Κέντρο Ερευνών Αστρονομίας
και Εφαρμοσμένων Μαθηματικών
της Ακαδημίας Αθηνών

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Our road to the reference 3D force-free solution

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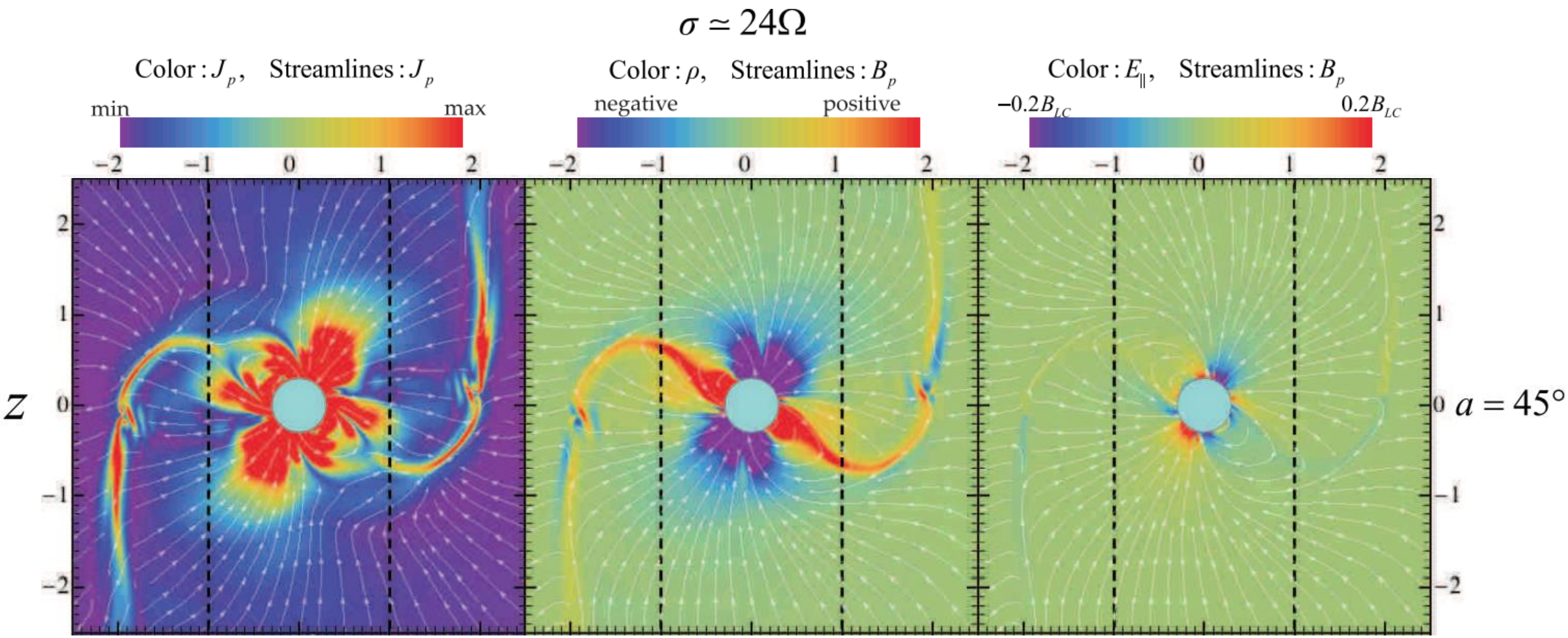
ISSI Bern, Wednesday September 14, 2022

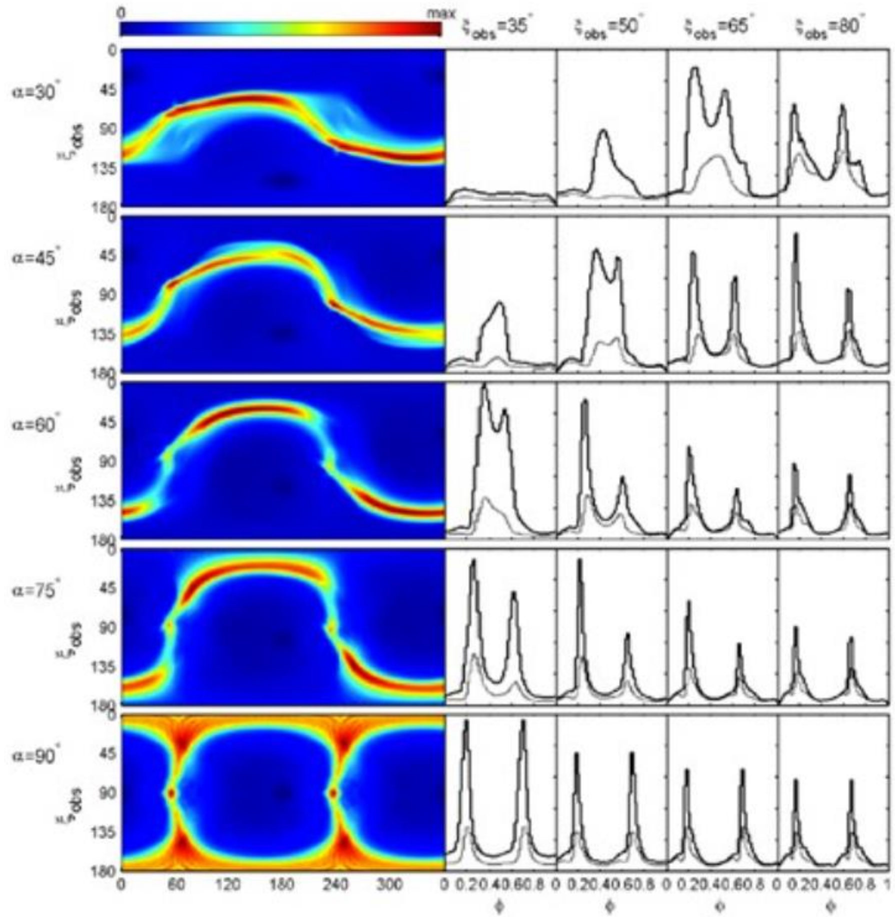
Progress in our understanding

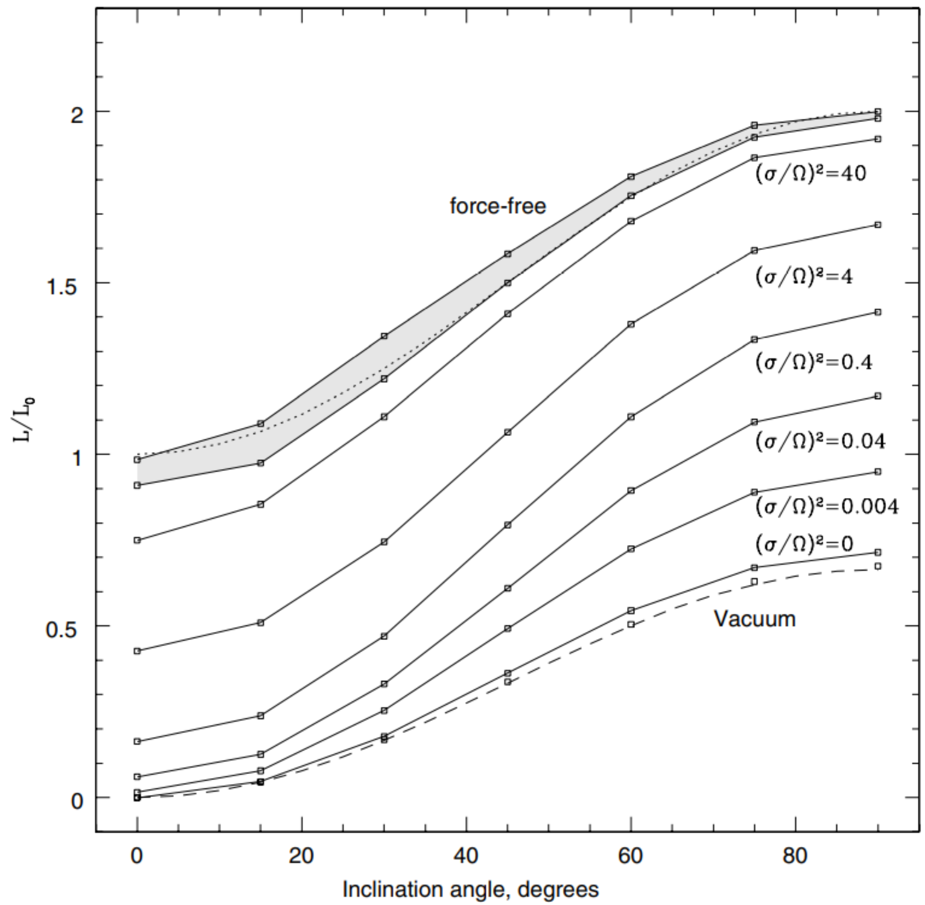
- Formulation of the problem (1969)
- 30 years of focusing on the light cylinder…
- The axisymmetric force-free solution (1999, 2003, 2006)
- The steady 3D force-free solution (2006, 2009)

- Towards a realistic pulsar magnetosphere: resistivity, radiation (2012, 2014)
- “Ab initio” simulations: particles, pair injection, radiation (2014, 2018)



$\sigma \simeq 24\Omega$
Color : J_p, Streamlines : J_p
min
max
Color : ρ, Streamlines : B_p
negative
positive
Color : $E_{\parallel}$, Streamlines : B_p
$-0.2B_{LC}$
$0.2B_{LC}$
−2 −1 0 1 2
Z
$a = 45°$


0
max
ξ_{obs}=35°
ξ_{obs}=50°
ξ_{obs}=65°
ξ_{obs}=80°
α=30°
α=45°
α=60°
α=75°
α=90°
ξ_{obs}
ϕ

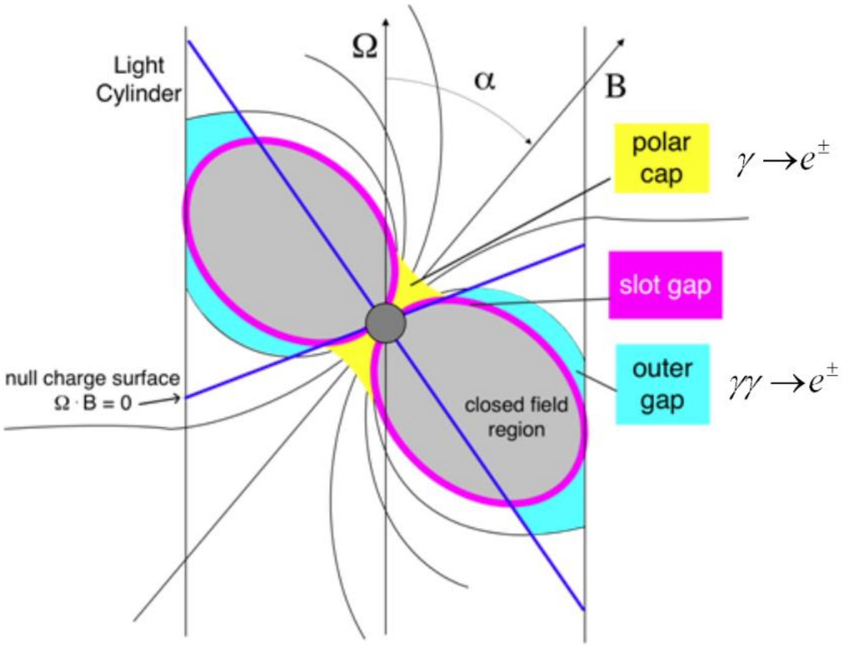

force-free
$(\sigma/\Omega)^2$=40
$(\sigma/\Omega)^2$=4
$(\sigma/\Omega)^2$=0.4
$(\sigma/\Omega)^2$=0.04
$(\sigma/\Omega)^2$=0.004
$(\sigma/\Omega)^2$=0
Vacuum
L/L_0
Inclination angle, degrees

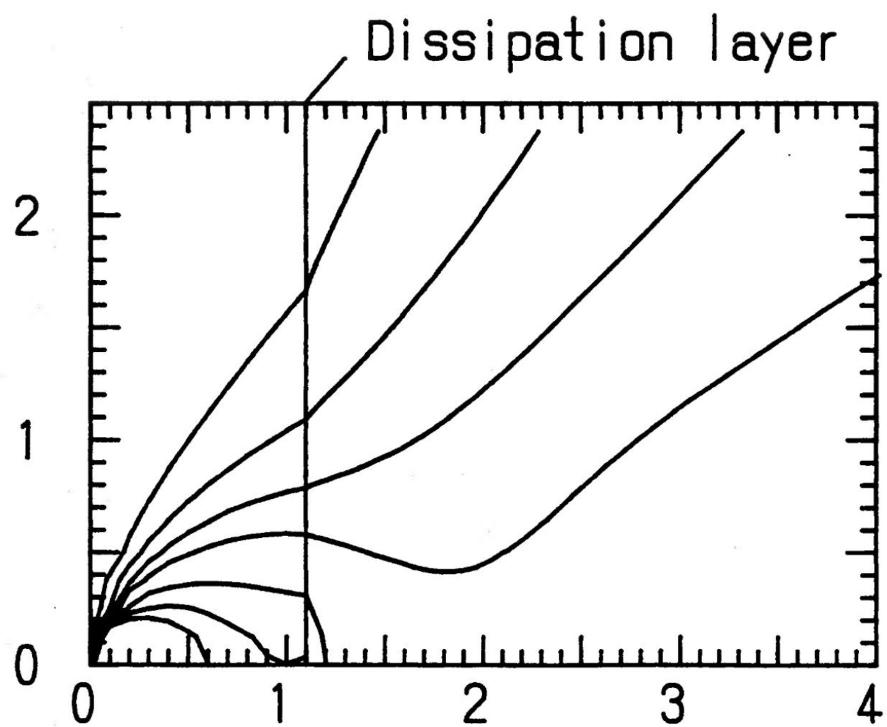
Progress in our understanding

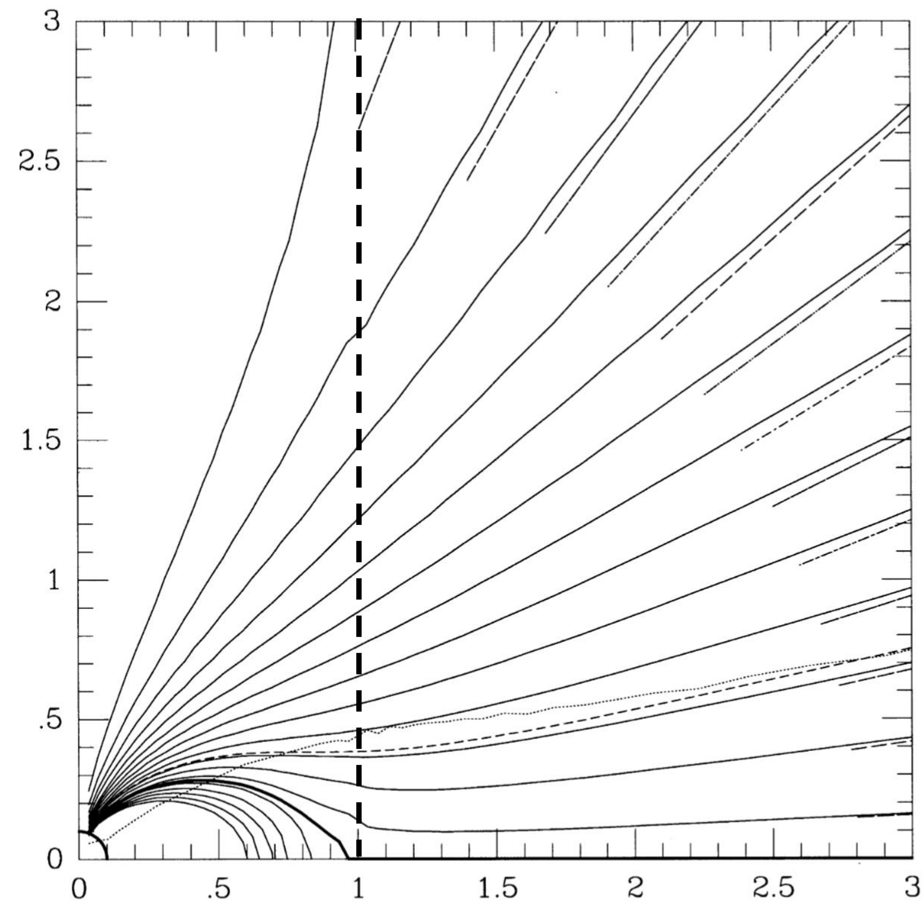
- We are all guided by observations
- Then why do we disagree?
  - Scale separation
    - “Your” interpretation/extrapolation is wrong!
    - ~~“Your” simulation is wrong!~~
- How do we make progress?
  - More particles, more physics, more HPC
    - Realistic simulations of MS pulsars in our lifetime
  - Return to the basics

