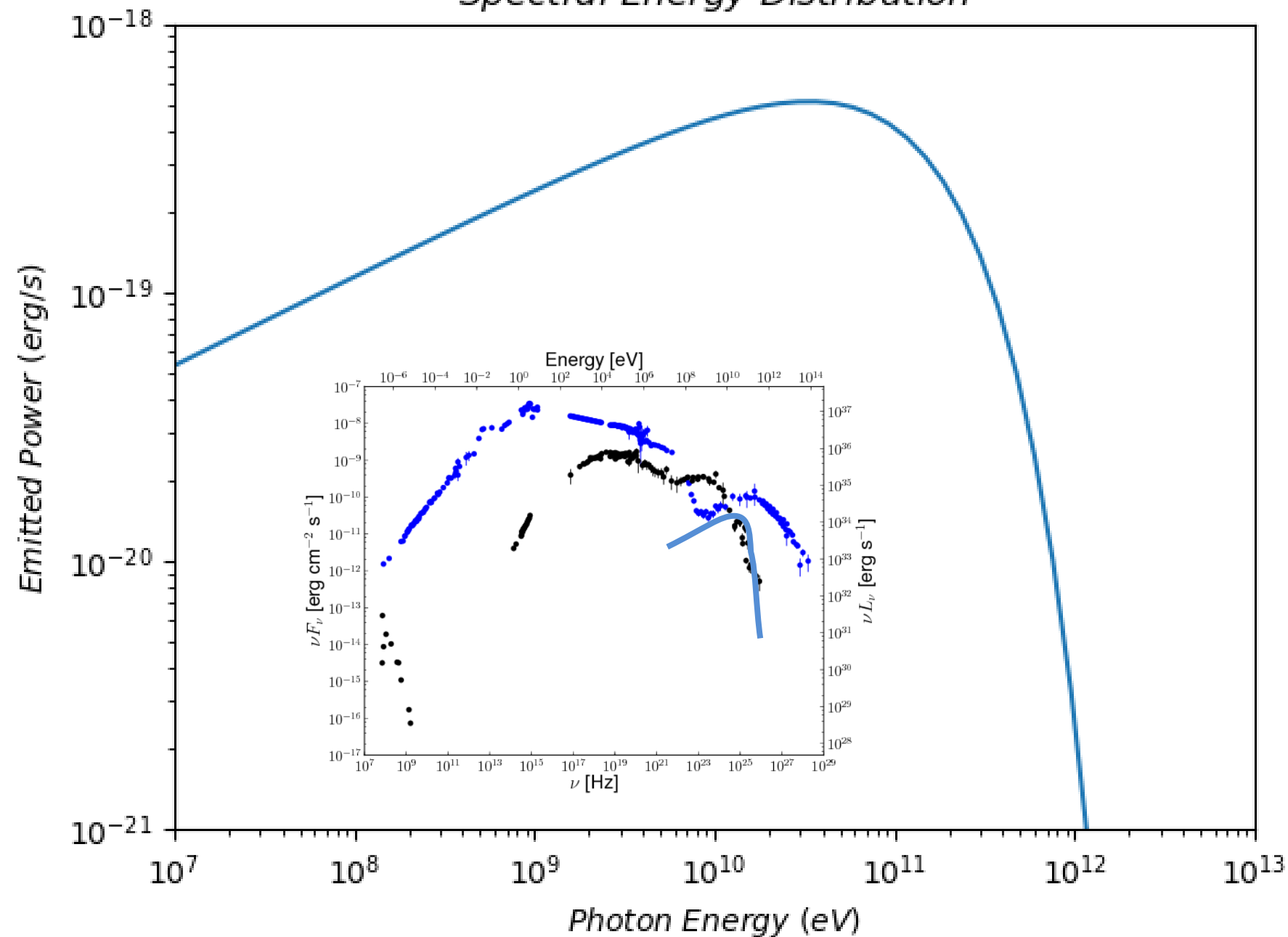
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{ecE_{\mathrm{acc}}}{m_e c^2} - \frac{2e^2\Gamma^4}{3r_{\mathrm{lc}}^2 m_e c}$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \frac{eB_{\mathrm{lc}}r_{\mathrm{lc}}}{m_e c^2} \left\{ \frac{3}{5\kappa x^3} \left(1 - \frac{1}{x^2}\right)^{0.2} - \frac{\Gamma^4/\Gamma_{\mathrm{rrl}}^4}{(R_{\mathrm{c}}/r_{\mathrm{lc}})^4} \left(1 - \frac{1}{x^2}\right)^{-0.5} \right\}$$

$$\Gamma_{\mathrm{rrl}} = \left(\frac{3r_{\mathrm{lc}}^2 E_{\mathrm{acc}}}{2e}\right)^{1/4} = 4 \times 10^7 \left(\frac{B_{*}}{10^{13}\mathrm{G}}\right)^{1/4} \left(\frac{P}{1\,\mathrm{s}}\right)^{-1/4}$$