Need to return to the basics

- In the early years (Goldreich & Julian 1969) the main focus of pulsar research was the light cylinder
- This is not a problem anymore
- The focus has shifted to the current sheet
  - Mathematically: a contact discontinuity
  - "Reconnection", "plasmoids", pair formation


Light
Cylinder
Ω
α
B
polar
cap
γ → e±
slot gap
outer
gap
γγ → e±
null charge surface
Ω · B = 0
closed field
region


Dissipation layer
0
1
2
0
1
2
3
4


3
2.5
2
1.5
1
.5
0
0
.5
1
1.5
2
2.5
3

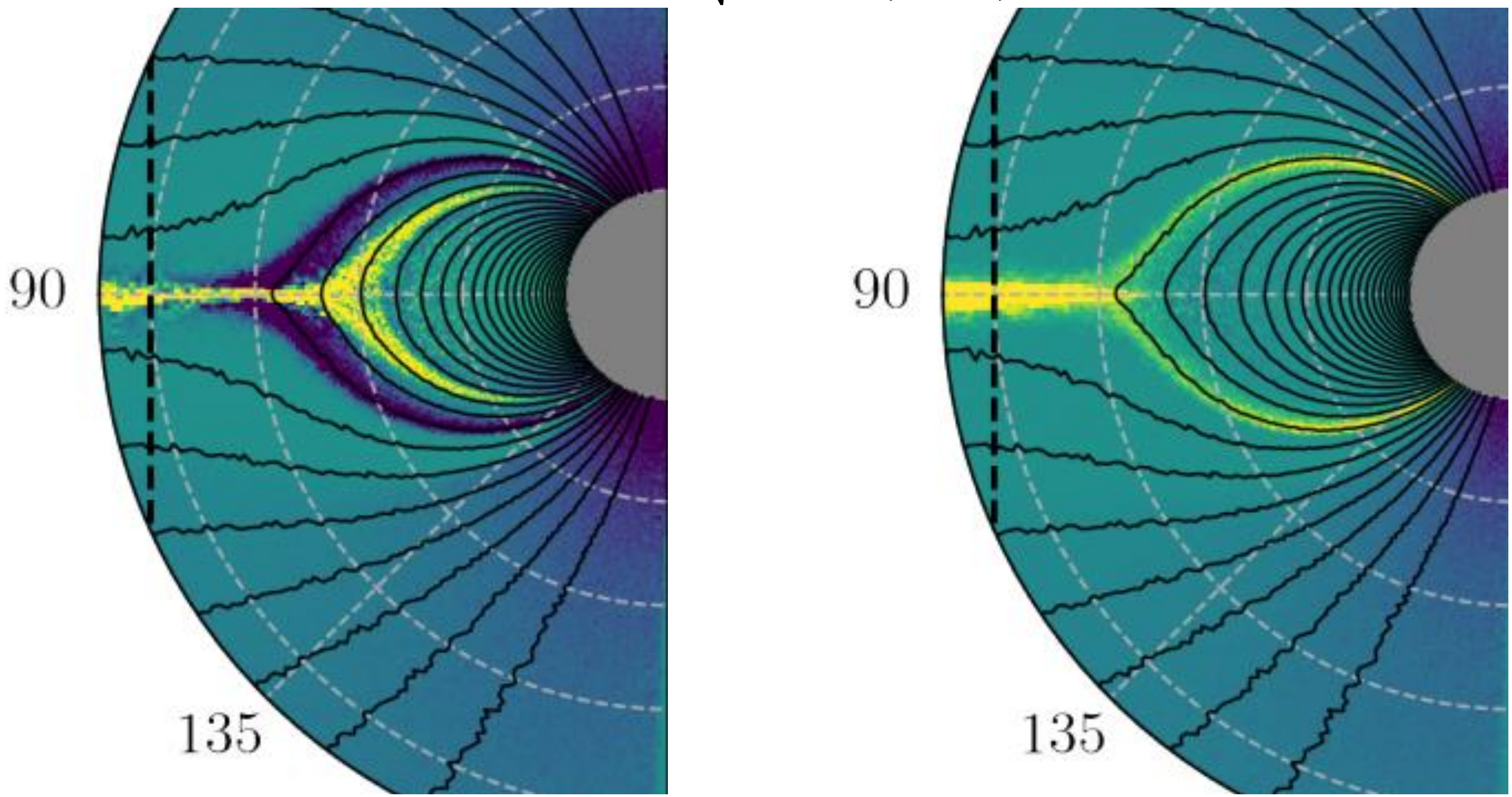
Need to return to the basics

- The current sheet beyond the Y-point seems to be the region of particle acceleration to extremely relativistic energies → VHE
- Is it? Not always
- What is its shape?
- Where does it start?
  - How many field lines are open?
  - What is the pulsar spindown rate?

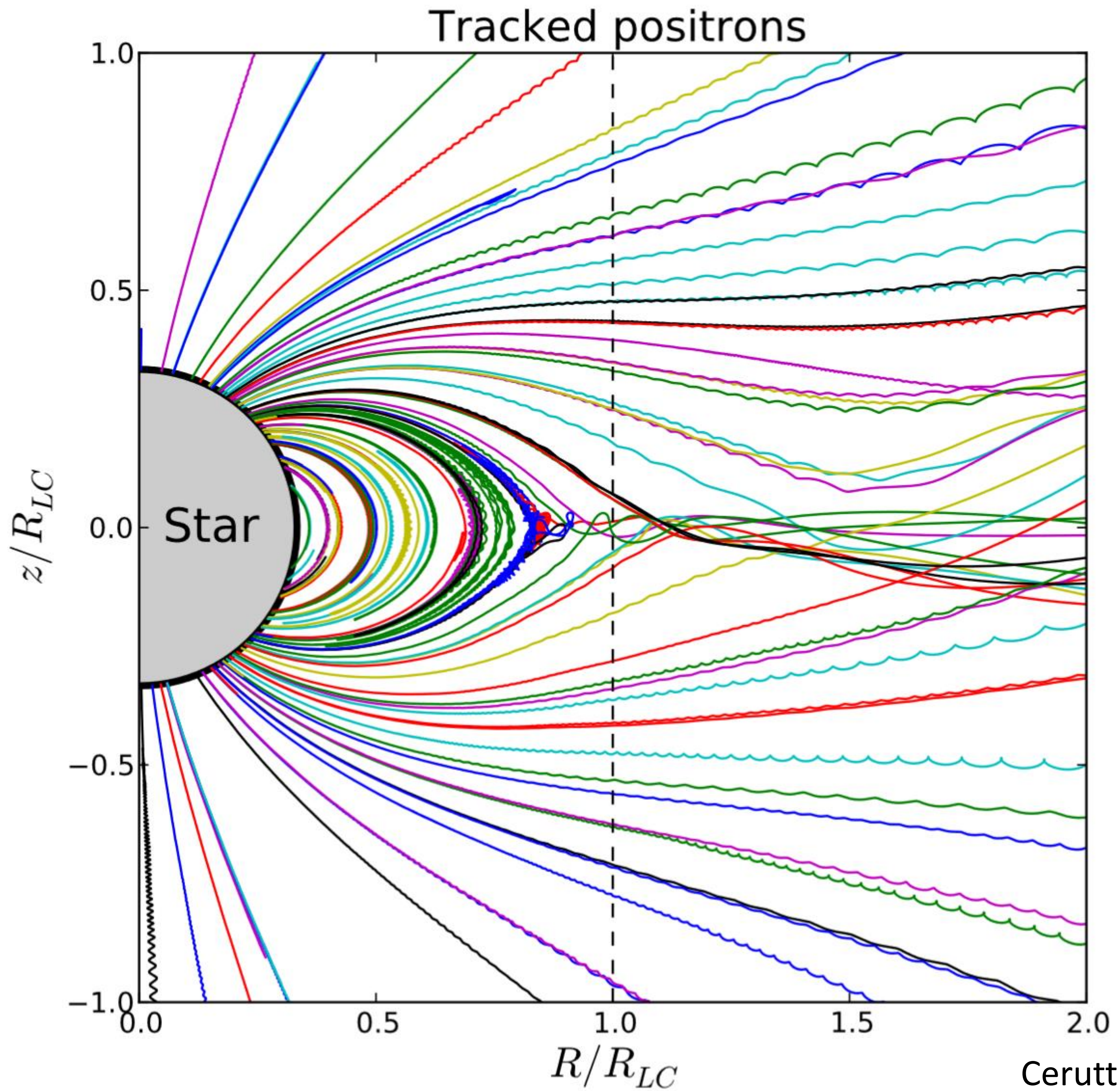
$$B^2 - E^2|_{in} = B^2 - E^2|_{out}$$

$$B_p^2(1 - x^2)|_{in} = B_p^2(1 - x^2)|_{out} + B_\varphi^2$$

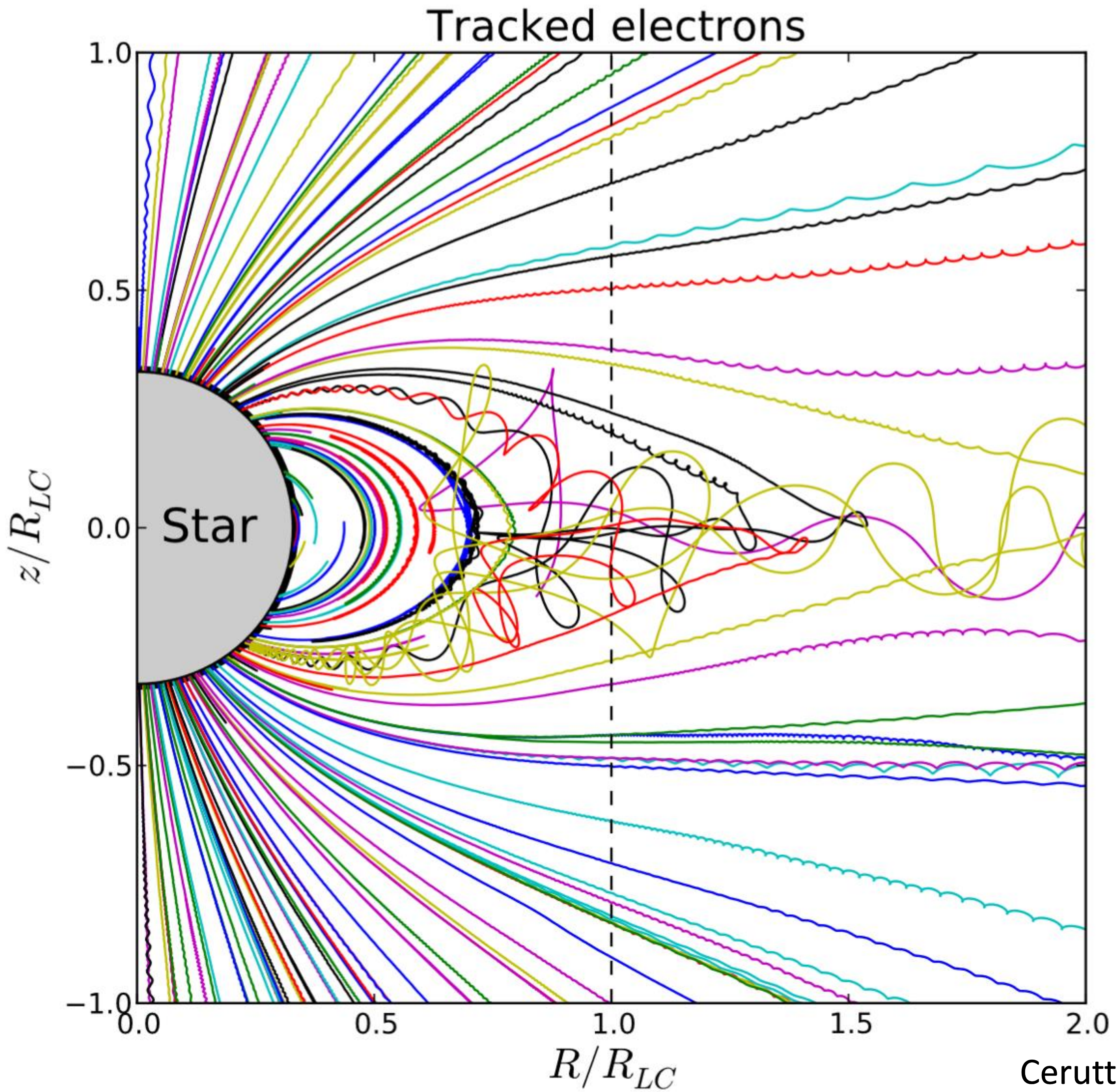
$$B_p|_{in} = \sqrt{B_p^2 + \frac{B_\varphi^2}{(1-x^2)}}\ |_{out}$$



$$r_{open} @\ 0.7\ r_{lc} \rightarrow \Psi_{open} \sim \frac{1}{r_{open}} \rightarrow \dot{E} \sim \frac{1}{r_{open}^2} = 2$$



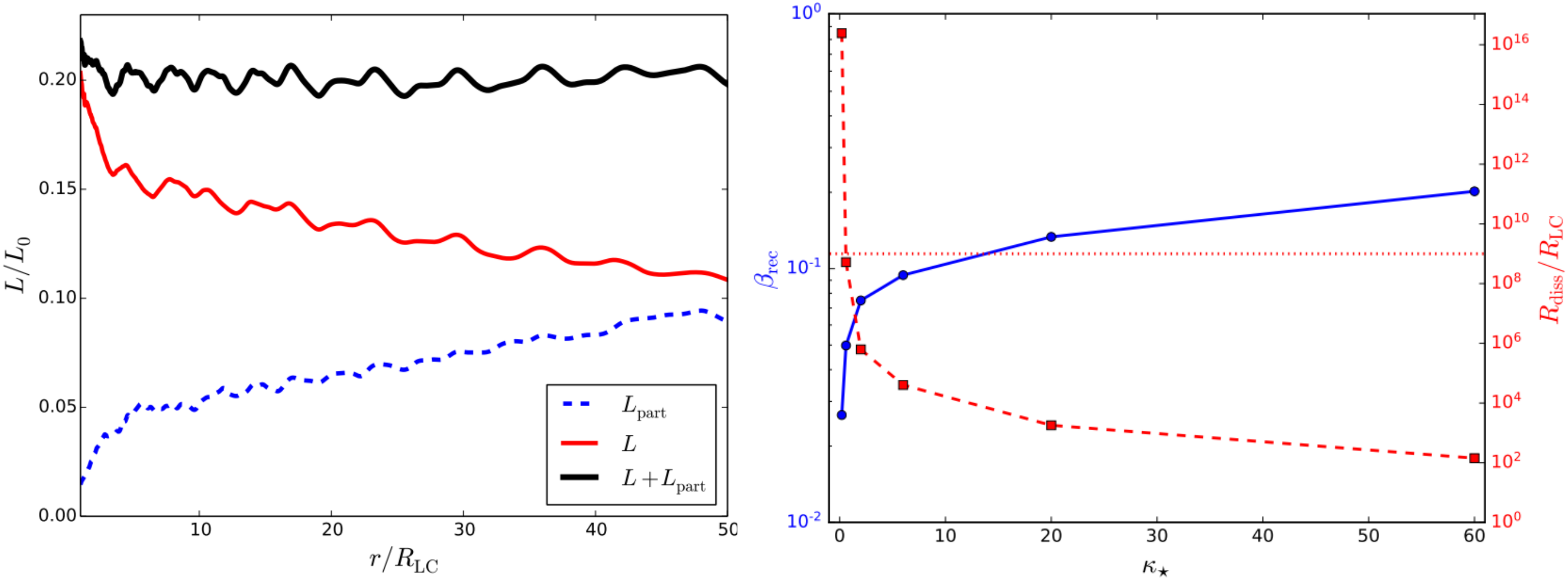
Cerutti et al. 2015



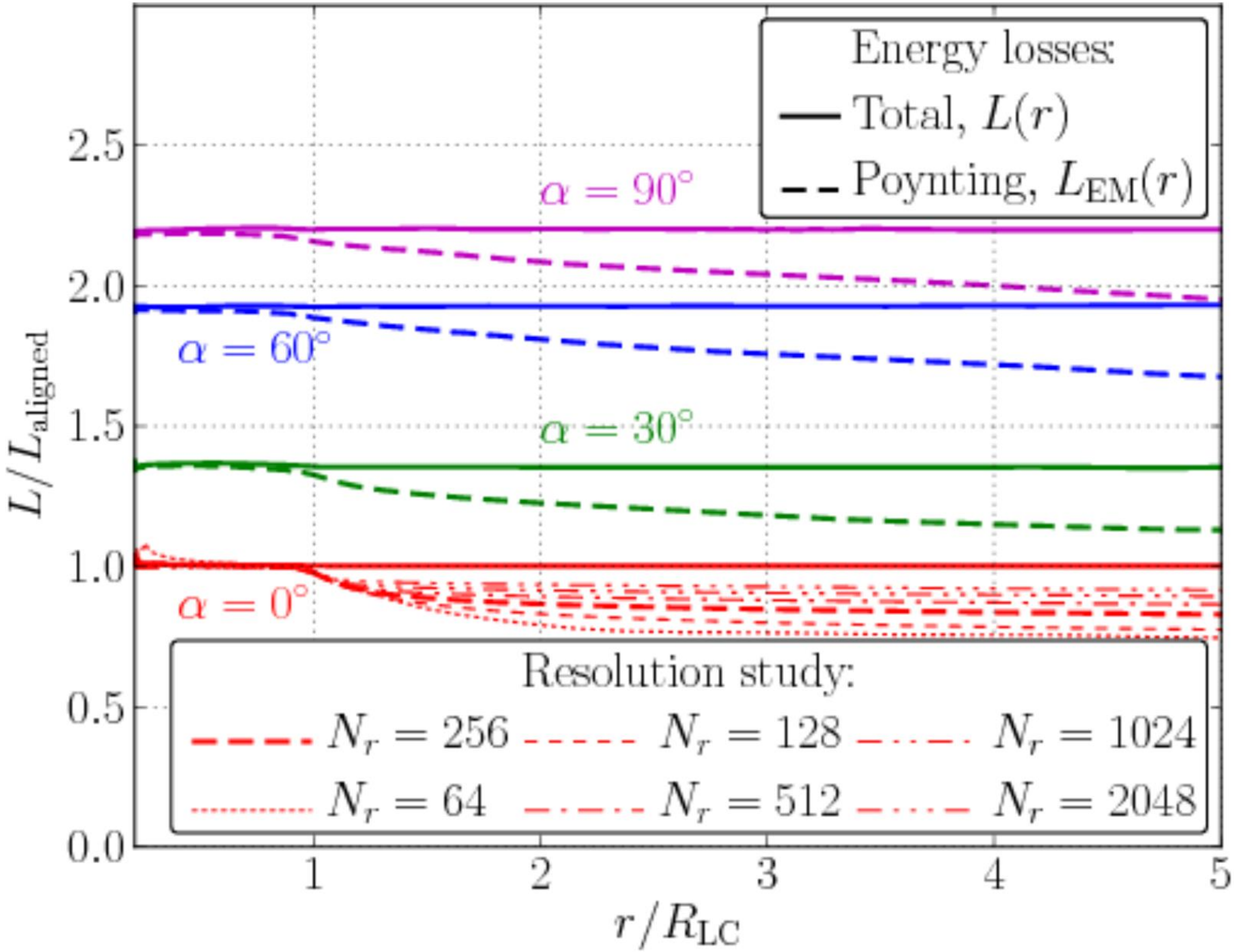
Cerutti et al. 2015

Need to return to the basics

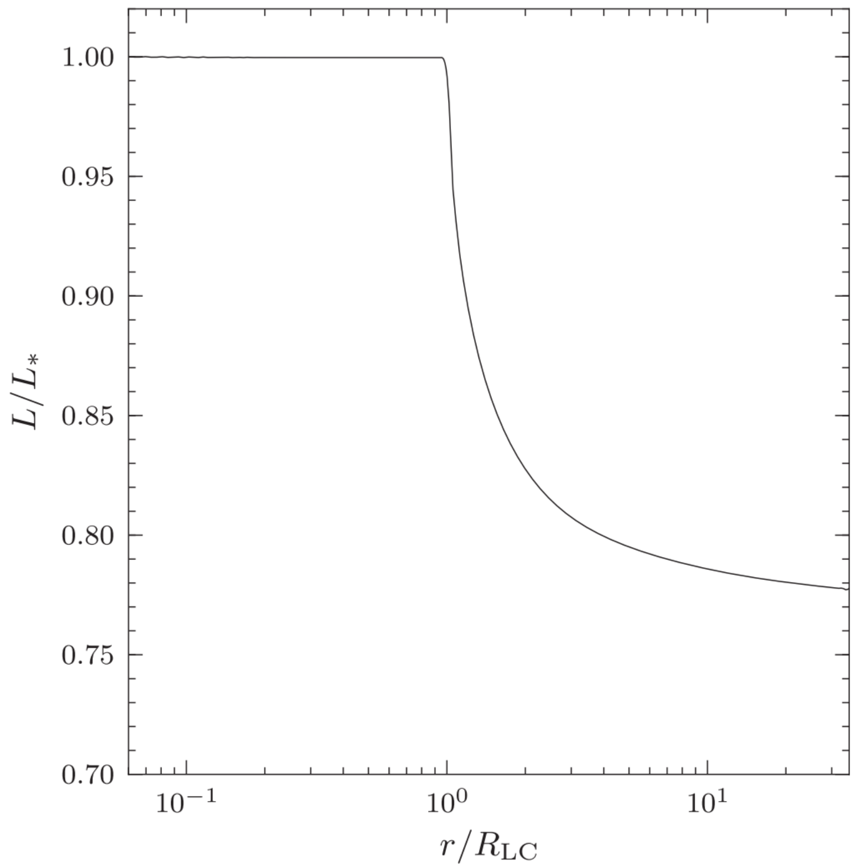
- Why reconnect?!
  - Is it spontaneous or forced?
  - At what rate? 0.1c everywhere? 0.1c max?



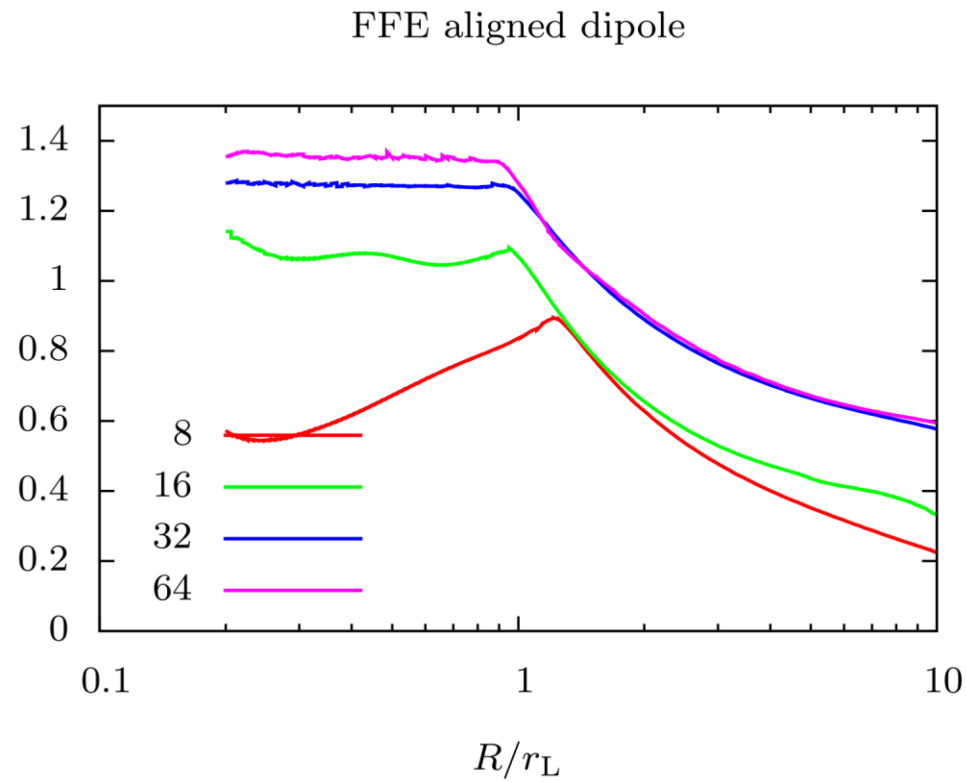
L/L_0
$r/R_{\rm LC}$
$L_{\rm part}$
L
$L+L_{\rm part}$
$\beta_{\rm rec}$
$R_{\rm diss}/R_{\rm LC}$
$\kappa_\star$