Spectral Energy Distribution

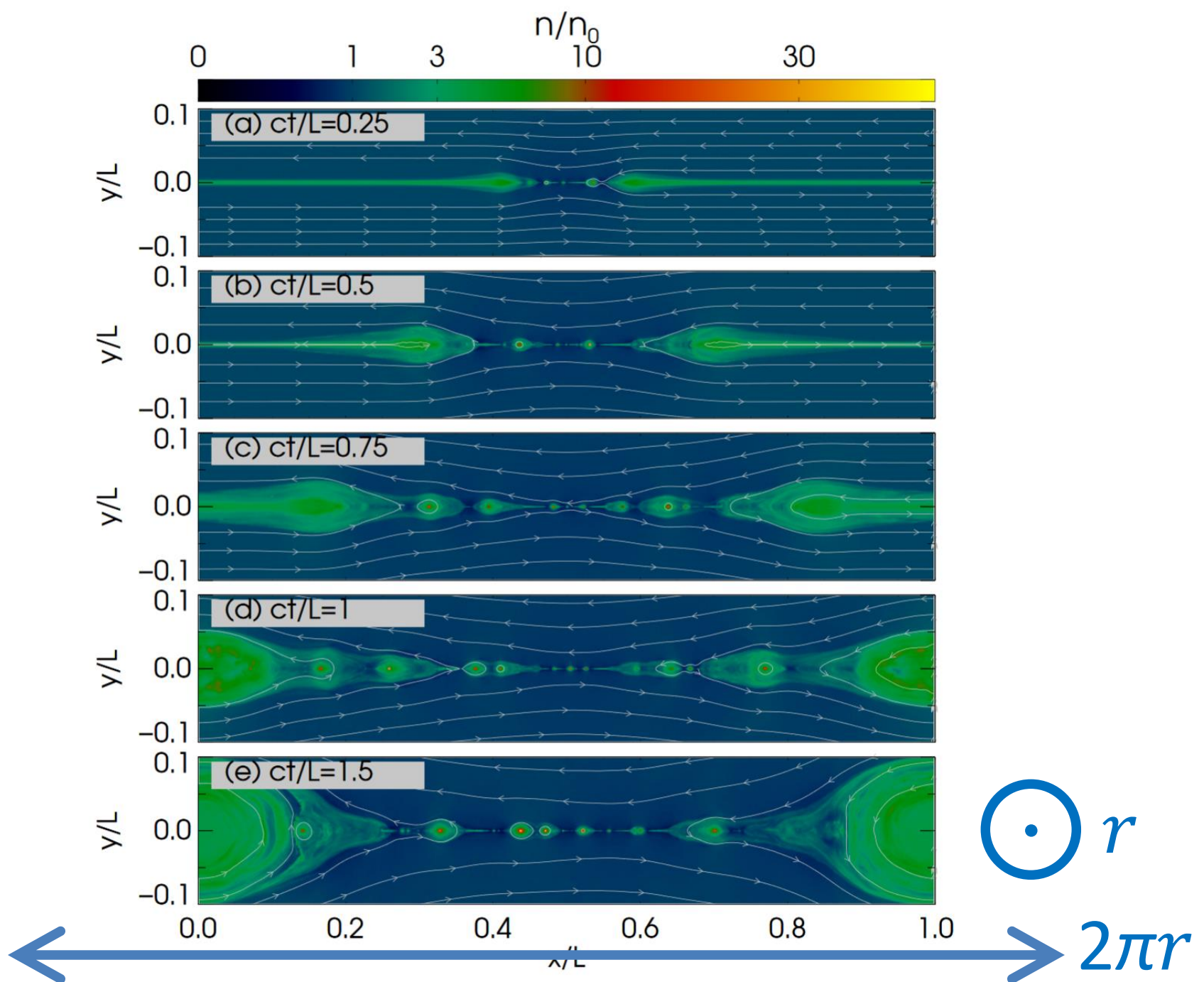


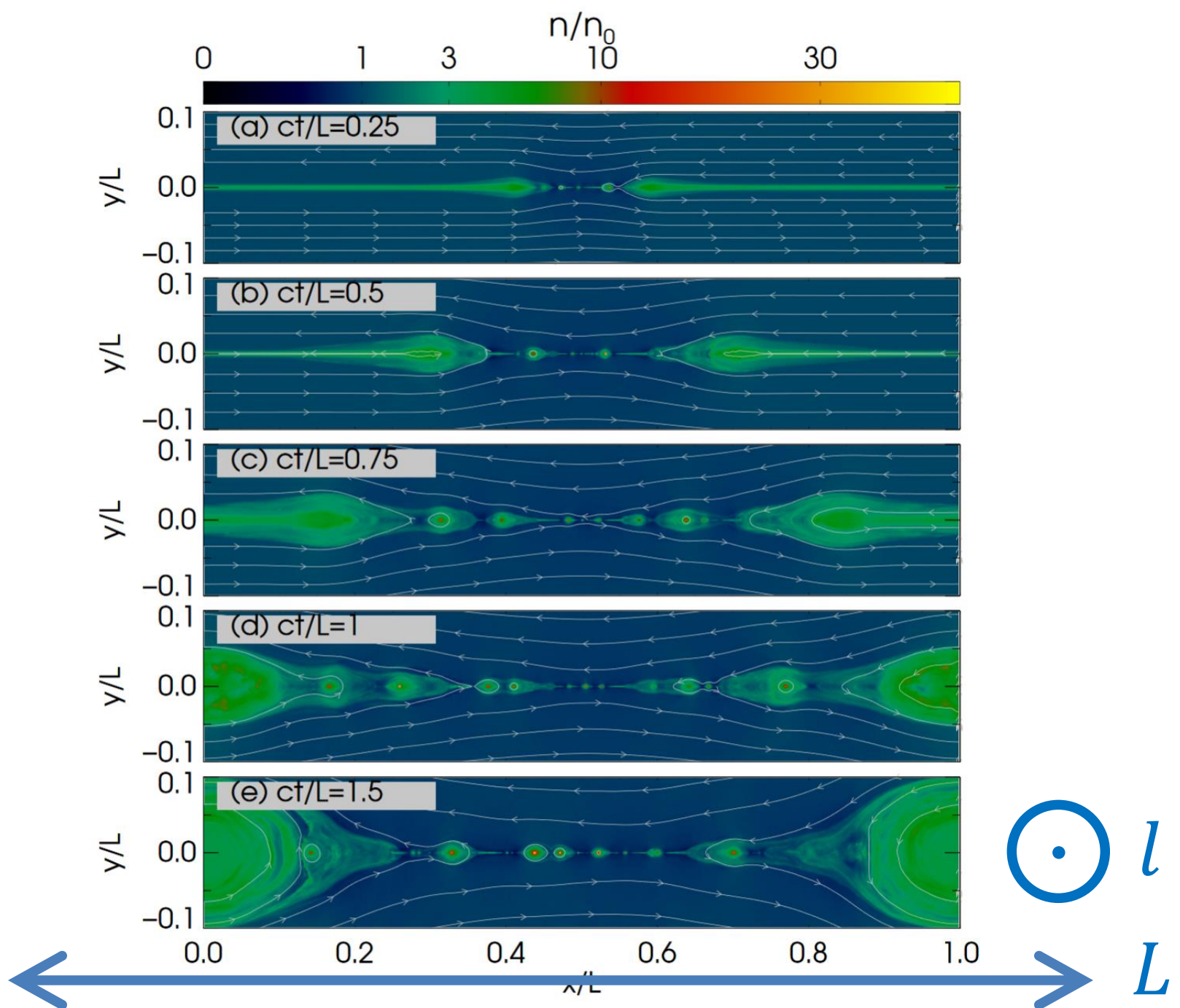
Conclusions

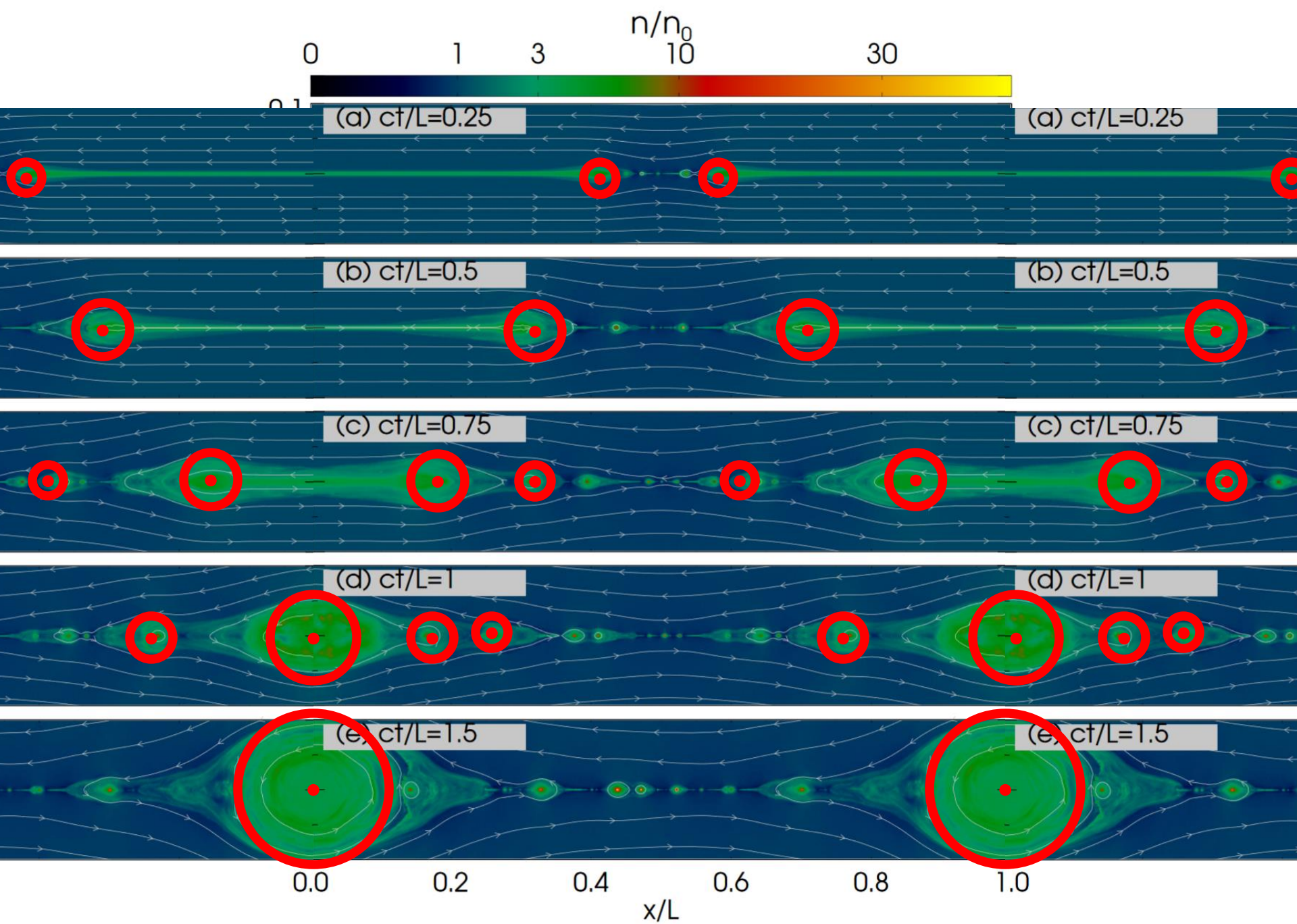
- “Ab initio” PIC simulations are very ambitious
- Current closure and charge replenishment
 - Ring of fire
- “Aristotelian” particle orbits when $E > B$ in the CS
- We can perform calculations with realistic parameters (Γ , magnetic field)
 - High energy spectra
 - Different spectra for e^- and e^+

Conclusions

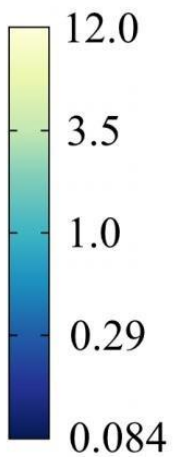
- Pair-formation multiplicities near the rim of the polar cap may be very low ($\kappa \rightarrow 0$?!)