"3D MHD pulsar magnetospheres", Tchekhovskoy, Spitkovsky & Li 2013



1.00
0.95
0.90
0.85
0.80
0.75
0.70
L/L_*
10^{-1}
10^0
10^1
r/R_{LC}

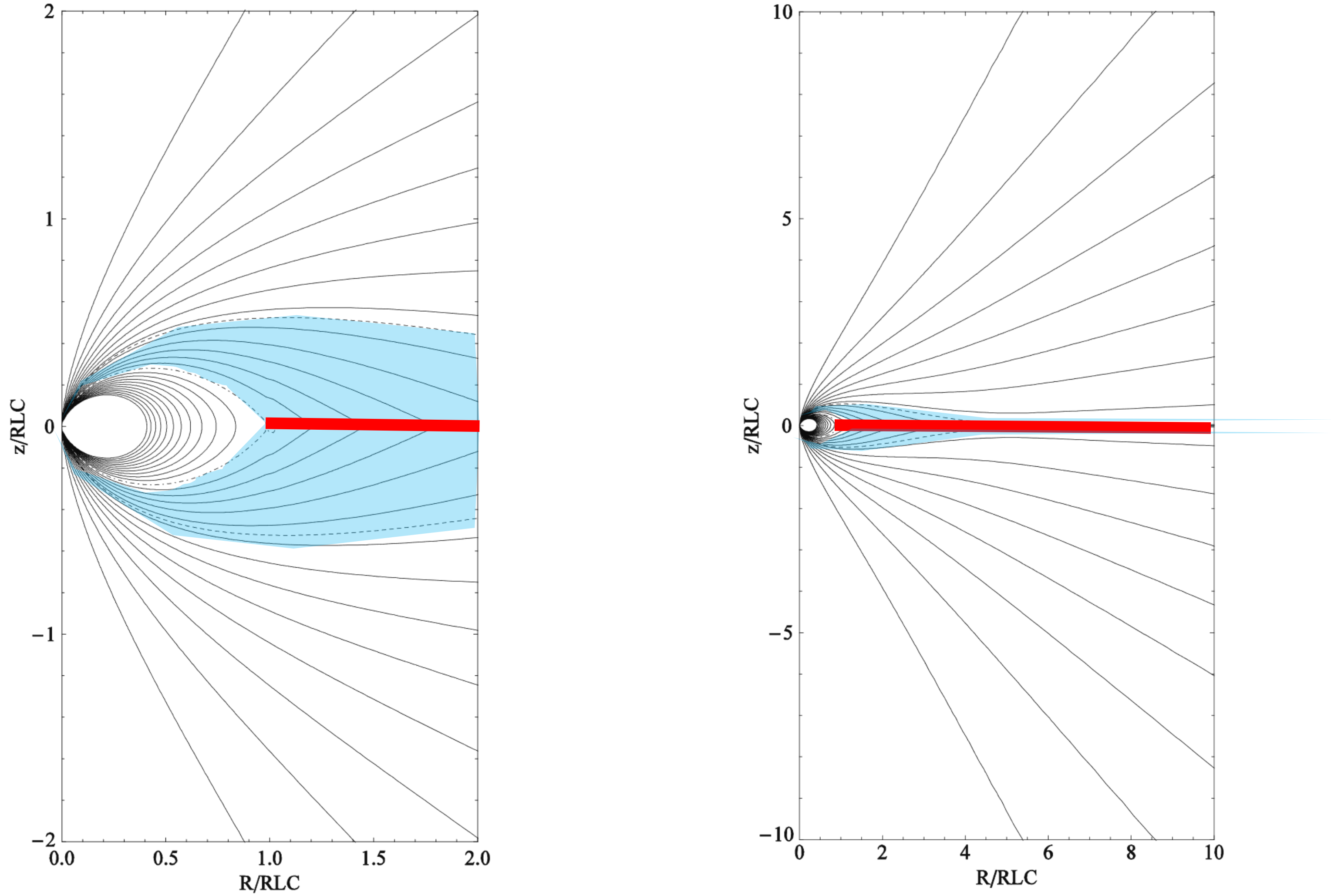


FFE aligned dipole
1.4
1.2
1
0.8
0.6
0.4
0.2
0
8
16
32
64
0.1
1
10
R/r_{L}

Need to return to the basics

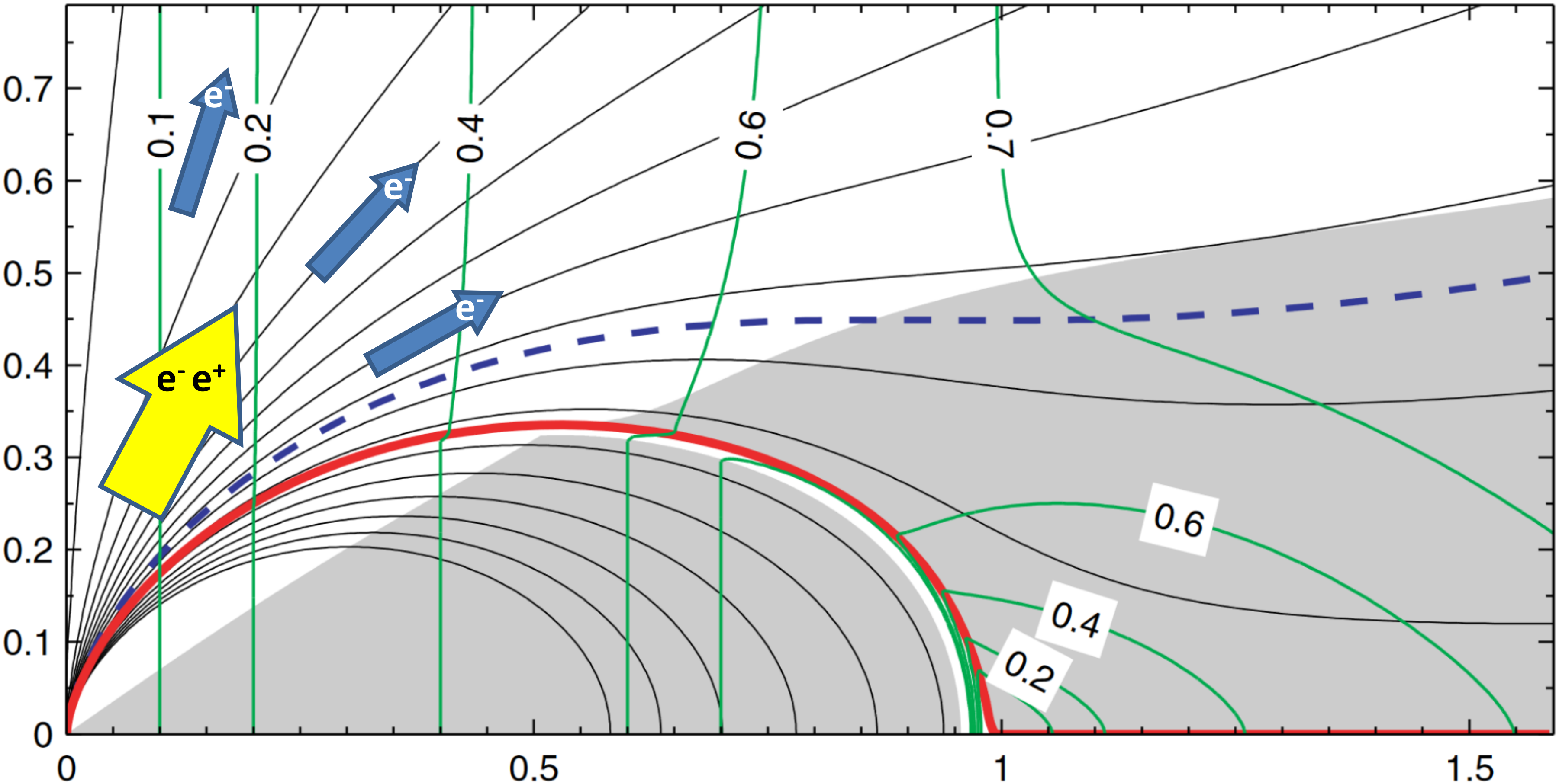
- The key may be global current closure
  - Dissipation related to the supply of charges?
  - Where are the required charges produced?
    - polar cap, near the light cylinder, in the current sheet

Need to return to the basics

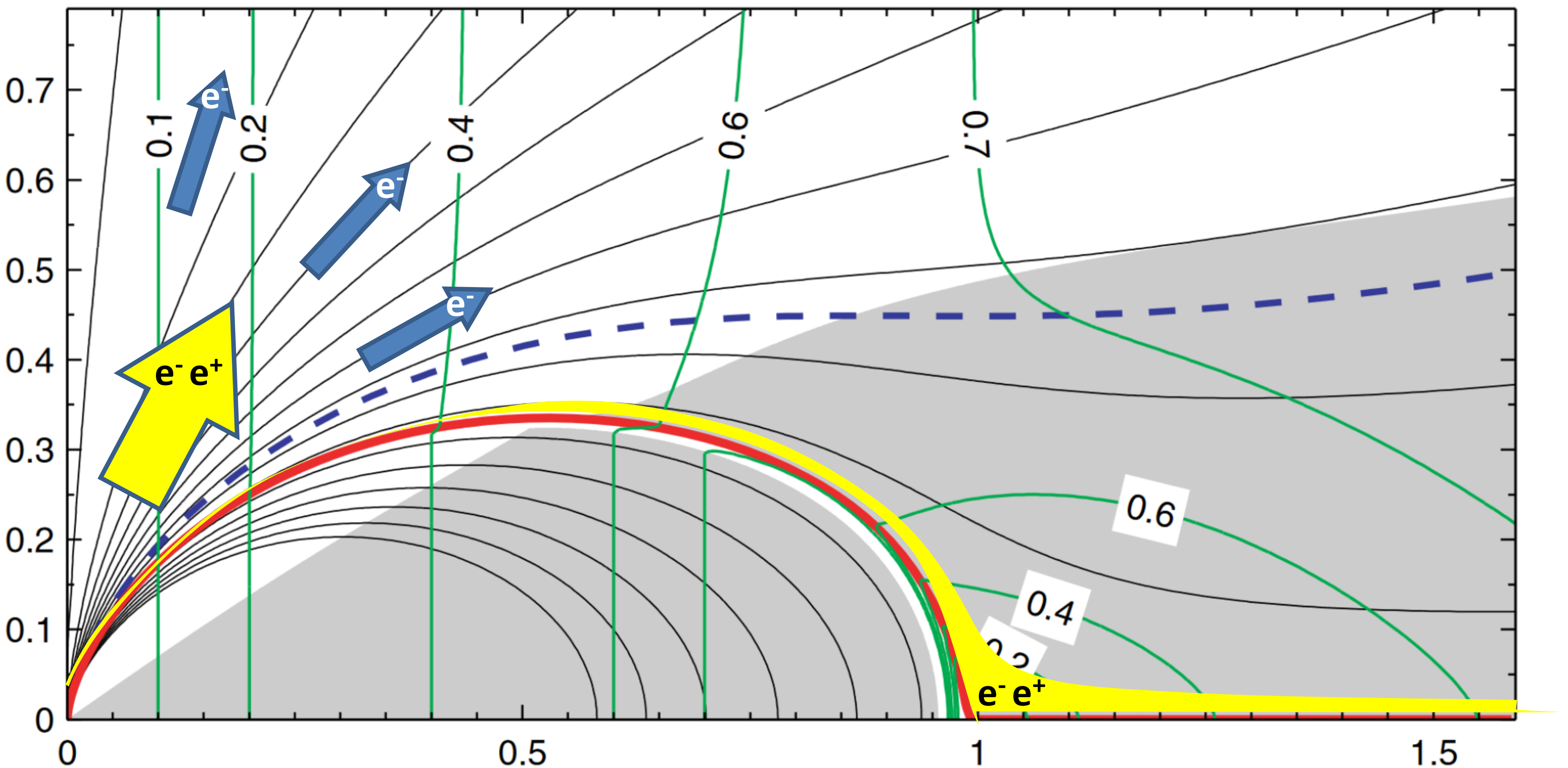


"A new standard pulsar magnetosphere", Contopoulos 2014

"The role of reconnection in the pulsar magnetosphere", Contopoulos 2007

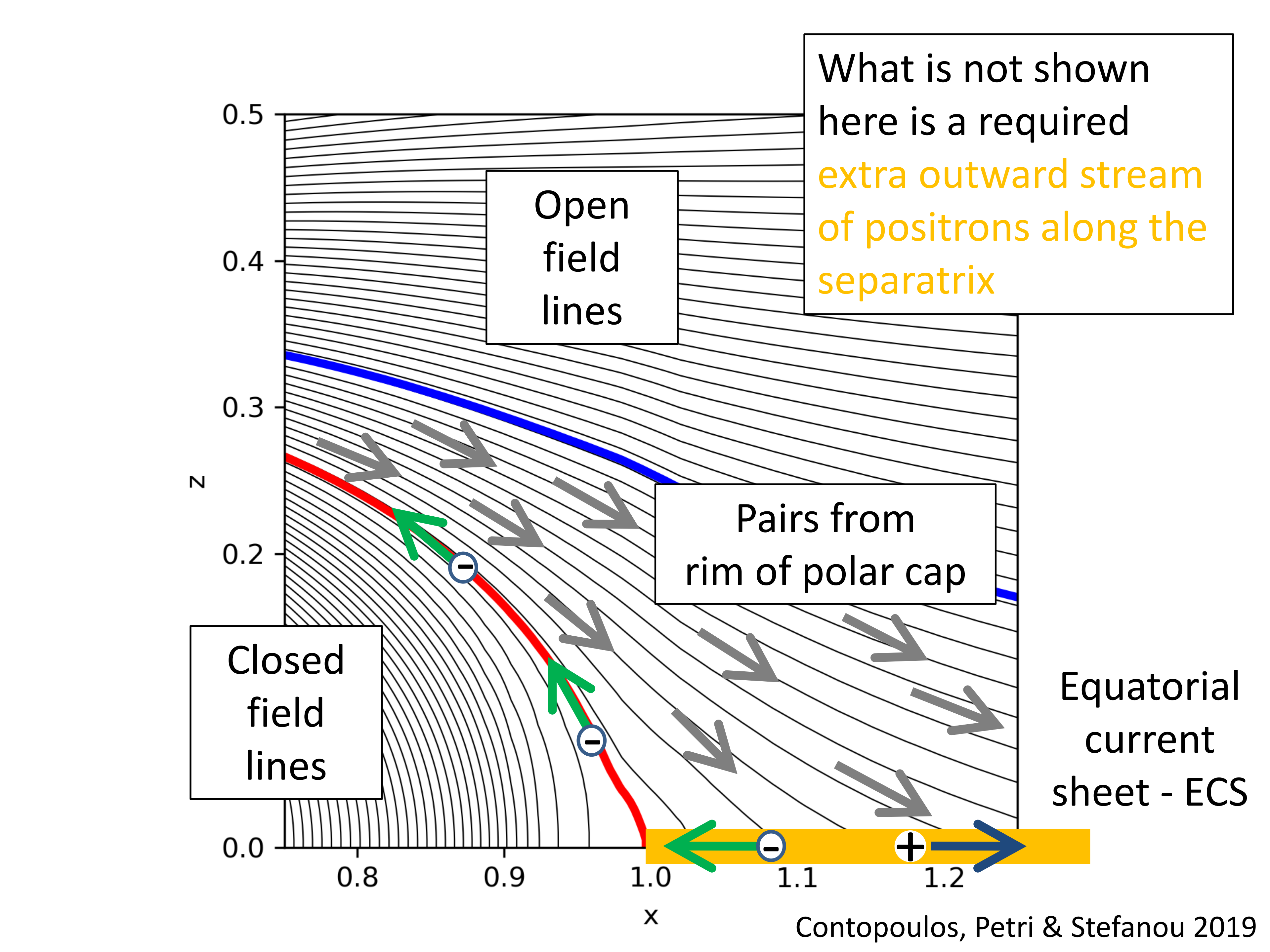


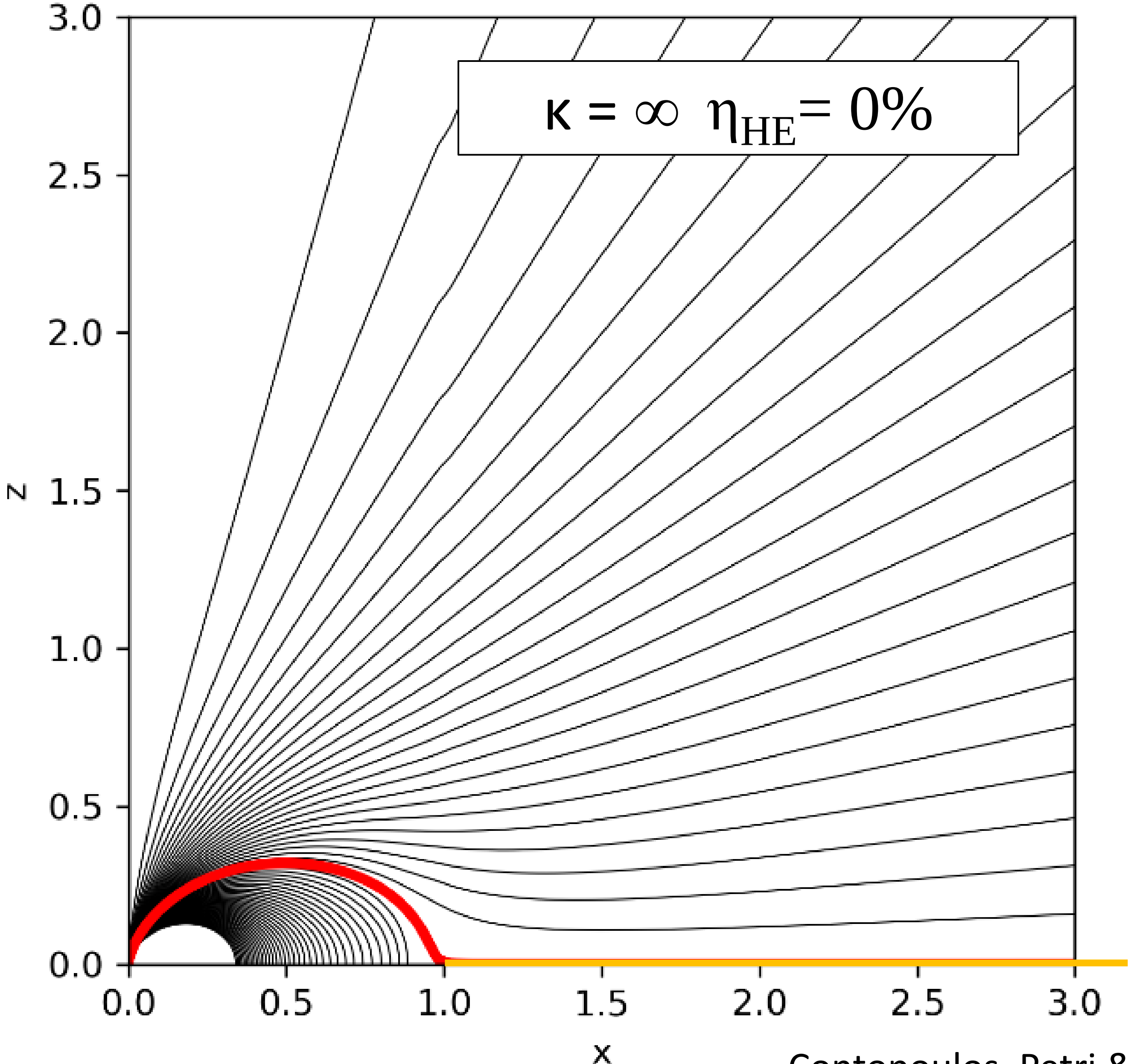
Timokhin 2006



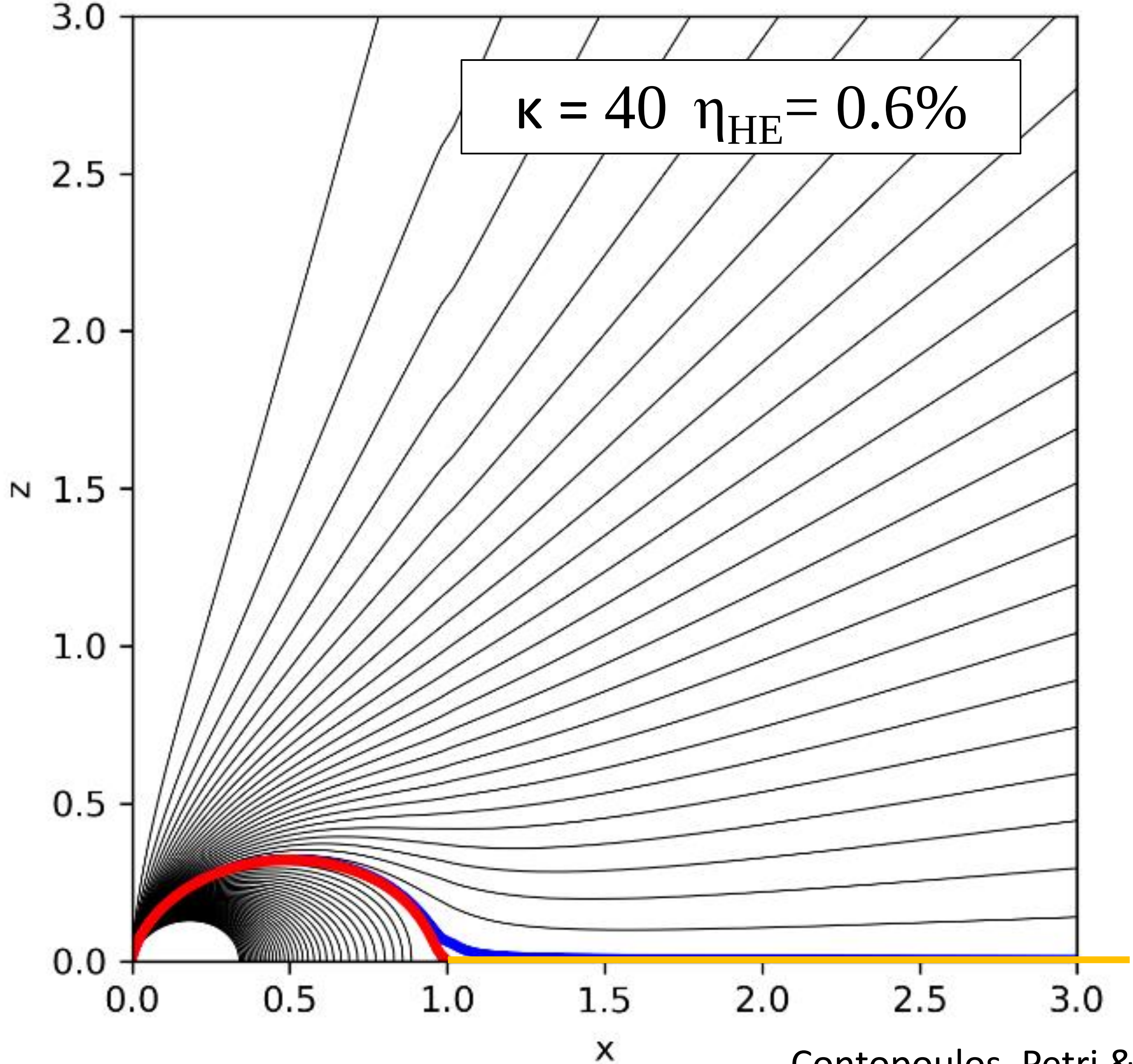
Timokhin 2006

Contopoulos 2019; Contopoulos & Stefanou 2019; Contopoulos, Petri & Stefanou 2019

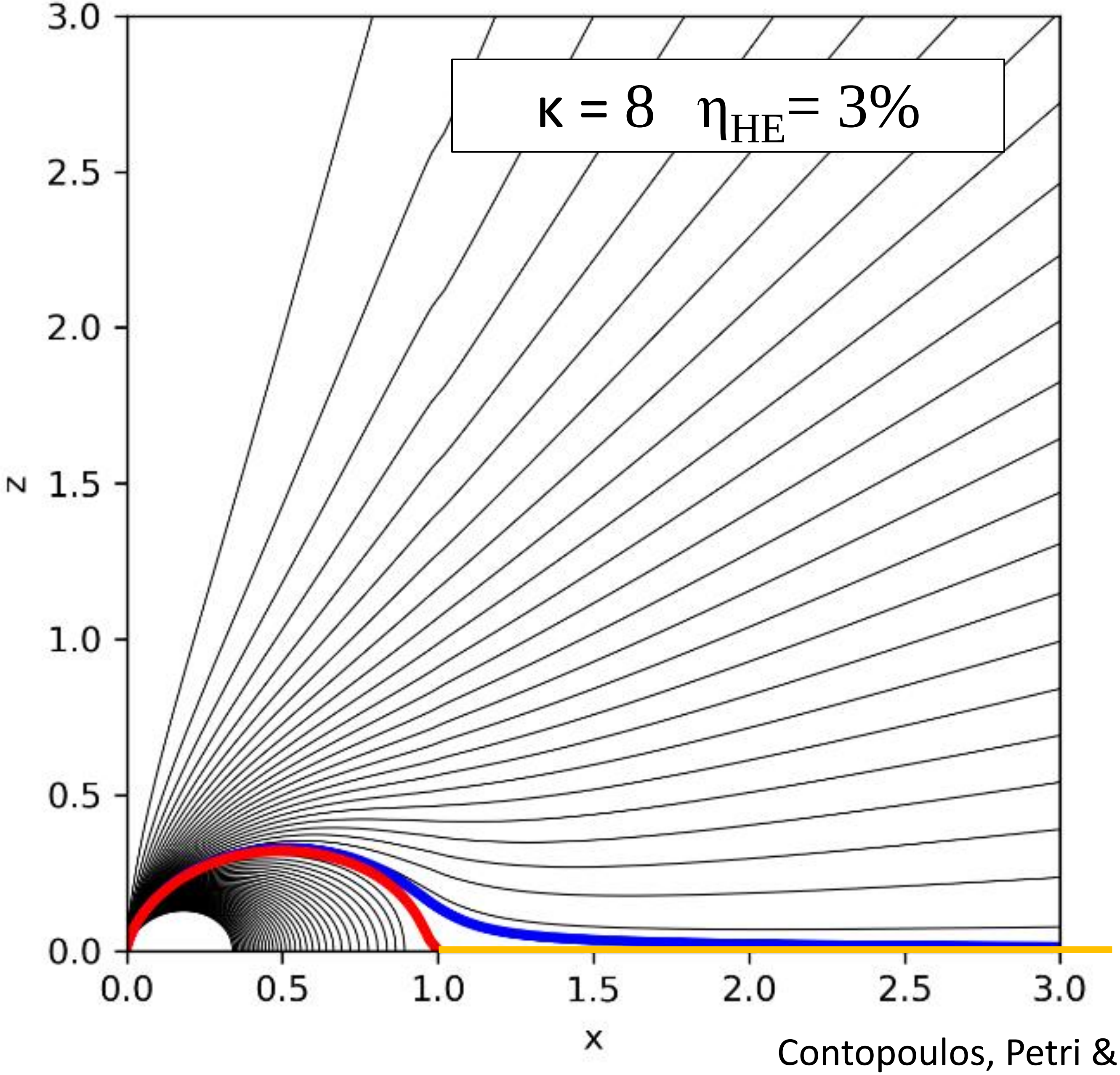

What is not shown here is a required extra outward stream of positrons along the separatrix
Open field lines
Pairs from rim of polar cap
Closed field lines
Equatorial current sheet - ECS
0.5
0.4
0.3
0.2
0.0
z
0.8
0.9
1.0
1.1
1.2
x
Contopoulos, Petri & Stefanou 2019