- Highly dissipative magnetospheres ?!
 - very different from CKF
- New type of global current sheet simulations:
 - $I \sim B_\varphi 2\pi r c \sim \rho_e v_z 2\pi r \cdot r$
 $\sim \kappa \rho_{\text{GJ}} \frac{E_r}{B_\varphi} c L \cdot l \rightarrow E_r \sim \frac{1}{\kappa}$







n/n_0



z/l

x/l

y/l

Conclusions

- Pair-formation multiplicities near the rim of the polar cap may be very low ($\kappa \rightarrow 0$?!)
- Highly dissipative magnetospheres ?!
 - very different from CKF
- New type of global current sheet simulations:
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 $\sim \kappa \rho_{\text{GJ}} \frac{E_r}{B_\varphi} c L \cdot l \rightarrow E_r \sim \frac{1}{\kappa}$
- More questions...
- Need hybrid simulations!

The multi-messenger physics and astrophysics of neutron stars

PHAROS Conference

30 March - 3 April, 2020

Porto Rio Hotel, Patras, Greece

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Thematic Areas

- Dense matter: equations of state, superfluidity and superconductivity
- Magnetic field formation, structure and evolution
- Neutron star observations: from radio to gamma-rays
- Neutron stars in the multimessenger era
- Magnetospheric high-energy emission
- Population studies
- Fast Radio Bursts



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