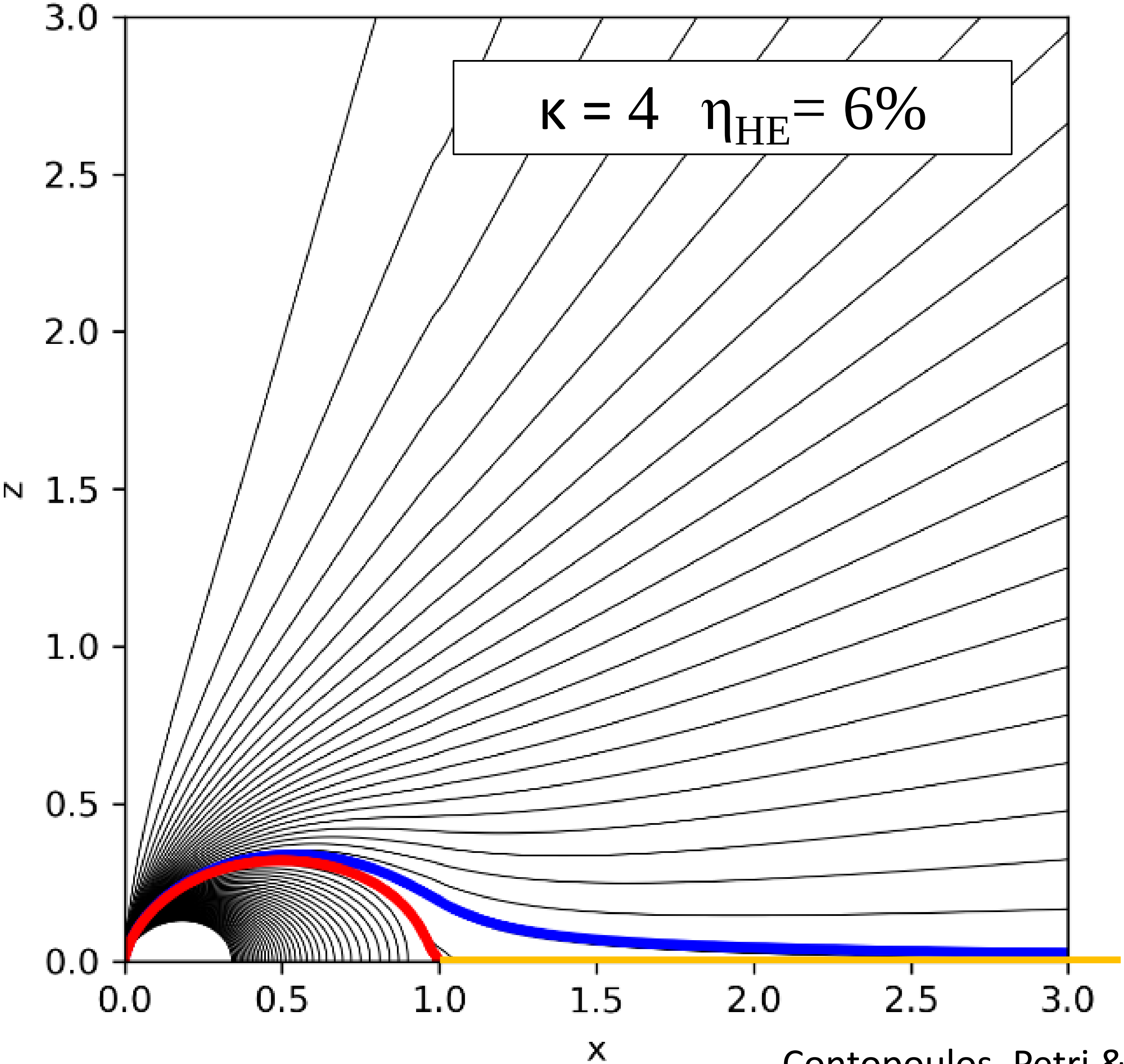
Contopoulos, Petri & Stefanou 2019



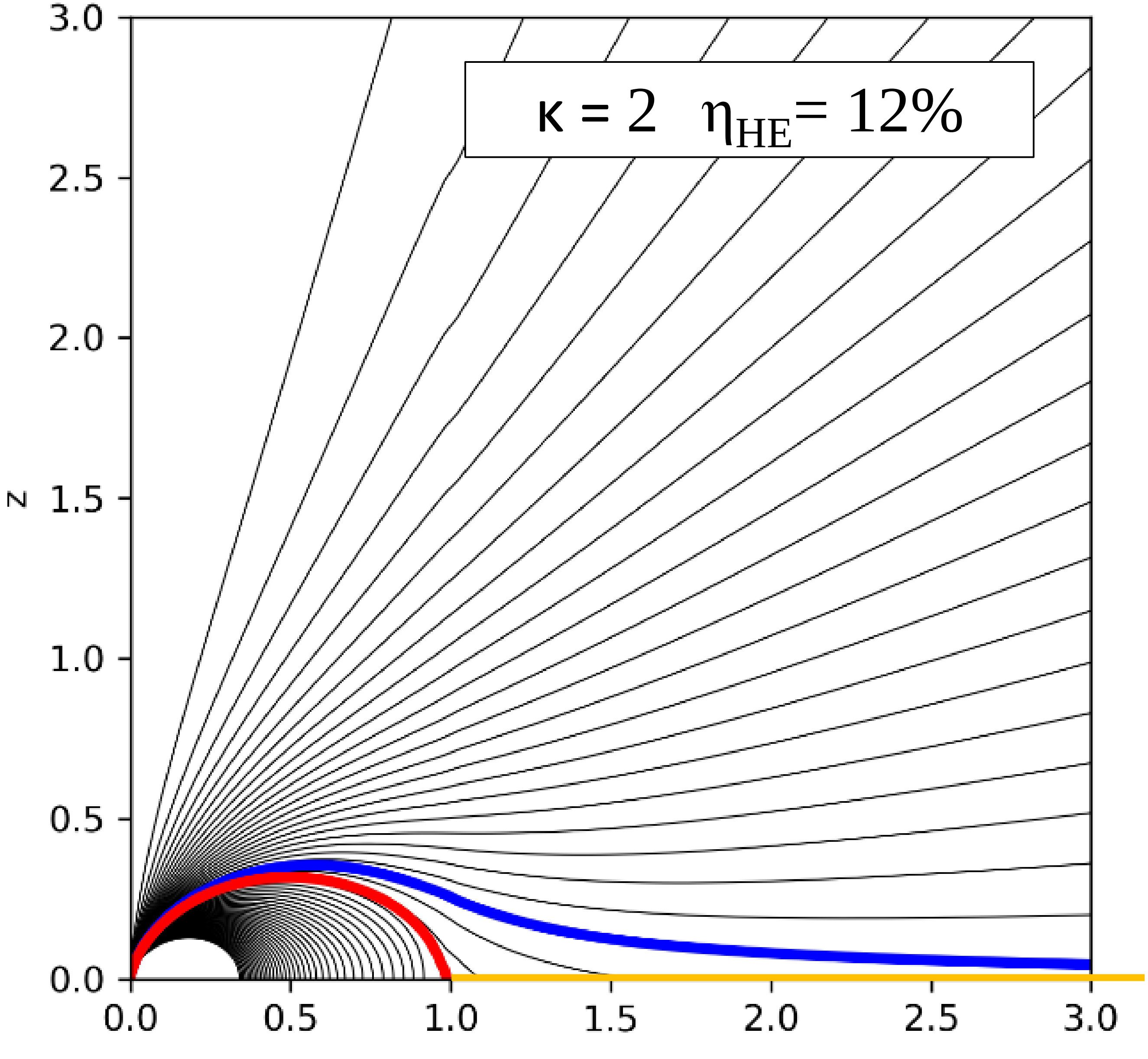
Contopoulos, Petri & Stefanou 2019



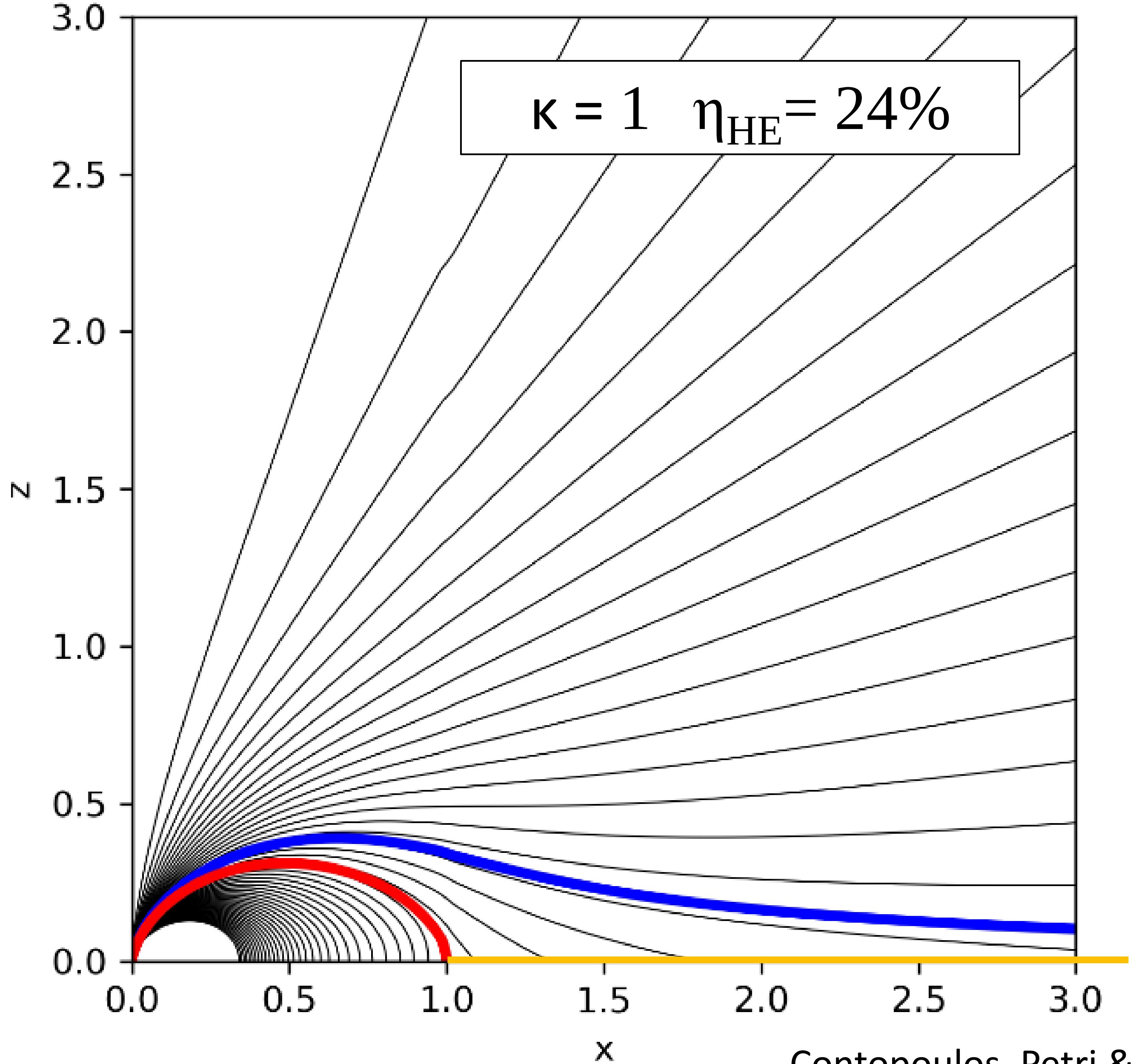
Contopoulos, Petri & Stefanou 2019



Contopoulos, Petri & Stefanou 2019



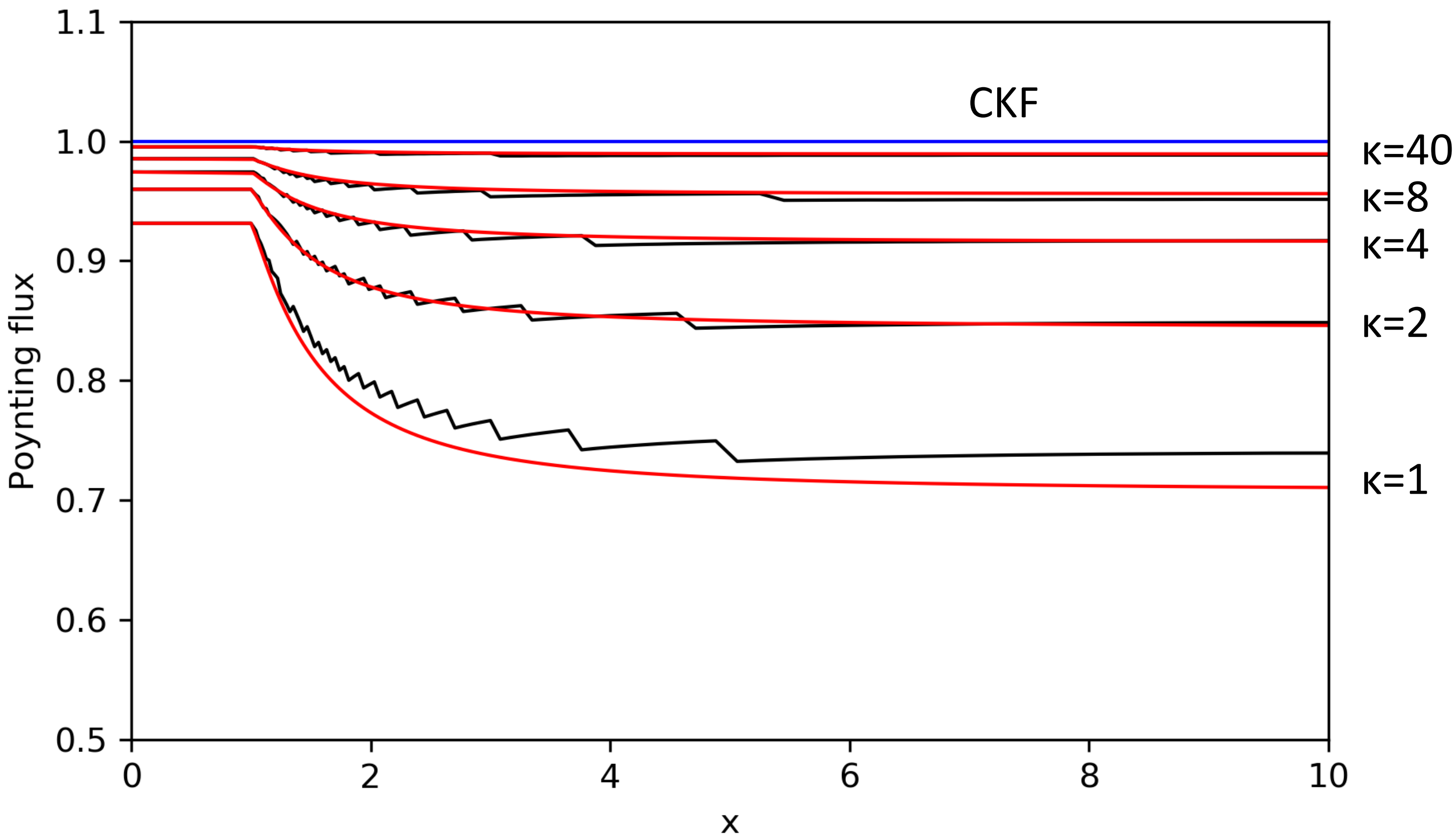
Contopoulos, Petri & Stefanou 2019



Contopoulos, Petri & Stefanou 2019

$$\dot{E}_{\rm Poynting}(x) = \dot{E} - \dot{E}_{\rm ECS}(x)$$

$$\approx \begin{cases} \dot{E}\left(1 - \frac{6}{25\kappa}\left(1 - \frac{1}{x^2}\right)^{1.2}\right) & \text{if } x \geq 1 \\ \dot{E} & \text{otherwise} \end{cases}$$



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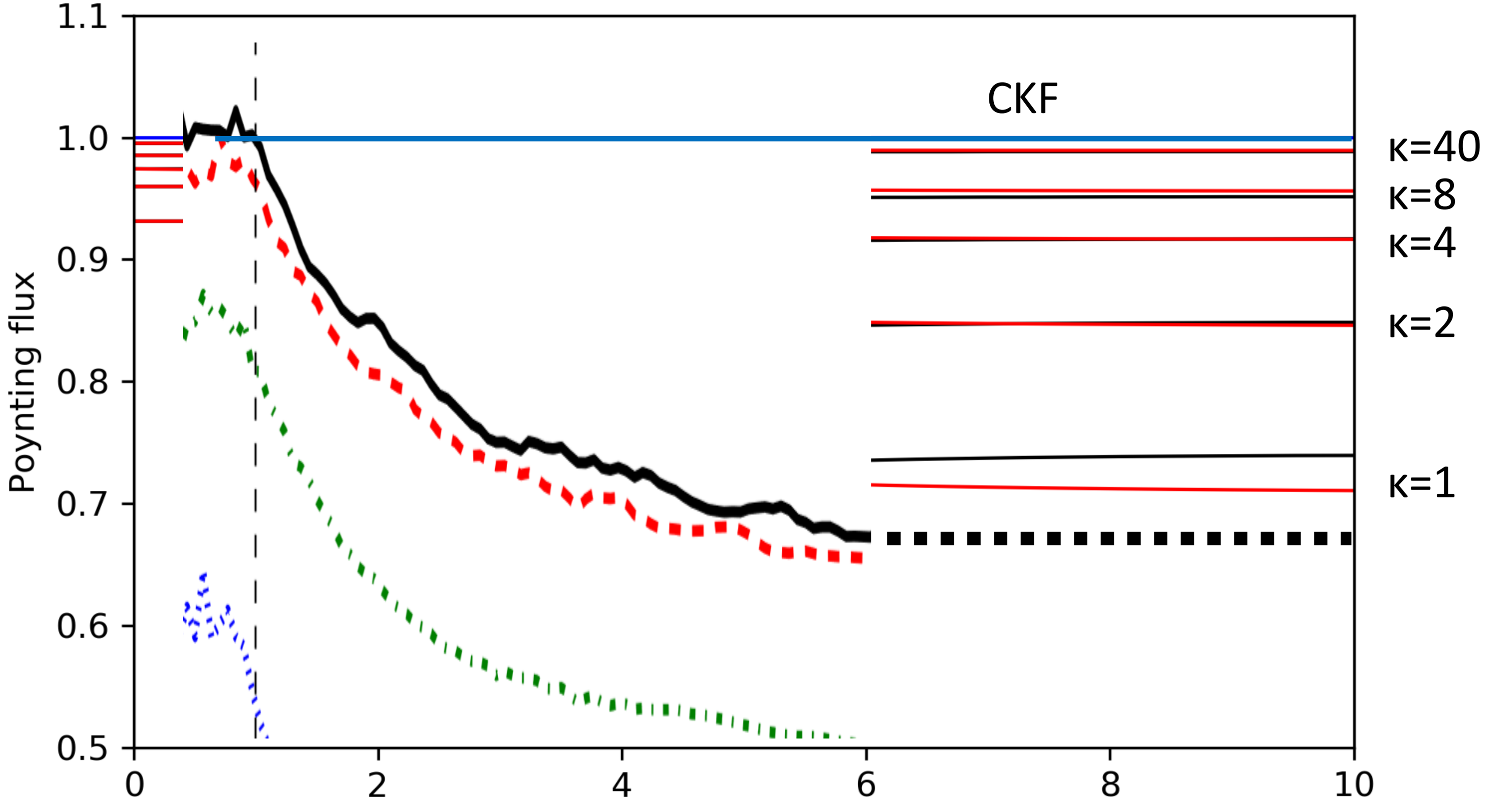
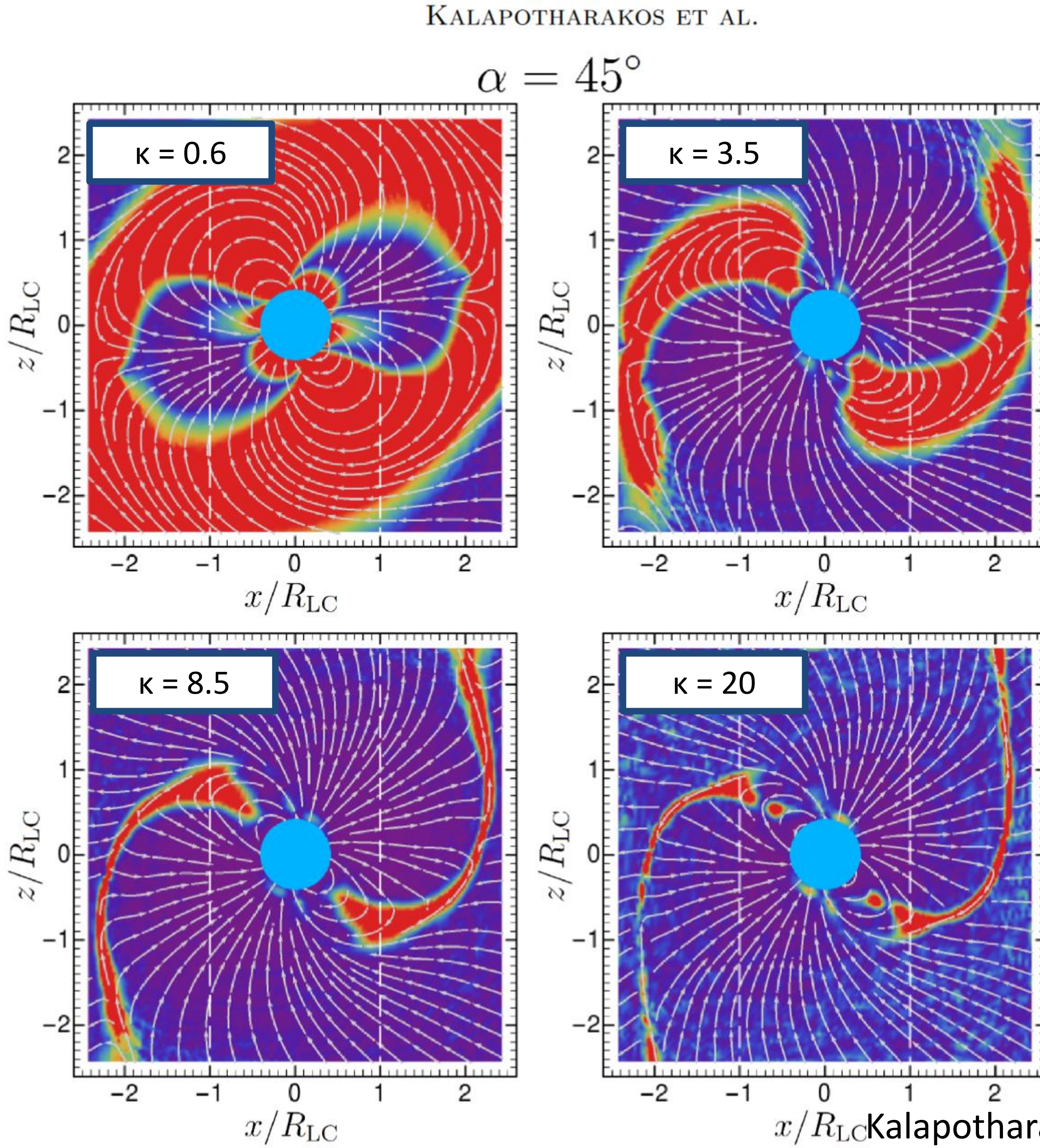
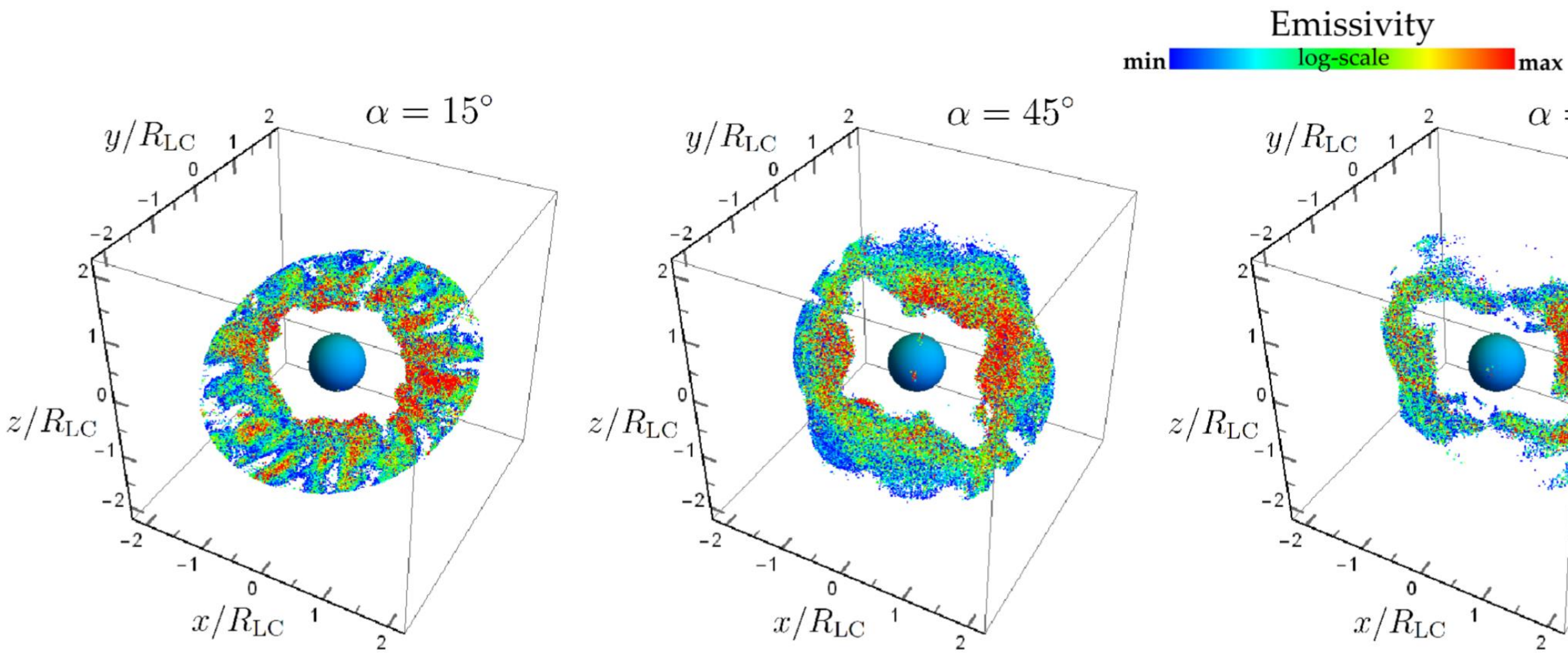


Philippov & Spitkovsky 2014; Cerutti et al. 2015

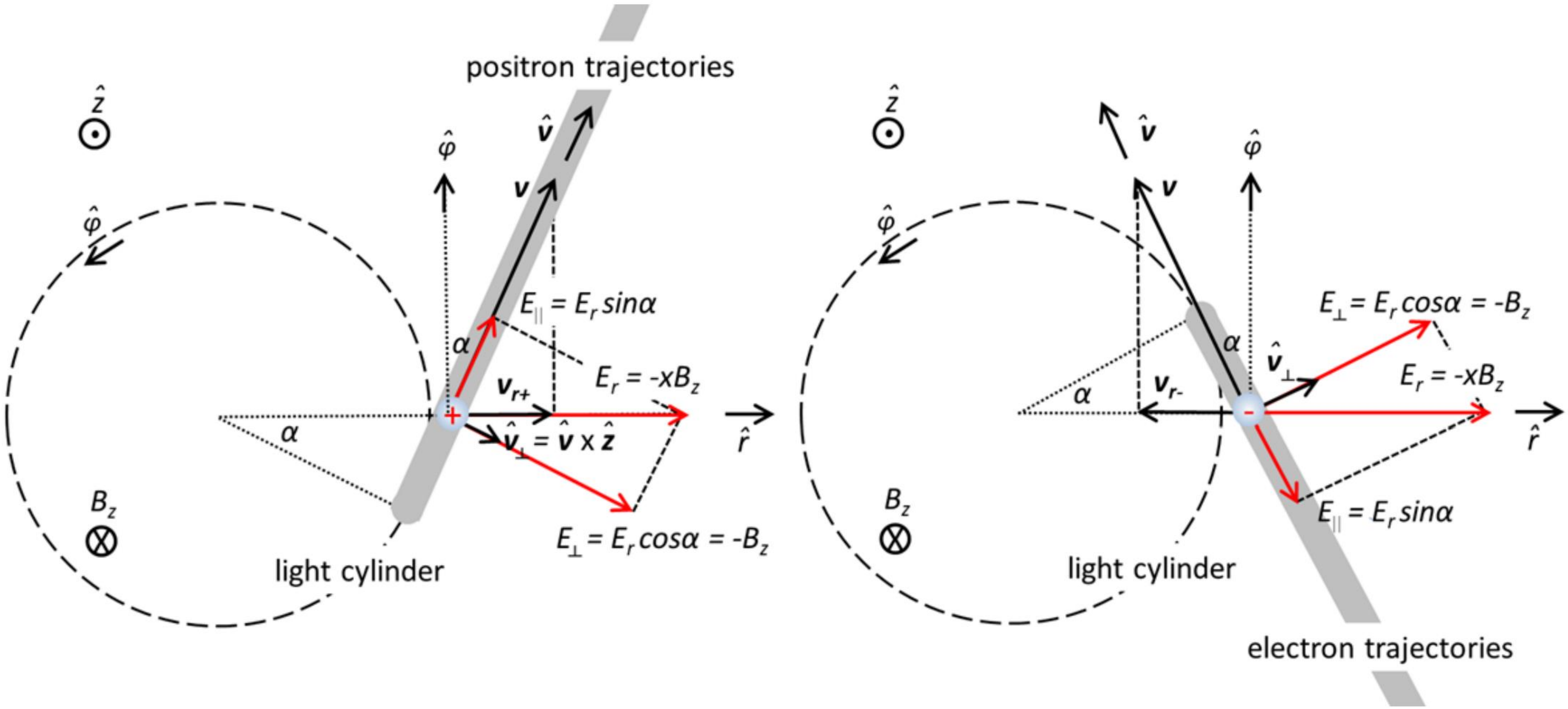


Kalapotharakos et al. 2018

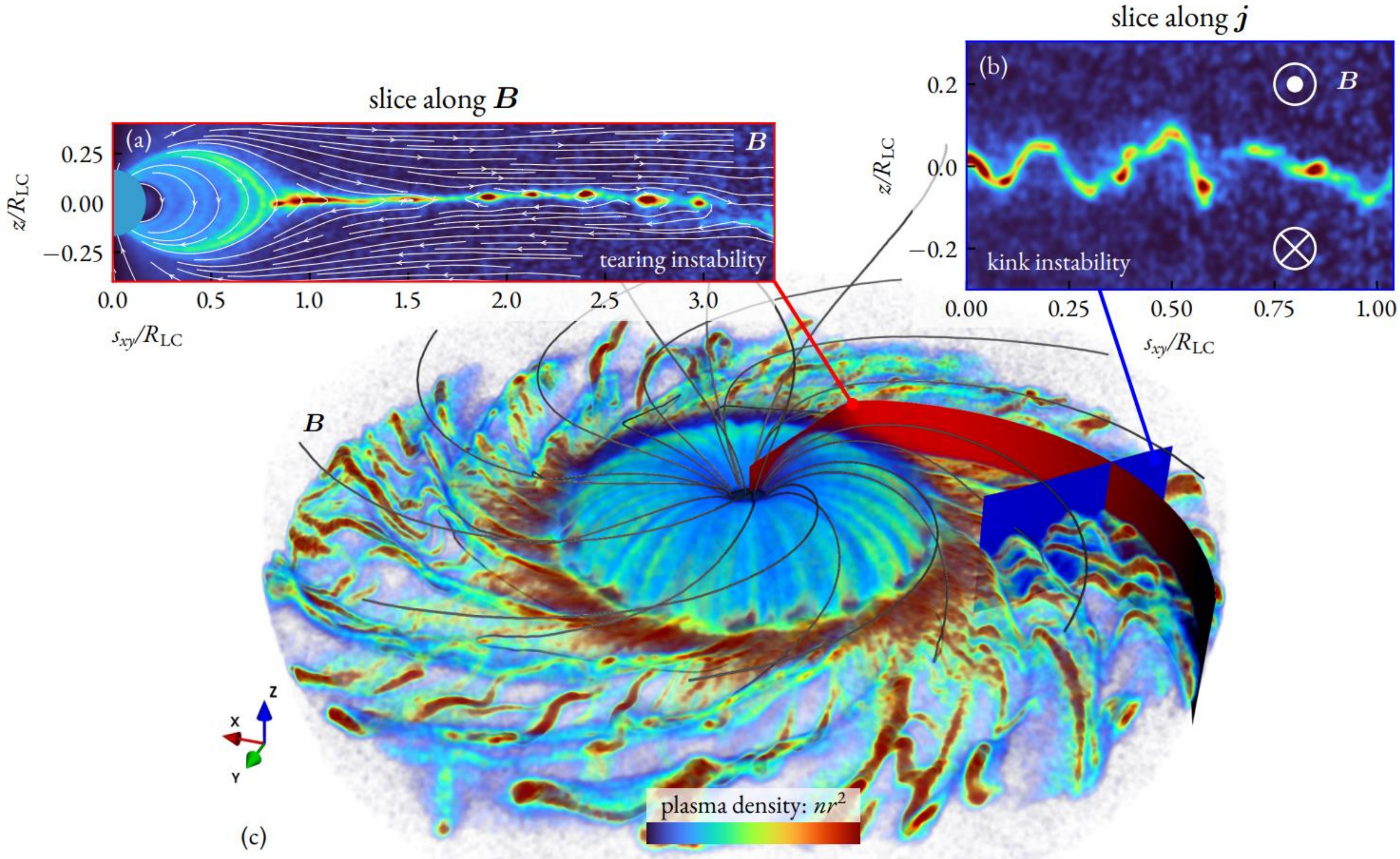


κ = 28

Kalapotharakos et al. 2018



positron trajectories
$\hat{z}$
$\hat{v}$
$\hat{\varphi}$
v
$\hat{\varphi}$
$E_{||} = E_r sin\alpha$
α
$E_r = -xB_z$
v_{r+}
α
$\hat{v}_{\perp} = \hat{v} \times \hat{z}$
$\hat{r}$
B_z
$E_{\perp} = E_r cos\alpha = -B_z$
light cylinder
$\hat{z}$
$\hat{v}$
$\hat{\varphi}$
v
$\hat{\varphi}$
$E_{\perp} = E_r cos\alpha = -B_z$
α
$\hat{v}_{\perp}$
$E_r = -xB_z$
α
v_{r-}
$\hat{r}$
B_z
$E_{||} = E_r sin\alpha$
light cylinder
electron trajectories



slice along $\boldsymbol{B}$
(a)
$\boldsymbol{B}$
tearing instability
$z/R_{\rm LC}$
0.25
0.00
−0.25
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$s_{xy}/R_{\rm LC}$
slice along $\boldsymbol{j}$
(b)
$\boldsymbol{B}$
kink instability
$z/R_{\rm LC}$
0.2
0.0
−0.2
0.00
0.25
0.50
0.75
1.00
$s_{xy}/R_{\rm LC}$
$\boldsymbol{B}$
Z
X
Y
plasma density: nr^2
(c)

Need to return to the basics

- We need the 3D force-free reference solution
  - No dissipation
  - Current sheet as a mathematical contact discontinuity

Our methodology

- We are looking for the steady-state 3D ideal force-free solution in the corotating frame
- “Steal” from the solar community

Our methodology

$$\left(\frac{\partial \boldsymbol{B}}{\partial t}\right)_{\text{corot}} = \left(\frac{\partial \boldsymbol{B}}{\partial t}\right)_{\text{lab}} - \nabla \times (\boldsymbol{u}_{\text{rot}} \times \boldsymbol{B}),$$

$$\left(\frac{\partial \boldsymbol{E}}{\partial t}\right)_{\text{corot}} = \left(\frac{\partial \boldsymbol{E}}{\partial t}\right)_{\text{lab}} - \nabla \times (\boldsymbol{u}_{\text{rot}} \times \boldsymbol{E}) + \boldsymbol{u}_{\text{rot}} \nabla \cdot \boldsymbol{E},$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{\text{corot}} = \left(\frac{\partial \rho}{\partial t}\right)_{\text{lab}} + \boldsymbol{u}_{\text{rot}} \cdot \nabla \rho,$$

where $\boldsymbol{u}_{\text{rot}}$ is the rotational velocity of the magnetosphere.

$$\nabla \times \boldsymbol{E} = -\nabla \times (\boldsymbol{\beta}_{\text{rot}} \times \boldsymbol{B}),$$

$$\nabla \times \boldsymbol{B} = \nabla \times (\boldsymbol{\beta}_{\text{rot}} \times \boldsymbol{E}) - \boldsymbol{\beta}_{\text{rot}} \nabla \cdot \boldsymbol{E} + \frac{4\pi}{c} \boldsymbol{j},$$

where $\boldsymbol{\beta}_{\text{rot}} = \boldsymbol{u}_{\text{rot}}/c$.

Our methodology

$$\boldsymbol{E} = -\,\boldsymbol{\beta}_{rot} \times \boldsymbol{B}_p = \boldsymbol{E}_p$$

$$\nabla \times \left(\boldsymbol{B} - \frac{\boldsymbol{u}_{\mathrm{rot}}}{c} \times \boldsymbol{E} \right) = \frac{4\pi}{c} (\boldsymbol{j} - \rho \boldsymbol{u}_{\mathrm{rot}}) \quad || \boldsymbol{B}$$

$$\times \boldsymbol{B}$$

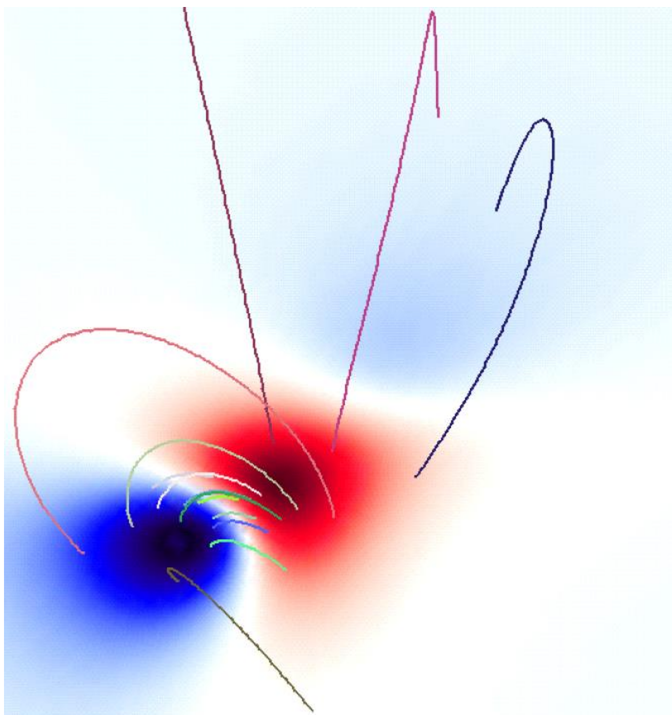
$$\nabla \times \left((1 - \beta_{rot}^2)\, \boldsymbol{B_p} + \boldsymbol{B_\varphi} \right) \times \boldsymbol{B} = 0$$

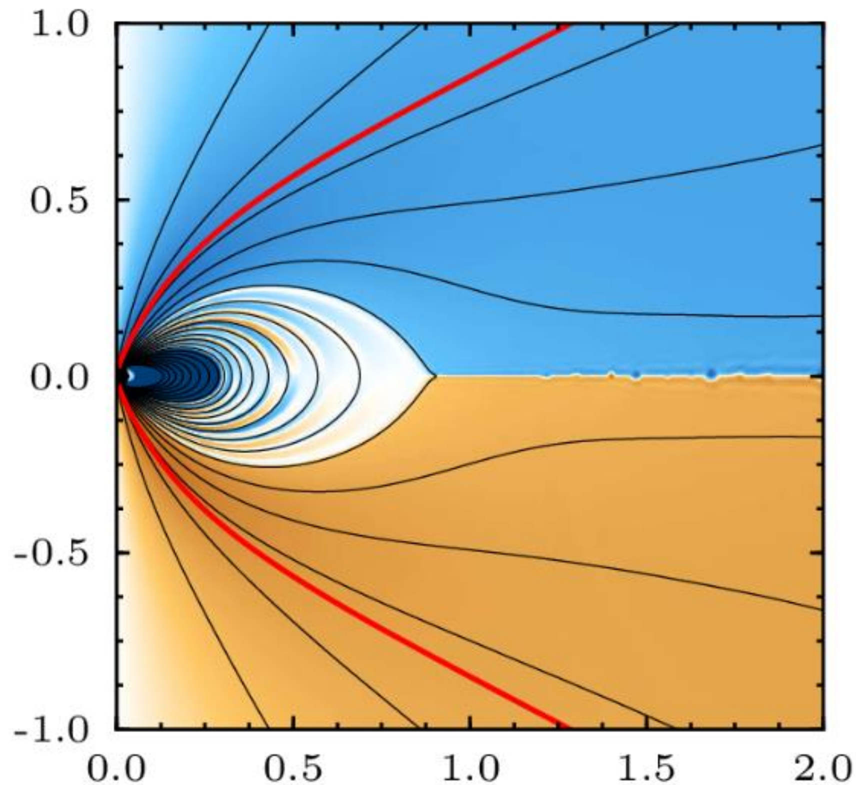
$$\nabla \times \left((1 - \beta_{rot}^2)\, \boldsymbol{B_p} + \boldsymbol{B_\varphi} \right) = a\boldsymbol{B}$$

Our methodology

$$\nabla \times \left(\left(1 - \beta_{rot}^2\right) \boldsymbol{B_p} + \boldsymbol{B_\varphi} \right) = a\boldsymbol{B}$$

$$\nabla \times \boldsymbol{B} = a\boldsymbol{B}$$





Our methodology

$$\nabla \times \left(\left(1 - \beta_{rot}^2\right) \boldsymbol{B_p} + \boldsymbol{B_\varphi} \right) = a\boldsymbol{B}$$

3. Grad-Rubin approach

Let us follow the approach that was proposed by Grad and Rubin (1958). The previous underlying mixed elliptic-hyperbolic structure of the system of equations is exploited by introducing the following sequences of hyperbolic and elliptic linear BVPs:

$$\mathbf{B}^{(n)}.\nabla\alpha^{(n)} = 0 \quad \text{in} \quad \Omega, \tag{7}$$

$$\alpha^{(n)}|_{\partial\Omega+} = \alpha_0\,, \tag{8}$$

and

$$\nabla \times \mathbf{B}^{(n+1)} = \alpha^{(n)}\mathbf{B}^{(n)} \quad \text{in} \quad \Omega, \tag{9}$$

$$\nabla \cdot \mathbf{B}^{(n+1)} = 0 \quad \text{in} \quad \Omega, \tag{10}$$

$$B_z^{(n+1)}|_{\partial\Omega} = b_0\,, \tag{11}$$

$$\lim_{|\mathbf{r}|\to\infty} |\mathbf{B}^{(n+1)}| = 0\,. \tag{12}$$

with $\mathbf{B}^0$ the unique solution of:

$$\nabla \times \mathbf{B}^0 = 0 \quad \text{in} \quad \Omega, \tag{13}$$

$$B_z^0|_{\partial\Omega} = b_0\,, \tag{14}$$

$$\lim_{|\mathbf{r}|\to\infty} |\mathbf{B}^0| = 0\,, \tag{15}$$

that is given by (Aly 1989):

$$\nabla \times \mathbf{A}^{(n)}.\nabla\alpha^{(n)} = 0 \quad \text{in} \quad \Omega, \tag{34}$$

$$\tag{35}$$

$$\alpha^{(n)}|_{\partial\Omega+} = \alpha_0\,. \tag{36}$$

and

$$-\Delta\mathbf{A}^{(n+1)} = \alpha^{(n)}\nabla \times \mathbf{A}^{(n)} \quad \text{in} \quad \Omega\,, \tag{37}$$

$$\mathbf{A}_t^{(n+1)} = \nabla^{\perp}\chi \quad \text{on} \quad \partial\Omega\,, \tag{38}$$

$$\partial_n\mathbf{A}_n^{(n+1)} = 0 \quad \text{on} \quad \partial\Omega \tag{39}$$

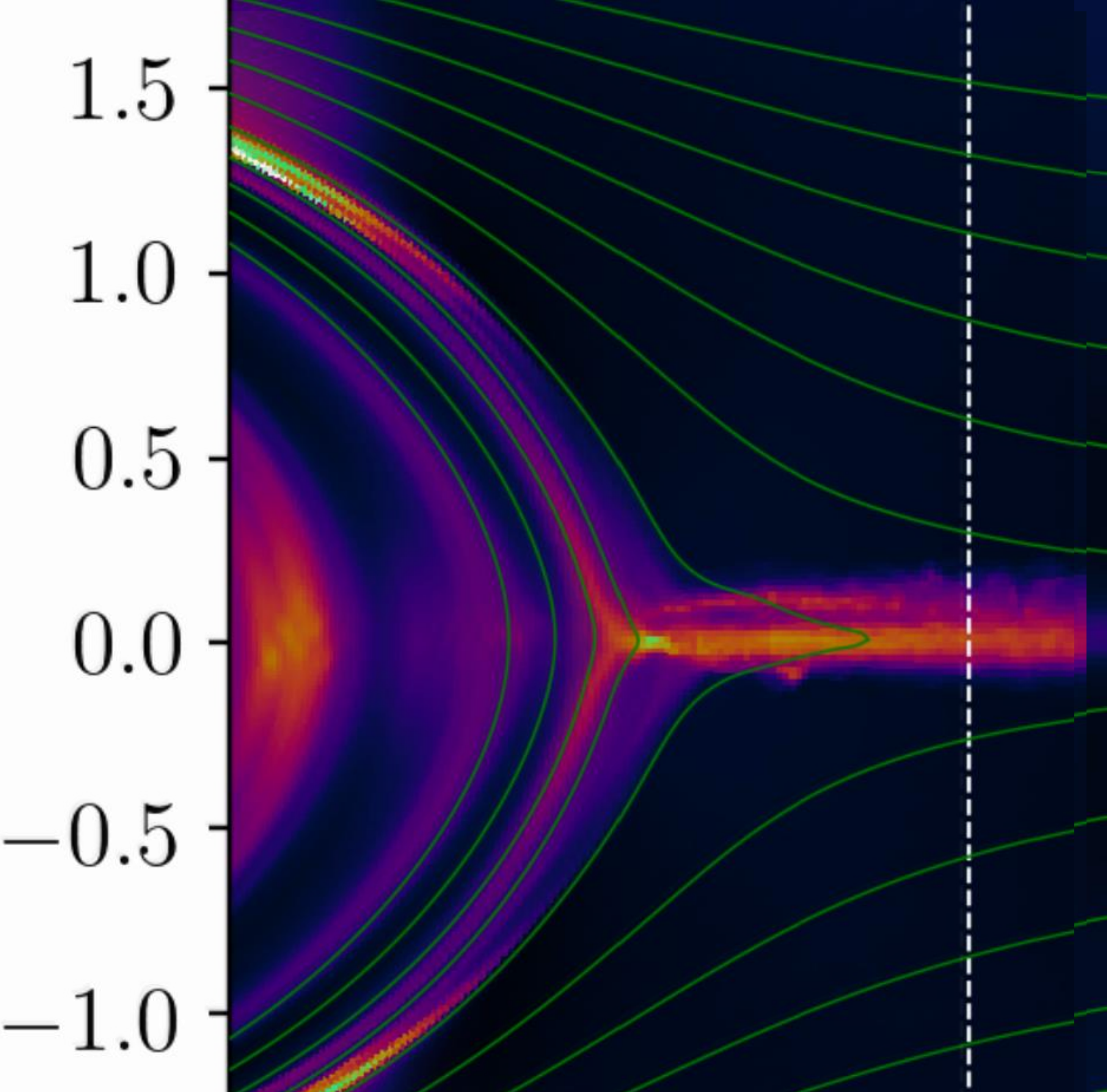
$$\lim_{|\mathbf{r}|\to\infty} |\mathbf{A}^{(n+1)}| = 0\,. \tag{40}$$

The solution $\mathbf{A}^{(n+1)}$ of the linear elliptic mixed Dirichlet-Neumann BVP is in general regular ($\mathbf{A}^{(n+1)} \in C^2(\Omega) \cup C^1(\partial\Omega)^3$).

$$\boldsymbol{a^{(n)}} = \boldsymbol{a^{(n)}}|_{\boldsymbol{lc}}$$

Questions that we will address

- Dissipation. Why?
- Y-point line inside the light cylinder and/or non-circular?
  - Spindown different from canonical
- γ-ray light curve shaped by the geometry of the inner equatorial current sheet
- Non-standard surface magnetic fields


1.5
1.0
0.5
0.0
−0.5
−1.0